

Asymptotic Higher Spin symmetry in gravity

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Based on [2410.15219](#) , [2409.12178](#) [2505.04327](#) with Nicolas Cresto

[2111.15607](#) [2112.15573](#), [2212.12469](#), [2306.02373](#).

with A. Raclariu and D. Pranzetti



W-symmetry in gravity

[Cachazo, Strominger '14];
[Zlotnikov '14];

The study of subleading soft theorems and its connection with asymptotic symmetries has opened up a new perspective on gravity and YM

[Campiglia, Laddha '16];
Conde, Mao '16]

The study of colinear limit revealed a tower of new symmetries encoded into $LW_{1+\infty}$

Strominger '21
Guevara, Himwich, Pate, Strominger 21

These results have been connected to classical results in twistor theory that relates integrability of Self-dual GR to the existence of $LW_{1+\infty}$ symmetries

[Adamo, Mason, Sharma '21];
[Ball, Narayanan, Salzer, Strominger '21]

These has led to a new perspective on holomorphic quantization of self-dual theories

[Costello, Paquette '22]
Bittleston, Skinner, Sharma 22

In canonical $LW_{1+\infty}$ was found in terms of an algebra of Noether charges build up from the knowledge of the asymptotic dynamics and expressed in terms of the gravitational phase space

Pranzetti, Raclariu, LF 21-23

Geiller 24

Donnay, Herfray, LF 24

W-symmetry in gravity

So far the connection between canonical symmetries and soft theorems Cresto, LF 24,25 was mostly perturbative and lacking a first principle derivation from Noether. The purpose of our work was to remedy this

Our results are deeply connected to the beautiful recent work which provided a non-perturbative twistor perspective on the construction of charges

Kmec, Mason, Ruzziconi, Srikanth 24

Loop corrections modifies the charge expressions and challenges our understanding of symmetries.

Sahoo, Sen et al. 18-24

Donnay, Nguyen, Ruzziconi '22-23

Choi Ladha, Puhm '24-25

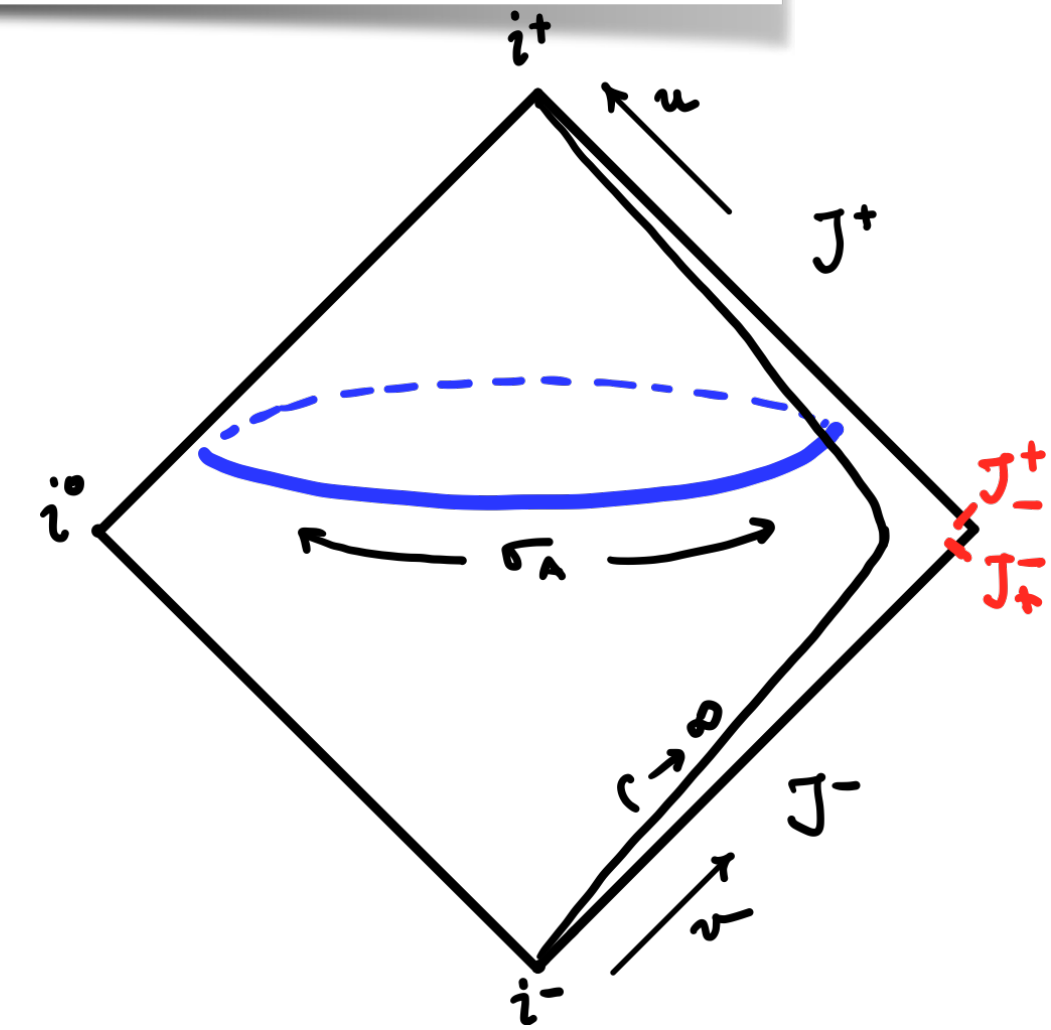
Asymptotically flat spacetimes

- In Bondi gauge, the metric near $\mathcal{I}^+$ takes the form:

$$ds^2 = -2e^{2\beta} du(dr + \Phi du) + g_{AB} \left(d\sigma^A + \frac{\Upsilon^A}{r^2} du \right) \left(d\sigma^B + \frac{\Upsilon^B}{r^2} du \right)$$

