

Gravity as a Noncommutative Gauge Theory

Fuzzy Gravity and Unification with Internal Interactions

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- We know that gravitational theories can be successfully described by gauge theories
*Utiyama '56, Kibble '61, McDowell, Mansuri '77
Chamseddine, West '77, Ivanov, Niederle '82
Kibble, Stelle '85, Wilczek '98, Ortin '04*
- Usually the dimension of the tangent space is considered to be equal to the dimension of the curved manifold. However, the tangent group of a manifold of dimension d is not necessarily SO_d .
Weinberg '84
- We aim to unify FG as a gauge theory with internal interactions under one unification gauge group.
*Similar ideas have been proposed in the past
(Percacci, Chamseddine, Mukhanov, et al.)*

4D Conformal Gravity as a Gauge Theory

- Group parameterizing the symmetry: $SO(2, 4)$
- 15 generators:

$$[\hat{M}_{AB}, \hat{M}_{CD}] = \eta_{AC} \hat{M}_{DB} - \eta_{BC} \hat{M}_{DA} - \eta_{AD} \hat{M}_{CB} + \eta_{BD} \hat{M}_{CA}$$

- Indices splitting $\rightarrow$ 6 LT M_{ab} , 4 translations P_a , 4 conformal boosts K_a and 1 dilatation D
- Action is taken of $SO(2, 4)$ invariant quadratic form
- Initial symmetry breaks spontaneously by introducing a scalar in the adjoint rep fixed in the dilatation direction

Roumelioti, S, Zoupanos '24

SSB by using a scalar in the adjoint representation

Gauge connection:

$$A_\mu = \frac{1}{2} \omega_\mu^{ab} M_{ab} + e_\mu^a P_a + b_\mu^a K_a + \tilde{a}_\mu D,$$

Field strength tensor:

$$F_{\mu\nu} = \frac{1}{2} R_{\mu\nu}^{ab} M_{ab} + \tilde{R}_{\mu\nu}^a P_a + R_{\mu\nu}^a K_a + R_{\mu\nu} D,$$

where

$$\begin{aligned} R_{\mu\nu}^{ab} &= \partial_\mu \omega_\nu^{ab} - \partial_\nu \omega_\mu^{ab} - \omega_\mu^{ac} \omega_{\nu c}^b + \omega_\nu^{ac} \omega_{\mu c}^b - 8e_{[\mu}^{[a} b_{\nu]}^{b]} \\ &= R_{\mu\nu}^{(0)ab} - 8e_{[\mu}^{[a} b_{\nu]}^{b]}, \end{aligned}$$

$$\begin{aligned} \tilde{R}_{\mu\nu}^a &= \partial_\mu e_\nu^a - \partial_\nu e_\mu^a + \omega_\mu^{ab} e_{\nu b} - \omega_\nu^{ab} e_{\mu b} - 2\tilde{a}_{[\mu} e_{\nu]}^a \\ &= T_{\mu\nu}^{(0)a}(e) - 2\tilde{a}_{[\mu} e_{\nu]}^a, \end{aligned}$$

$$\begin{aligned} R_{\mu\nu}^a &= \partial_\mu b_\nu^a - \partial_\nu b_\mu^a + \omega_\mu^{ab} b_{\nu b} - \omega_\nu^{ab} b_{\mu b} + 2\tilde{a}_{[\mu} b_{\nu]}^a \\ &= T_{\mu\nu}^{(0)a}(b) + 2\tilde{a}_{[\mu} b_{\nu]}^a, \end{aligned}$$

$$R_{\mu\nu} = \partial_\mu \tilde{a}_\nu - \partial_\nu \tilde{a}_\mu + 4e_{[\mu}^a b_{\nu]a},$$

→ $R_{\mu\nu}^{(0)ab}$, $T_{\mu\nu}^{(0)a}(e)$: *Curvature and Torsion of 4D Poincaré grav.*

We start with the following action, which is quadratic in terms of the field strength tensor and introduce a scalar in the adjoint rep.

