Quantum Geometry and Hawking Radiation

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[PMH-Imamura-Kawai-Shao 2309.12936]
[PMH-Kawai-Shao 2411.01105]
[Chang-PMH-Lee-Shao 2412.02577]
[Ho-Shao-Yoda …][Cheng-Ho-Kawai-Shao …]

spacetime uncertainty and UV-IR connection

Noncommutative Geometry:

$$[x, t] = i\ell_p^2 \quad \Rightarrow \quad \Delta x \Delta t \gtrsim \ell_p^2$$

$$\Rightarrow \qquad \Delta x > \ell_p^2 \Delta E > \ell_p^2 \Delta p$$

$\Delta x \gtrsim \ell_p^2 \Delta p$ (UV-IR connection)

Generalized Uncertainty Relation: [Amati-Ciafaloni-Veneziano 87, 89] [Konishi-Paffuti-Provero 90]

$$\Delta x \Delta p \gtrsim 1 + \ell_p^2 \Delta p^2$$

Large $\Delta p \quad \rightarrow \quad \Delta x \gtrsim \ell_p^2 \Delta p$

spacetime uncertainty and UV-IR connection

$$\Delta x \gtrsim \ell_p^2 \Delta p \qquad \text{(UV-IR connection)}$$

→ Large spatial uncertainty at trans-Planckian energies.

Dramatic UV corrections with IR effects.

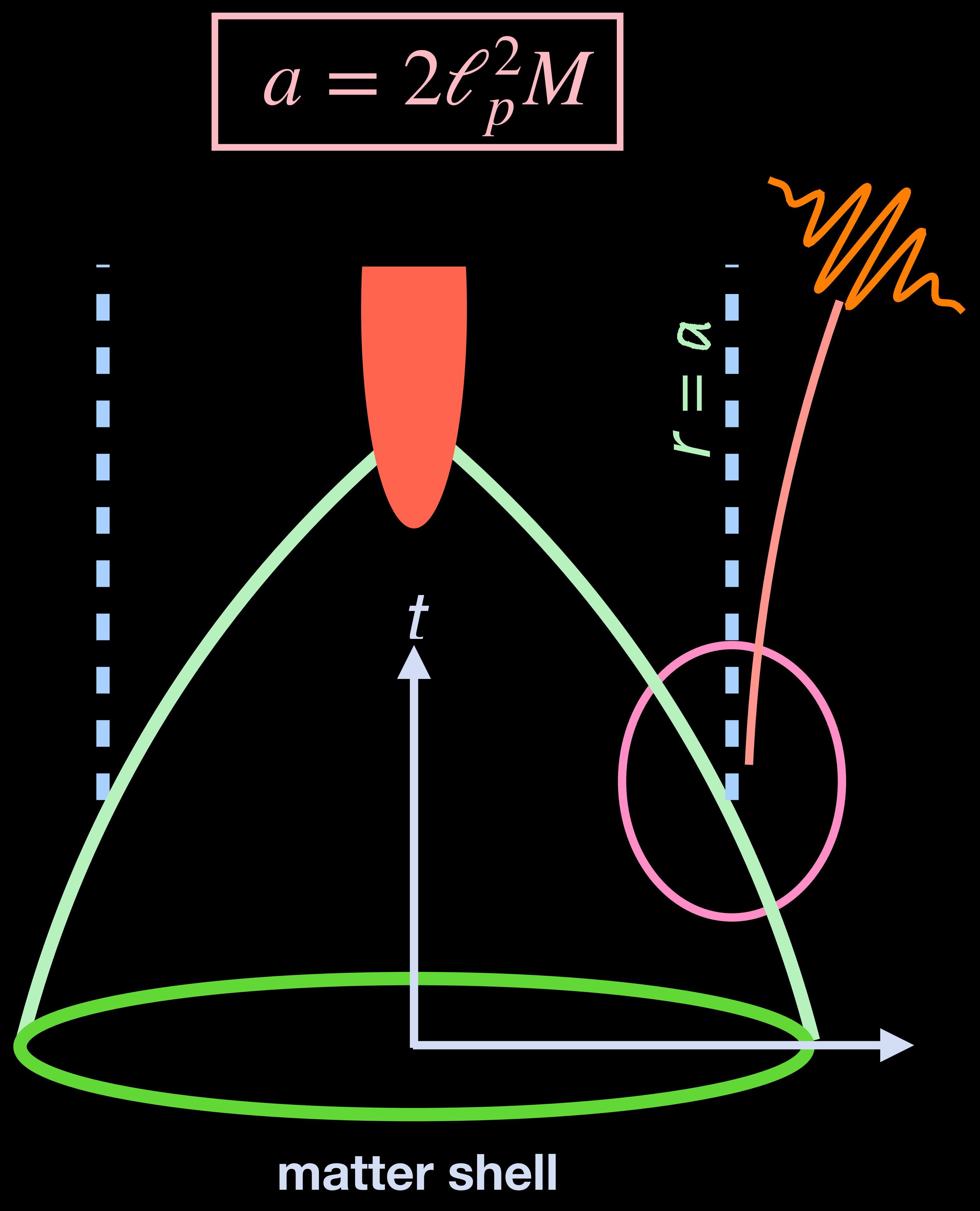
Hawking radiation is insensitive to generic UV physics.

But it is sensitive to UV-IR connection:

→ Hawking radiation stops around scrambling time.

black hole formation and evaporation: conventional model

[Hawking 76]



Hawking radiation: mostly geometric information

info. in matter $\rightarrow$ radiation?

