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Complex Langevin studies of the type IIB matrix model with deformations by “gauge-fixing” Lorentz symmetry

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1. Introduction

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Type IIB matrix model (a.k.a. **IKKT model**)

⇒ Promising candidate for nonperturbative string theory

[N. Ishibashi, H. Kawai, Y. Kitazawa, A. Tsuchiya, hep-th/9612115]

$$S = \underbrace{\frac{-N}{4} \text{tr}[A_\mu, A_\nu]^2}_{=S_b} + \underbrace{\frac{-N}{2} \text{tr} \bar{\psi}_\alpha (\Gamma^\mu)_{\alpha\beta} [A_\mu, \psi_\beta]}_{=S_f}$$

- Dimensional reduction of the **D=10** super-Yang-Mills theory to **0 dimension**
- $A_\mu (\mu=0, 1, \dots, 9)$, $\Psi_\alpha (\alpha=1, 2, \dots, 16 \text{ after Weyl projection})$
⇒ $N \times N$ Hermitian traceless matrices.
- $N=2$ supersymmetry ⇒ **eigenvalues of A_μ** are interpreted as the **spacetime coordinates**.

How does our (3+1)-dim spacetime emerge dynamically?

2. Lorentzian type IIB matrix model

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Sign problem of the type IIB matrix model

Euclidean version : Wick rotation $A_{10} = -iA_0$, $\Gamma_{10} = i\Gamma^0$
contracted by the Euclidean metric $\delta_{\mu\nu} = \text{diag}(1, 1, 1, \dots, 1)$

SSB $SO(10) \rightarrow SO(3)$
(3dim spacetime)

$$Z = \int dA \left(\underbrace{e^{-S_b}}_{\text{real}} \underbrace{\int d\psi e^{-S_f}}_{\text{complex}} \right)$$

[K.N. Anagnostopoulos, T. Azuma, Y. Ito J. Nishimura,
T. Okubo, S.K. Papadoudis arXiv:2002.07410]

$\mathcal{M}$ is a $16(N^2-1) \times 16(N^2-1)$ sparse matrix
 $\stackrel{\text{Pf}}{=}$

Lorentzian version : no Wick rotation
contracted by the **Lorentzian metric** $\eta_{\mu\nu} = \text{diag}(-1, 1, 1, \dots, 1)$

$$Z = \int dA \left(\underbrace{e^{iS_b}}_{\text{complex}} \underbrace{\int d\psi e^{iS_f}}_{\text{real}} \right)$$

$\stackrel{\text{Pf}}{=}$ $\mathcal{M}$

[J. Nishimura, A. Tsuchiya,
arXiv:1904.05919]

2. Lorentzian type IIB matrix model

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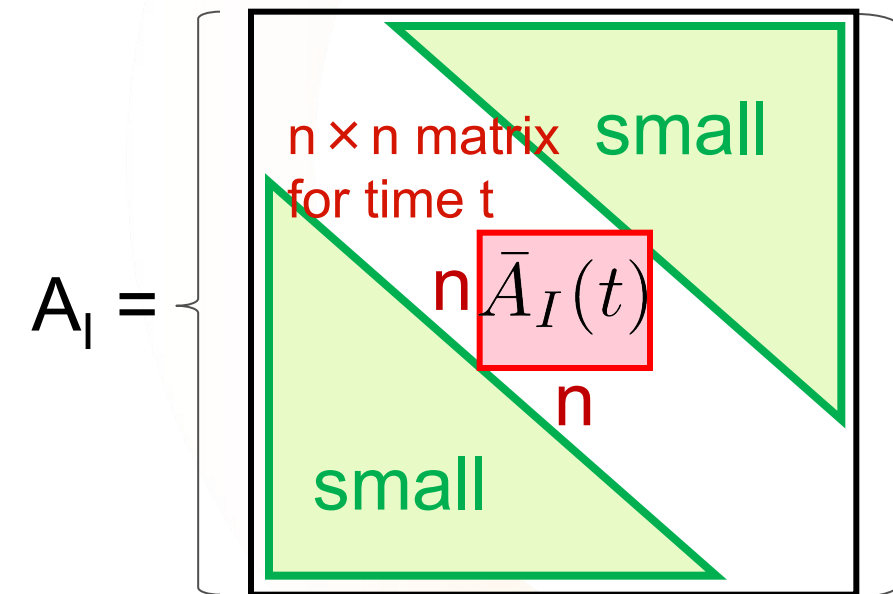
Time definition of the Lorentzian version:

⇒ gauge fixing to diagonalize A_0

[S.W. Kim, J. Nishimura, A. Tsuchiya, arXiv:1108.1540]

$A_0 = \text{diag}(\alpha_1, \alpha_2, \dots, \alpha_N)$, where $\alpha_1 < \alpha_2 < \dots < \alpha_N$.

