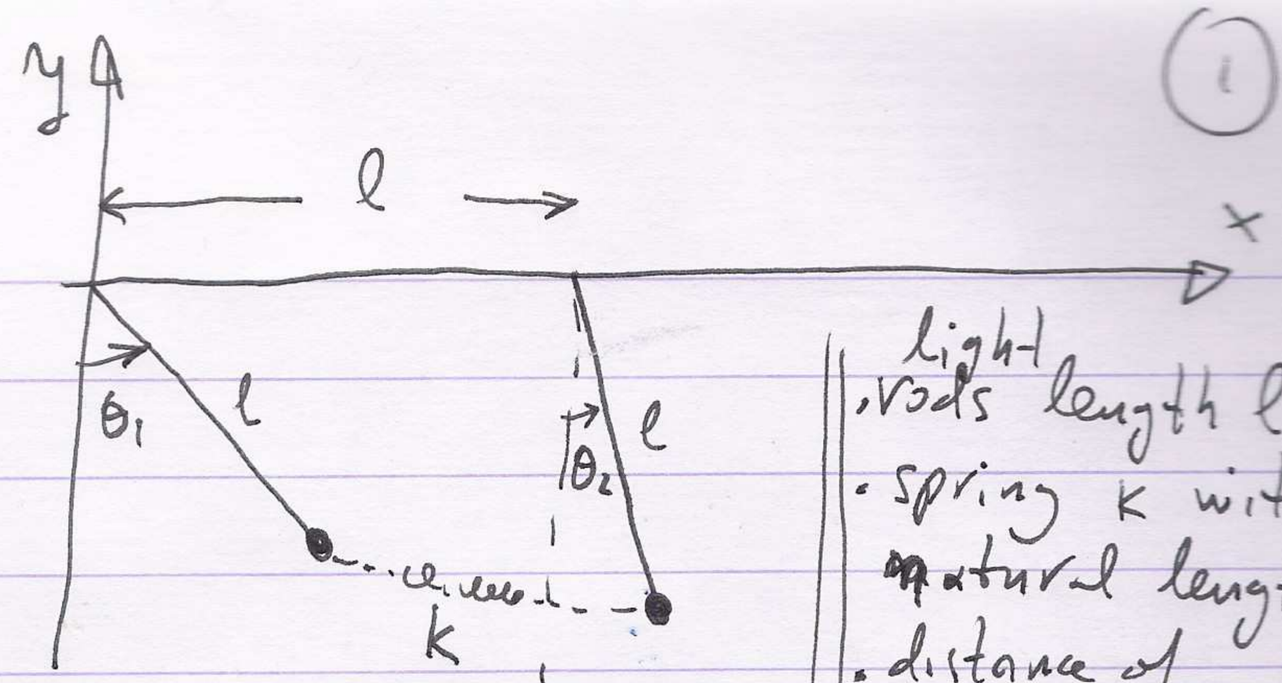

light rods length l
- spring k with natural length l
- distance of suspension points l

$$x_1 = l \sin\theta_1$$
$$y_1 = -l\cos\theta_1$$

$$x_2 = l + l\sin\theta_2$$
$$y_2 = -l\cos\theta_1$$

$$\mathcal{L} = \frac{1}{2} m l^2 (\dot\theta_1^2 + \dot\theta_2^2) - V_g - V_e$$

$$V_g = +mgy_1 + mgy_2 = -mgl(\cos\theta_1 + \cos\theta_2)$$

$$V_e = \frac{1}{2} k \cdot \Delta l^2 = \frac{1}{2} k (r - l)^2$$

$$r^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2 = l^2 [(1 + \sin\theta_2 - \sin\theta_1)^2 + (\cos\theta_1 - \cos\theta_2)^2]$$

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot\theta_1} = ml^2\ddot\theta_1$$

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot\theta_2} = ml^2\ddot\theta_2$$

$$\frac{\partial \mathcal{L}}{\partial \theta_1} = -\frac{\partial V_g}{\partial \theta_1} - \frac{\partial V_e}{\partial \theta_1}$$

$$-\frac{\partial V_g}{\partial \theta_1} = -\frac{\partial}{\partial \theta_1}(-mgl\cos\theta_1) = -mgl\sin\theta_1$$

$$-\frac{\partial V_g}{\partial \theta_2} = -\frac{\partial}{\partial \theta_2}(-mgl\cos\theta_2) = -mgl\sin\theta_1$$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\theta}_i} = \frac{\partial L}{\partial \theta_i} \Rightarrow m\ddot{\theta}_i = -\frac{\partial V}{\partial \theta_i}$$

