Instabilities in scale-separated Casimir vacua

Miquel Aparici

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Work done with Ivano Basile and Niccolò Risso

Scale separation

- Scale of new physics is separated from current HEP experiments.
- Scale of HEP $\gg$ Hubble scale

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  $\rightarrow$ **Scale separation** = Scale of new physics $\gg$ Hubble scale.
- In the case of extra dimensions [Courdachet 2023],

Scale separation condition

We say that a vacuum exhibits scale separation if

$$\frac{m_{\mathrm{KK}}^2}{|\Lambda|} \gg 1$$

Casimir vacua

- Scale separation is hard to achieve in Freund-Rubin vacua.
- An alternative is Casimir vacua.
[de Luca, de Ponti, Mondino, Tomasiello 2023; Bento and Montero 2025]
- Flux compactification where extra dimensions are compactified in a Riemaniann Flat Manifold $\rightarrow$ no internal curvature!
- Energy from the fluxes compensated by the Casimir effect.
- Non-vanishing Casimir achieved by breaking SUSY.
- Stability of these vacua is no longer SUSY protected!

Candidate scale-separated Casimir vacua (I)

- We study the simplest possible vacuum construction of this type.
[de Luca,de Ponti, Mondino, Tomasiello 2023]

- Consider M-Theory/11d SUGRA compactified on a 7-dimensional square torus

$$ds^2 = L^2 ds^2_{\mathrm{AdS}_4} + R^2 ds^2_{T^7} \,.$$

- SUSY is broken à la Scherk-Schwarz: periodic BCs for the bosons are chosen along torus cycles and antiperiodic for fermions.

- The Casimir energy can be estimated through a scaling argument.
  - It depends only on the internal manifold.
  - It has an overall negative sign from bosons.

On dimensional grounds

$$S_{\mathrm{Casimir}} = 2|\rho_c| \int d^{11}x \sqrt{-g}\, R^{-11} \,.$$

Candidate scale-separated Casimir vacua (II)

- The volume modulus is stabilized by introducing a four-flux

$$F_4 = f_4 \text{vol}_{\text{AdS}_4} \, ,$$

Quantization of the magnetic dual imposes

$$\frac{1}{(2\pi \ell_{\text{Pl},11})^6} \int_{T^7} F_7 = N \quad \Longrightarrow \quad f_4^2 = \frac{N^2}{4\pi^2} \ell_{\text{Pl},11}^{12} \frac{L^8}{R^{14}}$$

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- Solving the field equations with these sources,

$$L^2 = \ell_{\mathrm{Pl},11}^2 \left(\frac{N}{2\pi}\right)^{\frac{22}{3}} |\rho_c|^{-\frac{14}{3}} \frac{7^{14/3}}{2^{11} \cdot 3^{8/3}} \,, \quad R^{11} = \ell_{\mathrm{Pl},11}^{11} \left(\frac{N}{2\pi}\right)^{\frac{22}{3}} |\rho_c|^{-\frac{14}{3}} \frac{7^{11/3}}{2^{11} \cdot 3^{11/3}} \,.$$

When $N \gg 1$, the vacuum exhibits parametric scale separation

$$\frac{R^2}{L^2} \propto N^{-6} \ll 1$$

Potential instabilities

- We performed a detailed analysis of the stability of this vacuum.

In particular, we studied

  - runaway directions? $\times$
  - brane nucleation? $\checkmark$
  - tachyons? $\checkmark$

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  - runaway directions? ×
  - brane nucleation? ✓
  - tachyons? ✓

- For these purposes, it is necessary to understand how exactly the Casimir energy is computed. [Dall'Agata, Zwirner 2025]

Computation of the Casimir energy (I)

The Casimir potential can be evaluated from the usual trace-log box expression. For a single massless bosonic field on $\mathbb{R}^d \times T^q$

$$V = \frac{1}{2} \int \frac{d^d p}{(2\pi)^d} \frac{1}{2\pi R} \sum_{\vec{n} \in \mathbb{Z}^q} \log \left(p^2 + \frac{\vec{n}^2}{R^2} \right) = -\frac{\pi^2}{2(2\pi R)^{d+q}} \int_0^\infty \frac{ds}{s^{\frac{d}{2}+1}} \theta_3(e^{-s})^q \,.$$

with $\theta_3(e^{-s}) = \sum_n e^{-n^2 s}$.

This is a divergent expression. However, divergences cancel after adding contributions from all fields. Final result is

$$V = -\frac{128\pi^2}{2(2\pi R)^{d+q}} \int_0^\infty \frac{ds}{s^{\frac{d}{2}+1}} \left(\theta_3(e^{-s})^q - \theta_2(e^{-s})^q \right)$$

Computation of the Casimir energy (II)

Another method is to use $V = \langle T_{00} \rangle$. Contact divergence is regularized using point splitting. For a massless field **[Birrel and Davies 1982; Arkani-Hamed, Dubovski, Nicolis, Villadoro 2007]**

$$V = \langle T_{00} \rangle = \lim_{x \longrightarrow x'} \frac{\partial}{\partial x^0} \frac{\partial}{\partial x'^0} G(x, x')$$

where $G(x, x') = \langle \phi(x) \phi(x') \rangle$ is the Green's function. Divergences cancel after adding contributions from all fields.