Band-diagonal structure of A_I $t_\nu = \sum_{k=1}^{\nu} |\bar{\alpha}_{k+1} - \bar{\alpha}_k|$, $\bar{\alpha}_k = \frac{1}{n} \sum_{i=1}^n \alpha_{k+i}$



SSB of spatial $SO(9)$ symmetry

Eigenvalues of $T_{ij}(t) = \frac{1}{n} \text{tr}(\bar{A}_i(t) \bar{A}_j(t))$
 $\lambda_k(t)$ ($k = 1, 2, \dots, 9$)

2. Lorentzian type IIB matrix model

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Equivalence between the Lorentzian and Euclidean version

[Y. Asano, private communication]

Contour deformation $A_0 = e^{-3i\pi u/8} \tilde{A}_0$, $A_I = e^{i\pi u/8} \tilde{A}_I$

$$S_b \rightarrow \tilde{S}_b = \underbrace{e^{i\pi u/2}}_{\text{w.r.t. worldsheet}} \left\{ -\frac{N}{4} \text{tr}[\tilde{A}_I, \tilde{A}_J]^2 + \frac{N}{2} \underbrace{e^{-i\pi u}}_{\text{w.r.t. target space}} \text{tr}[\tilde{A}_0, \tilde{A}_I]^2 \right\}$$

u=0: Lorentzian, **u=1**: Euclidean

$$e^{iS_b(A)} = e^{-S(\tilde{A})}, \quad S(\tilde{A}) = -\frac{N}{4} \underbrace{e^{-i\pi(1-u)/2}}_{\text{real part is positive } (0 < u \leq 1)} \text{tr}[\tilde{A}_I, \tilde{A}_J]^2 - \frac{N}{2} \underbrace{e^{i\pi(1-u)/2}}_{\text{real part is positive } (0 < u \leq 1)} \text{tr}[\tilde{A}_0, \tilde{A}_I]^2$$

Cauchy's theorem:

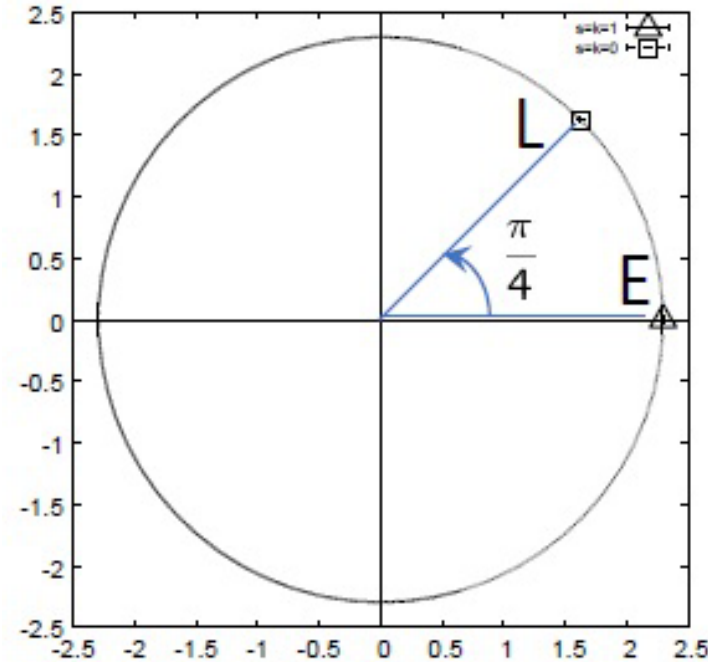
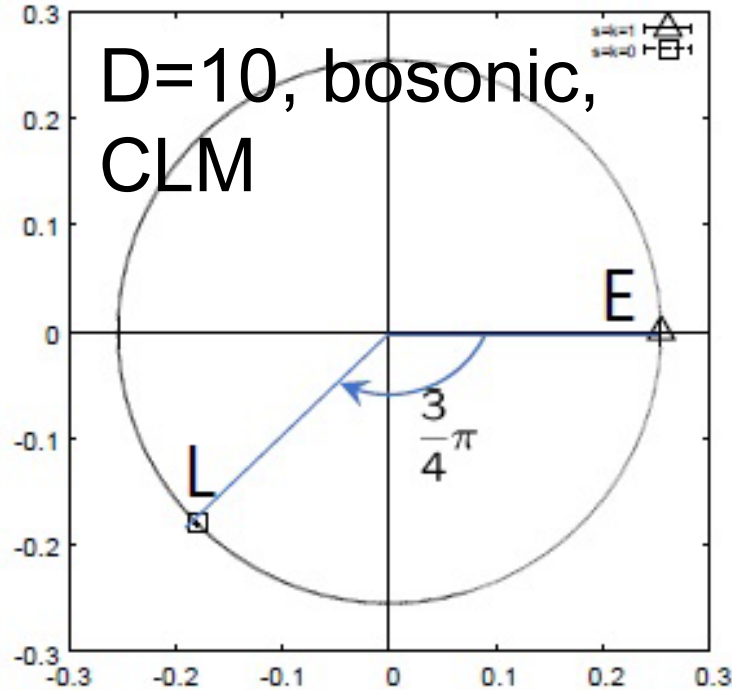
$\langle \mathcal{O}(e^{-3i\pi u/8} \tilde{A}_0, e^{i\pi u/8} \tilde{A}_I) \rangle_u$ is **independent of u**.

