

Emergence of expanding $(3+1)$ - dimensional spacetime in the Lorentzian type IIB matrix model

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Present status of superstring theory

Superstring theory is a promising candidate for a unified theory including quantum gravity

1. Perturbation theory with D-branes including a part of nonperturbative effects

→ numerous (meta-)stable vacua with different spacetime dimensionalities, gauge groups, matter contents and cosmological constants

In particular, 3+1 dimensions are not required, for instance 2+1 and 4+1 are possible

2. Singularity at the beginning of the Universe → Not resolved in perturbation theory

Liu-Moore-Seiberg (2002)

→ We need a nonperturbative formulation of superstring theory

Type IIB matrix model

Ishibashi Kawai, Kitazawa, AT (1996)

Proposed as a nonperturbative formulation of superstring theory

$$S = -N \text{Tr} \left(\frac{1}{4} [A_\mu, A_\nu] [A^\mu, A^\nu] + \frac{1}{2} \bar{\psi} \Gamma^\mu [A_\mu, \psi] \right)$$

$N \times N$ Hermitian matrices

A_μ : 10D Lorentz vector ($\mu = 0, 1, \dots, 9$)

SO(9,1) symmetry

ψ : 10D Majorana-Weyl spinor

The action takes the form of the dimensional reduction of 10D N=1 SYM to zero dimension

Space-time does not exist a priori,

but **emerges from the degrees of freedom of matrices.**

 Dimensionality of space-time can be predicted

Crucial properties: 10D N=2 SUSY

$$Q^{(1)} \begin{cases} \delta^{(1)} A_\mu = i\bar{\epsilon}_1 \Gamma_\mu \psi \\ \delta^{(1)} \psi = \frac{i}{2} \Gamma^{\mu\nu} [A_\mu, A_\nu] \epsilon_1 \end{cases} \quad Q^{(2)} \begin{cases} \delta^{(2)} A_\mu = 0 \\ \delta^{(2)} \psi = \epsilon_2 \mathbf{1}_N \end{cases} \quad P_\mu \begin{cases} \delta_{\text{T}} A_\mu = c_\mu \mathbf{1}_N \\ \delta_{\text{T}} \psi = 0 \end{cases}$$

dimensional reduction of
10D N=1 SUSY

$$\begin{cases} \tilde{Q}^{(1)} = Q^{(1)} + Q^{(2)} \\ \tilde{Q}^{(2)} = i(Q^{(1)} - Q^{(2)}) \end{cases} \quad \longrightarrow \quad [\bar{\epsilon}_1 \tilde{Q}^{(i)}, \bar{\epsilon}_2 \tilde{Q}^{(j)}] = -2\delta^{ij} \bar{\epsilon}_1 \Gamma^\mu \epsilon_2 P_\mu$$

10D N=2 SUSY if P_μ is identified
with momentum, which generates shift of A_μ

The space-time is represented as the eigenvalue distribution of A_μ .

The fact that the model has maximal SUSY suggests strongly that the model includes gravity.

Crucial properties: connection to the world sheet action

Green-Schwarz action of Schild-type for type IIB superstring with κ symmetry fixed

$$S_S = \int d\tau d\sigma \sqrt{-g} \left[\frac{1}{4} \{X_\mu, X_\nu\} \{X^\mu, X^\nu\} - \frac{i}{2} \bar{\Psi} \Gamma^\mu \{X_\mu, \Psi\} \right]$$

$$\{X, Y\} = \frac{1}{\sqrt{-g}} \left(\frac{\partial X}{\partial \tau} \frac{\partial Y}{\partial \sigma} - \frac{\partial X}{\partial \sigma} \frac{\partial Y}{\partial \tau} \right)$$

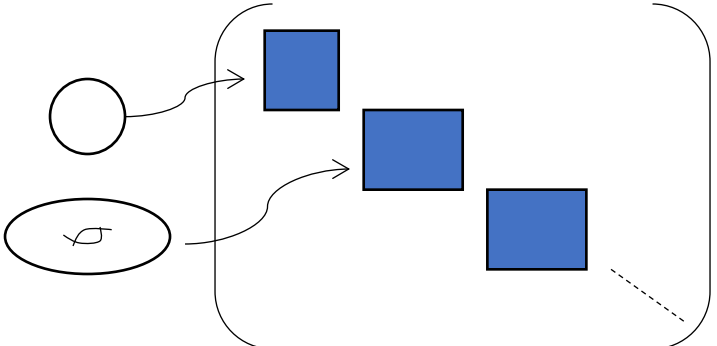
matrix regularization

$$\left[\begin{array}{l} X_\mu(\tau, \sigma) \rightarrow A_\mu \\ \Psi(\tau, \sigma) \rightarrow \psi \\ \{, \} \rightarrow \frac{1}{i} [,] \\ \int d\tau d\sigma \rightarrow \text{Tr} \end{array} \right.$$



type IIB matrix model

$$S = -N \text{Tr} \left(\frac{1}{4} [A_\mu, A_\nu] [A^\mu, A^\nu] + \frac{1}{2} \bar{\psi} \Gamma^\mu [A_\mu, \psi] \right)$$



multi strings
2nd quantized

Crucial properties (cont'd)

- Long distance behavior of interaction between D-branes is reproduced.
- Light-cone string field theory for type IIB superstring is reproduced from SD equations for Wilson loops under reasonable assumptions.