- Expansion to order $1/r$ of g_{ab}

$$\Phi = \frac{R(q)}{4} - \frac{m_B}{r} + o(r^{-1}),$$



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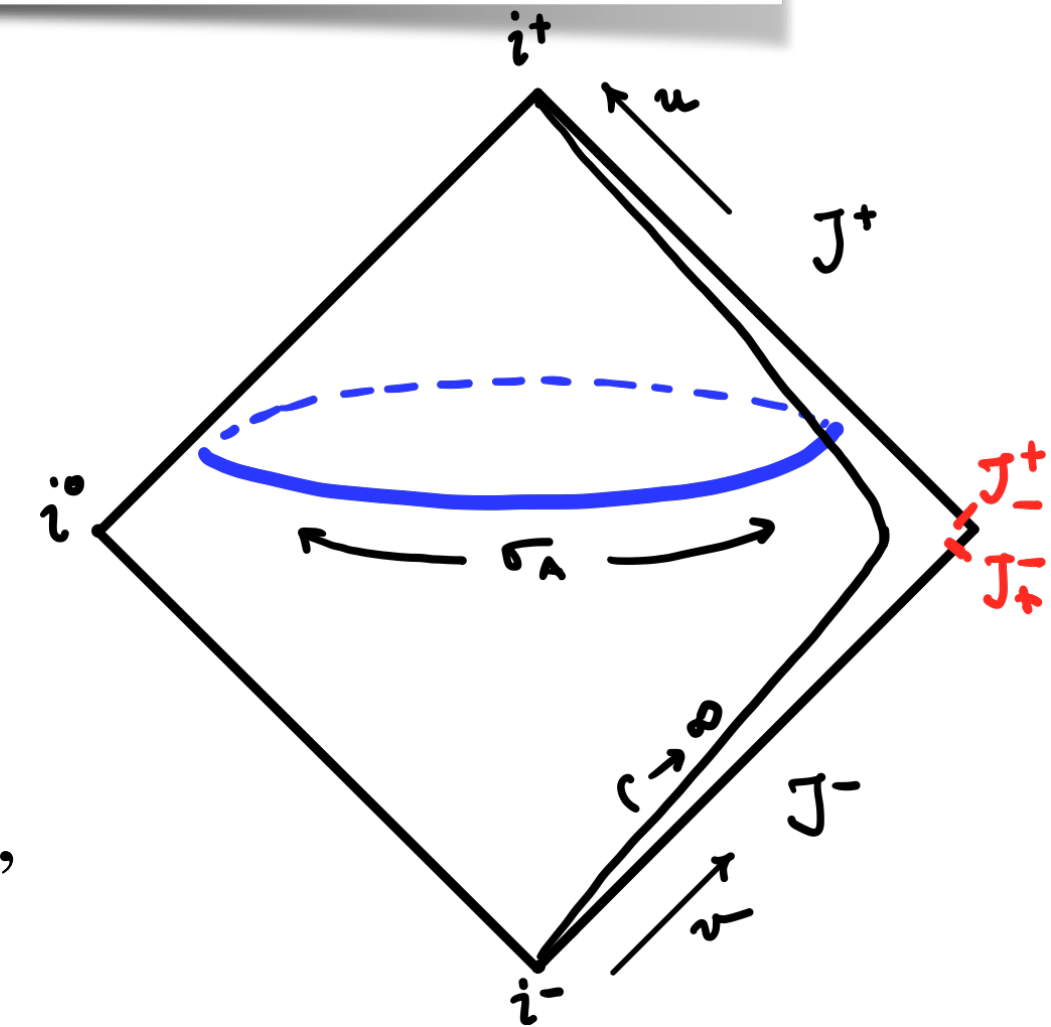
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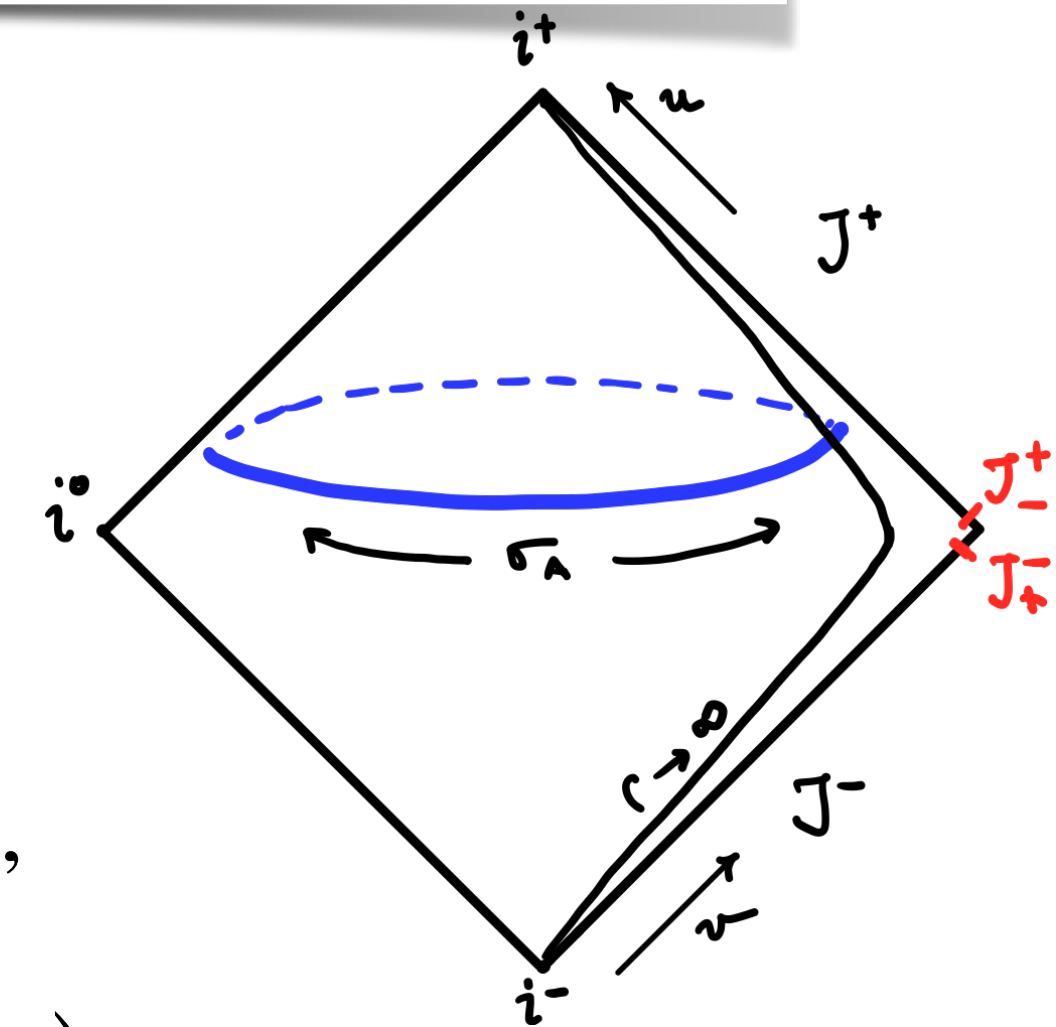
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$$\partial_u C_{AB} = N_{AB} \quad \text{free data}$$



