Non-relativistic Heterotic String Theory from a Target Space and Worldsheet Point of View

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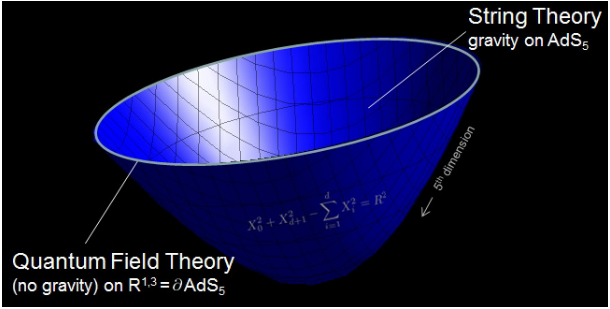
Why Non-relativistic ?

Holography



Lifshitz holography $\rightarrow$ Newton-Cartan gravity with twistless torsion

Christensen, Hartong, Kiritsis, Obers and Rollier (2013-2015)

new perspective on role of non-relativistic string theory within holography

Blair, Harmark, Lahnsteiner, Obers, Yan (2025); Guijosa, Rosas-López (2024)

Lambert, Smith (2024); Fontanella, Nieto-García (2024)

Remarks

- Non-Relativistic $\leftrightarrow$ Non-Lorentzian
- Limit $\neq$ Expansion
- generalized limits:
  1. string foliation
  2. cancellation of divergences
  3. controlling divergences via Hubbard-Stratonovich transformation

How to define a Limit

Step 1: make an invertible redefinition of the relativistic fields F in terms of "would-be" non-relativistic fields f and a contraction parameter c

Step 2: take the limit $c \to \infty$. The result is invariant under global dilatations

Outline

Heterotic Gravity

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The Heterotic Lagrangian

The bosonic sector of the $10D$ $\mathcal{N}=1$ Heterotic Supergravity Lagrangian in lowest-order of derivatives is given by

$$\mathcal{L}_{\text{heterotic}} \sim \mathcal{L}(R) + \beta\,\mathrm{Tr}\,F(V)^2 \qquad \text{with} \qquad \beta = 1/g^2 \sim \alpha' \quad \text{and}$$

$$\begin{aligned} \mathcal{L}(R) &= e^{-\phi}\Big[R(\omega(e)) + (H_3^{(1)})^2 + (\phi^{-1}d\phi)^2\Big]\,, \\ H_3^{(1)} &= dB_2 + \beta\,\mathrm{Tr}\Big[(VdV + \tfrac{1}{3}V^3\Big] \;:\quad \text{YM Chern} - \text{Simons term} \end{aligned}$$

We exploit a useful symmetry between Yang-Mills and supergravity where supergravity is similar to an SO(9,1) Yang-Mills multiplet in the sense that under supersymmetry

$$\delta_0\Omega_{\mu-}^{(0)ab} = \tfrac{1}{2}\bar{\epsilon}\Gamma_\mu\psi^{ab} \qquad \text{with} \qquad \Omega_{\mu-}^{(0)ab} \equiv \omega_\mu{}^{ab}(e) - H_\mu{}^{(0)ab}$$

Up to $O(\alpha)$ and $O(\beta)$, this leads to the quadratic effective action

$$\mathcal{L}_{\text{quadratic}} \sim \mathcal{L}(R) + e^{-\phi}\Big(\alpha\mathrm{Tr}\,R(\Omega_-^{(0)})^2 + \beta\,\mathrm{Tr}\,F(V)^2\Big) \qquad \text{with} \quad \alpha \sim \alpha'$$

and where $H_3^{(1)}$ now also involves a Lorentz Chern-Simons term

Higher-order corrections

de Roo + E.B. (1989)

The quadratic effective action is only supersymmetric up to $O(\alpha)$ and $O(\beta)$ because $\Omega_{\mu-}^{(0)ab}(e, B)$ is not an independent SO(9,1) gauge field and its transformation rule changes due to the fact that under supersymmetry

$$\big(\delta_\alpha + \delta_\beta\big) B_{\mu\nu} \neq 0 \ : \quad \text{Lorentz and Yang}-\text{Mills CS term}$$

We find that in the next order there are no cubic terms of the form

$$\mathcal{L}_{\text{cubic}} \sim \alpha^2 R^3 + \alpha\beta R F^2$$

but there is a non-trivial quartic effective action

$$\mathcal{L}_{\text{quartic}} \sim \alpha^3 R^4 + \alpha^2\beta R^2 F^2 + \alpha\beta^2 F^4$$

Note: there are other R^4 terms that do not follow from supersymmetry.