2. Lorentzian type IIB matrix model

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$$\left\langle \frac{1}{N} \text{tr}(A_0^{(L)})^2 \right\rangle = e^{-3i\pi/4} \left\langle \frac{1}{N} \text{tr}(A_0^{(E)})^2 \right\rangle$$

$$\left\langle \frac{1}{N} \text{tr}(A_I^{(L)})^2 \right\rangle = e^{i\pi/4} \left\langle \frac{1}{N} \text{tr}(A_I^{(E)})^2 \right\rangle$$



Equivalence of the **Euclidean** ($u=1$) and **Lorentzian** ($u \rightarrow +0$) model.

$\Rightarrow$ The spacetime is **Euclidean**.

[K.N.Anagnostopoulos, T.Azuma, M.Hirasawa, J.Nishimura, S.Papadoudis, A.Tsuchiya, arXiv:2604.19836]

2. Lorentzian type IIB matrix model

Adding the Lorentzian mass term

[K.N. Anagnostopoulos, T. Azuma, M. Hirasawa,
J. Nishimura, S. Papadoudis, A. Tsuchiya,
arXiv:2604.19836]

$$Z = \int dA d\psi e^{i(S+S'_\gamma)}, \quad S'_\gamma = \frac{-N\gamma}{2} \text{tr}(A^\mu A_\mu) = \frac{N\gamma}{2} \{\text{tr}(A_0)^2 - \text{tr}(A_I)^2\}$$

$$e^{iS_\gamma(A)} = e^{-S_\gamma(\tilde{A})}, \quad S_\gamma(\tilde{A}) = \frac{N\gamma}{2} \{e^{-i\pi(2+3u)/4} \text{tr}(\tilde{A}_0)^2 + e^{i\pi(2+u)/4} \text{tr}(\tilde{A}_I)^2\}$$

real part is negative ($0 < u \leq 1$).

At $\gamma > 0$, we cannot define the model by contour deformation

$\Rightarrow$ Equivalence to the Euclidean model ($u=1$) is violated.

We consider the limit $N \rightarrow \infty \Rightarrow \gamma \rightarrow 0$.

2. Lorentzian type IIB matrix model

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"Gauge fixing" of the Lorentz symmetry

[Y. Asano, J. Nishimura, W. Piensuk and N. Yamamori, arXiv:2404.14045]

Noncompact Lorentz group $\Rightarrow$ partition function is divergent.
Without fixing Lorentz sym. $\Rightarrow$ boosted configs. dominate.

SO(9,1) Lorentz invariant quantity $\underbrace{\text{tr}(A_0^2)}_{=\mathcal{T}} - \underbrace{\sum_{i=1}^9 \text{tr}(A_i^2)}_{=\mathcal{U}}$

Minimize $\mathcal{T}$ w.r.t the Lorentz transformation

$$\begin{pmatrix} A'_0 \\ A'_i \end{pmatrix} = \begin{pmatrix} \cosh \sigma & \sinh \sigma \\ \sinh \sigma & \cosh \sigma \end{pmatrix} \begin{pmatrix} A_0 \\ A_i \end{pmatrix}$$

$\text{tr}(A_0 A_j) = 0$ for all $j = 1, 2, \dots, 9$

Gauge-fix the Lorentz symmetry by the Faddeev-Popov procedure:

$$Z = \int dA d\psi e^{iS} \underbrace{\Delta_{\text{FP}}(A)}_{=\det \Omega_{ij}} \prod_{j=1}^9 \delta(\text{tr}(A_0 A_j))$$

$$\Omega_{ij} = \text{tr}(A_0^2) \delta_{ij} + \text{tr}(A_i A_j) \quad (i, j = 1, 2, \dots, 9)$$

3. Complex Langevin Method

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Complex Langevin Method (CLM)

⇒ Promising method to solve complex-action systems.

