

How curvature shapes phase structure in noncommutative matrix models

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phase transitions $\approx$ info on NC renormalizability

- adding curvature term to NC $\lambda\phi_\star^4$ shifts the model from an NC phase to a "commutative" phase
- the shift turns the NC model from nonrenormalizable to renormalizable
- we also see hints of new non-standard low- g_4 phase structure

- **GR + QM** suggest that spacetime is "granular"; one way to model this is with NC coordinates
- NC coordinates (Heisenberg algebra $\rightarrow$ **matrices**) realized, for example by Moyal $\star$ -product:

$$f \star g = f e^{i/2 \tilde{\partial} \theta \tilde{\partial}} g = fg + \frac{i\theta^{\mu\nu}}{2} \frac{\partial f}{\partial x^\mu} \frac{\partial g}{\partial x^\nu} + \dots \quad \Rightarrow \quad \boxed{[x^\mu, x^\nu]_\star = i\theta^{\mu\nu}}$$

- the simplest NC model $\lambda \phi_\star^4$ (**renormalizability issues**):

$$S = \int d^n x \left(\frac{1}{2} \partial_\mu \phi \star \partial^\mu \phi + \frac{m^2}{2} \phi \star \phi + \frac{\lambda}{4!} \phi \star \phi \star \phi \star \phi \right)$$

- propagator corrections & mass renormalization at one loop:



$$= \frac{\lambda}{4!} \int \frac{d^2 k}{(2\pi)^2} \frac{2}{k^2 + m^2} \sim \frac{\lambda}{96\pi} \log \frac{\Lambda_{\max}^2}{m^2}$$



$$= \frac{\lambda}{4!} \int \frac{d^2 k}{(2\pi)^2} \frac{\exp(ik_\mu \theta^{\mu\nu} p_\nu)}{k^2 + m^2} \sim \frac{\lambda}{96\pi} \log \frac{\Lambda_{\text{eff}}^2}{m^2}$$

$$\Lambda_{\text{eff}}^2 = \frac{1}{1/\Lambda_{\max}^2 + |\theta^{\mu\nu} p_\nu|^2}$$


 UV/IR

- 1PI 2-point function:

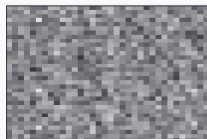
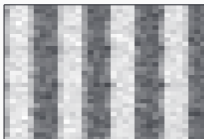
$$\Gamma^{(2)}(p) = p^2 + m_{\text{ren}}^2 - \#\lambda \log \tilde{p}^2 + \dots, \quad \tilde{p}_\mu = \theta_{\mu\nu} p^\nu$$

- $\Gamma^{(2)}$ is minimized by $p_*^2 \neq 0$, which gives rise to the stripe phase:

$$\phi(x) \sim \cos(p_* \cdot x)$$

- center plot represents the stripe phase in coordinate space

[MEJÍA-DÍAZ, BIETENHOLZ AND PANERO '14]



$$S_{GW} = \int d^{2n}x \left(\frac{1}{2} \partial_\mu \phi \star \partial^\mu \phi + \frac{m^2}{2} \phi \star \phi + \frac{\lambda}{4!} \phi \star \phi \star \phi \star \phi + \right. \\ \left. + \boxed{\frac{1}{2} \Omega^2 (\tilde{x}_\mu \phi) \star (\tilde{x}^\mu \phi)} \right)$$

$$\text{where } [x^\mu, x^\nu]_\star = i\theta^{\mu\nu} \quad \text{and} \quad \tilde{x}_\mu = 2(\theta^{-1})_{\mu\nu} x^\nu$$

-
- superrenormalizable in 2-dim [GROSSE AND WULKENHAAR '03]
 - **renormalizability** of the 2-dim $\lambda\phi_\star^4$ is **restored** by defining it as a $\Omega \rightarrow 0$ limit of the series of GW models labeled by their Ω
 - $\boxed{\Omega\text{-term}} \Leftrightarrow$ **curvature** [BURIĆ AND WOHLGENANT '10, **PREKRAT '26**]

- Weyl transform turns fields to Hermitian matrices and integrals to traces, giving the **matrix GW model**:

$$S_N = N \operatorname{tr} \left(\Phi \hat{K} \Phi - g_r R \Phi^2 - g_2 \Phi^2 + g_4 \Phi^4 \right)$$