Fukuma-Kawai-Kitazawa-AT (1997)

Developments

- Quantum (noncommutative) geometry arising from the model

Aoki-Ishibashi-Iso-Kawai-Kitazawa-Tada (1998), Steinacker,

- Covariant derivative interpretation

Cf.) talks by Tekel, Steinacker and Kovacic

Hanada-Kawai-Kimura (2005)

Cf.) Seko's talk in the afternoon

- Numerical simulation

Anagnostopoulos, Azuma, Nishimura, AT,

- Gauge group and matter content

Chatzistavrakidis-Steinacker-Zoupanos (2005), Aoki-Nishimura-AT (2014),

- Relation to cosmology

Kim-Nishimura-AT (2012), Sperling-Steinacker (2019), Brahma-Brandenberger-Laliberte (2022), Klinkhamer (2022),

- Holographic correspondence to type IIB sugra (Euclidean)

Hartnoll-Liu (2024), Komatsu-Martina-Penedones-Vuignier-Zhao (2024), Ciceri-Samtleben (2025)

- Relation to causality Asano's talk

⋮

Plan of the present talk

- ✓ 1. Introduction
- 2. Lorentzian vs Euclidean
- 3. How to investigate the model
- 4. Results of numerical simulations
- 5. Summary and outlook

Lorentzian vs Euclidean

Euclidean model

$$Z = \int dA d\psi e^{-S} = \int dA \text{Pf} \mathcal{M}_E(A) e^{-S_b} \quad \mathcal{M}_E(A) \cdot = \Gamma^\mu [A_\mu, \cdot] : \text{Dirac operator}$$

connection to worldsheet theory

$$S_b = \frac{N}{4} \sum_{\mu, \nu=0}^9 \text{Tr}(-[A_\mu, A_\nu]^2) \quad : \text{positive semi-definite} \quad \text{SO}(10) \text{ symmetry}$$

$\text{Pf} \mathcal{M}_E(A) : \text{complex} \longrightarrow \text{sign problem}$

The Euclidean model is well-defined without cutoff.

Krauth, Nicolai, Staudacher ('98) Austing, Wheeler ('01)

Numerical simulations showed SSB of SO(10) to SO(3) due to less fluctuations of the complex phase of Pfaffian for lower dimensions

Nishimura, Vernizzi (2000) Anagnostopoulos, Azuma, Ito, Nishimura, Okubo, Papadoudis (2020)

3D space emerges, but time does not emerge \longrightarrow study the Lorentzian model

Partition function of Lorentzian model

Kim-Nishimura-AT (2011)

$$S = -N \text{Tr} \left(\frac{1}{4} [A_\mu, A_\nu] [A^\mu, A^\nu] + \frac{1}{2} \bar{\psi} \Gamma^\mu [A_\mu, \psi] \right)$$

$$Z = \int dA d\psi e^{iS} = \int dA \text{Pf} \mathcal{M}(A) e^{iS_b} \quad \text{phase factor} \rightarrow \text{sign problem}$$

polynomial in A_μ , real

connection to
worldsheet theory

$$S_b = -\frac{N}{4} \text{Tr} ([A^\mu, A^\nu] [A_\mu, A_\nu])$$

gauge-volume of Lorentz symmetry is infinite

Asano, Piensuk, Nishimura, Yamamori (2024)

\rightarrow Gauge-fixing of Lorentz symmetry

$$\Delta_{\text{FP}} = \det \Omega, \quad \Omega_{ij} = \text{Tr}(A_0)^2 \delta_{ij} + \text{Tr}(A_i A_i)$$

$$S_{\text{gf}} = \frac{\alpha}{2} (\text{Tr}(A_0 A_i))^2$$

How to investigate the model

Complex Langevin method

Parisi (1983), Klauder (1984)

We use the **complex Langevin method** to overcome the sign problem.

We take the gauge in which A_0 is diagonal.

We make a change of variables to introduce a time-ordering preserving holomorphy

$$\alpha_1 = 0, \alpha_2 = e^{\tau_1}, \alpha_3 = e^{\tau_1} + e^{\tau_2}, \dots, \alpha_N = \sum_{a=1}^{N-1} e^{\tau_a} \quad \alpha_1 < \alpha_2 < \dots < \alpha_N$$

Nishimura, AT (2019)

➔ **complexify τ_a and A_i**

complex Langevin equation

$$\left\{ \begin{array}{l} \frac{d\tau_a}{dt_L} = -\frac{\partial S_{\text{eff}}}{\partial \tau_a} + \eta_a(t_L) \\ \frac{d(A_i)_{ab}}{dt_L} = -\frac{\partial S_{\text{eff}}}{\partial (A_i)_{ba}} + (\eta_i(t_L))_{ab} \end{array} \right. \quad \begin{array}{l} S_{\text{eff}} = -iS_b - \log \text{Pf} \mathcal{M}(A) - 2 \log \Delta(\alpha) \\ - \sum_{a=1}^{N-1} \tau_a + S_{\text{gf}} + \text{tr} \ln \Omega \end{array} \quad \rightarrow \quad \begin{array}{l} \text{zero eigenvalue of } \mathcal{M}(A) \\ \text{singular drift problem} \end{array}$$

t_L : Langevin time ~discretized in practice

η_a, η_i : Gaussian noises

expectation value of holomorphic observables can be calculated by taking samples around sufficiently large t_L

Avoiding the singular drift problem

To avoid the singular drift problem, we add a mass term to the fermionic action.

$$S_f = -\frac{N}{2} \text{Tr} (\bar{\psi} \Gamma^\mu [A_\mu, \psi] + im_f \bar{\psi} \Gamma^7 \Gamma^{8\dagger} \Gamma^9 \psi)$$

unique mass term

Break SUSY

Break $SO(9,1)$ to $SO(6,1) \times SO(3)$

The effect of fermions is weakened for finite m_f

m_f should be as small as possible

Controlling the quantum fluctuation of bosonic matrices

We add a bosonic mass term to control the quantum fluctuation of bosonic matrices, because the fermionic mass term weakens the effect of the quantum fluctuations of fermionic matrices


$$S_{\text{bm}} = \frac{1}{2} N \gamma \text{Tr} \left(\text{Tr}(A_0)^2 - \sum_{i=1}^6 \text{Tr}(A_i)^2 - \xi \sum_{i=7}^9 \text{Tr}(A_i)^2 \right)$$

Keep $SO(6,1) \times SO(3)$

γ, ξ : parameters that can control the quantum fluctuations of bosonic matrices

For large ξ , the bosonic degrees of freedom reduces effectively to (6+1)-dimensional one.