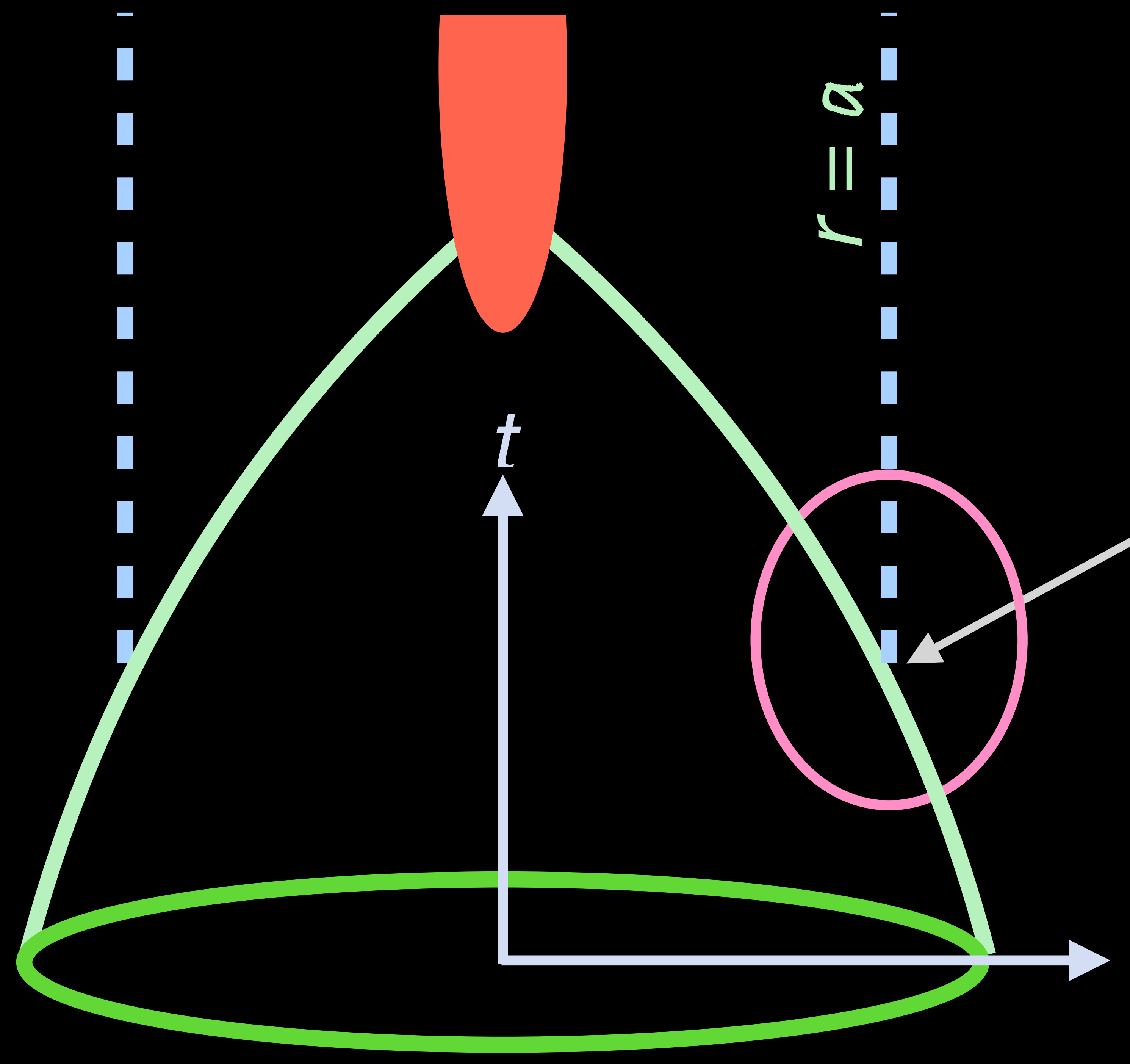
same as burning a piece of coal?

$G_N = \ell_p^2, \quad \hbar = 1, \quad c = 1$

black hole formation and evaporation: conventional model

[Hawking 76]

$$a = 2\ell_p^2 M$$



matter shell

Equivalence principle:

We may have just crossed the horizon.

General Relativity ⇒ "uneventful" horizon

black-hole information paradox

Hawking's calculation in low-energy effective theory (LEET)

→ Hawking radiation carries (almost) no information
  and persists until (almost) complete evaporation

  → information paradox

Q: Is Hawking's LEET calculation reliable?

black-hole information paradox

[Quanta magazine "*The Fuzzball Fix for a Black Hole Paradox*" 2015]

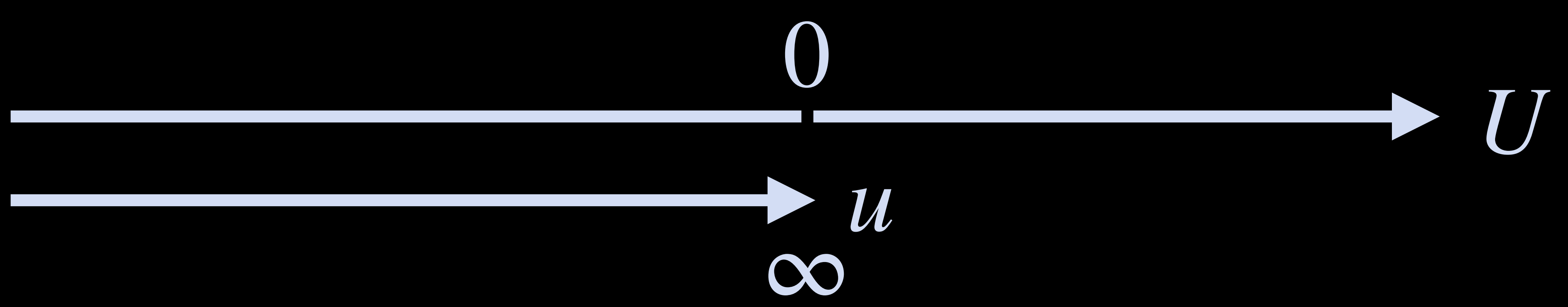
Polchinski said of Hawking's original premise:

"… a situation that seems to violate the laws of quantum mechanics. … 'Quantum mechanics is modified. Find my mistake.' And nobody found his mistake."

There have been countless papers on how Hawking radiation is robust, insensitive to UV modifications of LEET.