$$\mathcal{S}_{SO(2,4)} = a_{CG} \int d^4x \left[\text{tr} \epsilon^{\mu\nu\rho\sigma} m \phi F_{\mu\nu} F_{\rho\sigma} + \lambda (\phi^2 - m^{-2} \mathbb{1}_4) \right],$$

The scalar expanded on the generators is:

$$\phi = \phi^{ab} M_{ab} + \tilde{\phi}^a P_a + \phi^a K_a + \tilde{\phi} D,$$

We pick the specific gauge in which ϕ is only in the direction of the dilatation generator D :

$$\phi = \phi^0 = \tilde{\phi} D \xrightarrow{\phi^2 = m^{-2} \mathbb{1}_4} \phi = -2m^{-1} D.$$

The resulting broken action is (after employing anticommutator relations and the traces over the generators):

$$\mathcal{S}_{SO(1,3)} = \frac{a_{CG}}{4} \int d^4x \epsilon^{\mu\nu\rho\sigma} \epsilon_{abcd} R_{\mu\nu}{}^{ab} R_{\rho\sigma}{}^{cd}$$

→ *Similar results are obtained using two scalars in the vector rep instead.*

The $\tilde{a}_\mu$ and $R_{\mu\nu}$ are not present in the action, so we can set both equal to zero.

$$R_{\mu\nu} = \partial_\mu \tilde{a}_\nu - \partial_\nu \tilde{a}_\mu + 4e_{[\mu}{}^a b_{\nu]a} = 0 \xrightarrow{\tilde{a}_\mu=0}$$

$$e_\mu{}^a b_{\nu a} - e_\nu{}^a b_{\mu a} = 0$$

We examine two possible solutions of the above equation:

- $b_\mu{}^a = a e_\mu{}^a$, *Chamseddine '03*
- $b_\mu{}^a = -\frac{1}{4} \left(R_\mu{}^a + \frac{1}{6} R e_\mu{}^a \right)$ *Kaku, Townsend, van Nieuwenhuizen, 78*
Freedman, Van Proyen 'Supergravity' '12

The first choice leads to the [Einstein-Hilbert](#) action, while the second leads to [Weyl](#) action.

Einstein-Hilbert action

- When $b_\mu{}^a = a e_\mu{}^a$, the broken action becomes:

$$\begin{aligned}\mathcal{S}_{\text{SO}(1,3)} &= \frac{a_{\text{CG}}}{4} \int d^4x \epsilon^{\mu\nu\rho\sigma} \epsilon_{abcd} R_{\mu\nu}{}^{ab} R_{\rho\sigma}{}^{cd} \implies \\ \mathcal{S}_{\text{SO}(1,3)} &= \frac{a_{\text{CG}}}{4} \int d^4x \epsilon^{\mu\nu\rho\sigma} \epsilon_{abcd} \left[R_{\mu\nu}^{(0)ab} R_{\rho\sigma}^{(0)cd} - 16m^2 a R_{\mu\nu}^{(0)ab} e_\rho{}^c e_\sigma{}^d + \right. \\ &\quad \left. + 64m^4 a^2 e_\mu{}^a e_\nu{}^b e_\rho{}^c e_\sigma{}^d \right]\end{aligned}$$

This action consists of three terms: one G-B topological term, the E-H term, and a cosmological constant term. For $a < 0$ it describes GR in AdS space.

Weyl action

- When $b_\mu{}^a = -\frac{1}{4}(R_\mu{}^a + \frac{1}{6}R e_\mu{}^a)$, the broken action becomes

$$\begin{aligned} \mathcal{S} = \frac{a_{CG}}{4} \int d^4x \epsilon^{\mu\nu\rho\sigma} \epsilon_{abcd} \Big[R_{\mu\nu}^{(0)ab} - \frac{1}{2} \left(\tilde{e}_\mu{}^{[a} R_{\nu}{}^{b]} - \tilde{e}_\nu{}^{[a} R_\mu{}^{b]} \right) + \\ + \frac{1}{3} R \tilde{e}_\mu{}^{[a} \tilde{e}_\nu{}^{b]} \Big] \\ \Big[R_{\rho\sigma}^{(0)cd} - \frac{1}{2} \left(\tilde{e}_\rho{}^{[c} R_{\sigma}{}^{d]} - \tilde{e}_\sigma{}^{[c} R_\rho{}^{d]} \right) + \\ + \frac{1}{3} R \tilde{e}_\rho{}^{[c} \tilde{e}_\sigma{}^{d]} \Big], \end{aligned}$$