By choosing γ and ξ appropriately, we can expect to realize a situation in which the effects of fluctuations of bosons and fermions are balanced

Eventually, we want to take $N \rightarrow \infty, m_f \rightarrow 0, \xi \rightarrow 1, \gamma \rightarrow 0$  target theory

Supersymmetric deformation

Bonelli (2002)

$$S = S_0 + S_{\text{fm}} + S_{\text{bm}} + S_{\text{Myers}}$$

$$S_0 = -N \text{Tr} \left(\frac{1}{4} [A_\mu, A_\nu] [A^\mu, A^\nu] + \frac{1}{2} \bar{\psi} \Gamma^\mu [A_\mu, \psi] \right)$$

$$S_{\text{fm}} = -i \frac{N}{2} m_f \text{Tr}(\bar{\psi} \Gamma^7 \Gamma^8 \Gamma^9 \psi)$$

$$S_{\text{bm}} = \frac{1}{2} N \gamma \text{Tr} \left(\text{Tr}(A_0)^2 - \sum_{i=1}^6 \text{Tr}(A_i)^2 - \xi \sum_{i=7}^9 \text{Tr}(A_i)^2 \right)$$

$$S_{\text{Myers}} = -i N \zeta \text{Tr}(A_7 [A_8, A_9])$$

$$\text{SO}(9,1) \longrightarrow \text{SO}(6,1) \times \text{SO}(3)$$

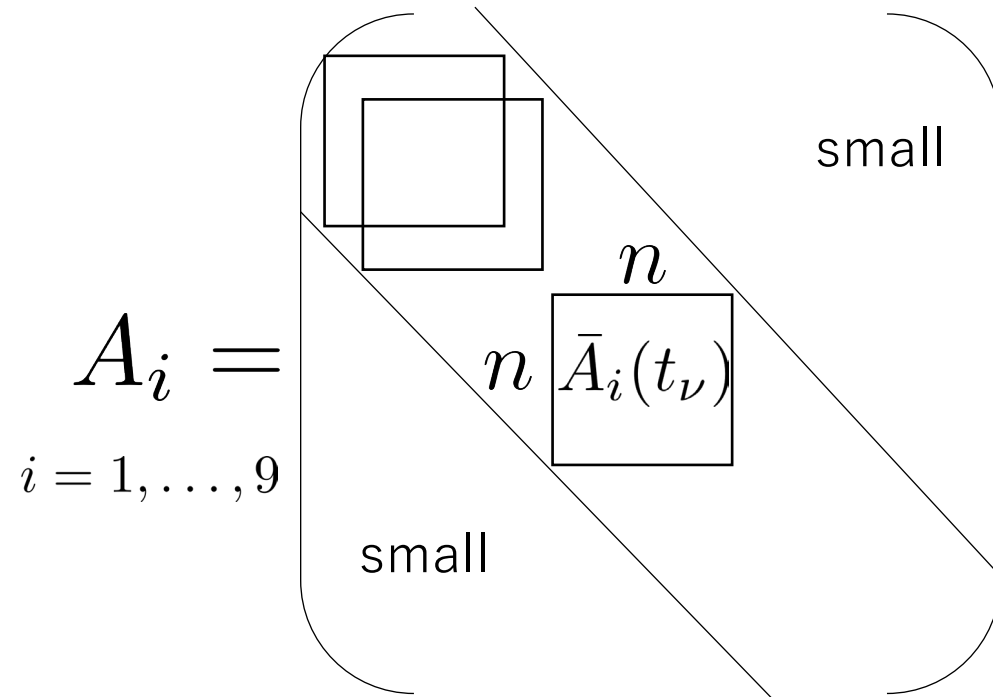
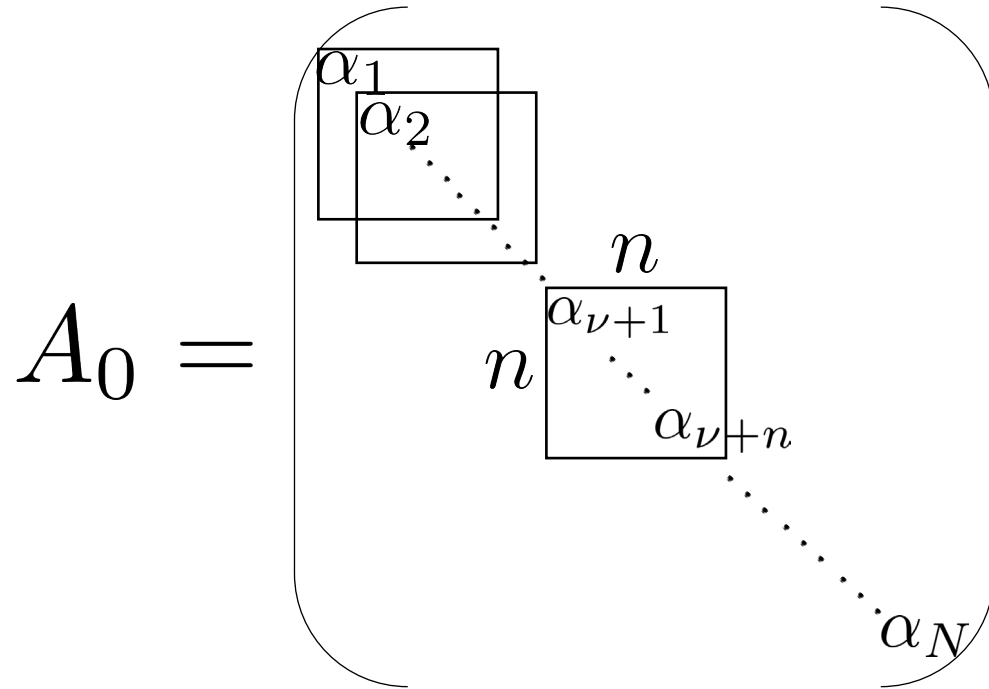
{	$m_f = \frac{1}{4} \mu$	SUSY deformation
	$\gamma = -\frac{1}{32} \mu^2$	
	$\xi = 3$	
	$\zeta = \mu$	
		16 SUSY preserved

Our present deformation ($\zeta = 0$) is akin to SUSY deformation

Extracting the time evolution

Kim-Nishimura-AT (2011)

We take the gauge in which A_0 is diagonal.