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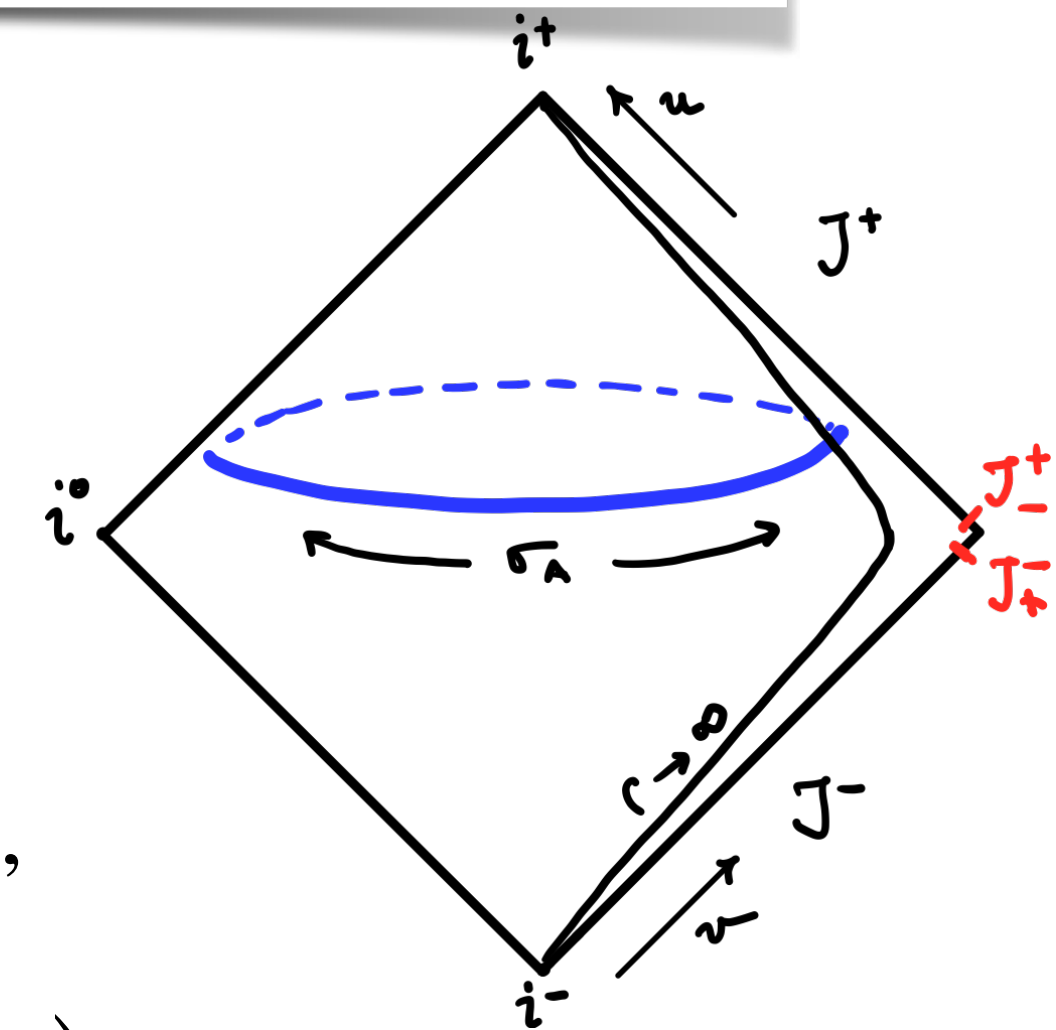
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- Symmetric traceless tensor can be encoded in pair of complex scalars $C_{AB} \rightarrow (C, \bar{C})$
- What are the primary fields? $(m_B, J_A, T_{AB}) \rightarrow (Q_0, Q_1, Q_2)$

Sub-leading theorems and Charge conservation

The gravitational phase space can be defined at $\mathcal{I}^\pm$

It possesses two types of data:

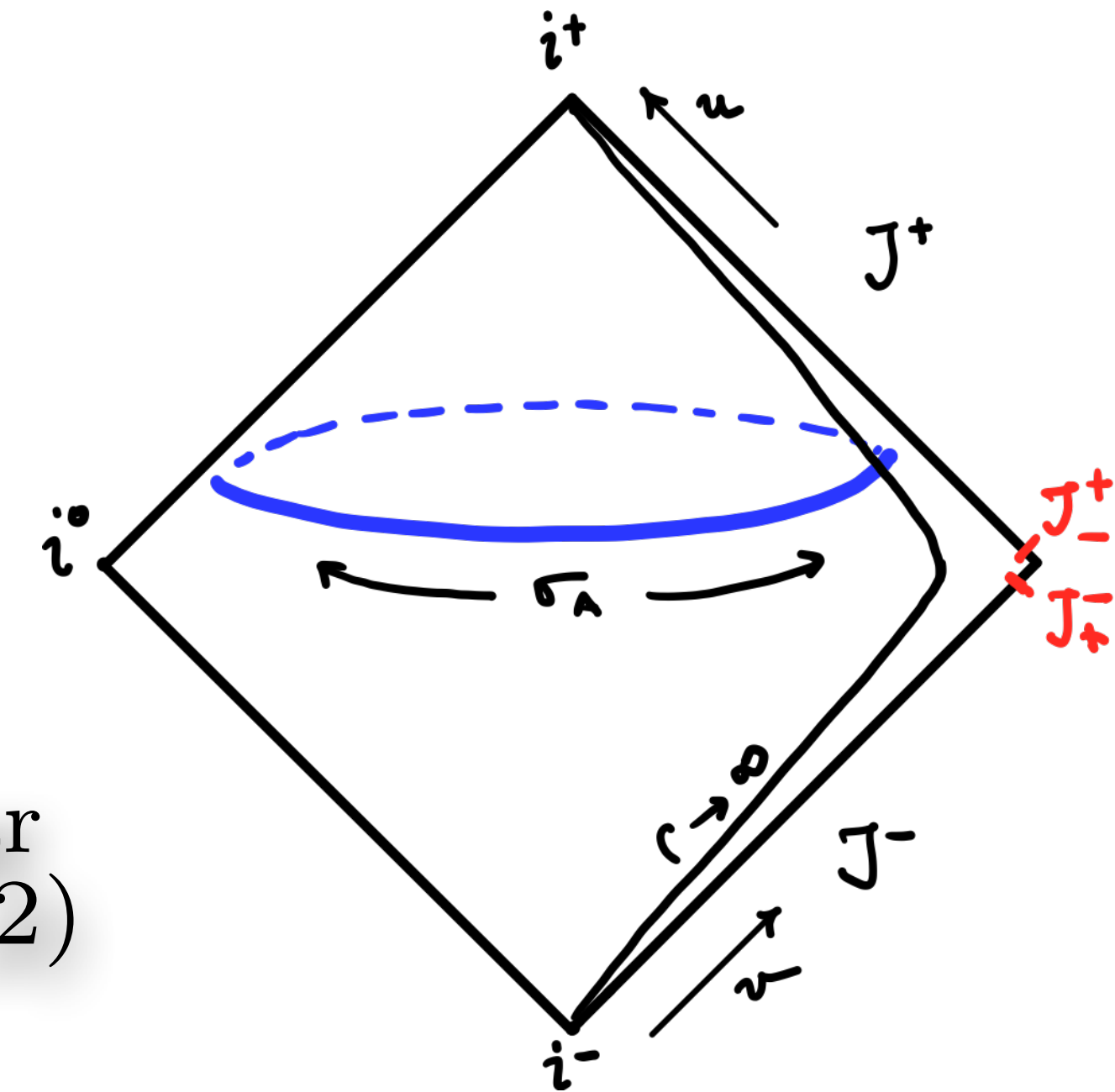
radiative: complex shear $(C, \bar{C})(u, z, \bar{z})$ and news $N = \partial_u C$