where $\tilde{e}_\mu{}^a = m e_\mu{}^a$ is the rescaled vierbein. The above action is equal to

$$\mathcal{S} = \frac{a_{CG}}{4} \int d^4x \epsilon^{\mu\nu\rho\sigma} \epsilon_{abcd} C_{\mu\nu}{}^{ab} C_{\rho\sigma}{}^{cd},$$

where $C_{\mu\nu}{}^{ab}$ is the Weyl conformal tensor.

The NC framework & gauge theories

- Noncommutative space $\rightarrow$ replace coordinates with operators X^i ($\in \mathcal{A}$) satisfying: $[X^i, X^j] = i\Theta^{ij}(X)$

Connes '94, Madore '99

- Antisymmetric tensor $\Theta^{ij}(X)$ - defines the NC of the space
- Introduction of *covariant NC coordinate*:

$$\mathcal{X}_\mu = X_\mu + A_\mu$$

Madore, Schraml, Schupp, Wess '00

- Obeys a covariant gauge transformation rule: $\delta\mathcal{X}_\mu = i[\epsilon, \mathcal{X}_\mu]$
- Definition of a NC *covariant field strength tensor*:

$$F_{ab} = [\mathcal{X}_a, \mathcal{X}_b] - i\Theta_{ab}$$

Non-Abelian case

- Let us consider the commutator of two elements of an algebra:

$$[\epsilon, A] = [\epsilon^A T^A, A^B T^B] = \frac{1}{2} \{ \epsilon^A, A^B \} [T^A, T^B] + \frac{1}{2} [\epsilon^A, A^B] \{ T^A, T^B \}$$

- Not possible to restrict to a matrix algebra:

▷ last term neither *vanishes* in NC nor is an *algebra element*

- There are two options to overpass the difficulty:

▷ Consider the universal enveloping algebra

▷ Fix the rep and expand algebra so that the anticommutators close

Aschieri, Castellani '09

Ćirić, Gočanin, Konjik, Radovanović '18

▷ *We will later employ the second option*

The 4d covariant noncommutative space

- Constructing field theories on NC spaces is non-trivial: NC deformations break Lorentz invariance
- Such an example is the Fuzzy Sphere (2d space) - coords are identified as rescaled SU(2) generators
Madore '92, Hammou, Lagraa, Sheikh Jabbari '02
Vitale, Wallet '13, Vitale '14, Jurman, Steinacker '14
Chatzistavrakidis, Jonke, Jurman, Manolakos, Manousselis, Zoupanos '18
- We will need a 4d covariant NC space to construct a gravity gauge theory
- We will aim for a NC version of dS₄, described by the embedding
 $\eta^{AB} X_A X_B = R^2$ into M_5

- The $SO(1,4)$ generators, J_{mn} , $m, n = 0, \dots, 4$, satisfy the commutation relation:

$$[J_{mn}, J_{rs}] = i(\eta_{mr}J_{ns} + \eta_{ns}J_{mr} - \eta_{nr}J_{ms} - \eta_{ms}J_{nr})$$

- Consider decomposition of $SO(1,4)$ to maximal subgroup, $SO(1,3)$
- Introduce a length parameter λ and convert the generators to physical quantities by identifying $\Theta_{ij} = \hbar J_{ij}$, $X_i = \lambda J_{i4}$
- Thus, the commutation relations regarding the operators $\Theta_{\mu\nu}$ and X_μ are:

$$[\Theta_{ij}, \Theta_{kl}] = i\hbar (\eta_{ik}\Theta_{jl} + \eta_{jl}\Theta_{ik} - \eta_{jk}\Theta_{il} - \eta_{il}\Theta_{jk}),$$

$$[\Theta_{ij}, X_k] = i\hbar (\eta_{ik}X_j - \eta_{jk}X_i),$$

$$[X_i, X_j] = \frac{i\lambda^2}{\hbar} \Theta_{ij}$$

- The noncommutativity of coordinates becomes manifest

Yang's Model '47

- Extending covariance, also including momenta as generators $\rightarrow$ use a group with larger symmetry $\rightarrow$ minimum extension: $SO(1,4) \subset SO(1,5)$