The state of the universe at time t_ν

definition of time

$$\bar{\alpha}_k = \frac{1}{n} \sum_{i=1}^n \alpha_{k+i} \in \mathbb{C}, \quad t_\nu = \sum_{k=1}^{\nu} |\bar{\alpha}_{k+1} - \bar{\alpha}_k|$$

A_i has band-diagonal structure, which is nontrivial dynamical property.

locality of time is guaranteed.
~ emergence of time evolution

Results of numerical simulations

Set-up

$$S = S_0 + S_{\text{fm}} + S_{\text{bm}} + S_{\text{Myers}}$$

$$S_0 = -N \text{Tr} \left(\frac{1}{4} [A_\mu, A_\nu] [A^\mu, A^\nu] + \frac{1}{2} \bar{\psi} \Gamma^\mu [A_\mu, \psi] \right)$$

$$S_{\text{fm}} = -i \frac{N}{2} m_f \text{Tr} (\bar{\psi} \Gamma^7 \Gamma^{8\dagger} \Gamma^9 \psi)$$

$$S_{\text{bm}} = \frac{1}{2} N \gamma \text{Tr} \left(\text{Tr}(A_0)^2 - \sum_{i=1}^6 \text{Tr}(A_i)^2 - \xi \sum_{i=7}^9 \text{Tr}(A_i)^2 \right)$$

$$S_{\text{Myers}} = -i N \zeta \text{Tr}(A_7 [A_8, A_9])$$

$$N = 32, m_f = 2, \gamma = 6,$$

$$\xi = 10, \zeta = 0, n = 8$$

with various initial conditions:

(2+1)d, (3+1)d, (4+1)d configurations

← bosonic model with (d+1) bosonic matrices

CLM cannot sample all the relevant saddle points in one simulation.

→ See what happens when we change initial configs.

Important questions

Is real spacetime obtained?

Spacetime dimensionality?

← weight e^{iS} is complex

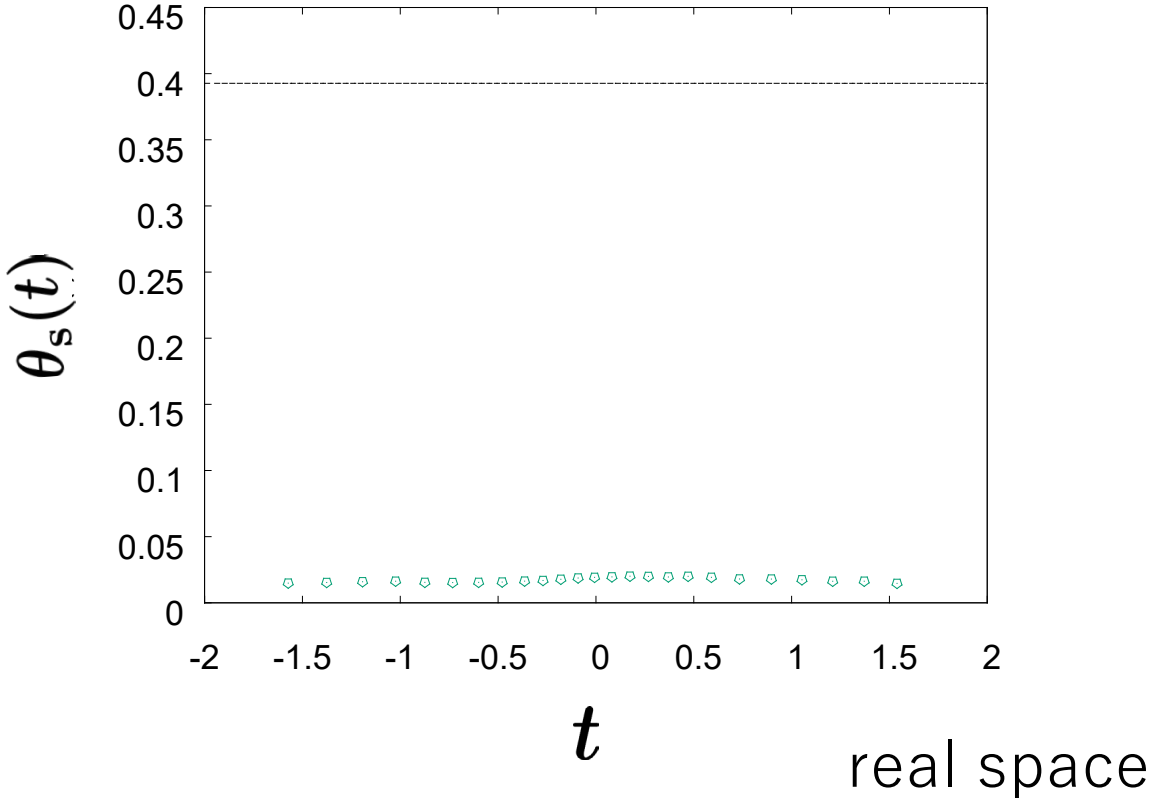
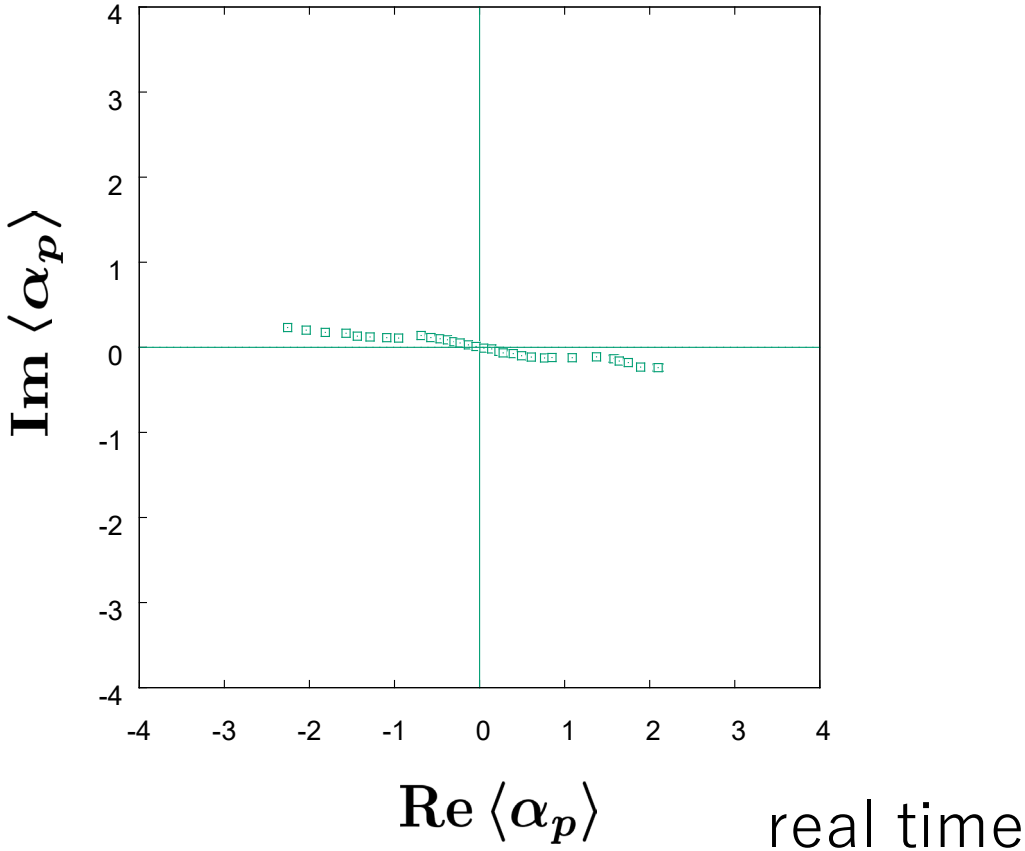
Emergence of real spacetime