-
- kinetic operator:

$$\hat{K} = [X, [X, \cdot]] + [Y, [Y, \cdot]]$$

- curvature:

$$N \cdot R \approx -\mathbf{16} \operatorname{diag}(1, 2, \dots, N)$$

- unscaled (G 's) vs. scaled (g 's) parameters:

$$G_2 = N g_2 \qquad G_4 = N g_4$$

- EOM:

$$2\hat{K}\Phi - g_r(R\Phi + \Phi R) + 2\Phi(2g_4\Phi^2 - g_2\mathbb{1}) = 0$$

- classical vacuum "solutions":

$$\Phi = \frac{\text{tr } \Phi}{N} \mathbb{1} \quad \Phi = 0 \quad \Phi^2 = \begin{cases} 0 & \text{for } g_2 \leq 0 \\ \frac{g_2 \mathbb{1}}{2g_4} & \text{for } g_2 > 0 \end{cases}$$

- 3 phases:

- disordered/symmetric 1-cut phase (S1):
- striped/2-cut phase (M2, S2):**
- ordered/asymmetric 1-cut phase (A1):

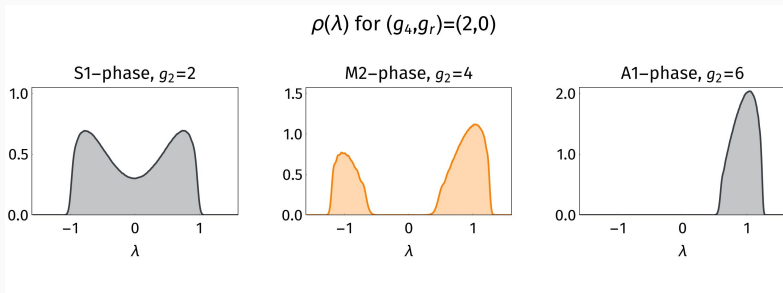
$$\Phi_{S1} = 0$$

$$\Phi_{M2} \propto U \mathbb{1}_{\pm} U^{\dagger}$$

$$\Phi_{A1} \propto \mathbb{1}$$

- modified ordered phases (and other beasts):**

$$\Phi_R^2 = \frac{g_2 \mathbb{1} - g_r |R|}{2g_4}$$



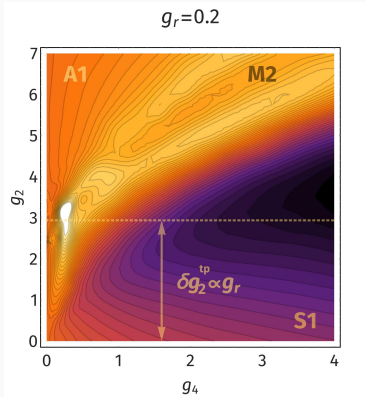
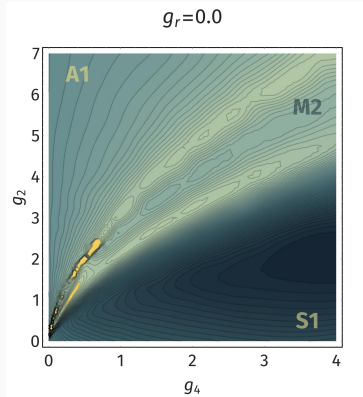
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- we locate the transitions with Hamiltonian Monte Carlo, using eigenvalue distributions, susceptibility χ and heat capacity C



replacing $|R|$ by its max eigenvalue gives the observed shift $g_2^{\text{tp}} \approx 16g_r$

$$S_N \ni \text{tr}(g_r |R| \Phi^2) \Rightarrow \text{tr}(16g_r \Phi^2) \Rightarrow \text{tr}(-(g_2 - 16g_r) \Phi^2)$$

- triple-point shift:

$$\delta g_2^{\text{tp}} \propto g_r \Rightarrow \delta G_2^{\text{tp}} \propto N g_r \Rightarrow \boxed{G_2^{\text{tp}} \sim \frac{N}{\log^2 N} \rightarrow \infty}$$

- mass shift (δm_{ren}^2 : e.g. Franchino-Viñas and Pisani '14):

$$m_{\text{bare}}^2 + \delta m_{\text{ren}}^2 = m_{\text{ren}}^2 \leftrightarrow m_{\text{ph}}^2$$

$$\delta m_{\text{ren}}^2 = \frac{\lambda}{12\pi(1 + \Omega^2)} \log \frac{\Lambda_{\text{max}}^2 \theta}{\Omega} \quad \text{and} \quad \Lambda_{\text{max}}^2 \sim N$$