Coulombic: Charge aspects $\tilde{Q}_s(u, z, \bar{z})$

Carrollian weight: $\tilde{Q}_s \in C_{(3,s)}^{\text{Car}}$ $C \in C_{(1,2)}^{\text{Car}}$ $N \in C_{(2,2)}^{\text{Car}}$

where $\Phi \in C_{(\delta,s)}^{\text{Car}}$ when

$$\Phi(|a|^2 u, a \lambda_\alpha, \bar{a} \bar{\lambda}_{\dot{\alpha}}) = a^{-(\delta+s)} \bar{a}^{-(\delta-s)} \Phi(u, \lambda_\alpha, \bar{\lambda}_{\dot{\alpha}})$$



Sub-leading theorems and Charge conservation

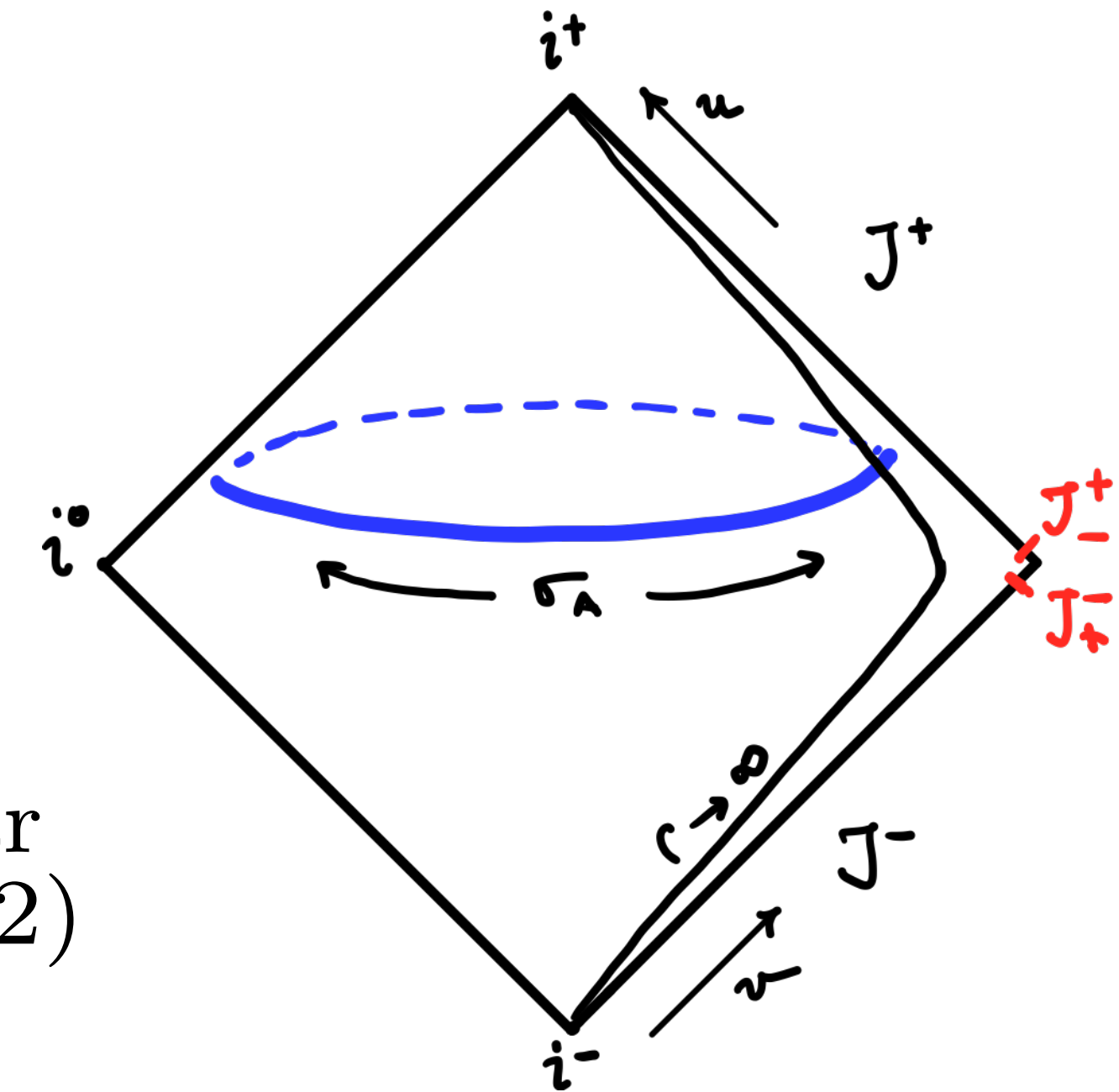
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The charge aspects encodes the subradiative components of the asymptotic Weyl tensor