starting with (3+1)d initial config.

complex phase of space

$$\left\langle \sum_{i=1}^9 \frac{1}{n} \text{tr} (\bar{A}_i(t))^2 \right\rangle \sim e^{2i\theta_s(t)}$$

eigenvalues of A_0



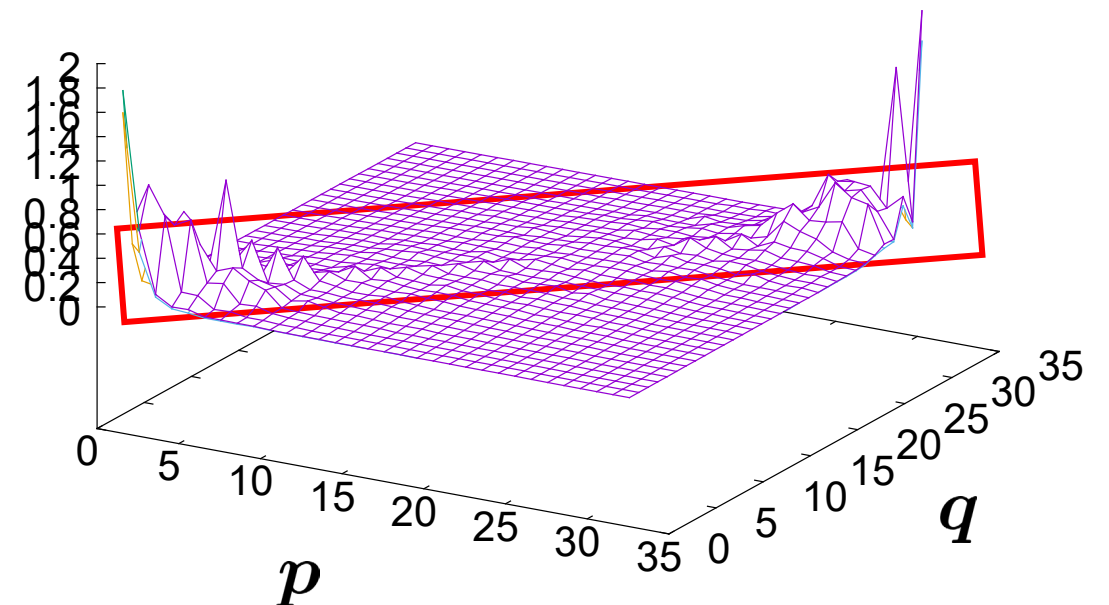
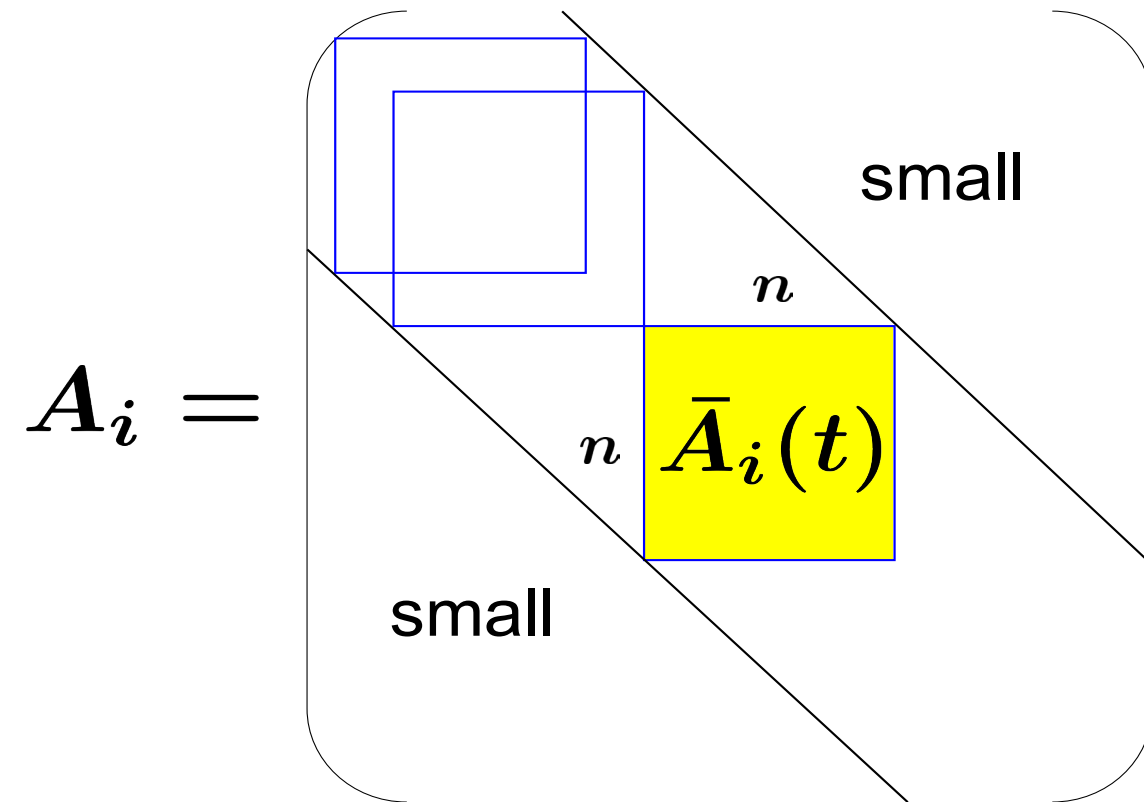
We obtain similar results in the cases in which we start with (2+1)d and (4+1)d configs

Emergence of real spacetime

Band-diagonal structure

starting with (3+1)d initial config.