Yang '47

Kimura '02, Heckman, Verlinde '15

Burić, Madore '14, '15

Steinacker '16

Sperling, Steinacker '17, '19

Manousselis, Manolakos, Zoupanos '19, '21

- The $SO(1,5)$ generators, $J_{MN}, M, N = 0, \dots, 5$, satisfy the commutation relation:

$$[J_{MN}, J_{P\Sigma}] = i(\eta_{MP}J_{N\Sigma} + \eta_{N\Sigma}J_{MP} - \eta_{NP}J_{M\Sigma} - \eta_{M\Sigma}J_{NP})$$

- Employ a 2-step decomposition $SO(1,5) \supset SO(1,4) \supset SO(1,3)$
- Introducing a length parameter λ (like in Snyder's case) we convert the generators to physical quantities by identifying $\Theta_{ij} = \hbar J_{ij}, X_i = \lambda J_{i5}, P_i = \frac{\hbar}{\lambda} J_{i4}, h = J_{45}$

Yang's Model '47 (Continued)

- Thus, the commutation relations regarding all the operators $\Theta_{\mu\nu}, X_\mu, P_\mu, h$ are:

$$[\Theta_{\mu\nu}, \Theta_{\rho\sigma}] = i\hbar(\eta_{\mu\rho}\Theta_{\nu\sigma} + \eta_{\nu\sigma}\Theta_{\mu\rho} - \eta_{\nu\rho}\Theta_{\mu\sigma} - \eta_{\mu\sigma}\Theta_{\nu\rho}),$$

$$[\Theta_{\mu\nu}, X_\rho] = i\hbar(\eta_{\mu\rho}X_\nu - \eta_{\nu\rho}X_\mu)$$

$$[\Theta_{\mu\nu}, P_\rho] = i\hbar(\eta_{\mu\rho}P_\nu - \eta_{\nu\rho}P_\mu)$$

$$[P_\mu, P_\nu] = i\frac{\hbar}{\lambda^2}\Theta_{\mu\nu}, \quad [X_\mu, X_\nu] = i\frac{\lambda^2}{\hbar}\Theta_{\mu\nu},$$

$$[P_\mu, h] = -i\frac{\hbar}{\lambda^2}X_\mu, \quad [X_\mu, h] = i\frac{\lambda^2}{\hbar}P_\mu,$$

$$[P_\mu, X_\nu] = i\hbar\eta_{\mu\nu}h, \quad [\Theta_{\mu\nu}, h] = 0$$

- Momenta are seamlessly included in algebra
 - ▷ Momentum space becomes quantized
 - ▷ Heisenberg type CR between momenta and coords
- The above relations describe the noncommutative space

Noncommutative gauge theory of 4d gravity

- We want to formulate gravitation theory on the above space
- We make use of NC gauge theory toolbox combined with the procedure described in the 4d conformal gravity case

Kimura '02, Heckman, Verlinde '15

- Begin by gauging the isometry group of the space, $SO(1,4)$
- Anticommutators do not close $\rightarrow$ fix the representation + enlargement of the algebra