$$\Psi_2 = \sum_{s=0}^{\infty} \frac{(-1)^s \bar{D}^s Q_s}{r^{3+s}} + \dots$$

Newman-Penrose 62

LF, Pranzetti, Raclariu 22-23

Geiller. 24

Sub-leading theorems and Charge conservation

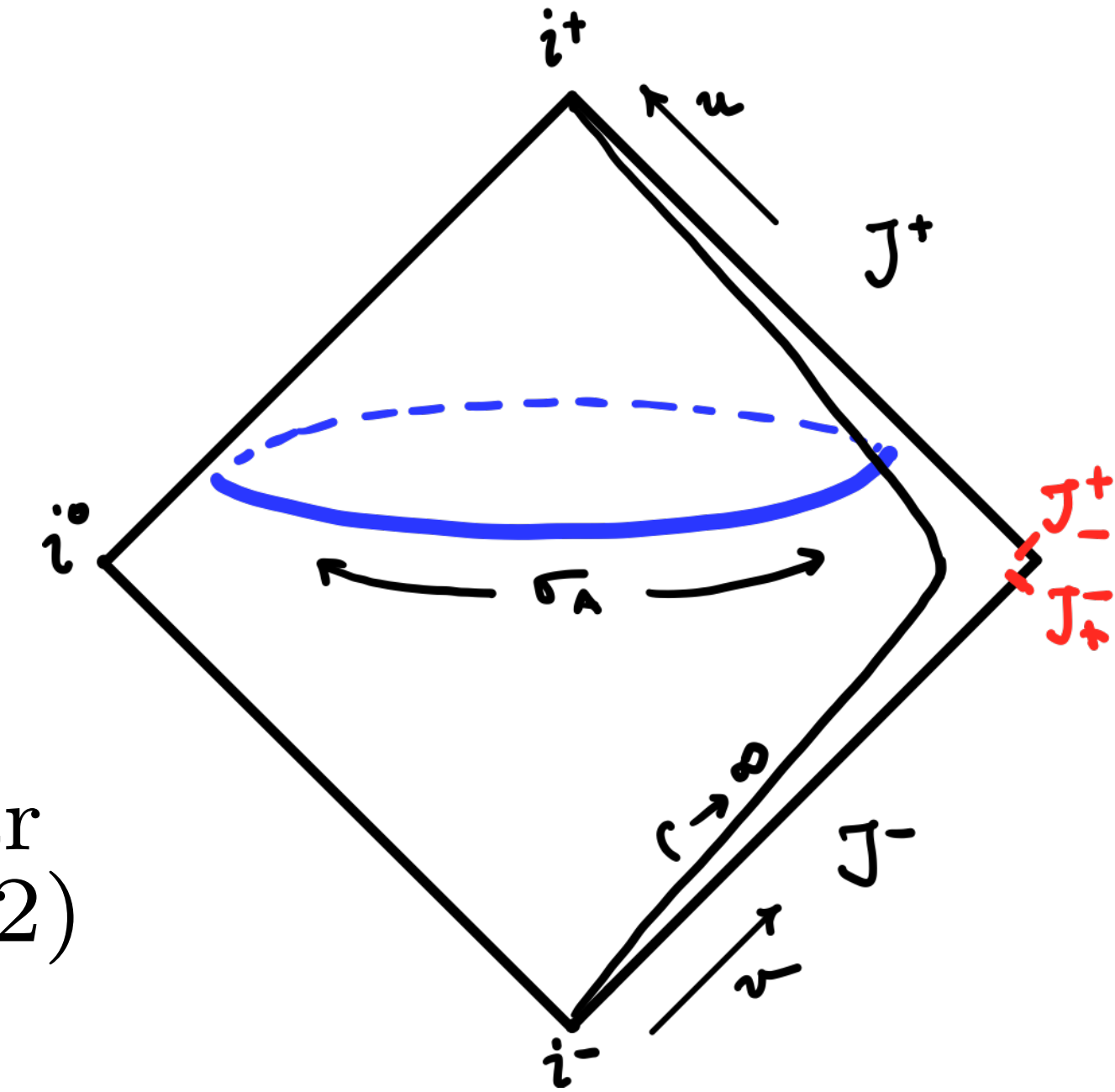
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LF, Pranzetti, Raclariu 22-23
Geiller. 24

The data are not independent: They satisfy **constraint** equations $D = D_z$

$$\boxed{\partial_u \tilde{Q}_s = D \tilde{Q}_{s-1} + (s+1) C \tilde{Q}_{s-2}} \quad \in C_{(4,s)}^{\text{Car}}$$