Hawking radiation

[Hawking 74]



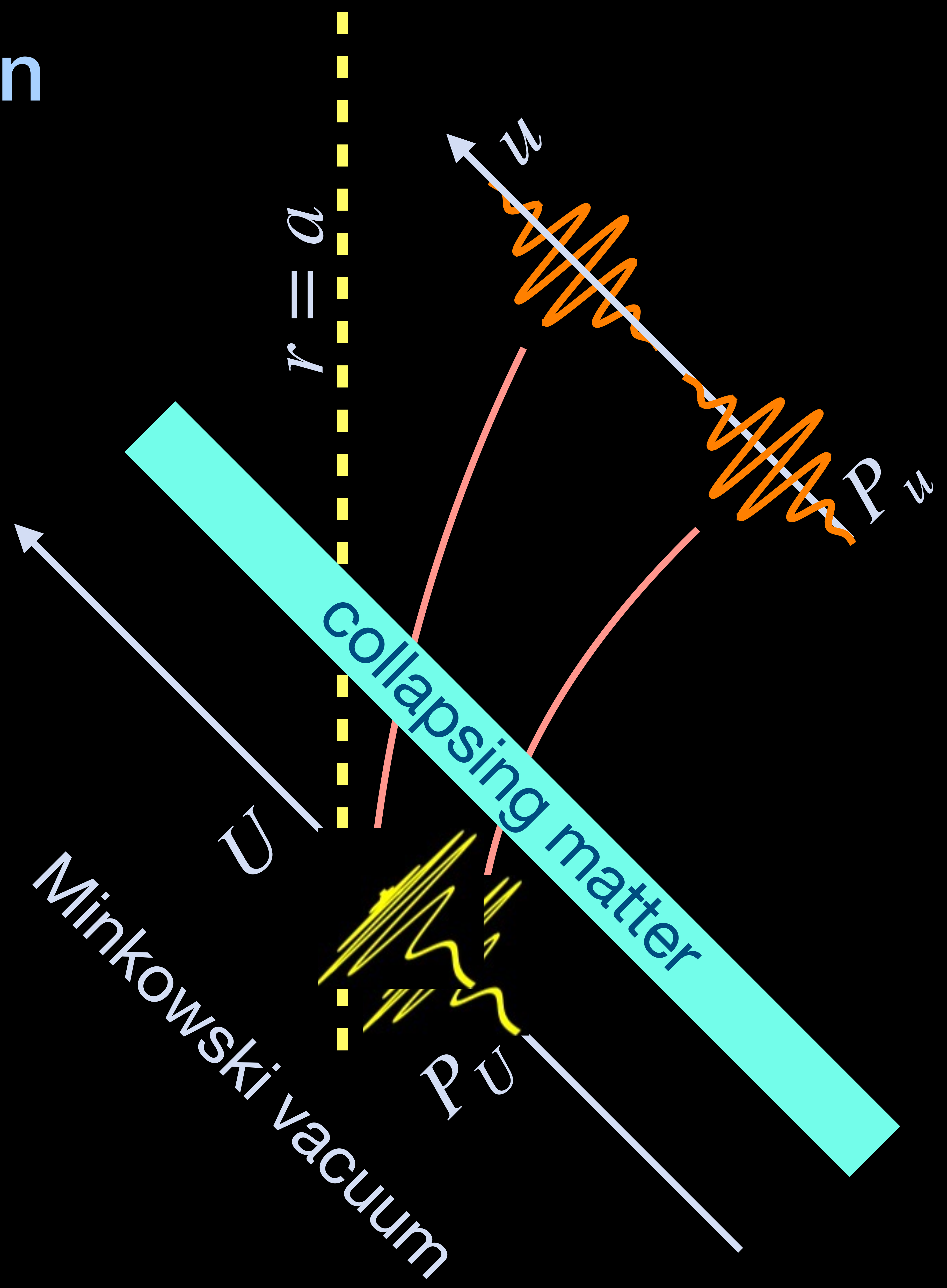
$$U = -2ae^{-u/2a}$$

$$\langle P_U \rangle \sim \langle P_u \rangle \, e^{u/2a}$$

Minkowski vacuum $|\,0\rangle$ of the infinite past

$\simeq$ Hawking radiation at large distance

Q: Is there a trans-Planckian Lorentz-invariant?



Hawking radiation

[Hawking 74]

outgoing massless scalar

$$\phi = \int_0^\infty \frac{d\Omega}{2\pi\sqrt{\Omega r}} \left(a_\Omega e^{-i\Omega U} + a_\Omega^\dagger e^{i\Omega U} \right) = \int_0^\infty \frac{d\omega}{2\pi\sqrt{\omega r}} \left(b_\omega e^{-i\omega u} + b_\omega^\dagger e^{i\omega u} \right)$$

Hawking radiation

[Hawking 74]

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$$\Rightarrow \quad b_\omega = \int_0^\infty d\Omega \left(\alpha_{\omega\Omega} a_\Omega + \beta_{\omega\Omega} a_\Omega^\dagger \right), \qquad b_\omega^\dagger = \int_0^\infty d\Omega \left(\alpha^*_{\omega\Omega} a_\Omega^\dagger + \beta^*_{\omega\Omega} a_\Omega \right).$$

Hawking radiation

[Hawking 74]

outgoing massless scalar

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Unruh vacuum (for freely falling observers): $a_\Omega|0\rangle = 0 \quad (\forall \Omega > 0)$

→ No Hawking radiation for <u>freely falling observers.</u>

Hawking radiation for <u>distant observers</u>: $\langle 0|b_\omega^\dagger b_{\omega'}|0\rangle > 0.$

higher-derivative interactions

[PMH-Yokokura 20][PMH-Kawai-Yokokura 21]