- from their comparison:

$$\boxed{|\delta G_2^{\text{ren}}| \sim \log N < G_2^{\text{tp}}} \Rightarrow \text{bare model stays in S1-phase!}$$

$$S_N(\Phi) = N \operatorname{tr}(\Phi \hat{K} \Phi - g_r R \Phi^2 - g_2 \Phi^2 + g_4 \Phi^4)$$

$$\Phi = U \Lambda U^\dagger, \quad \Lambda = \operatorname{diag}(\lambda_1, \dots, \lambda_N)$$

$$S_N(\Lambda, U) = N \operatorname{tr}((U \Lambda U^\dagger) \hat{K} (U \Lambda U^\dagger) - g_r R U \Lambda^2 U^\dagger - g_2 \Lambda^2 + g_4 \Lambda^4)$$

$$Z = \int [d\Phi] e^{-S_N(\Phi)} = \int [d\Lambda][dU] e^{-(S_N(\Lambda, U) - \log \Delta^2(\Lambda))} \stackrel{?}{=} \int [d\Lambda] e^{-\text{S}_{\text{eff}}(\Lambda)}$$

$$\Delta^2(\Lambda) = \prod_{i < j} (\lambda_i - \lambda_j)^2$$

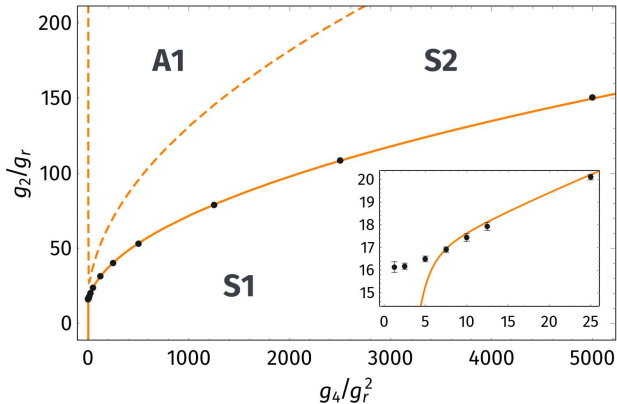
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- expanding Z in g_r and integrating out U gives the eff. action

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$$\begin{aligned} S_{\text{eff}}(\Lambda) = & - (g_2 - 8g_r) N \text{tr} \Lambda^2 + \left(g_4 - \frac{(8g_r)^2}{6} \right) N \text{tr} \Lambda^4 + \frac{(8g_r)^2}{6} \text{tr}^2 \Lambda^2 \\ & + \frac{(8g_r)^4}{180} N \text{tr} \Lambda^8 + \frac{(8g_r)^4}{60} \text{tr}^2 \Lambda^4 - \frac{(8g_r)^4}{45} \text{tr} \Lambda^6 \text{tr} \Lambda^2 \\ & - \frac{(8g_r)^6}{2835} N \text{tr} \Lambda^{12} + \frac{2(8g_r)^6}{945} \text{tr} \Lambda^2 \text{tr} \Lambda^{10} - \frac{(8g_r)^6}{189} \text{tr} \Lambda^4 \text{tr} \Lambda^8 + \frac{2(8g_r)^6}{567} \text{tr}^2 \Lambda^6 \\ & - \log \Delta^2(\Lambda) \end{aligned}$$

