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Quantum space visualisation

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What geometry is encoded in matrices?



$$X_1, X_2, X_3 \in \text{Mat}_N$$

$$[X_i, X_j] \neq 0$$



$$Z = \int dA e^{-S}$$

$$S[A] = N \operatorname{tr} \left[-\frac{1}{4} [A_i, A_j]^2 + \frac{2}{3i\alpha} \epsilon_{ijk} A_i A_j A_k \right],$$

where A_i ($i = 1, 2, 3$) are $N \times N$ traceless Hermitian matrices

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Accretion, mergers, and metastability of fuzzy spheres in a three-matrix model

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ABSTRACT: A three-matrix model that contains a plethora of fuzzy sphere solutions was defined twenty years ago. It has been shown that in a certain regime of the model, combinations of fuzzy spheres become classical solutions of the model. In the original paper and subsequent works, the stability of specific states has been discussed, showing that only a single largest fuzzy-sphere configuration is stable. Here, we focus on a complementary question: starting from some initial configuration, is the thermodynamically preferred state approached rapidly, or are there other long-lived, metastable configurations which may support interesting physics? We find that the answer in the large-matrix limit depends on the dispersion of the initial configuration. We identify the elementary transition as an accretion, in which one sphere absorbs a single unit from another, and show that it is driven by fluctuations of the sphere centers. By measuring these fluctuations in long simulations, we find quantitative agreement with the one-loop effective action, with no free parameters. For multi-sphere states, we find that the stiffness of an inner sphere grows logarithmically with the number of surrounding layers, so that concentric onion-like configurations – describing an emergent three-dimensional quantum space – become increasingly long-lived.

Fig.: Hrmo et al., 2026

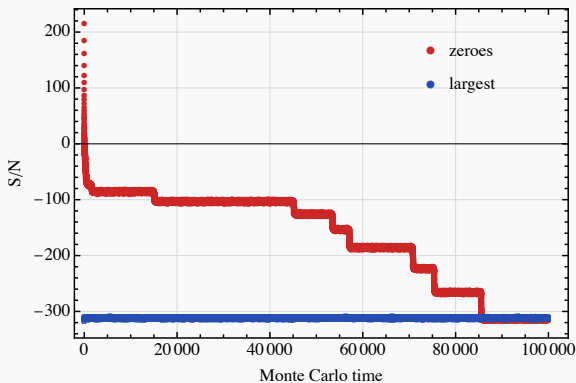


Fig.: Simulations with $N = 20$



tools of quantum mechanics

+

noncommutative spaces

data $\Rightarrow$ geometry

Quantum Geometry of Data

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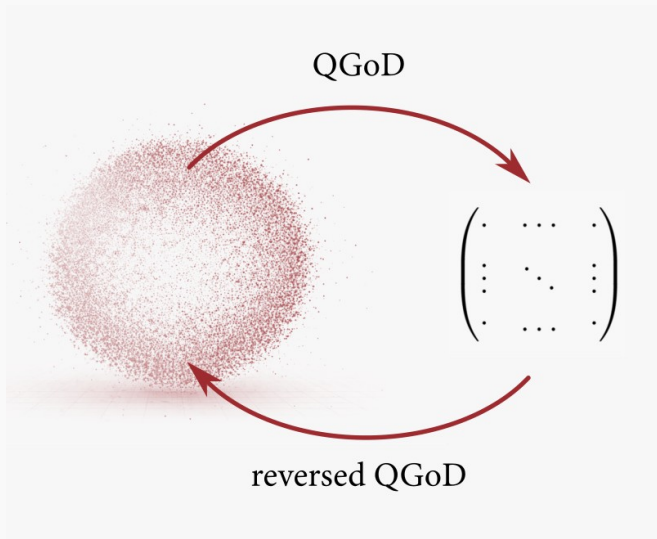
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ABSTRACT

We demonstrate how Quantum Cognition Machine Learning (QCM) encodes data as quantum geometry. In QCM, features of the data are represented by learned Hermitian matrices, and data points are mapped to states in Hilbert space. The quantum geometry description endows the dataset with rich geometric and topological structure—including intrinsic dimension, quantum metric, and Berry curvature—derived directly from the data. QCM captures global properties of data, while avoiding the curse of dimensionality inherent in local methods. We illustrate this on a number of synthetic and real-world examples. Quantum geometric representation of QCM could advance our understanding of cognitive phenomena within the framework of quantum cognition.