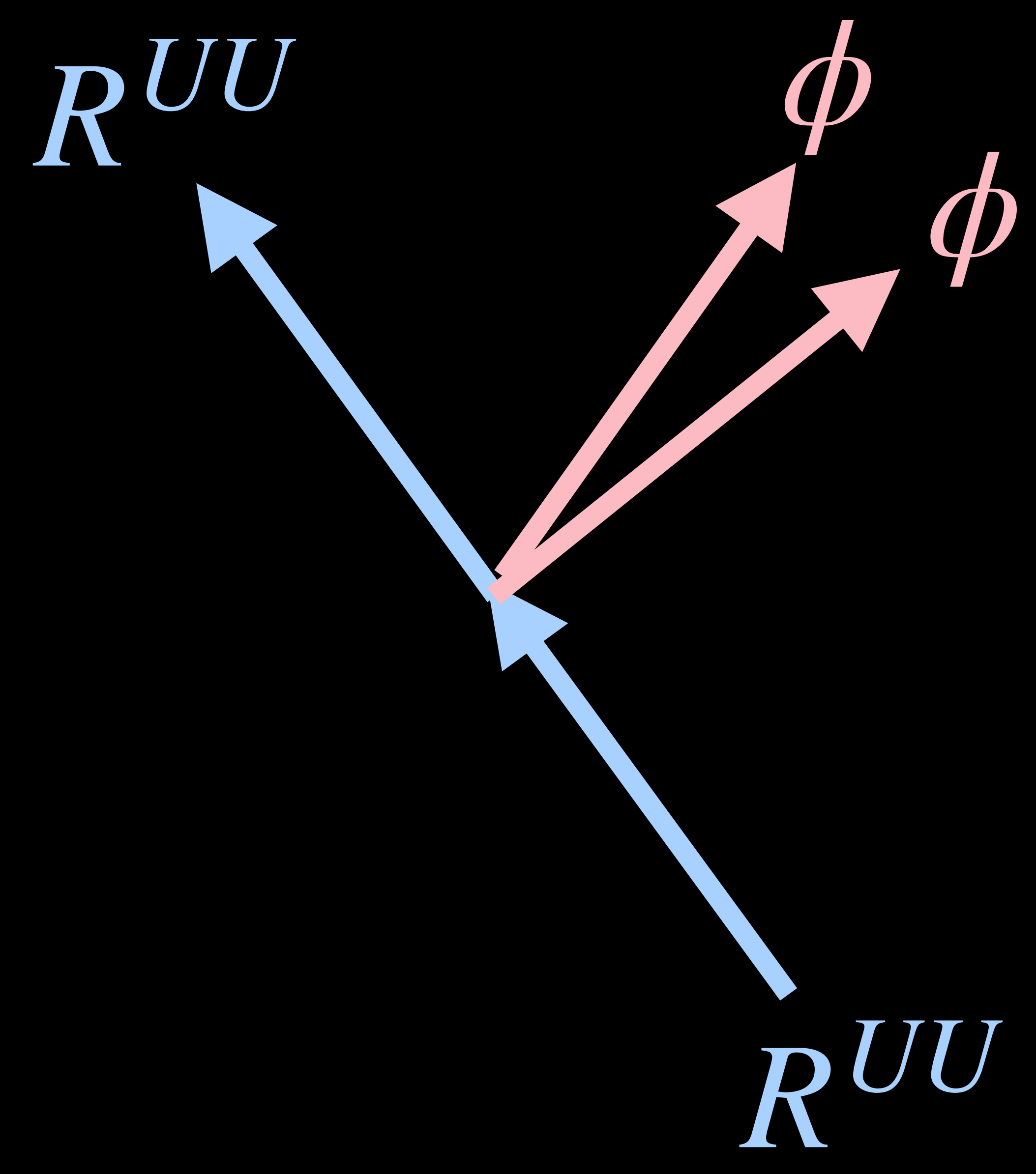
Ex:

$$S_n \equiv \frac{\lambda_n}{M_p^{4n-2}} \int d^4x \sqrt{-g}\, R^{\mu_1\nu_1}\dots R^{\mu_n\nu_n} \left(\nabla_{\mu_1}\cdots\nabla_{\mu_n}\phi\right)\left(\nabla_{\nu_1}\cdots\nabla_{\nu_n}\phi\right)$$

$$\sim \lambda_n \left(\frac{s}{M_p^2}\right)^{2n-1}$$

$$s \equiv P_U P_V \sim \frac{1}{a^2} e^{u/2a}$$

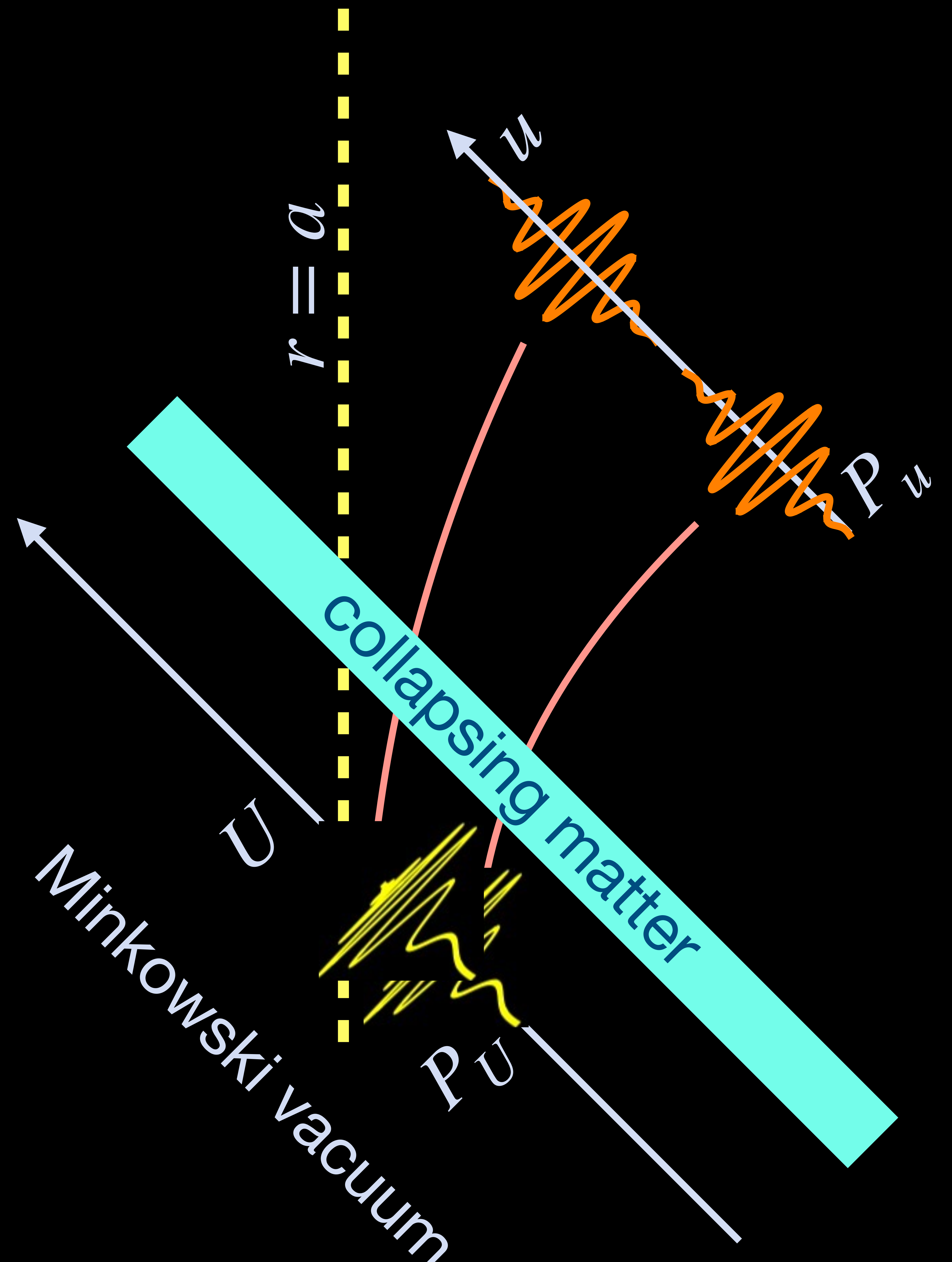
$$u \sim a \log(a/\ell_p) \quad \Rightarrow \quad Ampl. \sim \mathcal{O}(1)$$



black hole as a microscope

The detection of Hawking quanta explores trans-Planckian regions of the spacetime in Minkowski vacuum.