Gross, Witten (1986); Grisaru, van de ven, Zanon (1986); Nilsson, Tollstén (1986)

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Heterotic T-Duality from a Target Space Point of View

Janssen, Ortín + E.B. (1989)

The heterotic T-duality transformations correspond to a $\mathbb{Z}_2$ transformation of the dimensionally reduced heterotic supergravity Lagrangian where the 'momentum' KK vector field coming from the metric is inter-changed with the 'winding' vector field coming from the NS-NS 2-form field

Remarkably, we found that the heterotic T-duality rules simplified when formulated in terms of the following not gauge-invariant generalized metric

$$G_{MN} = g_{MN} + B_{MN} + \tfrac{1}{2}\beta\,\mathrm{Tr}\,V_M V_N$$

Comparing the duality rules of $\Omega^{(0)}$ and V^I we argued that at order $O(\alpha)$ and $O(\beta)$ the duality rules could be formulated in terms of the generalized metric

$$G_{MN} = g_{MN} + B_{MN} + \tfrac{1}{2}\alpha\mathrm{Tr}\,\Omega^{(0)}_M\Omega^{(0)}_N + \tfrac{1}{2}\beta\mathrm{Tr}\,V_M V_N$$

This was much later confirmed by explicit dimensional reduction

Elgood, Ortín (2020)

At the time, we had no simple explanation for this!

Heterotic T-Duality from a Worldsheet Point of View

Grosvenor, Romano, Yan + E.B. (2025)

Ignoring the dilaton, the Heterotic Sigma Model is given by

$$S_{\text{heterotic}} = \frac{1}{4\pi}\int \mathrm{d}^2 z \left\{ \partial_z X^{\mathrm{M}}\, \partial_{\bar{z}} X^{\mathrm{N}} \left[g_{\mathrm{MN}}(X) + B_{\mathrm{MN}}(X) \right] + \psi_a \nabla_z \psi^a \right.$$
$$\left. + \mathrm{tr}\left(\lambda\, \nabla_{\bar{z}} \lambda\right) + \frac{i}{2}\,\mathrm{tr}\left[\lambda\, \psi^{\mathrm{M}}\, \psi^{\mathrm{N}}\, F_{\mathrm{MN}}(X)\, \lambda\right] \right\}$$

The heterotic fermions λ are Majorana-Weyl spinors leading to a YM gauge anomaly

Hull, Witten (1985); Hull, Townsend (1986)

One can obtain a gauge-invariant path-integral by adding to the above sigma model a counterterm which has the effect that $g_{\mathrm{MN}}(X) + B_{\mathrm{MN}}(X)$ is replaced by the generalised metric $G_{MN}(X)$ that we found in our old work.

One can include gravitational gauge anomalies too

A simple world-sheet duality transformation in the thus obtained not gauge-invariant heterotic sigma model leads to the heterotic T-duality rules derived many years ago!

A NR Heterotic Sigma Model from 'Relativistic' T-duality

Our starting point is the relativistic Heterotic Sigma Model at $O(\alpha)$ and $O(\beta)$

T-dualizing along a spacelike isometry direction x with $G_{xx} = G_{MN}k^M k^N \neq 0$ or along a lightlike isometry direction x with $G_{xx} = G_{MN}\ell^M \ell^N = 0$ can be done as follows:

Write $X^M = (X^\mu, x)$ and construct a parent sigma model involving the terms

$$\tilde{x}\left(\partial_z \chi_- - \partial_{\bar{z}} \chi_+\right) \quad \text{and} \quad \chi_+ \chi_- G_{xx} \quad (\underline{\text{only}} \text{ for spacelike isometry direction})$$

Integrating out $\tilde{x}$ leads to $\chi_+ = \partial_z x\,, \chi_- = \partial_{\bar{z}} x$ which leads back to the original sigma model

Integrating out $\chi_\pm$, one can solve for $\chi_\pm$ in the case of a spacelike isometry direction leading to a relativistic heterotic sigma model in the T-dual frame whereas $\chi_\pm$ remain Lagrange multipliers in the case of a lightlike isometry direction leading to a NR Heterotic Sigma Model

compare to Gomis, Ooguri (2001)

Non-relativistic Heterotic T-Duality

The NR Heterotic sigma model is characterized by a string Newton-Cartan geometry that distinguishes between 2 longitudinal and 8 transverse directions

There are three different ways of T-dualizing:

(i) T-dualizing in a longitudinal spatial direction leads back to the heterotic string in DLCQ $\longrightarrow$

NR Heterotic String theory can be used to describe Heterotic String theory in the DLCQ

(ii) T-dualizing in a transverse spatial direction leads to non-relativistic Buscher-like T-duality rules

(iii) One can also T-dualize in a longitudinal null direction

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The Heterotic NR Limit from a Target Space Perspective

Romano + E.B. (2025); see also Lescano (2025)