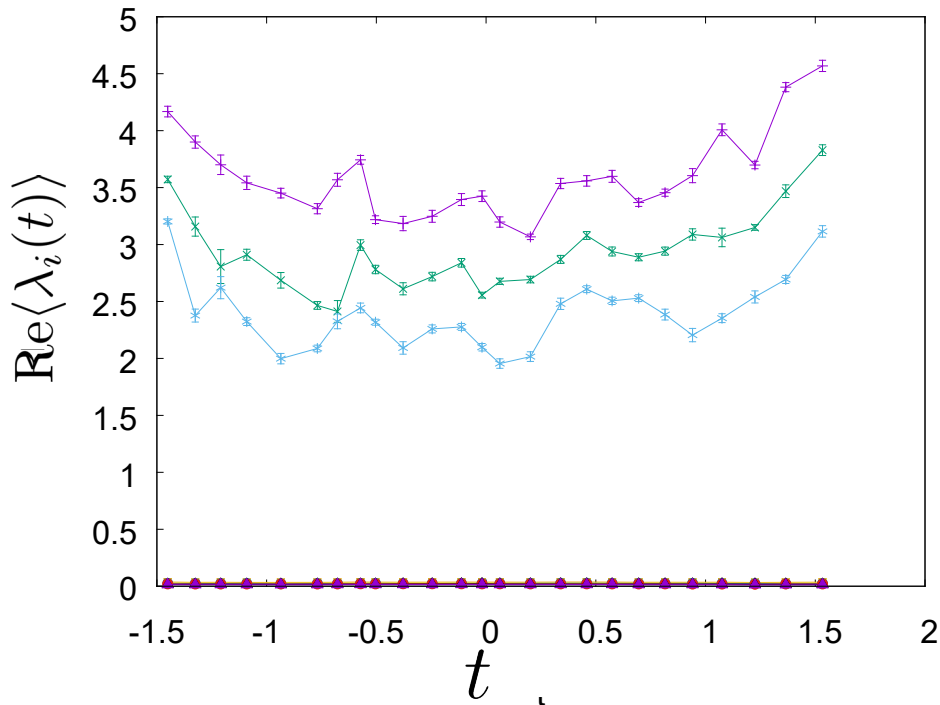
$$\mathcal{A}_{pq} = \frac{1}{9} \text{Re} \left\langle \sum_{i=1}^9 (A_i)_{pq} (A_i)_{qp} \right\rangle$$



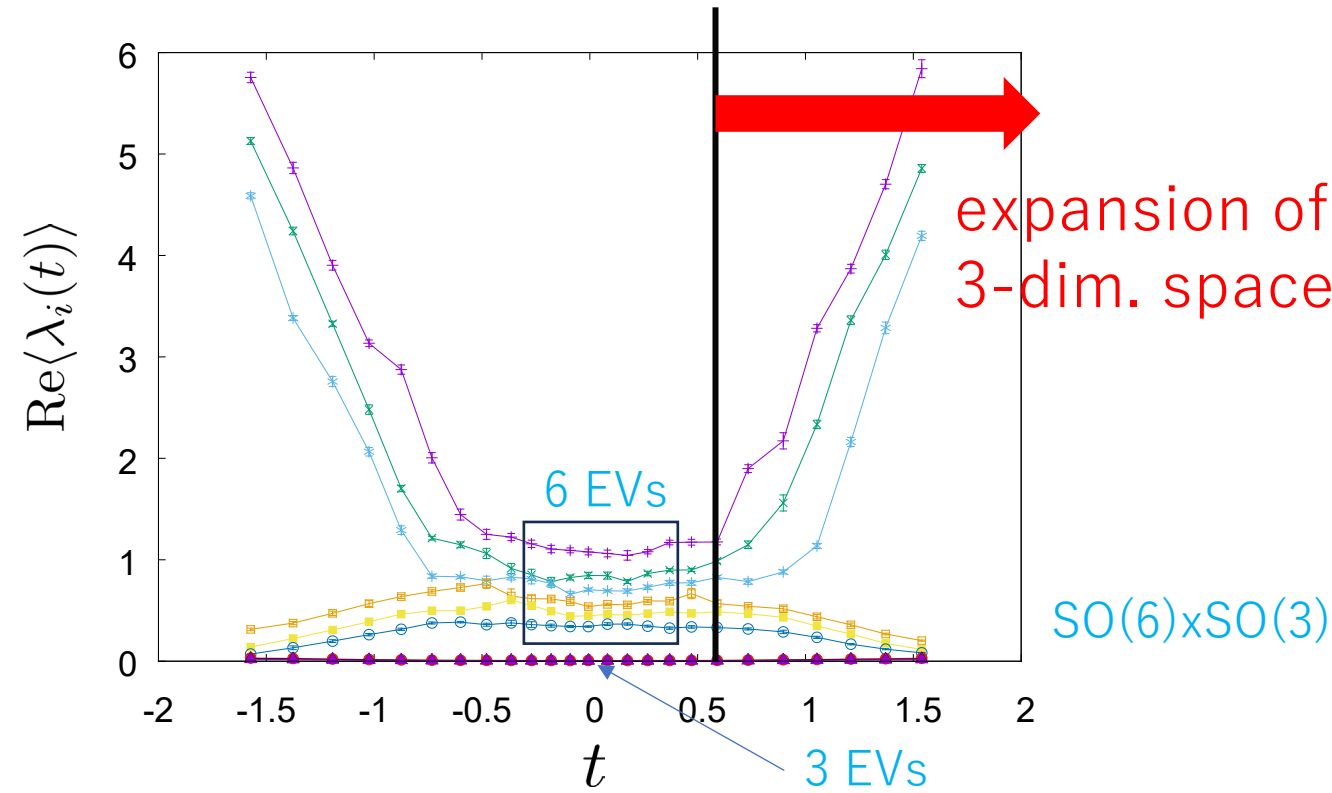
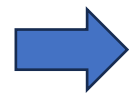
We obtain similar results in the cases in which we start with (2+1)d and (4+1)d configs