It needs UV theories for reliable predictions.



black-hole information paradox

Q: Is Hawking's calculation reliable?

No. ← Trans-Planckian local Lorentz inv. after scrambling time.

black-hole information paradox

Q: Is Hawking's calculation reliable?

No. $\leftarrow$ Trans-Planckian local Lorentz inv. after scrambling time.

$\rightarrow$ UV physics is relevant.

How does a given UV theory change HR?

1. HR as in LEET $\Rightarrow$ still a paradox. $\leftarrow$ literature
2. HR enhanced by informative radiation.

Q: Other logical possibilities?

alternative resolution

[Chau-PMH-Kawai-Shao-Wang 23][PMH-Imamura-Kawai-Shao 23][PMH-Kawai-Shao 24]

3. Hawking radiation stops well before the Page time, say, around scrambling time:

$$t_{scr} \sim a \log(a/\ell_p) \quad \ll \quad t_{Page} \sim a^3/\ell_p^2$$

$\rightarrow$ HR is negligible.

alternative resolution

[Chau-PMH-Kawai-Shao-Wang 23][PMH-Imamura-Kawai-Shao 23][PMH-Kawai-Shao 24]

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$\rightarrow$ HR is negligible.

However, in the literature, "*HR is insensitive to UV.*"

Are there loopholes in the literature?

Departure from the literature

In the literature, typically,

- the collapsing matter is ignored.

  Time translation symmetry $\Rightarrow$ Hawking radiation persists.

  Hartle-Hawking or Boulware vacuum.

- time-dependent amplitude of Hawking radiation is not derived.

  Hawking temperature is indeed robust.

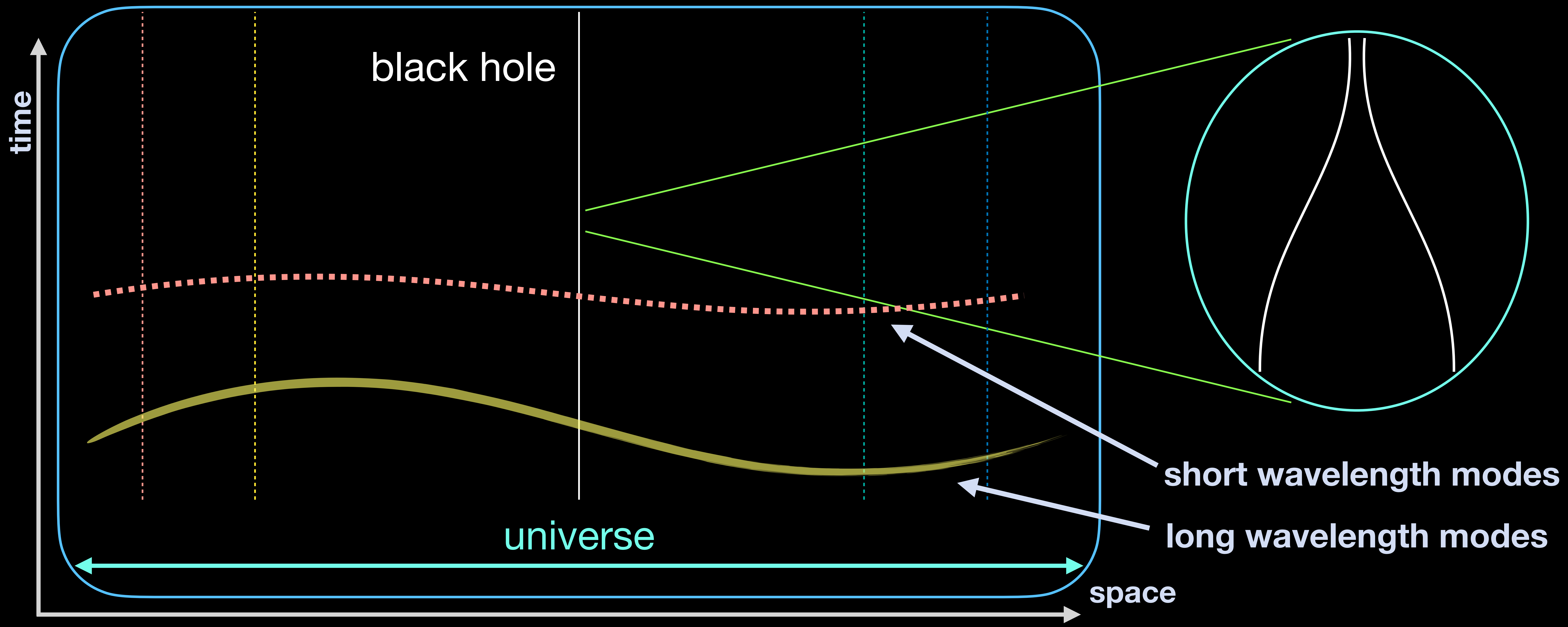
- there is no UV-IR relation in UV physics.

  e.g. UV dispersion relations do not turn off Hawking radiation.

  e.g. 2D or 3D theories.

Small-scale strcutures are irrelevant to low-energy modes.

UV-IR relation $\rightarrow$ also to trans-Planckian modes.