Aschieri, Castellani '09

Chatzistavrakidis, Jonke, Jurman, Manolakos, Manousselis, Zoupanos '18

- Noncommutative gauge theory of $SO(2,4) \times U(1)$

Manolakos, Manousselis, Zoupanos '19, '21

Roumelioti, S, Zoupanos '24

- The generators of $SO(2, 4) \times U(1)$ are represented by combinations of the 4×4 gamma matrices:
 - six Lorentz rotation generators: $M_{ab} = -\frac{i}{4} [\gamma_a, \gamma_b]$
 - four generators for conformal boosts: $K_a = \frac{1}{2} \gamma_a (1 + \gamma_5)$
 - four generators for translations: $P_a = -\frac{1}{2} \gamma_a (1 - \gamma_5)$
 - one generator for special conformal transformations: $D = -\frac{1}{2} \gamma_5$
 - one $U(1)$ generator: $\mathbb{1}$

- The above expressions of the generators allow the calculation of the algebra they satisfy:

$$\begin{aligned}[M_{ab}, M_{cd}] &= \eta_{bc}M_{ad} + \eta_{ad}M_{bc} - \eta_{ac}M_{bd} - \eta_{bd}M_{ac}, \\ [K_a, P_b] &= -2(\eta_{ab}D + M_{ab}), \quad [P_a, D] = P_a, \quad [K_a, D] = -K_a, \\ [M_{ab}, K_c] &= \eta_{bc}K_a - \eta_{ac}K_b, \quad [M_{ab}, P_c] = \eta_{bc}P_a - \eta_{ac}P_b\end{aligned}$$

- Generators satisfy the following anticommutation relations:

Smolin '03

$$\begin{aligned}\{M_{ab}, M_{cd}\} &= \frac{1}{2}(\eta_{ac}\eta_{bd} - \eta_{bc}\eta_{ad}) - i\epsilon_{abcd}D, \\ \{M_{ab}, P_c\} &= +i\epsilon_{abcd}P^d, \\ \{M_{ab}, K_c\} &= -i\epsilon_{abcd}K^d, \\ \{M_{ab}, D\} &= 2M_{ab}D, \\ \{P_a, K_b\} &= 4M_{ab}D + \eta_{ab}, \\ \{K_a, K_b\} &= \{P_a, P_b\} = -\eta_{ab}, \\ \{P_a, D\} &= \{K_a, D\} = 0.\end{aligned}$$

Noncommutative Gauge Theory

- Since the gauge group is determined to be $SO(2,4) \times U(1)$, we can move on with the gauging procedure.

Manolakos, Manousselis, Zoupanos '21

- Consider the *covariant coordinate* $\mathcal{X}_\mu = X_\mu + A_\mu$
- Determine appropriate *covariant field strength tensor*

$$\mathcal{R}_{\mu\nu} = [\mathcal{X}_\mu, \mathcal{X}_\nu] - i \frac{\lambda^2}{\hbar} \hat{\Theta}_{\mu\nu},$$

where $\hat{\Theta}_{\mu\nu} = \Theta_{\mu\nu} + \mathcal{B}_{\mu\nu}$, the *covariant noncommutative tensor*

For the SSB to take place we:

- ↪ Introduce scalar field $\Phi(X)$ belonging in the **2nd rank antisym.** of $SO(4)$, *charged* under $U(1) \rightarrow U(1)$ breaks and doesn't appear in final action
- ↪ Gauge fix $\Phi(X)$ in the direction that leads to Lorentz group

Gauge connection and field strength tensor decompose as:

$$A_\mu(X) = e_\mu^a \otimes P_a + \frac{1}{2} \omega_\mu^{ab} \otimes M_{ab} + b_\mu^a \otimes K_a + \tilde{a}_\mu \otimes D + a_\mu \otimes \mathbb{1}_4.$$

$$\mathcal{R}_{\mu\nu}(X) = \tilde{R}_{\mu\nu}^a \otimes P_a + \frac{1}{2} R_{\mu\nu}^{ab} \otimes M_{ab} + R_{\mu\nu}^a \otimes K_a + \tilde{R}_{\mu\nu} \otimes D + R_{\mu\nu} \otimes \mathbb{1}_4.$$