$$-\frac{\partial V_e}{\partial \theta_1} = -k\cdot\Delta l\,\frac{\partial \Delta l}{\partial \theta_1} = -k\,\Delta l\,\frac{\partial r}{\partial \theta_1} = -k\,\Delta l\,\frac{1}{2r}\frac{\partial r^2}{\partial \theta_1}$$

$$= -\frac{k\,\Delta l}{2r}\cdot l^2\left[2(1+\sin\theta_2-\sin\theta_1)\cdot(-\cos\theta_1)+2(\cos\theta_1-\cos\theta_2)(-\sin\theta_1)\right]$$

$$= +k\frac{l^2\Delta l}{r}\left[(1+\sin\theta_2-\sin\theta_1)\cos\theta_1+(\cos\theta_1-\cos\theta_2)\sin\theta_1\right]$$

$$-\frac{\partial V_e}{\partial \theta_2} = -k\,\Delta l\,\frac{1}{2r}\frac{\partial r^2}{\partial \theta_2}$$

$$= -\frac{k\,\Delta l}{2r}\,l^2\left[2(1+\sin\theta_2-\sin\theta_1)\cos\theta_2+2(\cos\theta_1-\cos\theta_2)\sin\theta_2\right]$$

$$= -k\frac{l^2\Delta l}{r}\left[(1+\sin\theta_2-\sin\theta_1)\cos\theta_2+(\cos\theta_1-\cos\theta_2)\sin\theta_2\right]$$

$$\Delta l \equiv r-l \qquad r^2 = l^2\left[(1+\sin\theta_2-\sin\theta_1)^2+(\cos\theta_1-\cos\theta_2)^2\right]$$

$\underline{\theta_1}$ $\quad \frac{\Delta l}{l} = \frac{r}{l}-1 \qquad \frac{r}{l} = \left[(1+\sin\theta_2-\sin\theta_1)^2+(\cos\theta_1-\cos\theta_2)^2\right]^{1/2}$

$$ml^2\ddot{\theta}_1 = -mgl\sin\theta_1 + k\frac{l^2\Delta l}{r}\left[(1+\sin\theta_2-\sin\theta_1)\cos\theta_1+(\cos\theta_1-\cos\theta_2)\sin\theta_1\right]$$

$$\Rightarrow \boxed{\ddot{\theta}_1 = -\frac{g}{l}\sin\theta_1 + \frac{k\,\Delta l}{mr}\left[(1+\sin\theta_2-\sin\theta_1)\cos\theta_1+(\cos\theta_1-\cos\theta_2)\sin\theta_1\right]}$$

$\underline{\theta_2}$

$$ml^2\ddot{\theta}_2 = mgl\sin\theta_2 \; \frac{kl^2\Delta l}{mr}\left[(1+\sin\theta_2-\sin\theta_1)\cos\theta_2+(\cos\theta_1-\cos\theta_2)\sin\theta_2\right]$$

$$\Rightarrow \boxed{\ddot{\theta}_2 = -\frac{g}{l}\sin\theta_2 - \frac{k(\Delta l/l)}{m(r/l)}\left(1+\sin\theta_2-\sin\theta_1\right)\cos\theta_2+(\cos\theta_1-\cos\theta_2)\sin\theta_2\Big]}$$

$$k_1 = \frac{g}{l} = \omega_0^2,\; k_2 = \frac{k}{m}$$

$$E = \frac{1}{2} m l^2 (\dot{\theta}_1^2 + \dot{\theta}_2^2) + V_g + V_e =$$

$$= \frac{1}{2} m l^2 (\dot{\theta}_1^2 + \dot{\theta}_2^2) - mgl(\cos\theta_1 + \cos\theta_2) + \frac{1}{2} k (r-l)^2$$

$$\frac{E}{ml^2} = \frac{1}{2}(\dot{\theta}_1^2 + \dot{\theta}_2^2) - \frac{g}{l}(\cos\theta_1 + \cos\theta_2) + \frac{1}{2}\left(\frac{k}{m}\right)\left(\frac{r}{l} - 1\right)^2$$

$$(\theta_1, \theta_2, \omega_1, \omega_2) \longleftrightarrow (x_1, x_2, y_1, y_2, v_{1x}, v_{1y}, v_{2x}, v_{1y})$$

$x_1 = l \sin\theta_1$ $\quad v_{1x} = (\omega_1 \cdot l) \cdot \cos\theta_1$

$y_1 = -l\cos\theta_1$ $\quad v_{1y} = (\omega_1 l) \cdot \sin\theta_1$

$x_2 = l(1 + \sin\theta_2)$ $\quad v_{2x} = (\omega_2 l) \cdot \cos\theta_2$

$y_2 = -l\cos\theta_2$ $\quad v_{2y} = (\omega_2 l) \cdot \sin\theta_2$