Generalized Uncertainty Principle (GUP)

[Amati-Ciafaloni-Veneziano 87, 89] [Konishi-Paffuti-Provero 90]

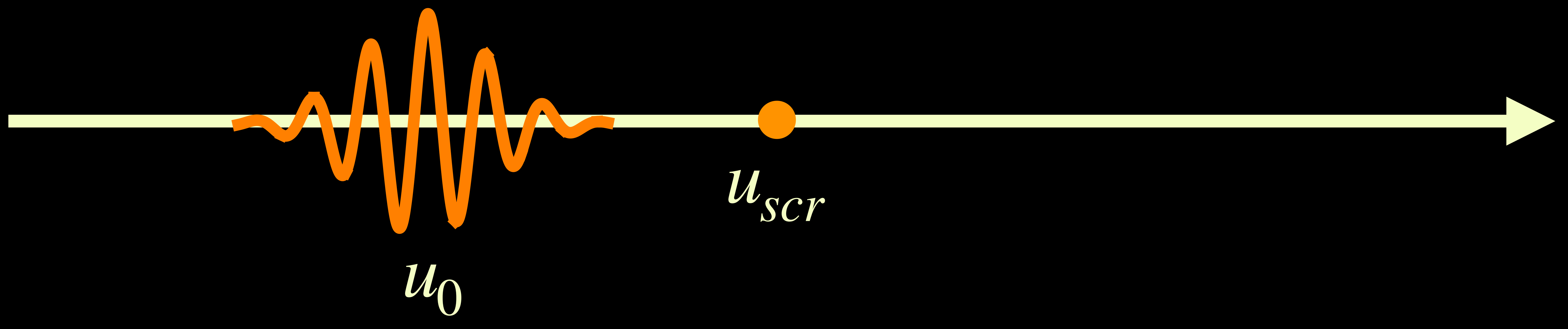
$$\Delta x \Delta p \gtrsim 1 + \ell_p^2 \Delta p^2$$

$\Rightarrow$ minimal length $\Delta x \gtrsim 2\ell_p$

$$\Delta p \gg \ell_p^{-1} \quad \Rightarrow \quad \Delta x \gtrsim \ell_p^2 \Delta p$$

Robustness of HR with GUP noted repeatedly in many works.

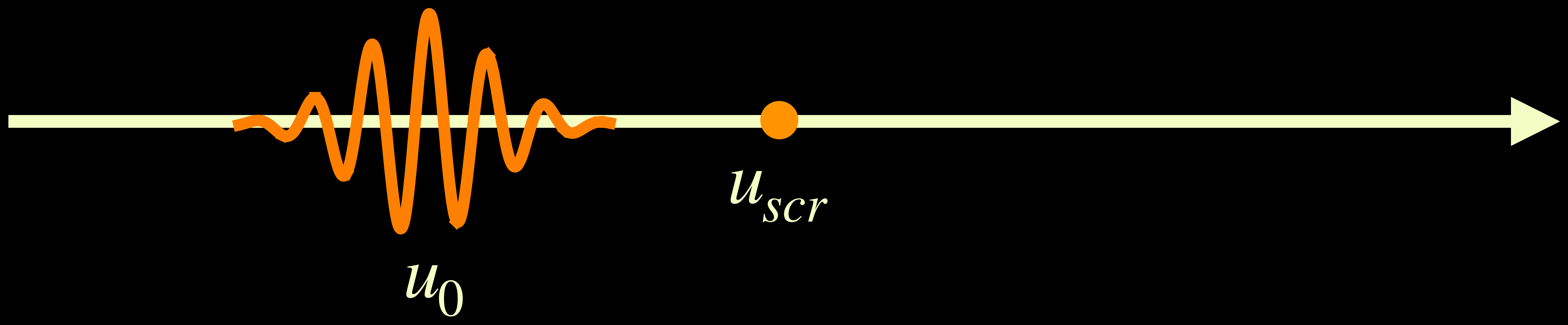
[0506110, 1212.6591, 1410.4115, 1410.5065, 1501.03256, 1501.06025, 1602.04304, 1704.03536, 1709.00637, 1903.01382, 2112.13573, 2306.03077].



Hawking radiation = the VEV of the number operator for this wave packet:

$$\langle 0 \mid b_\Psi^\dagger b_\Psi \mid 0 \rangle \simeq \frac{1}{2}\frac{\omega_0}{e^{4\pi a \omega_0} - 1} \int_{-\infty}^{u_{scr}} du \left| \Psi_{(\omega_0, u_0)}(u) \right|^2$$

$$u_{scr} = 4a \log(a/\ell_p)$$



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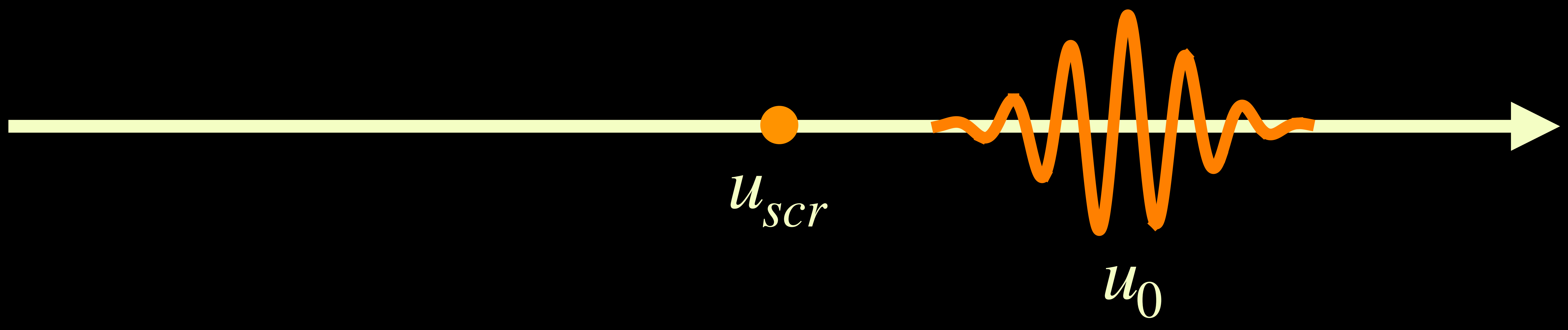
$$u_{scr} = 4a \log(a/\ell_p)$$

Hawking temperature is the same.



Hawking radiation = the VEV of the number operator for this wave packet:

$$\langle 0 \mid b_\Psi^\dagger b_\Psi \mid 0 \rangle \simeq \frac{1}{2} \frac{\omega_0}{e^{4\pi a \omega_0} - 1} \int_{-\infty}^{u_{scr}} du \left| \Psi_{(\omega_0, u_0)}(u) \right|^2$$

$\longrightarrow 0$ when $u_0 \gg u_{scr} \equiv 2a \log(a/\ell_p)$ (scrambling time).