The component curvatures:

$$R_{\mu\nu} = [X_\mu, a_\nu] - [X_\nu, a_\mu] + [a_\mu, a_\nu] + [b_\mu^a, b_\nu^a] + [\tilde{a}_\mu, \tilde{a}_\nu] + \frac{1}{2} [\omega_\mu^{ab}, \omega_\nu^{ab}]$$

$$+ [e_{\mu a}, e_{\nu}^a] - \frac{i\hbar}{\lambda^2} B_{\mu\nu}$$

$$\tilde{R}_{\mu\nu} = [X_\mu, \tilde{a}_\nu] + [a_\mu, \tilde{a}_\nu] - [X_\nu, \tilde{a}_\mu] - [a_\nu, \tilde{a}_\mu] - i\{b_{\mu a}, e_{\nu}^a\} + i\{b_{\nu a}, e_{\mu}^a\}$$

$$+ \frac{1}{2} \epsilon_{abcd} [\omega_\mu^{ab}, \omega_\nu^{cd}] - \frac{i\hbar}{\lambda^2} \tilde{B}_{\mu\nu}$$

$$R_{\mu\nu}^a = [X_\mu, b_\nu^a] + [a_\mu, b_\nu^a] - [X_\nu, b_\mu^a] - [a_\nu, b_\mu^a] + i\{b_{\mu b}, \omega_\mu^{ab}\} - i\{b_{\nu b}, \omega_\mu^{ab}\}$$

$$+ i\{\tilde{a}_\mu, e_{\nu}^a\} - i\{\tilde{a}_\nu, e_{\mu}^a\} + \epsilon_{abcd} ([e_\mu^b, \omega_\nu^{cd}] - [e_\nu^b, \omega_\mu^{cd}]) - \frac{i\hbar}{\lambda^2} B_{\mu\nu}^a$$

$$\tilde{R}_{\mu\nu}^a = [X_\mu, e_{\nu}^a] + [a_\mu, e_{\nu}^a] - [X_\nu, e_{\mu}^a] - [a_\nu, e_{\mu}^a] + i\{b_\mu^a, \tilde{a}_\nu\} - i\{b_\nu^a, \tilde{a}_\mu\}$$

$$- ([b_\mu^b, \omega_\nu^{cd}] - [b_\nu^b, \omega_\mu^{cd}]) \epsilon_{abcd} - i\{\omega_\mu^{ab}, e_{\nu b}\} + i\{\omega_\nu^{ab}, e_{\mu b}\} - \frac{i\hbar}{\lambda^2} \tilde{B}_{\mu\nu}^a$$

$$R_{\mu\nu}^{ab} = [X_\mu, \omega_\nu^{ab}] + [a_\mu, \omega_\nu^{ab}] - [X_\nu, \omega_\mu^{ab}] - [a_\nu, \omega_\mu^{ab}] + 2i\{b_\mu^a, b_\nu^b\} + ([b_\mu^c, e_\nu^d]$$

$$- [b_\nu^c, e_\mu^d]) \epsilon_{abcd} + \frac{1}{2} ([\tilde{a}_\mu, \omega_\nu^{cd}] - [\tilde{a}_\nu, \omega_\mu^{cd}]) \epsilon_{abcd} + 2i\{\omega_\mu^{ac}, \omega_\nu^{bc}\}$$

$$+ 2i\{e_\mu^a, e_\nu^b\} - \frac{i\hbar}{\lambda^2} B_{\mu\nu}^{ab}$$

Symmetry Breaking

Introduction of auxiliary field $\Phi(X)$ charged under $U(1)$:

$$\Phi = \tilde{\phi}^a \otimes P_a + \phi^{ab} \otimes M_{ab} + \phi^a \otimes K_a + \phi \otimes \mathbb{1}_4 + \tilde{\phi} \otimes D$$

into the action:

$$S = \text{Tr} \text{tr}_G \lambda \Phi(X) \mathcal{R}_{\mu\nu} \mathcal{R}_{\rho\sigma} \varepsilon^{\mu\nu\rho\sigma} + \eta(\Phi(X)^2 - \lambda^{-2} \mathbb{1}_N \otimes \mathbb{1}_4),$$

when the auxiliary field is gauge fixed as:

$$\Phi(X) = \tilde{\phi}(X) \otimes D|_{\tilde{\phi} = -2\lambda^{-1}} = -2\lambda^{-1} \mathbb{1}_N \otimes D$$

it induces a symmetry breaking:

$$S_{br} = \text{Tr} \left(\frac{\sqrt{2}}{4} \varepsilon_{abcd} R_{\mu\nu}{}^{ab} R_{\rho\sigma}{}^{cd} - 4 R_{\mu\nu} \tilde{R}_{\rho\sigma} \right) \varepsilon^{\mu\nu\rho\sigma}$$