For a bosonic field, the method of images yields

$$G(x, x') = \frac{1}{(d+q-2)\Omega_{d+q-1}} \sum_{n \in \mathbb{Z}^q} \frac{1}{|x - x' + 2\pi R \, \vec{n}|^{d+q-2}}$$

The Casimir potential is

$$V_{T^q}(R) = -\frac{1}{\Omega_{d+q-1}} \sum_{n \neq 0} \frac{1}{|2\pi R \, \vec{n}|^{d+q}} = -\frac{\zeta_{\mathbb{Z}^q}(d+q)}{\Omega_{d+q-1}(2\pi R)^{d+q}}$$

This is for square torus, now we turn to deformations.

Casimir energy on a deformed torus

Consider traceless metric perturbations of the torus:

$$ds^2 = \eta_{\mu\nu}\, dx^\mu \otimes dx^\nu + \left(\delta_{ij} + h_{ij}(y)\right) dy^i \otimes dy^j \,,$$

Casimir energy is computed from GF's and point splitting. For that,

$$L = L_0 + L_1, \quad G = G_0 + G_1$$

where

$$L_0 = \partial_\mu \partial^\mu + \partial_i \partial^i, \quad L_1 = -\,\partial_i (h^{ij}\,\partial_j)$$

Then

$$LG = (L_0 + L_1)(G_0 + G_1) = \delta \quad \rightarrow \quad L_0 G_1 = -L_1 G_0$$

Solve this expanding in a basis of eigenfunctions of L_0, $\{f_i(x,y)\}$,

$$G_1 = \sum_{ij} (L_1)_{ij}\, \frac{f_i(x)^*\, f_j(y)}{\lambda_i \lambda_j}\,.$$

Casimir energy on a deformed torus

More explicitly

$$G_1((x,y),(x',y')) = \int \frac{d^d p}{(2\pi)^d} \sum_{n,m\in\mathbb{Z}^q} \frac{1}{(2\pi R)^{2q}} \frac{n_i\, m_j}{R^2} \frac{\widetilde{h}^{ij}_{n-m}\, e^{-ip\cdot(x-x')-i\frac{n\cdot y-m\cdot y'}{R}}}{\left(p^2+\frac{n^2}{R^2}\right)\left(p^2+\frac{m^2}{R^2}\right)}\,.$$

where $\widetilde{h}_n \equiv \int_{T^q} d^q y\, h(y)\, e^{i\frac{n}{R}\cdot y}$ are the Fourier modes of the metric perturbation.

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where $\widetilde{h}_n \equiv \int_{T^q} d^q y\, h(y)\, e^{i\frac{n}{R}\cdot y}$ are the Fourier modes of the metric perturbation. The perturbed Casimir potential is

$$\delta V = -\int \frac{d^d p}{(2\pi)^d} \frac{p^2}{d} \sum_{n,m\in\mathbb{Z}^q} \frac{n_i\, m_j\, \widetilde{h}^{ij}_{n-m}}{(2\pi R)^{2q} R^2} \frac{e^{-i(n-m)\cdot\frac{y}{R}}}{\left(p^2+\frac{n^2}{R^2}\right)\left(p^2+\frac{m^2}{R^2}\right)}\,.$$

This expression implies that flat deformations are on-shell since, in that case, $\widetilde{h}^{ij}_{n-m} = (2\pi R)^q\, \delta_{n,m}\, h^{ij}$. This, combined with rotational invariance, implies that the result will be $\propto h^i_i = 0$.

The vacuum is on-shell

• It is enough that flat deformations vanish for the vacuum to be on-shell. Corrections to the potential take the form

$$\delta V = \sum_{\vec{m}} a_{\vec{m}} h_{\vec{m}} e^{i \frac{\vec{m}\cdot\vec{y}}{R}} .$$

Performing the integral in the internal coordinates

$$\int d^{d+q}x \sum_{\vec{m}} a_{\vec{m}} h_{\vec{m}} e^{i \frac{\vec{m}\cdot\vec{y}}{R}} = \sum_{\vec{m}} a_{\vec{m}} \int d^{d+q}x \, h_{\vec{m}} e^{i \frac{\vec{m}\cdot\vec{y}}{R}} = (2\pi R)^7 a_{\vec{0}} h_{\vec{0}} = 0 .$$

• This argument works provided that the Fourier series of δV is well-defined, i.e. the sequence $\{a_{\vec{m}}\}$ is square-summable. We proved this in the paper. [MA, Basile, Risso 2025]

Non-perturbative instabilities: brane nucleation

- Non-SUSY AdS vacua are expected to decay via flux tunneling.