Decomposing $\widehat{A} = (A, a) = (+, -, a)$: $\mathrm{SO}(1,1)^{\pm} \times \mathrm{SO}(8)$ we find that a Heterotic Limit up to $O(\beta)$ can be defined by the following invertible redefinitions:

$$\begin{aligned} E_M{}^- &= c\tau_M{}^- , \qquad E_M{}^a = e_M{}^a , \qquad \Phi = \phi + \log c , \\ E_M{}^+ &= -c^3 \frac{(v_-)^2}{2} \tau_M{}^- + c\tau_M{}^+ , \\ B_{MN} &= c^2 \tau_M{}^A \tau_N{}^B \epsilon_{AB} (1 + v_+^I v_{-I}) + 2c^2 \tau_{[M}{}^- e_{N]}{}^a v_{-a} + b_{MN} , \\ V_M^I &= c^2 \tau_M{}^- v_-^I + \tau_M{}^+ v_+^I + e_M{}^a v_a^I , \end{aligned}$$

where $\tau_M{}^{\pm}, e_M{}^a$ and b_{MN} describe a string Newton-Cartan geometry

We obtained the above redefinitions by requiring that the Heterotic Limit leads to

1. finite transformation rules
2. a finite target space heterotic action
3. finite heterotic T-duality rules

The Hubbard-Stratonovich Transformation

We already saw that the NR Heterotic sigma model, as opposed to the relativistic one, contains Lagrange multipliers $\chi_\pm$

From the heterotic NR limit point of view, these Lagrange multipliers are the result of controlling divergences using a Hubbard Stratonovich transformation

These divergences are a feature of the fact that the regular Riemannian geometry is converted into a singular String Newton-Cartan geometry

The Hubbard-Stratonovich transformation

1. $\mathcal{L} \sim c^2 X^+ X^- \qquad \longleftrightarrow \qquad \mathcal{L} \sim -\frac{1}{c^2}\chi_+\chi_- - \chi_+ X^+ - \chi_- X^-$

with auxiliary fields $\chi_\pm$: $\quad \chi_\pm = -c^2 X^\mp$

2. $c \to \infty \quad \Rightarrow \quad \mathcal{L} = -\chi_+ X^+ - \chi_- X^-$,

where $\chi_\pm$ have become Lagrange multipliers

For the Heterotic string we have

$$X^+ = \partial_{\bar{z}} X^M \tau_M{}^+ \quad \text{and} \quad X^- = \partial_z X^M \tau_M{}^- + i\,\mathrm{tr}\,\lambda v_+ \lambda$$

The Heterotic NR limit from a Sigma-model Perspective

Grosvenor, Romano, Yan + E.B. (2025)

The NR heterotic sigma model contains the following Lagrange multiplier terms:

$$\mathcal{L}_{\rm heterotic} \sim \chi_+ \partial_{\bar{z}} X^M {\tau_M}^+ + \chi_- \Big[\partial_z X^M {\tau_M}^- + i\,{\rm tr}\lambda v_+ \lambda\Big]$$

Applying an inverse Hubbard-Stratonovich transformation, we convert the Lagrange multipliers $\chi_\pm$ into auxiliary fields by adding the deformation

$$\mathcal{L}_{\rm deformation} \sim \frac{1}{c^2}\chi_+\chi_-$$

Gomis, Oh, Yan (2019)

Solving for the auxiliary fields lead to a standard relativistic Polyakov action upon making an inverse field redefinition. These redefinitions are precisely the ones that define the NR Heterotic Limit discussed before!

N.B. The NR formulation with the Lagrange multipliers is invariant under an emerging local dilatation symmetry that is absent in the relativistic case

Outline

Summary

We showed that earlier target space derivations of heterotic T-duality and the heterotic NR limit can be conveniently be re-derived from a world-sheet point of view

We found, after taking the limit, an emergent dilatation symmetry that was earlier observed for non-heterotic strings from a target space point of view

The local dilatation symmetry implies that the target space effective action does not give rise to the Poisson equation corresponding to String Newton-Cartan geometry

Open Issues

1. extending the target space NR heterotic limit to $O(\alpha)$ including the gravitational Chern-Simons term

Grosvenor, Romano, Yan + E.B., work in progress

2. extension to supergravity leads to large multiplets of constraints due to infinities in the susy transformation rules. A notable exception is the string limit of ten-dimensional minimal supergravity

Lahnsteiner, Romano, Rosseel, Şimşek + E.B. (2021); Rosseel + E.B. (2022)
see also talk by S. Zeko

3. properties of (heterotic or non-heterotic) NR brane solutions

(i) role of local dilatations

(ii) elementary or stacked ($\rightarrow$ scaling or smearing)

(iii) p-branes aligned or not aligned to the q-brane limit

Take Home Message

Taking non-Lorentzian limits is often non-trivial and

leads to several interesting questions !