At late times, the wave packet has a large Δp, GUP implies a large Δx.

When Δx is much larger than a, it does not contribute to Hawking radiation.

[Chau-PMH-Kawai-Shao-Wang 23]

comments

- Entanglement Island proposal [Almheiri-Engelhardt-Marolf-Maxfield 19, Penington 19]
  nonlocal but low-energy effective description (lower dimensions)

  UV description for $D \geq 4$?
- AdS/CFT duality compatibility

Applications:

- Primordial black holes as dark matter candidates. [Che-Yu's talk]
- Trans-Planckian Censorship Conjecture
  [Blamart-Laliberte-Brandenberger 23][Brandenberger-PMH-Kawai-Shao 24]

conclusion

LEET breaks down at scrambling time for Hawking radiation.

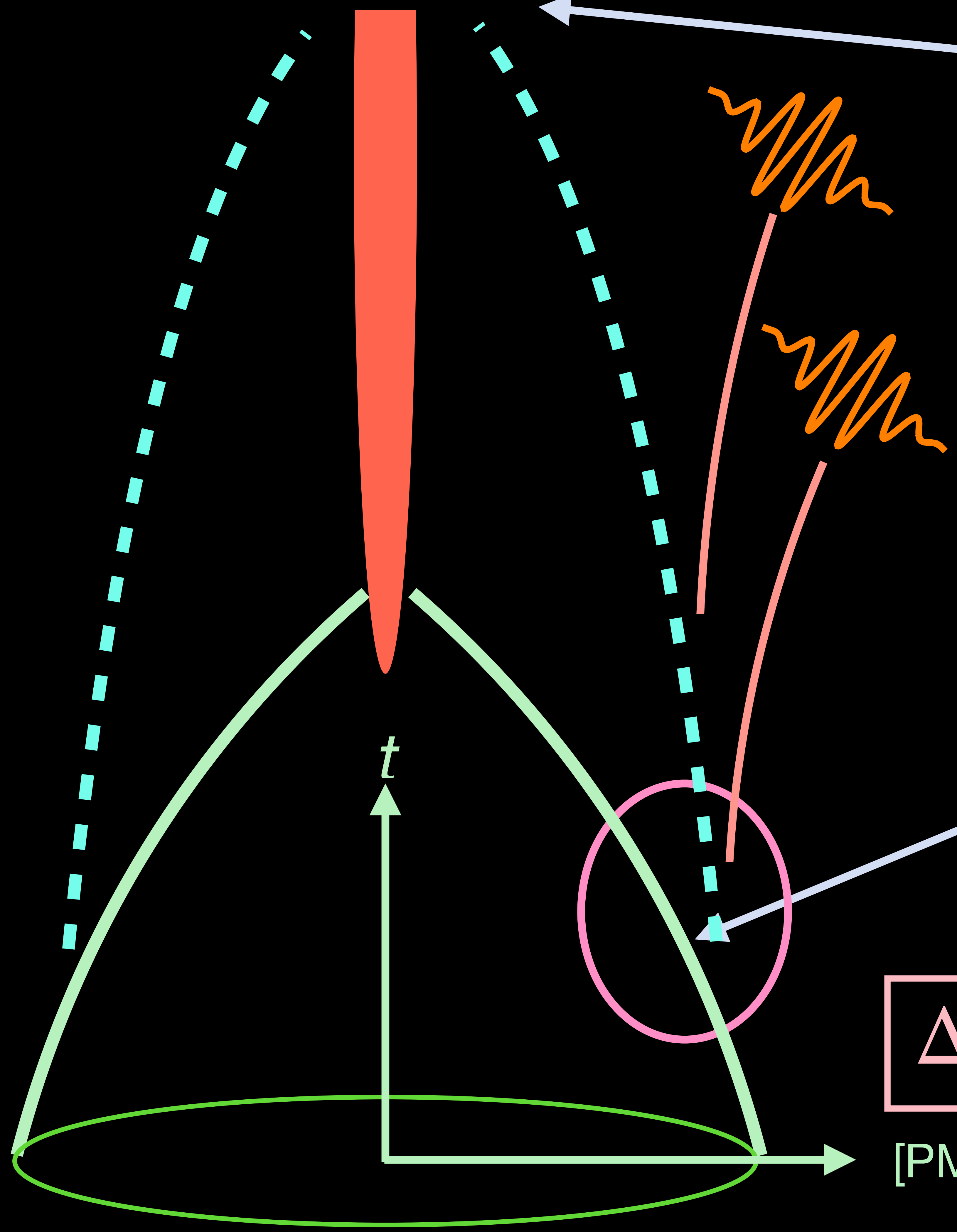
Alternative proposal

- Spacetime uncertainty turns off Hawking radiation at scrambling time.
- Black holes are essentially classical.
- Hawking temperature not modified $\rightarrow$ same BH entropy, etc.
- Spacetime uncertainty blurs the horizon.

  $\rightarrow$ Other channels of informative radiation (cf. fuzzball)

Thank you!

Information transfer in the conventional model



M/100

M

$\Delta t \sim a^3/\ell_p^2 \;=\; \text{Page time}$

Through Hawking rad. ⇒
99% of mass/information in the collapsing matter comes out within $1000\,\ell_p$!!!

→ firewall or nonlocality!

$\Delta\tau \lesssim 1000\,\ell_p$

[PMH-Matsuo-Yokokura 20]

Is UV theory relevant?

Nice-slice argument*

Near-horizon region remains approximately vacuum in low-energy effective theory.

nice-slice argument

[Lowe-Polchinski-Sussking-Thorlacius-Uglum 95]

The curvature $\sim \mathcal{O}(1/a)$.