[Horowitz, Orgera, Polchisnki 2007; Brown, Dahlen 2010; Ooguri, Vafa 2016; Antonelli, Basile 2019; Dibitetto, Petri, Schillo 2020]

- M2-brane decay rate per unit volume is proportional to $e^{-S^E_{M2}}$, with

$$S^E_{M2} = T\Omega_3 L^3 (x^3 - 2\beta \mathcal{V}(x)),$$

and

$$\mathcal{V}(x) = \int_0^{\frac{\rho}{L}} d\tilde{\rho} \frac{\tilde{\rho}^3}{\sqrt{1+\tilde{\rho}^2}} \ , \ x = \frac{\rho}{L} \ , \ \beta = \frac{f_4}{2L^3} \frac{\mu}{T}$$

- This decay channel is allowed as long as $\beta > 1$. Here, $\beta = 2\sqrt{2} > 1$.

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• The vacuum exhibits non-perturbative instabilities!

• Exponentially suppressed decay channel, $S^E_{M2} \sim N^{11} \to \frac{\Gamma}{\text{Vol}} \sim e^{-N^{11}}$.

• Brane nucleation near boundary of AdS. Vacuum decays in an AdS time for an observer in the bulk.

Perturbative instabilities: presence of tachyons (I)

Consider the 11d effective potential, now adding the dependence of the Casimir in moduli orthogonal to the volume h

$$V_{11}(R_7) = \frac{1}{8\pi^2 M_{Pl,11}^3} \frac{N^2}{R^{14}} - \frac{2v(h)}{R^{11}} \, .$$

The original solution is chosen to be at $h = 0$, i.e. $v(0) = |\rho_c|$. From the preceding discussion $v'(0) = 0$.

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Lower dimensional potential:

$$V_4(R) = (2\pi)^7 R_*^{14} \left(\frac{1}{8\pi^2 M_{Pl,11}^3} \frac{N^2}{R^{21}} - \frac{2v(h)}{R^{18}} \right),$$

Masses can be read off from the hessian, once the fields are canonically normalized.

Perturbative instabilities: presence of tachyons (II)

In particular, consider the following (single) flat torus deformation ϕ

$$R^2 ds^2_{T^7} = R_*^2 e^{2\rho} \left(e^{2\phi} dy_1^2 + e^{-2\phi} dy_2^2 + \sum_{i=3}^{7} dy_i^2 \right) ,$$

The masses are found to be

$$m_\rho^2 = \left. \frac{\partial^2 V_4}{\partial \rho^2} \right|_{\rho=0} = \frac{36}{L_*^2} > 0 , \quad m_\phi^2 = \left. \frac{\partial^2 V_4}{\partial \phi^2} \right|_{\rho=0} = -\frac{42}{L^2} \frac{v''(0)}{v(0)} .$$

Importantly, m_ϕ^2 could be negative!

Perturbative instabilities: presence of tachyons (III)

In AdS, tachyonic particles might still be fine, provided that they are above the BF bound [Breitenlohner, Freedman 1982]

$$m^2 L^2 \geq -\frac{9}{4}\,.$$

It is crucial to compute $m_\phi^2 L^2 = -42\frac{v''(0)}{v(0)}$. For that, we consider

$$\frac{v(x)}{v(0)} = \frac{\displaystyle\int_0^\infty \frac{ds}{s^3}\left[\theta_3\left(e^{-e^x s}\right)\theta_3\left(e^{-e^{-x}s}\right)\theta_3\left(e^{-s}\right)^5 - \theta_2\left(e^{-e^x s}\right)\theta_2\left(e^{-e^{-x}s}\right)\theta_2\left(e^{-s}\right)^5\right]}{\displaystyle\int_0^\infty \frac{ds}{s^3}\left[\theta_3\left(e^{-s}\right)^7 - \theta_2\left(e^{-s}\right)^7\right]}$$

Numerically computing the second derivative we find

$$m_\phi^2 L^2 = -42\frac{v''(0)}{v(0)} \approx -685.46\ldots < -\frac{9}{4}\,,$$

The vacua suffer from perturbative instabilities!

Conclusion and outlook

- Casimir vacua provide a simple setting to look for AdS vacua with parametric scale separation.
- We studied the simplest compactification of this type, confirming the presence of both perturbative and non-perturbative instabilities.
- Even though the vacuum is unstable under brane nucleation, an observer in the bulk can survive up to an AdS time, which is parametrically larger than the EFT cutoff.
- In the presence of tachyons, the dimensionally reduced EFT lacks a perturbative vacuum and unitarity is violated.
- Perturbative instabilities might be avoidable in more refined constructions. We will further explore this in the future.