[Parisi, Phys.Lett. 131B (1983) 393, Klauder, Phys.Rev. A29 (1984) 2036]

• Time order $\alpha_1 < \alpha_2 < \dots < \alpha_N$ ($a=2, \dots, N$)

$$\alpha_1 = 0, \quad \alpha_a = \sum_{i=1}^{a-1} e^{\tau_i} \quad A_0 = \text{diag}(\alpha_1, \dots, \alpha_N)$$

[J. Nishimura, A. Tsuchiya, arXiv:1904.05919]

$$S_{A\text{diag}} = -\log \prod_{1 \leq a < b \leq N} (\alpha_a - \alpha_b)^2 - \sum_{a=1}^{N-1} \tau_a.$$

$$\frac{d(A_i)_{ab}(\sigma)}{d\sigma} = - \frac{dS}{d(A_i)_{ba}} + \eta_{i,ab}(\sigma), \quad \frac{d\tau_a(\sigma)}{d\sigma} = - \frac{dS}{d\tau_a} + \eta_a(\sigma)$$

fictitious Langevin time

drift term

Hermitian-matrix

white noise

real-number

white noise

• A_i : Hermitian → general complex traceless matrices.

• τ_a : Real number → complex number.

3. Complex Langevin Method

• The drift term involves $\frac{d}{d(A_I)_{ba}} \{-\log \text{Pf } \mathcal{M}\} = -\frac{1}{2} \text{Tr} \left(\frac{d\mathcal{M}}{d(A_I)_{ba}} \mathcal{M}^{-1} \right)$

⇒ **Mass term** to avoid the near-zero modes of the

Dirac operator: $\text{Pf } \mathcal{M} = \int d\psi e^{iS_{m_f}}$ ($m_f \rightarrow \infty$: bosonic model)

$$S_{m_f} = \frac{-N}{2} \text{tr} \{ \bar{\psi}_\alpha (\Gamma^\mu)_{\alpha\beta} [A_\mu, \psi_\beta] + im_f \bar{\psi}_\alpha (\Gamma^7 \Gamma^{8\dagger} \Gamma^9)_{\alpha\beta} \psi_\beta \}$$

• Inspired by the SUSY deformation

⇒ Anisotropy in the bosonic mass term

[K.N. Anagnostopoulos, T. Azuma, M. Hirasawa, J. Nishimura, S. Papadoudis, A. Tsuchiya, arXiv:2604.19836]

$$S_\xi = \frac{N\gamma}{2} \left\{ \text{tr}(A_0)^2 - \sum_{I=1}^{\tilde{d}} \text{tr}(A_I)^2 - \xi \sum_{I=\tilde{d}+1}^9 \text{tr}(A_I)^2 \right\} \quad (\tilde{d} = 4, 5, 6)$$

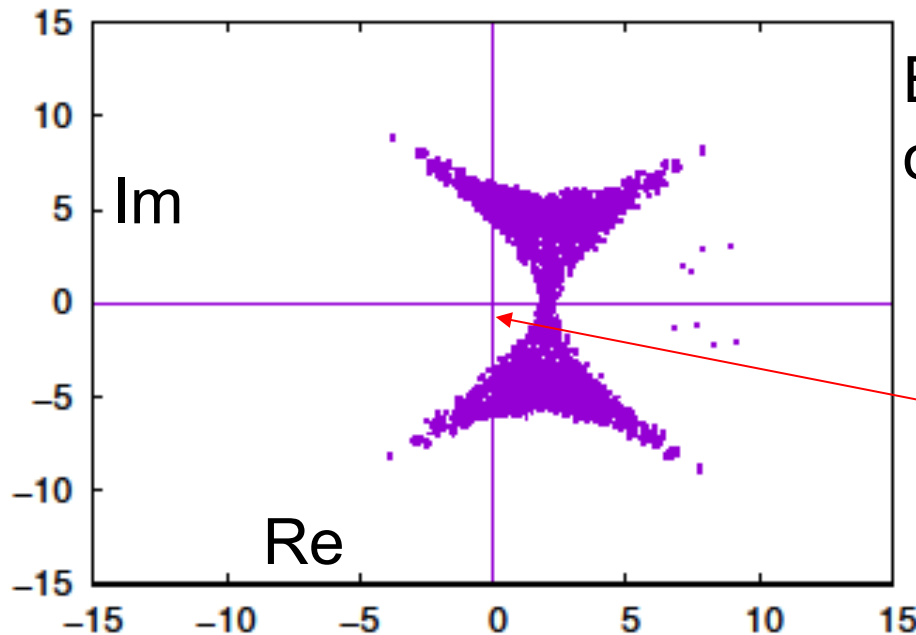
Break spatial symmetry $SO(9) \rightarrow SO(\tilde{d})$