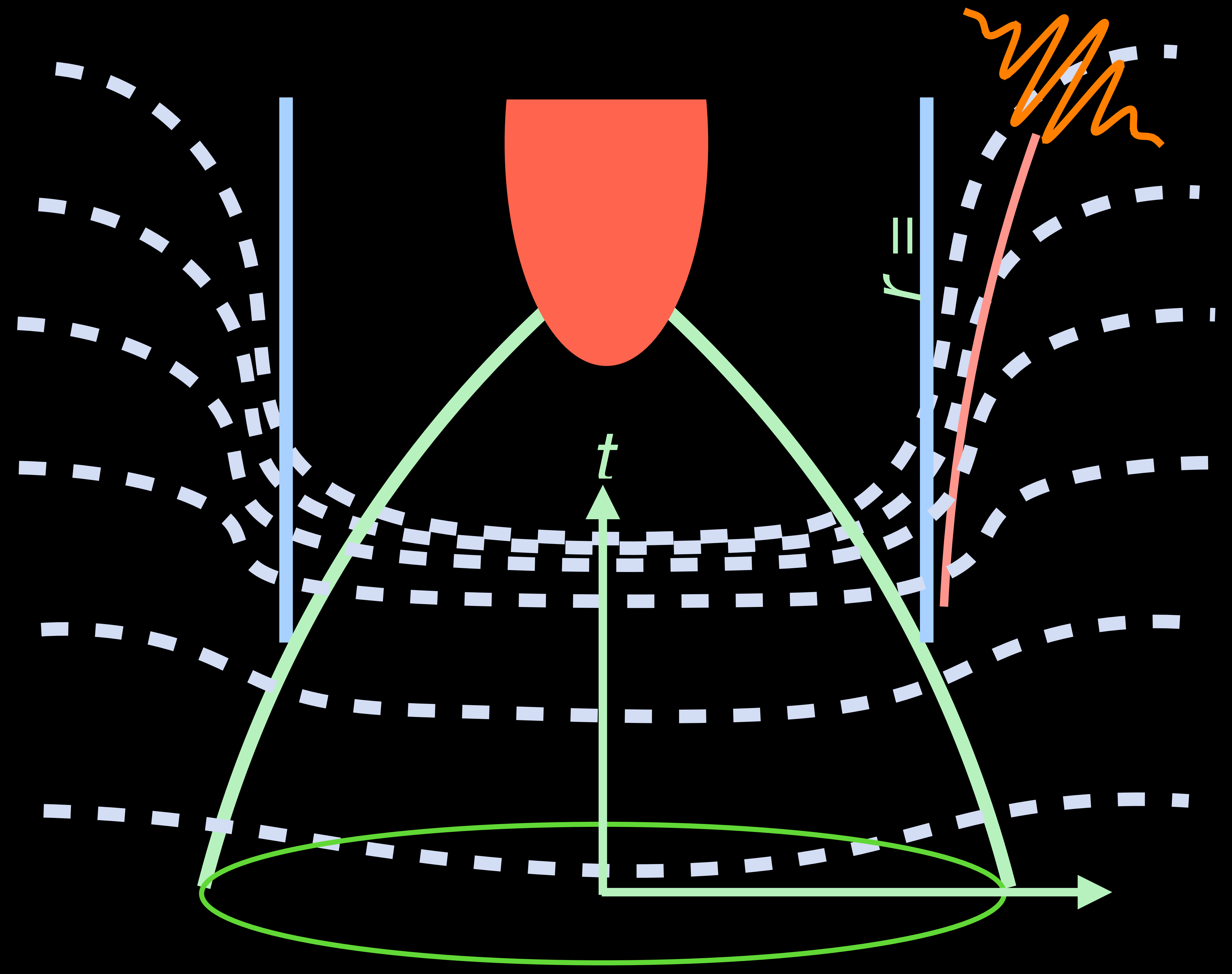
Adiabatic theorem $\Rightarrow$

If the initial state is vacuum,

excitations of energies $\lesssim \mathcal{O}(1/a)$

from time evolution.

$\Rightarrow$ Effective theory remains valid.



Decoupling theorem

Effective theory breaks down for trans-Planckian observations!

Is UV theory relevant?

Nice-slice argument

Near-horizon region remains approximately vacuum

in low-energy effective theory. (OK)

→ UV physics is irrelevant?

LEET is not good for predicting UV events.

Q: Is Hawking radiation UV physics?

Q: Trans-Planckian local Lorentz invariant?

another example: SFT

[PMH-Imamura-Kawai-Shao 23] [Chang-PMH-Lee-Shao 24]

$$S_{SFT} = \int d^D x \left[\frac{1}{2}\phi_\alpha(\partial^2 - m_\alpha^2)\phi_\alpha + g_{\alpha\beta\gamma}\tilde{\phi}_\alpha\tilde{\phi}_\beta\tilde{\phi}_\gamma + \cdots \right],$$

$$\tilde{\phi}_\alpha \equiv e^{\frac{1}{2}\ell^2\partial_\mu\partial^\mu}\phi_\alpha \qquad\qquad \eta = (-++\cdots+)$$

→ exponential suppression of UV interactions in string theory.

$$S_{SFT} = \int d^D x \left[\tilde{\phi}_\alpha (\partial^2 - m_\alpha^2) e^{-\ell^2\partial^2}\tilde{\phi}_\alpha + g_{\alpha\beta\gamma}\tilde{\phi}_\alpha\tilde{\phi}_\beta\tilde{\phi}_\gamma + \cdots \right]$$

→ coupling to background suppressed for large-k modes.

→ $\Delta U \Delta V \gtrsim \ell_E^2$ (spacetime uncertainty relation, cf. [Yoneya])

Hawking radiation turned off at scrambling time.

compatibility with AdS/CFT duality

- Small AdS black hole

Before scrambling time: same classical black-hole geometry in all models.

↔ a thermal state with T_H at scrambling time [Danielsson et al 99])

After scrambling time: (before Hawking-Page transition)

- Conventional model: complete evaporation via HR in conventional model
- Our scenario: HR stops. If there are other decay channels …

↔ Hawking temp increases towards Planck scale but details are unknown.

Entanglement entropy formula cannot predict Page time.

It is possible to have the “same” Page curve, but different Page time.

- Large AdS black hole [Maldacena 01]

No obvious paradox if the causal barrier (horizon) is removed.

compatibility with AdS/CFT duality

	initial time	$\mathcal{O}(a)$	$\mathcal{O}(a\log(a))$ scrambling time		$\mathcal{O}(a^3)$	Hawking-Page transition $\to \infty$ as $R_{AdS} \to \infty$
CFT	pure initial state		thermal state at T_H		thermal state at T_P or at T_H ? ?	thermal state at $T \sim 0$
AdS	collapsing matter $r - a \sim \mathcal{O}(a)$	HR @ T_H $r - a \ll \mathcal{O}(a)$	$T \to T_H$ HR stops	$T_H \to T_P$	complete evaporation ?	thermal gas at $T \sim 0$