35v1 [cs.LG] 22 Jul 2025

Fig.: Quantum Geometry of Data,
A.Abanov et al., 2025

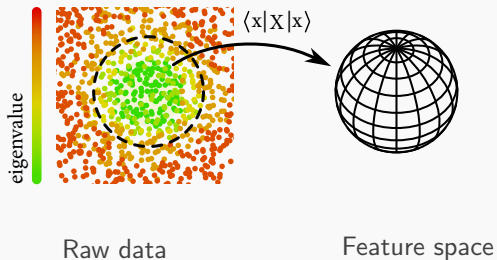




$$x \in \mathbb{R}^3 \implies H(x) = \frac{1}{2} \sum_{a=1}^D (X_a - x_a I_N)^2$$

$$H(x)|x\rangle = \lambda|x\rangle$$

if λ small $\implies$ well localised quantum state $|x\rangle$



- λ small
- $\Downarrow$
- take $|x\rangle$
(groundstate of H)
- $\Downarrow$
- $(X(x))_i = \langle x|X_i|x\rangle$
- $\Downarrow$
- QCML point cloud

Raw vs. expectations

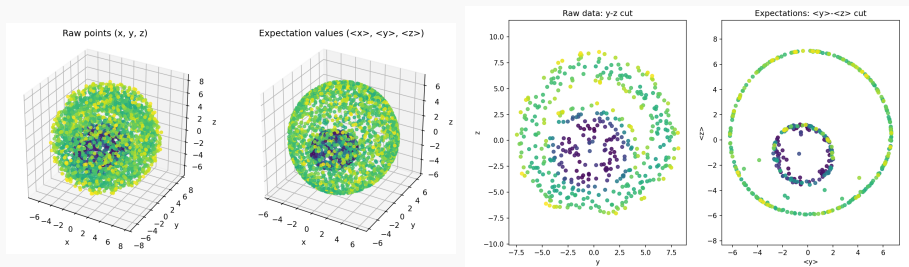


Fig.: Comparison of points with lowest eigenvalues and their computed expectation values $\langle x|X_i|x\rangle$.

Dynamics of a three-matrix model

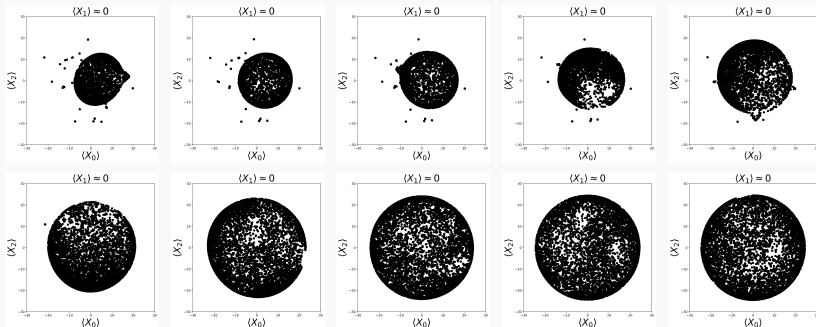


Fig.: Dynamics of a system initialised in a hot state; $y - z$ plane projection

Dynamics of a three-matrix model

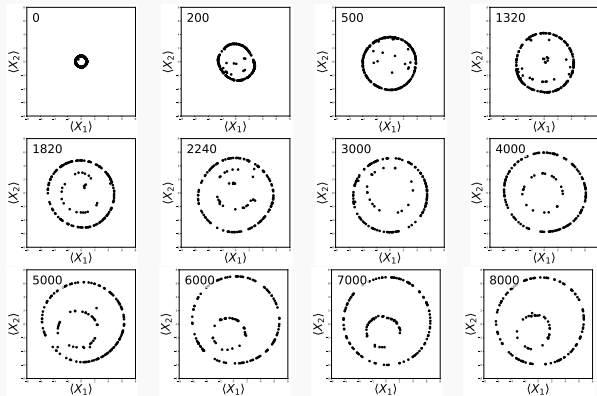


Fig.: Dynamics of a system initialised in cold state; X -cut visualisation.

What geometry is encoded in matrices?



A lot.



- The displacement Hamiltonian $\rightarrow$ non-commuting matrix coordinates
- Expectation values $\rightarrow$ point of the quantum space
- Three matrix model : accretion, merges, dynamics
- What next? (ABNN, IKKT)

THANK YOU



- [Abanov et al., 2025] Abanov, A. G., Candelori, L., Steinacker, H. C., Wells, M. T., Busemeyer, J. R., Hogan, C. J., Kirakosyan, V., Marzari, N., Pinnamaneni, S., Villani, D., Xu, M., and Musaelian, K. (2025). Quantum geometry of data.
- [Hrmo et al., 2026] Hrmo, M., Kováčik, S., Magdolenová, K., Manta, A., Nedel'ková, K., Rusnák, P., Steinacker, H. C., and Tekel, J. (2026). Accretion, mergers, and metastability of fuzzy spheres in a three-matrix model.
- [Schneiderbauer and Steinacker, 2016] Schneiderbauer, L. and Steinacker, H. C. (2016). Measuring finite quantum geometries via quasi-coherent states. *J. Phys. A*, 49(28):285301.