Residual symmetry: $SO(1,3) \times U(1)$

▷ At commutative limit *the action reduces to the Palatini action*, equivalent to Einstein gravity with a cosmological constant term.

Manousselis, Manolakos, Zoupanos '18

Unification of FG with Internal Interactions

- Fuzzy gravity is based on gauging $SO(2,4) \times U(1)$.
- Internal Interactions by $SO(10)$ (GUT).
- Spontaneous symmetry breaking is used to reach wanted gauge groups.

Roumelioti, S., Zoupanos '24

Fermions should be chiral in the original theory to have a chance to survive in low energies.

In order to have a chiral theory we need an $SO(4n+2)$ group. The smallest unification group in which we can accommodate chiral fermions is $SO(2,16)$ from which:

$$SO(2,16) \xrightarrow{SSB} SO(2,4) \times SO(12)$$

and

$$SO(12) \xrightarrow{SSB} SO(10) \times U(1).$$

Breakings and branching rules

- We start from $SO(2, 16) \sim SO(18)$ (Euclidean signature)
- For FG we gauge $SO(2, 4) \times U(1) \sim SO(6) \times U(1) \sim U(4)$
- For internal interactions we require $SO(10)$ GUT

$$SO(18) \supset SU(4) \times SO(12)$$

$$\mathbf{18} = (\mathbf{6}, \mathbf{1}) + (\mathbf{1}, \mathbf{12}) \quad \text{vector}$$

$$\mathbf{153} = (\mathbf{15}, \mathbf{1}) + (\mathbf{6}, \mathbf{12}) + (\mathbf{1}, \mathbf{66}) \quad \text{adjoint}$$

$$\mathbf{256} = (\mathbf{4}, \overline{\mathbf{32}}) + (\overline{\mathbf{4}}, \mathbf{32}) \quad \text{spinor}$$

$$\mathbf{170} = (\mathbf{1}, \mathbf{1}) + (\mathbf{6}, \mathbf{12}) + (\mathbf{20}', \mathbf{1}) + (\mathbf{1}, \mathbf{77}) \quad \text{2nd rank symmetric}$$

Giving VEV in the $\langle \mathbf{1}, \mathbf{1} \rangle$ component of a scalar in $\mathbf{170}$ leads to $SU(4) \times SO(12)$.

Breakings and branching rules (Continued)

- Moving on with the $SO(12)$, we break it to $SO(10) \times U(1)$ given the branching rule:

$$SO(12) \supset SO(10) \times U(1)$$
$$\mathbf{66} = (\mathbf{1})(0) + (\mathbf{10})(2) + (\mathbf{10})(-2) + (\mathbf{45})(0)$$

by giving VEV to the $\langle (\mathbf{1})(0) \rangle$ of the $\mathbf{66}$ rep.

Breakings and branching rules (Continued)

Lastly, regarding $SU(4)$, we break it in 2 steps:

- First step: breaking $SU(4) \supset Sp_4$:

$$SU(4) \supset Sp_4$$

$$\mathbf{4} = \mathbf{4}$$

$$\mathbf{6} = \mathbf{1} + \mathbf{5}$$

Giving VEV to a scalar in the $\mathbf{6}$ rep, in the $\langle \mathbf{1} \rangle$ component, $SU(4)$ breaks down to Sp_4 .