⇒ explore the remaining symmetry

3. Complex Langevin Method

$\text{Pf } \mathcal{M} = \int d\psi e^{iS_{m_f}}$, where

$$S_{m_f} = \frac{-N}{2} \text{tr} \{ \bar{\psi}_\alpha (\Gamma^\mu)_{\alpha\beta} [A_\mu, \psi_\beta] + im_f \bar{\psi}_\alpha (\Gamma^7 \Gamma^{8\dagger} \Gamma^9)_{\alpha\beta} \psi_\beta \}$$



Eigenvalues of $\mathcal{M}$
on the complex plane.

$N=16, \gamma=6, m_f=2, \xi=18, \tilde{d} = 6$

This avoids the origin.

3. Complex Langevin Method

- Comment on the SUSY deformation of the Lorentzian model

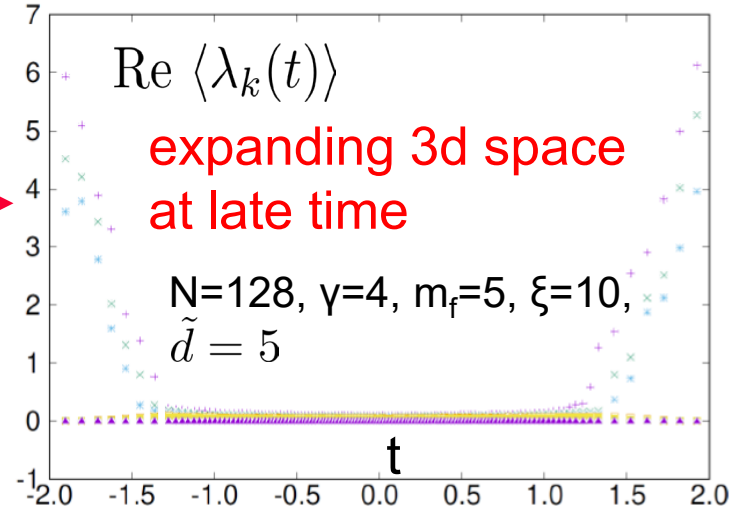
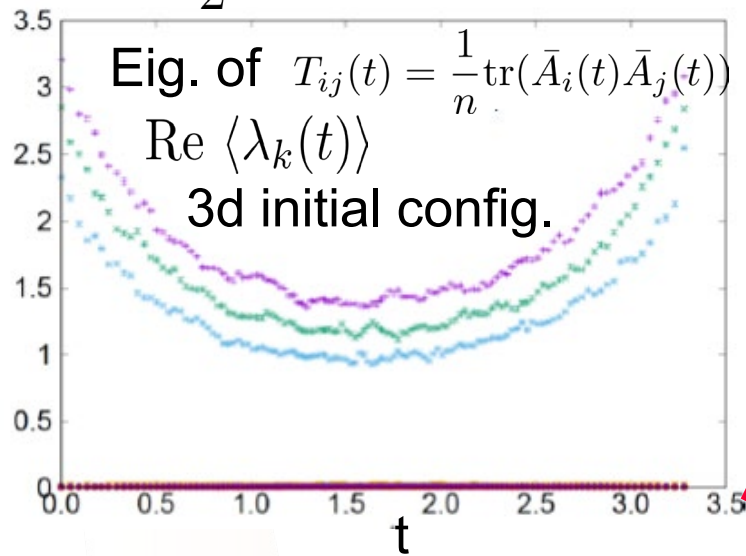
$$S_{m_f} = \frac{-N}{2} \text{tr} \{ \bar{\psi}_\alpha (\Gamma^\mu)_{\alpha\beta} [A_\mu, \psi_\beta] + i m_f \bar{\psi}_\alpha (\Gamma^7 \Gamma^{8\dagger} \Gamma^9)_{\alpha\beta} \psi_\beta \}$$
$$S_\xi = \frac{N\gamma}{2} \left\{ \text{tr}(A_0)^2 - \sum_{I=1}^{\tilde{d}} \text{tr}(A_I)^2 - \xi \sum_{I=\tilde{d}+1}^9 \text{tr}(A_I)^2 \right\}$$

Add the Myers term $S_{\text{Myers}} = -iN\mu \text{tr}(A_7[A_8, A_9])$
and set $\gamma = \frac{-\mu^2}{32}, m_f = \frac{\mu}{4}, \xi = 3, \tilde{d} = 6.$