Emergence of (3+1)-dimensional expanding space-time

$\lambda_i(t)$ ($i = 1, \dots, 9$): eigenvalues of $T_{ij}(t) = \frac{1}{n} \text{tr}(\bar{A}_i(t)\bar{A}_j(t))$ ~ analog of moment of inertia tensor



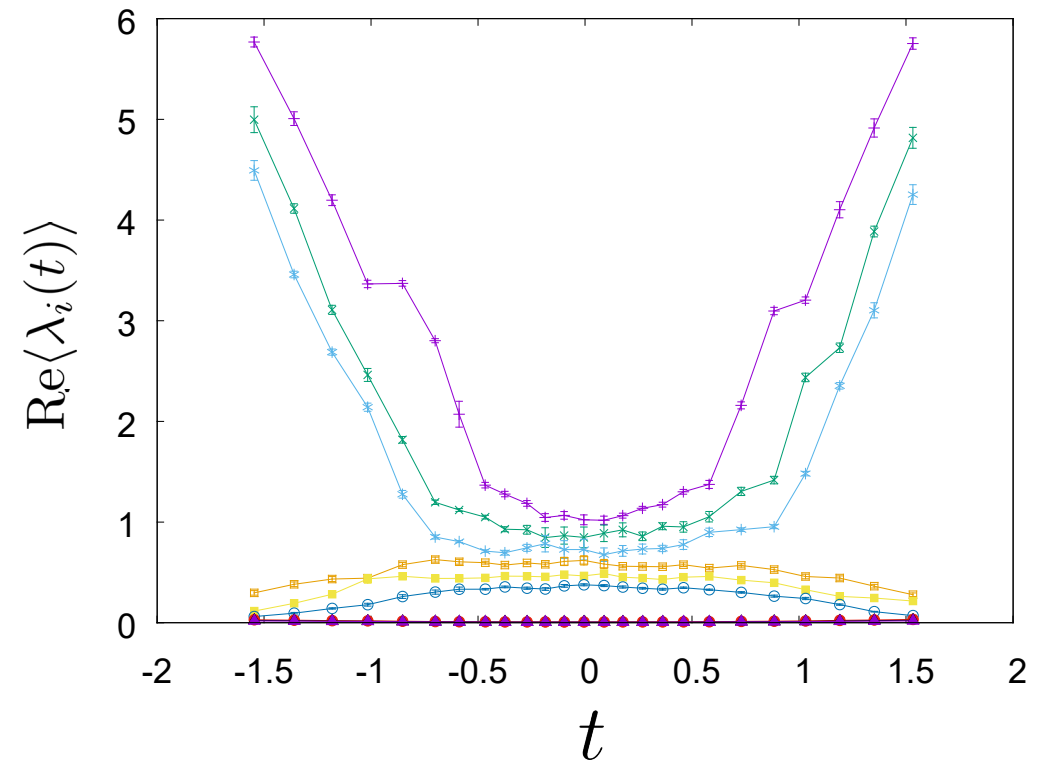
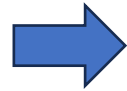
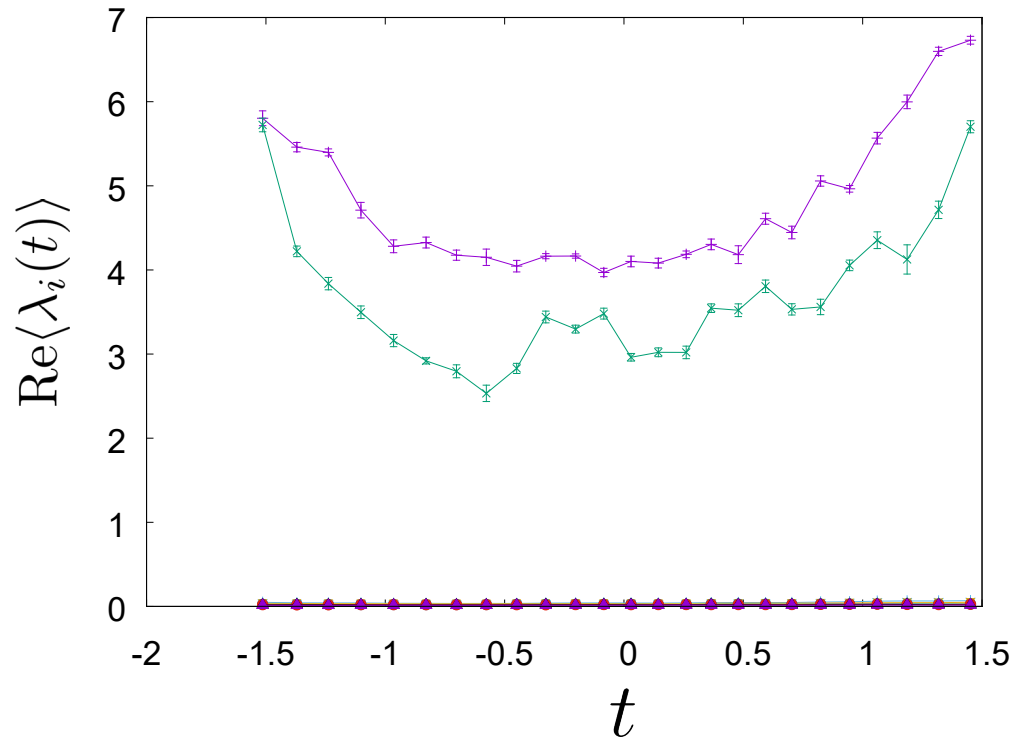
Initial configuration: (3+1)d space-time