Sub-leading theorems and Charge conservation

The data are not independent: They satisfy evolution equations

$$\partial_u \tilde{Q}_s = D \tilde{Q}_{s-1} + (s+1) C \tilde{Q}_{s-2}$$

$s = -1, 0, 1, 2, 3$ represent the first 5 Asymptotic Einstein's equations

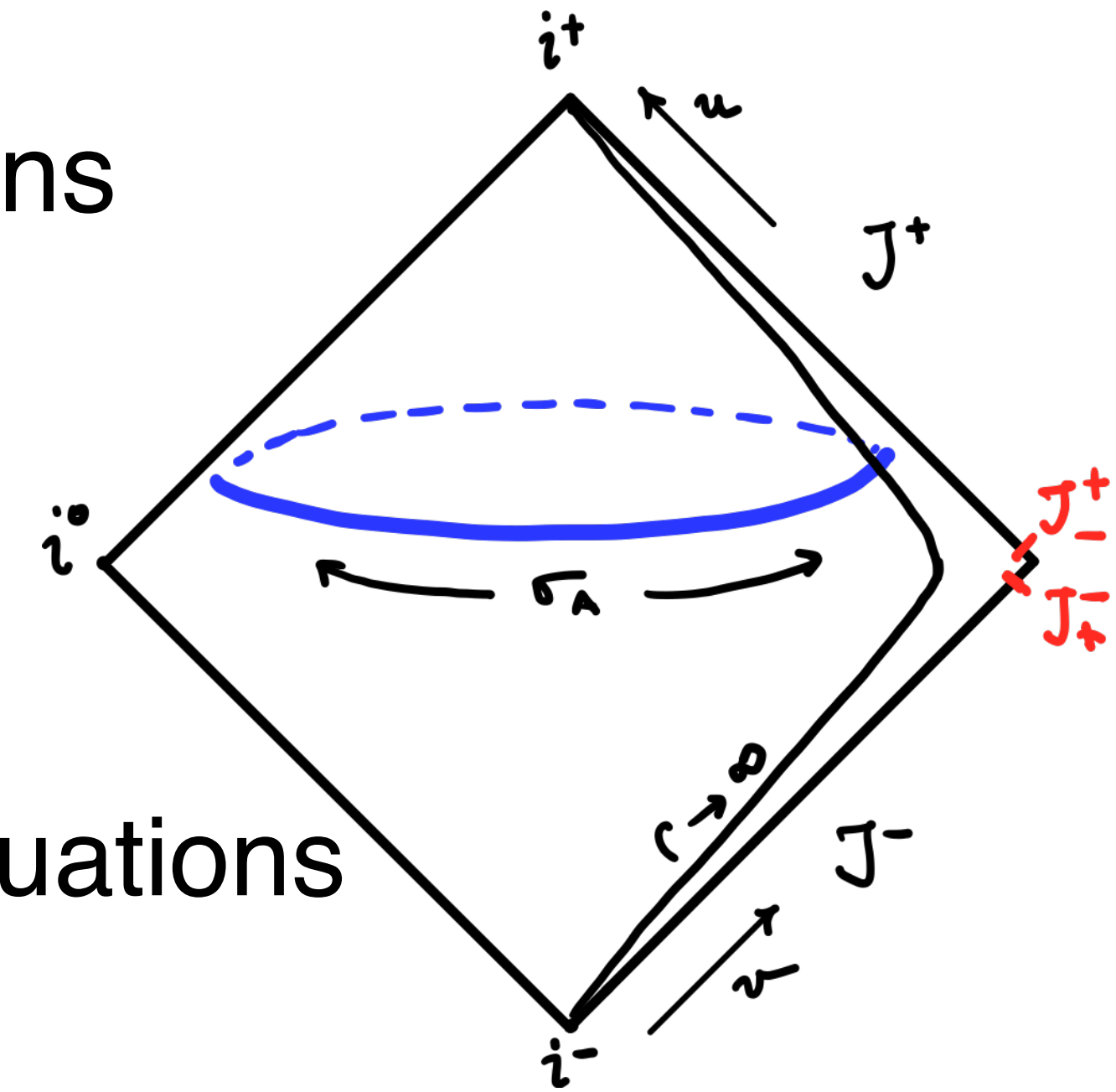
Initial value $\tilde{Q}_{-2} = \frac{\dot{N}}{4\pi G}$ $\tilde{Q}_{-1} = \frac{D\bar{N}}{4\pi G}$ radiative data

$\tilde{Q}_0$ mass aspect $\tilde{Q}_1$ angular-momentum aspect

$\tilde{Q}_2, \tilde{Q}_3$ spin 2,3 charges

In pure gravity these equation are initialized by the conditions that $\tilde{Q}_s(i^\pm) = 0$

In the presence of massive matter $\tilde{Q}_s(i^\pm) = \tilde{Q}_s(\rho_\pm)$



Newman-Adamo 09
Pranzetti, LF 21

Sub-leading theorems and Charge conservation

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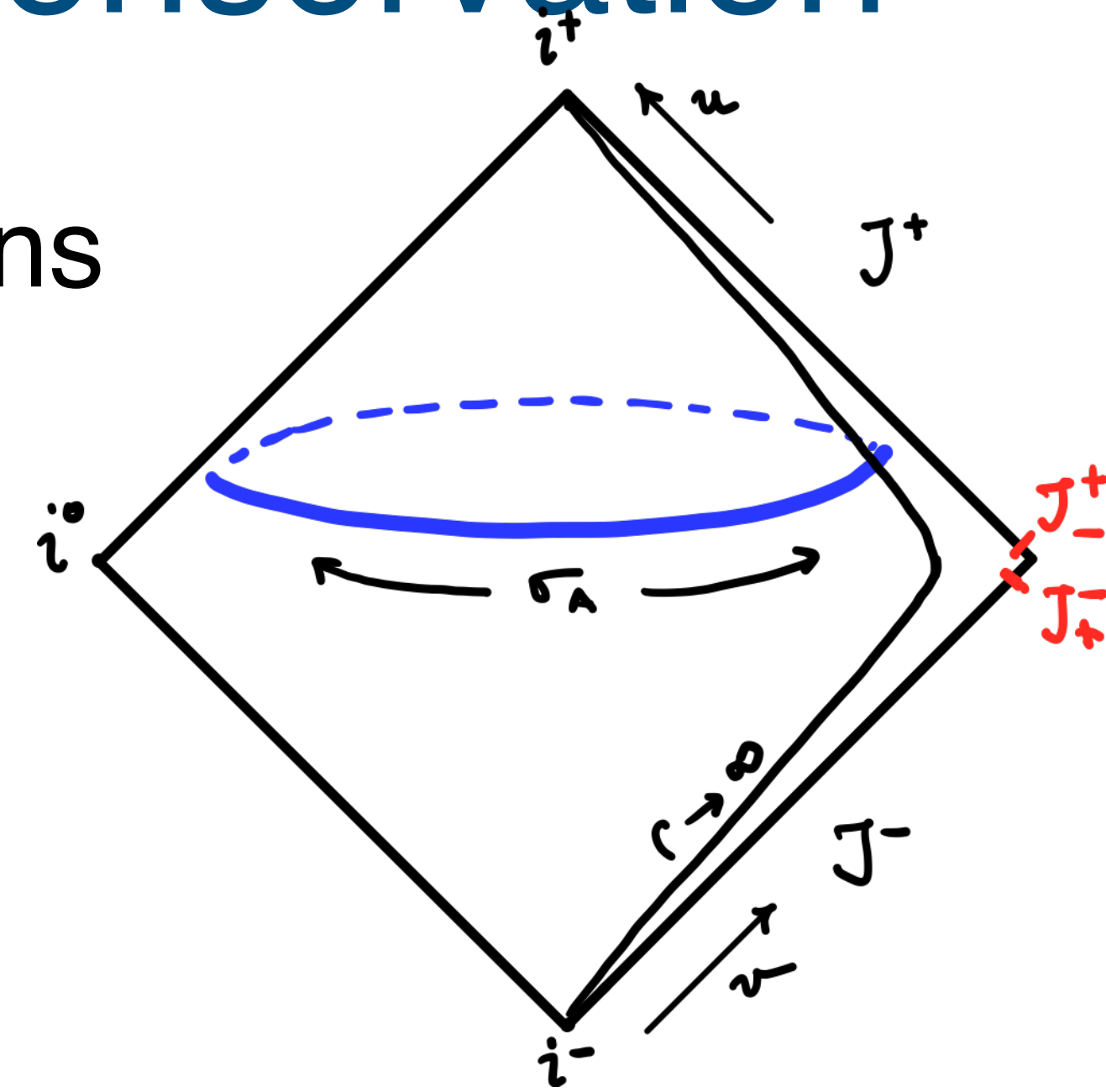
In pure gravity using $\tilde{Q}_s(i^\pm) = 0$