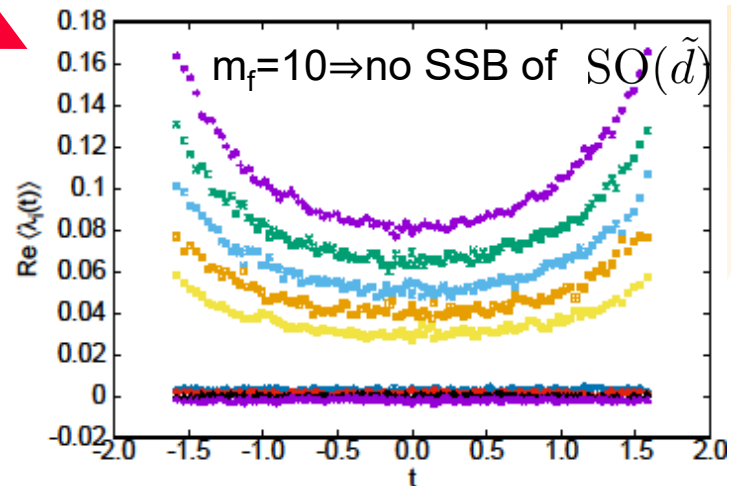
4. Result [work in progress]

(1) $N=128, \gamma=4, m_f=5, \xi=10, \tilde{d} = 5$ $S_\xi = \frac{N\gamma}{2} \left\{ \text{tr}(A_0)^2 - \sum_{I=1}^{\tilde{d}} \text{tr}(A_I)^2 - \xi \sum_{I=\tilde{d}+1}^9 \text{tr}(A_I)^2 \right\}$

$$S_{m_f} = \frac{-N}{2} \text{tr} \{ \bar{\psi}_\alpha (\Gamma^\mu)_{\alpha\beta} [A_\mu, \psi_\beta] + i m_f \bar{\psi}_\alpha (\Gamma^7 \Gamma^{8\dagger} \Gamma^9)_{\alpha\beta} \psi_\beta \}$$



At large m_f , the $\text{SO}(\tilde{d})$ symmetry is not broken.

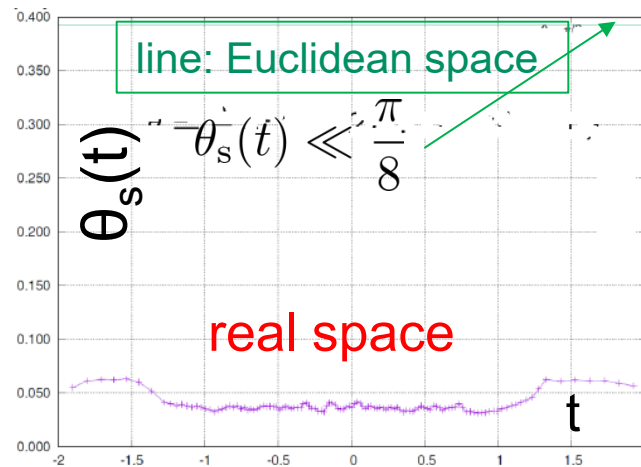
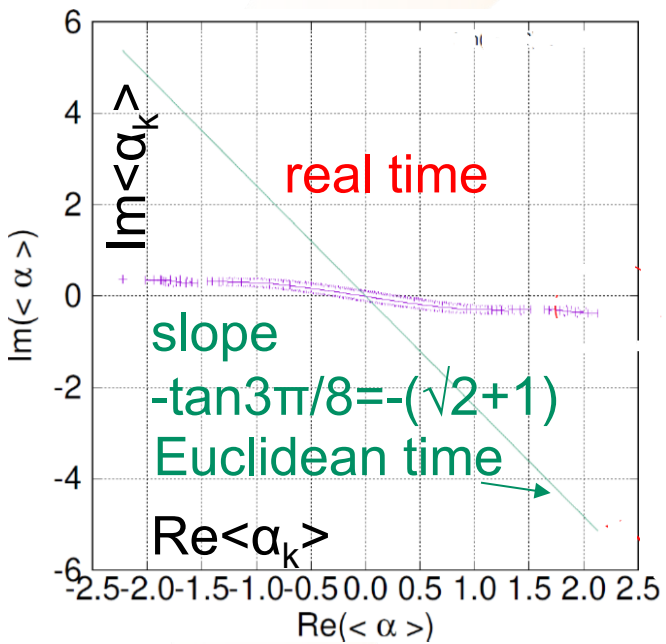