After a critical time, 3 out of 6 directions are expanding ~ SSB of SO(6) occurs

Emergence of (3+1)-dimensional expanding space-time

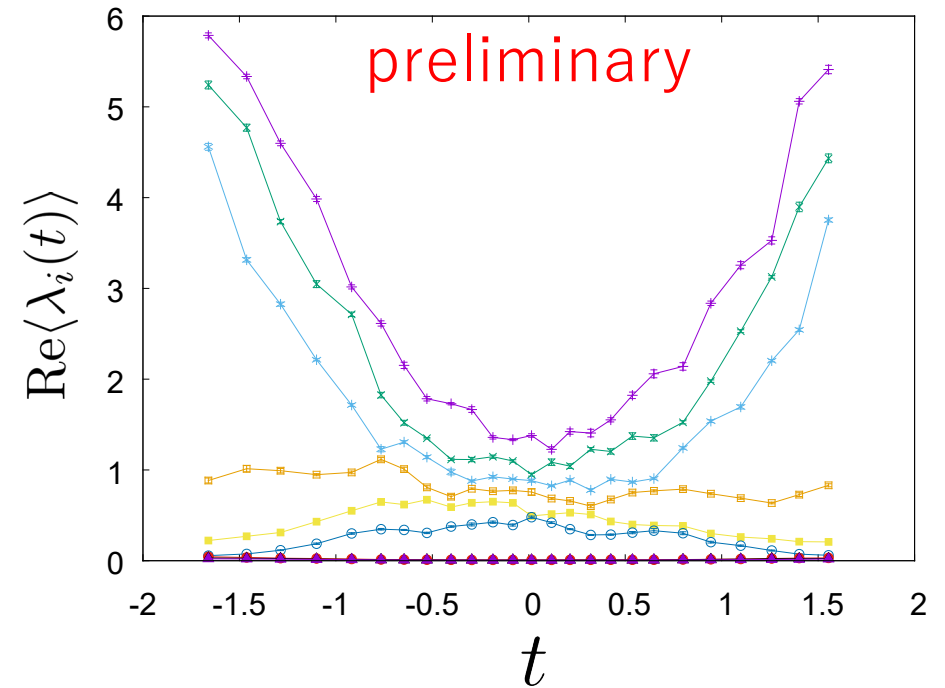
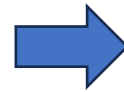
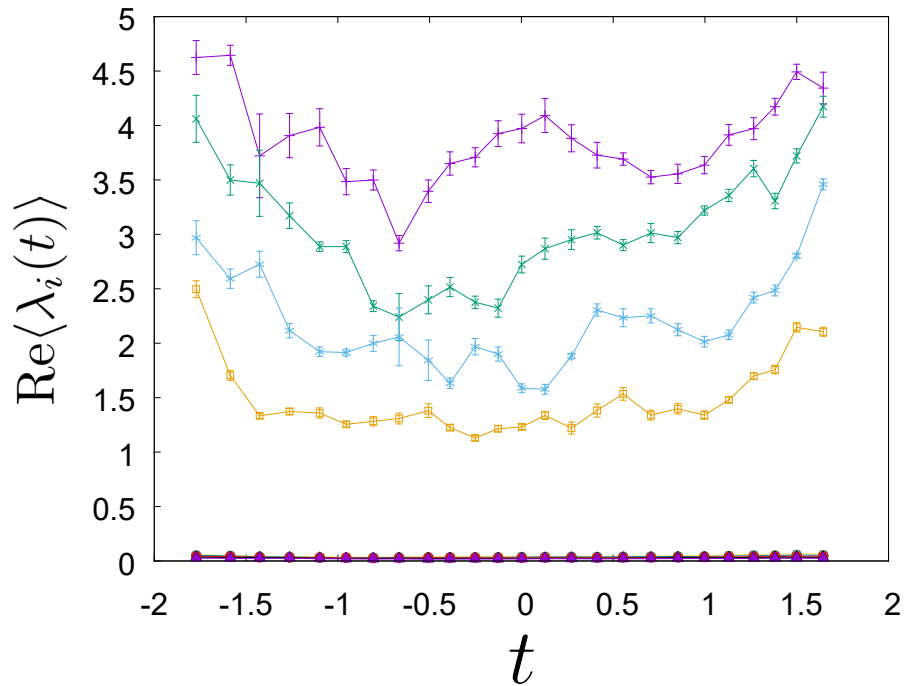
$\lambda_i(t)$ ($i = 1, \dots, 9$): eigenvalues of $T_{ij}(t) = \frac{1}{n} \text{tr}(\bar{A}_i(t) \bar{A}_j(t))$ ~ analog of moment of inertia tensor



Initial configuration: (2+1)d space-time

Emergence of (3+1)-dimensional expanding space-time

$\lambda_i(t)$ ($i = 1, \dots, 9$): eigenvalues of $T_{ij}(t) = \frac{1}{n} \text{tr}(\bar{A}_i(t)\bar{A}_j(t))$ ~ analog of moment of inertia tensor



Initial configuration: (4+1)d space-time

Thermalization is under the way

Speculation on mechanism of SSB

$\text{Pf}\mathcal{M}(A_0, A_1, \dots, A_9) = 0$ if there are only two nonzero A_μ at $m_f = 0$

Krauth, Nicolai, Staudacher (1998)

For sufficiently small m_f , it is expected that spacetimes with at least 3 expanding directions are enhanced

Conclusion and outlook

- We performed complex Langevin simulations of the Lorentzian type IIB matrix model
- We introduced a mass term for fermions to avoid the singular drift problem
- We introduced the mass term for bosons with γ and ξ to balance the effects of fluctuations of bosons and fermions
- This deformation breaks $SO(9,1)$ to $SO(6,1) \times SO(3)$ and is akin to SUSY deformation
- We found that the $SO(6)$ rotational symmetry is spontaneously broken and $(3+1)d$ expanding spacetime appears at some point in time
- We observed independence of initial configurations. Namely, we observed $(3+1)d$ expanding space-time starting with $(2+1)d$, $(3+1)d$ and $(4+1)d$ initial configs.
- In order to investigate whether the $(3+1)$ -dimensional spacetime emerges in the original model, we need to take the limits of $N \rightarrow \infty$, $m_f \rightarrow 0$, $\xi \rightarrow 1$, $\gamma \rightarrow 0$ eventually
- We would like to perform simulations of the SUSY deformed model and take $N \rightarrow \infty$, $\mu \rightarrow 0$ limit