We can integrate these charges as functionals $\tilde{Q}_s(C, \bar{C})$

Convenient to introduce an holomorphic perturbative counting $(C, \bar{C}) \rightarrow (g_N C, \bar{g}_N \bar{C})$

$$\tilde{Q}_s = \sum_{k=0}^{1+[s/2]} Q_s^{(k)} \quad \text{with} \quad Q_s^{(k)} \sim \bar{g}_N g_N^k$$

the charge are quadratic $\sim \bar{g}_N g_N$ for spin 0,1 cubic $\sim \bar{g}_N g_N^2$ for spin 2,3



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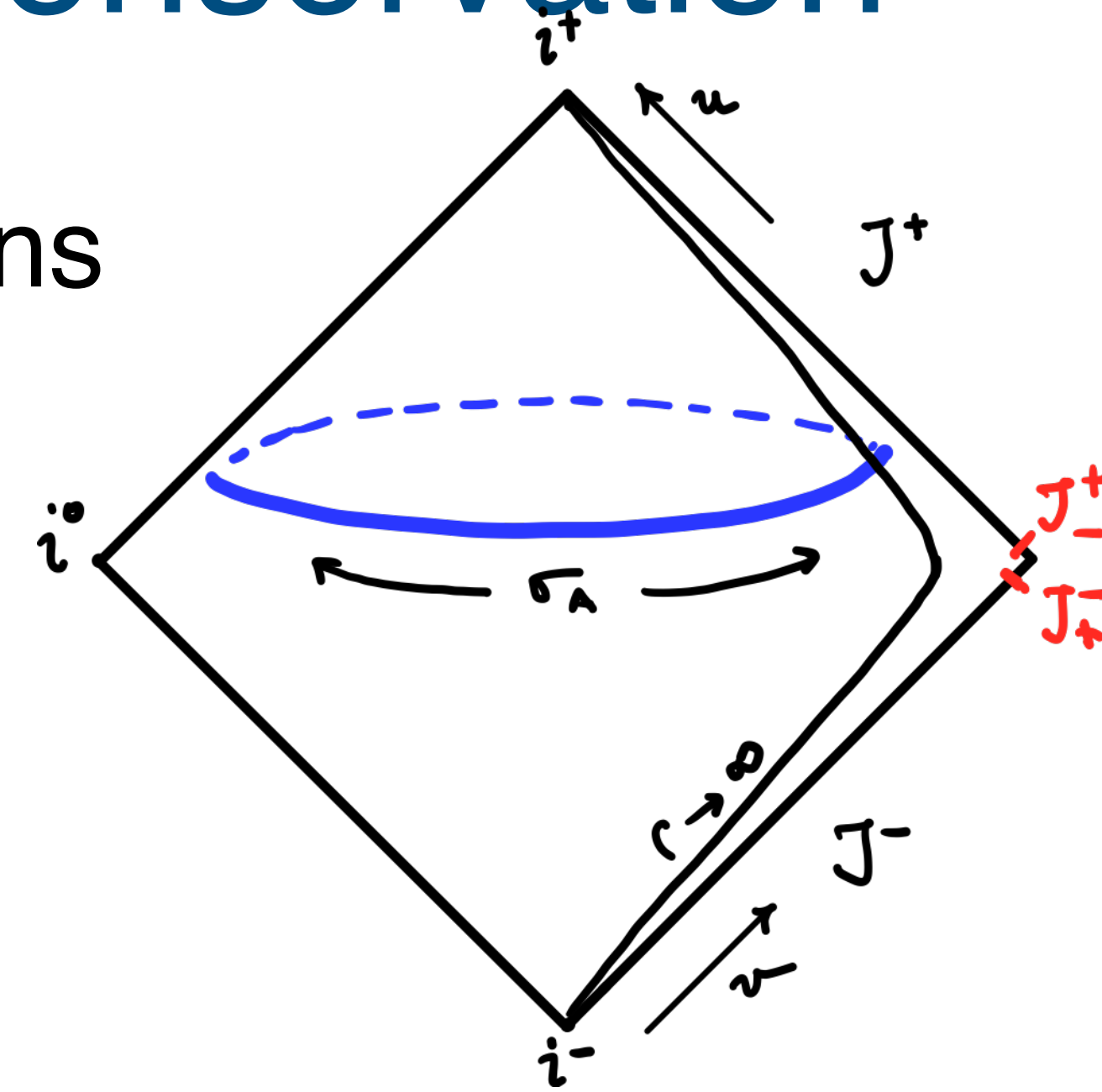
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This leads, beyond spin 2, to an expansion $\tilde{Q}_s = \tilde{Q}_s^S + \tilde{Q}_s^H + \tilde{Q}_s^{SH}$