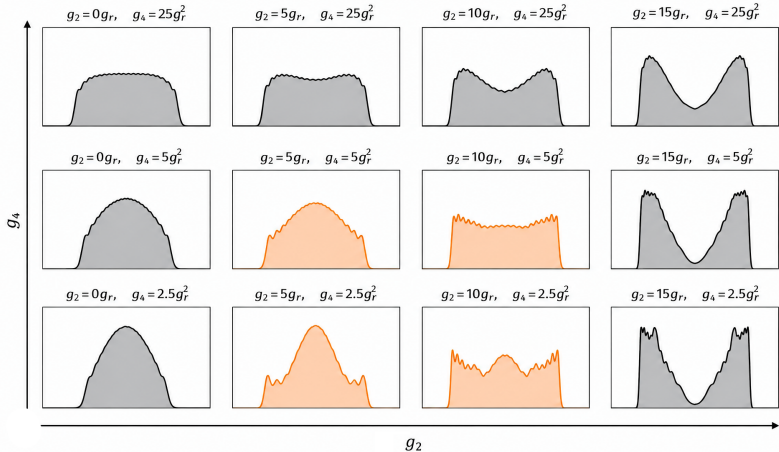
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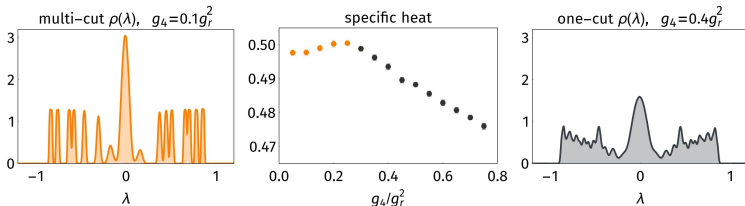


- **solid orange line** — 6th order analytical expression
- **black dots** — HMC simulation data extrapolated to $N \rightarrow \infty$

$N = 20$



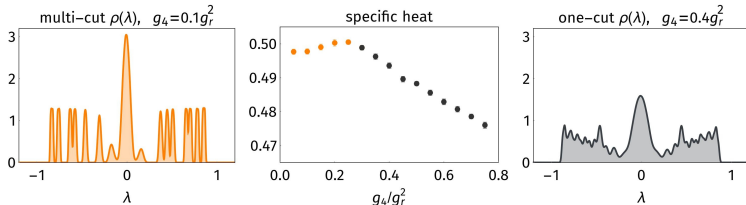
Multi-cut transition for $N=20$, $g_2=10g_r$, $g_r=0.02$



- multi-cut transition to $\Phi_R^\pm$ expected perturbatively at some $g_4^\star$, because up to $O(g_r^2)$ we have

$$\left. \frac{S_{\text{eff}}^\Delta}{S_{\text{eff}} - S_{\text{eff}}^\Delta} \right|_{\Phi_R^\pm} \propto -g_4 \ln g_4 \rightarrow 0 \quad \text{when} \quad g_4 \rightarrow 0$$

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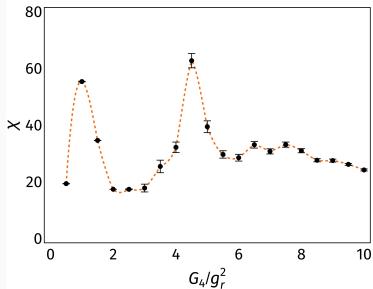


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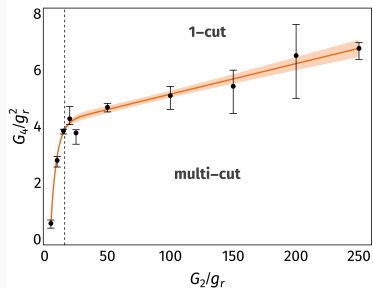
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- disappears*** in the large- N limit: $g_4^\star(N) \rightarrow 0$ as $N \rightarrow \infty$

$N = 50, \quad G_2 = 100g_r$



$N = 20$



$$\Phi_R^2 \geq 0 \quad \Rightarrow \quad G_2 - 16 \cdot N_{\text{eigen}} \cdot g_r \geq 0 \quad \Rightarrow \quad N_{\text{eigen}}^{\text{max}} = \left\lfloor \frac{G_2}{16g_r} \right\rfloor$$

Transition looks stable

- no significant large N -dependence resolved in the transition position from eigenvalue distribution gaps ($N = 10, 20, 30, 40, 50$)
- no significant N -dependence resolved in the χ peak position and their heights show no clear qualitative N -scaling
- C profile is not useful at current precision

But!

- eigenvalue distribution approaches a "renormalized" 1-cut: stable off-bulk peaks but their height decreases as $N \rightarrow \infty$:

$$N_{\text{eigen}}^{\text{max}}/N \rightarrow 0$$

Is this a proper transition?

phase transitions $\approx$ info on NC renormalizability

- hypothesis: **no stripe-phase** $\leftrightarrow$ **renormalizability**
 - connection is **demonstrated** between the **GW model** renormalizability and its phase structure
 - preliminary evidence for the **new "phase"** in the GW model
-
- numerical simulation of phase transitions could tell us **in advance about** the renormalization properties of **new models**

Thank you for your attention.