4. Result [work in progress]

(1) $N=128, \gamma=4, m_f=5, \xi=10, \tilde{d} = 5$

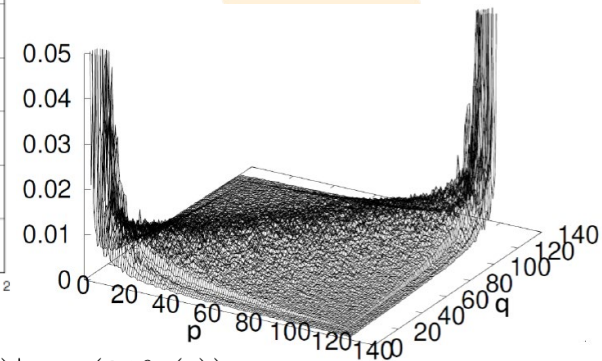
$$S_\xi = \frac{N\gamma}{2} \left\{ \text{tr}(A_0)^2 - \sum_{I=1}^{\tilde{d}} \text{tr}(A_I)^2 - \xi \sum_{I=\tilde{d}+1}^9 \text{tr}(A_I)^2 \right\}$$

$$S_{m_f} = \frac{-N}{2} \text{tr} \{ \bar{\psi}_\alpha (\Gamma^\mu)_{\alpha\beta} [A_\mu, \psi_\beta] + i m_f \bar{\psi}_\alpha (\Gamma^7 \Gamma^{8\dagger} \Gamma^9)_{\alpha\beta} \psi_\beta \}$$



$$A_{pq} = \frac{1}{9} \text{Re} \left\langle \sum_{i=1}^9 (A_i)_{pq} (A_i)_{qp} \right\rangle$$

band-diagonal structure



$$R^2(t) = \left\langle \frac{1}{N} \text{tr} \sum_{i=1}^9 (\bar{A}_i(t))^2 \right\rangle = |R^2(t)| \exp(2i\theta_s(t))$$