- Second step: breaking $Sp_4 \supset SU(2) \times SU(2)$:

$$Sp_4 \supset SU(2) \times SU(2)$$

$$\mathbf{5} = (\mathbf{1}, \mathbf{1}) + (\mathbf{2}, \mathbf{2})$$

$$\mathbf{4} = (\mathbf{2}, \mathbf{1}) + (\mathbf{1}, \mathbf{2})$$

Giving VEV in $\langle \mathbf{1}, \mathbf{1} \rangle$ of a scalar in the $\mathbf{5}$ rep, we obtain the Lorentz group $SU(2) \times SU(2) \sim SO(1, 3)$.

Fermions in Fuzzy Gravity and Unification with Internal Interactions

- Fermions should be chiral in the original theory and also appear in a matrix representation since FG is a matrix model
- ▷ Instead of using fermions in fundamental, spinor or adjoint reps of an $SU(N)$, we can use bi-fundamental reps of cross product of gauge groups.

Chatzistavrakidis, Steinacker, Zoupanos '10

Interesting example $N = 1$, $SU(N)^k$ models:

$$SU(N)_1 \times SU(N)_2 \times \dots \times SU(N)_k$$

with matter content

$$(N, \bar{N}, 1, \dots, 1) + (1, N, \bar{N}, \dots, 1) + \dots + (\bar{N}, 1, 1, \dots, N)$$

with successful phenomenology, $N = 1$, $SU(3)^3$.

Ma, Mondragon, Zoupanos '04

Fermions in Fuzzy Gravity and Unification with Internal Interactions (Continued)

- ▷ In FG choosing to start with the $SU(4) \times SO(12)$ as the initial gauge theory with fermions in the $(\mathbf{4}, \overline{\mathbf{32}}) + (\overline{\mathbf{4}}, \mathbf{32})$ we satisfy the criteria to obtain chiral fermions in tensorial representation.
- ▷ The gauge $U(1)$ of FG due to the anticommutation relations, is identified with the one appearing in the $SO(12) \supset SO(10) \times U(1)$.

Fermions

We start with fermions in the $(\mathbf{4}, \overline{\mathbf{32}}) + (\overline{\mathbf{4}}, \mathbf{32})$ of the $SU(4) \times SO(12)$.

Then

$$\begin{aligned}SO(12) &\supset SO(10) \times U(1) \\ \mathbf{32} &= (\overline{\mathbf{16}})(1) + (\mathbf{16})(-1)\end{aligned}$$

On the other hand

$$\begin{aligned}SU(4) &\supset Sp_4 \\ \mathbf{4} &= \mathbf{4},\end{aligned}$$

and

$$\begin{aligned}Sp_4 &\supset SU(2) \times SU(2) \\ \mathbf{4} &= (\mathbf{2}, \mathbf{1}) + (\mathbf{1}, \mathbf{2})\end{aligned}$$

Following the full sequence of symmetry breakings, by imposing the Weyl condition, we will be left with four families of fermions:

$$4 \times \mathbf{16}_L(-1)$$

Further studies (Works in

- Cosmological studies treating the heavy spin-2 graviton as DM with a possible solution of the Boulware-Deser ghost by fixing its mass at the Higuchi bound (work in progress).
- Possible path of solving the Ostrogradsky's ghost problem by:
addition of specific boundary conditions
and bimetric theory (work in progress).

*Maldacena; Hell, Lust, Zoupanos
Hassan, Rosen*

Thank you!

The commutative limit

- The 2-form field $B_{\mu\nu}$ and a_μ decouple
- The commutators of functions vanish: $[f(x), g(x)] \rightarrow 0$
- The anticommutators of functions reduce to the product: $\{f(x), g(x)\} \rightarrow 2f(x)g(x)$
- The inner derivation becomes: $[X_\mu, f] \rightarrow \partial_\mu f$
- The trace reduces to integration: $\frac{\sqrt{2}}{4} \text{Tr} \rightarrow \int d^4x$
- Together with the reparametrizations $e_\mu^a \rightarrow i m e_\mu^a$, $\omega_\mu^{ab} \rightarrow -\frac{i}{2} \omega_\mu^{ab}$ (and the matching ones for P_a, M_{ab}),
▷ *the action reduces to the Palatini action*, equivalent to Einstein gravity with a cosmological constant term.

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