4. Result

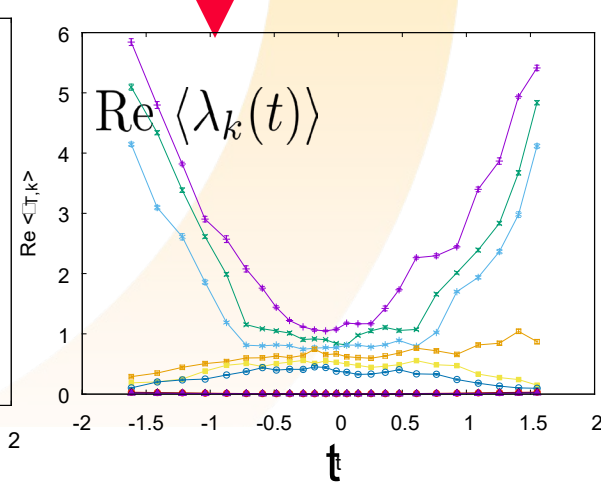
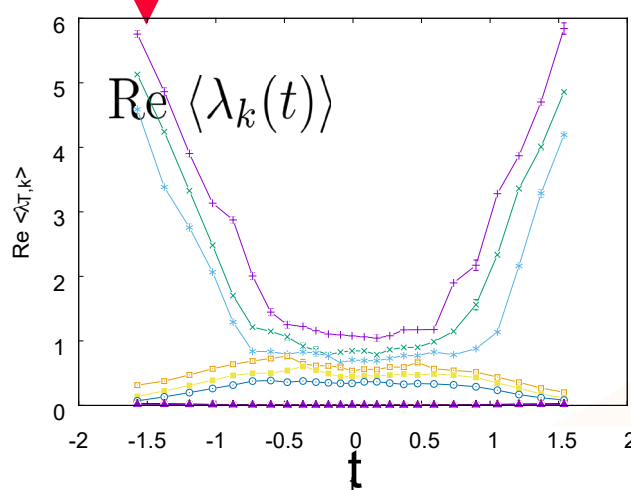
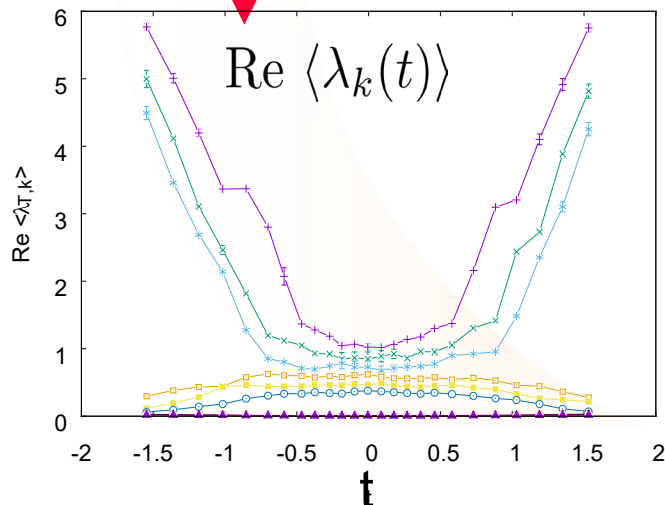
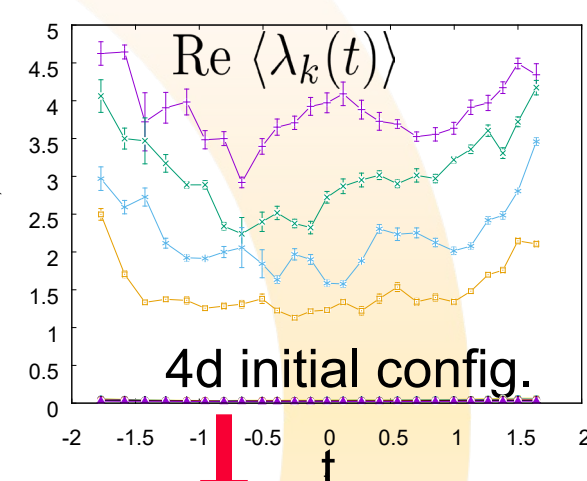
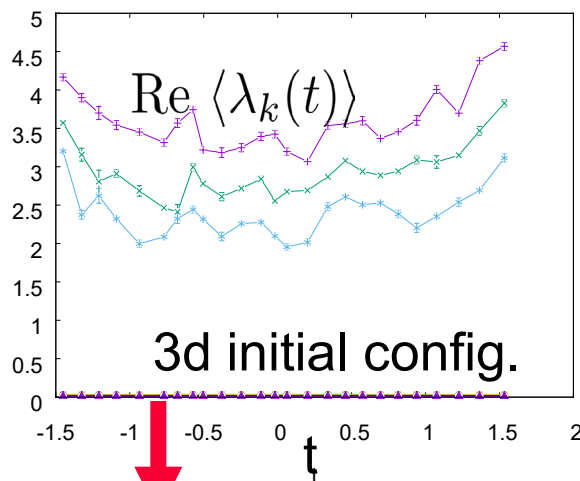
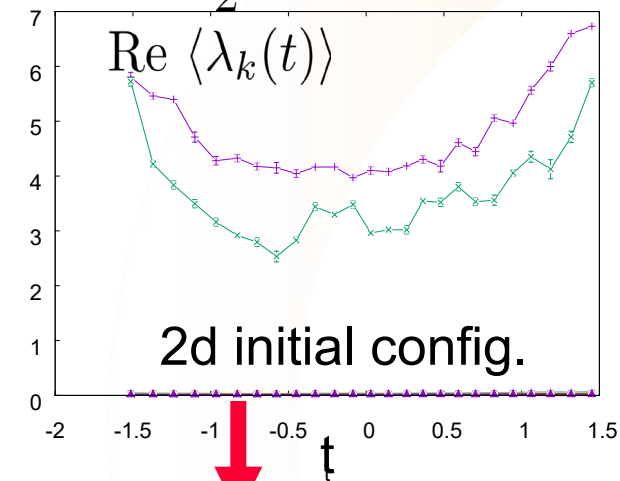
[K.N.Anagnostopoulos, T.Azuma, M.Hirasawa,
J.Nishimura, A.Tsuchiya, N. Yamamori arXiv:2604.25564]

(2) Dependence on initial config.

$N=32, \gamma=6, m_f=2, \xi=10, \tilde{d}=6$

$$S_\xi = \frac{N\gamma}{2} \left\{ \text{tr}(A_0)^2 - \sum_{I=1}^{\tilde{d}} \text{tr}(A_I)^2 - \xi \sum_{I=\tilde{d}+1}^9 \text{tr}(A_I)^2 \right\}$$

$$S_{m_f} = \frac{-N}{2} \text{tr} \{ \bar{\psi}_\alpha (\Gamma^\mu)_{\alpha\beta} [A_\mu, \psi_\beta] + i m_f \bar{\psi}_\alpha (\Gamma^7 \Gamma^{8\dagger} \Gamma^9)_{\alpha\beta} \psi_\beta \}$$



5. Conclusion

Complex Langevin Method (CLM) of the Lorentzian type IIB matrix model.

Lorentz symmetry is "gauge-fixed" by Faddeev-Popov procedure.

Emergence of **expanding 3d space**.

Future prospects

- $N \rightarrow \infty \Rightarrow \gamma \rightarrow 0, m_f \rightarrow 0$ limit: does the 3d space emerge?
- Lefschetz thimble method on the model.