→ a non-linear generalization of the soft theorems



Sub-leading theorems and Charge conservation

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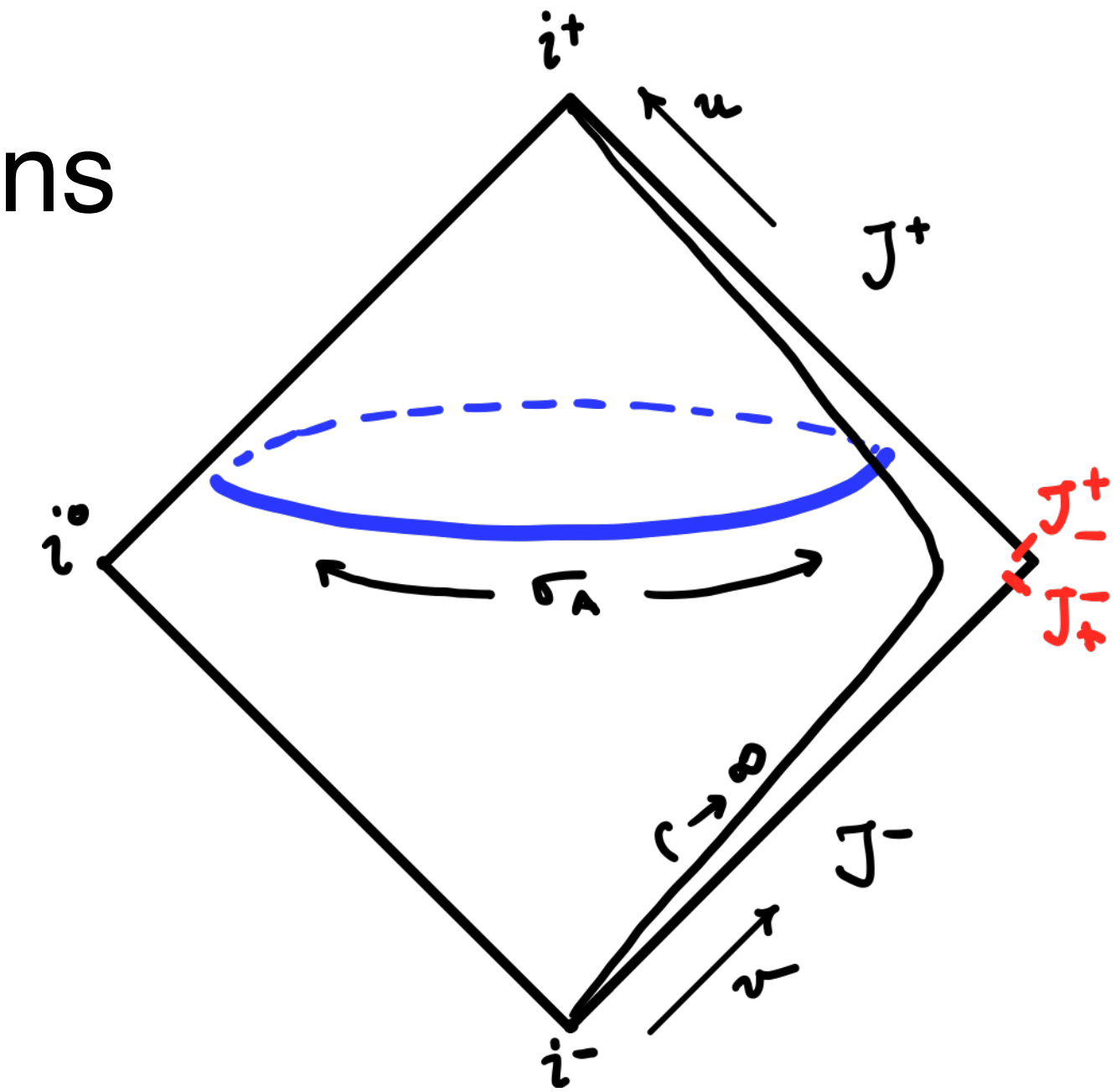
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Beyond spin4 they represent a **truncation** of EE linear in $\bar{g}_N$ **non-linear** in g_N

They represent the **Gauss conservation law** associated with self-dual gravity



Subleading Theorem from Symmetry

The goal of this talk is to show that the system of equations

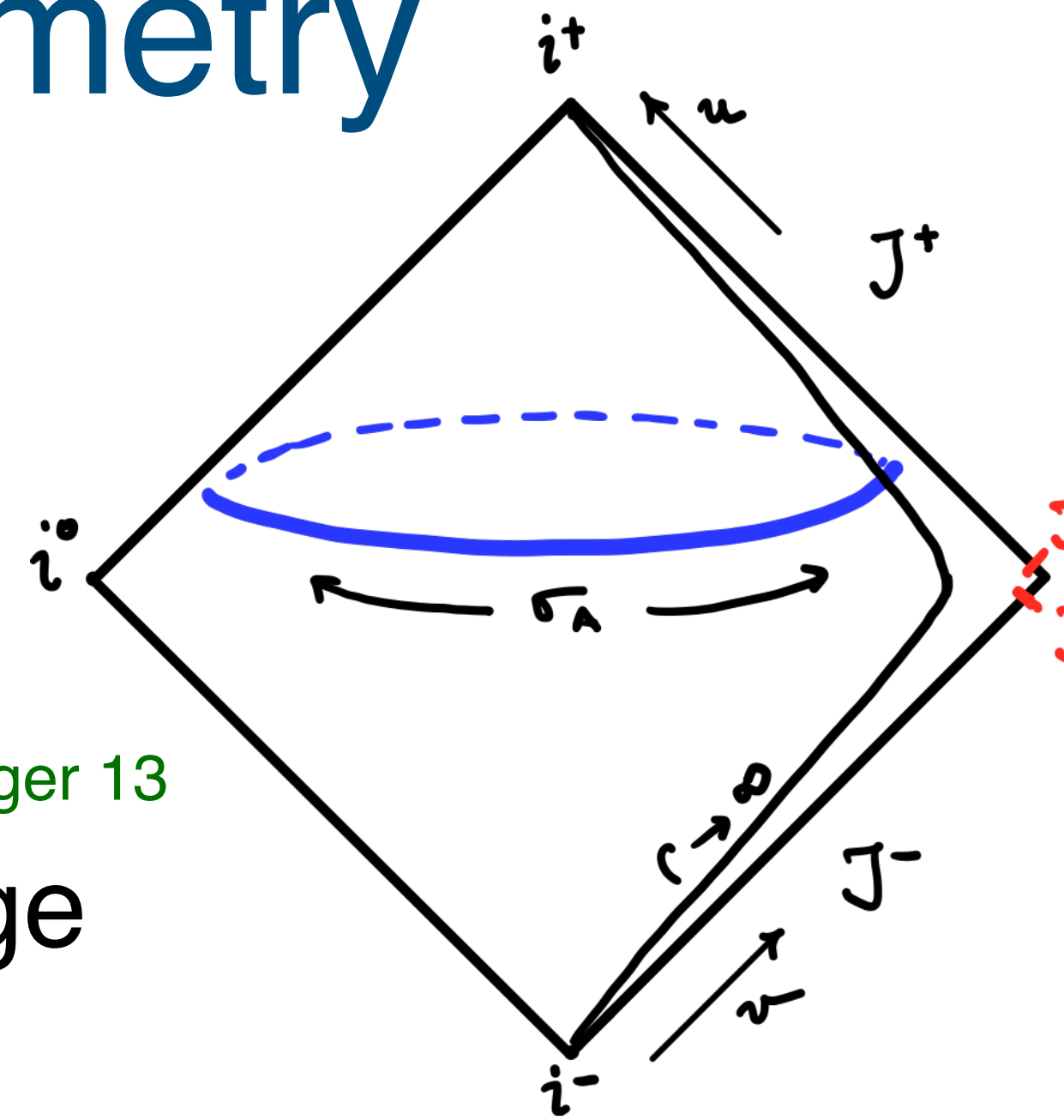
$$\partial_u \tilde{Q}_s = D \tilde{Q}_{s-1} + (s+1) C \tilde{Q}_{s-2}$$

supplemented by the conservation law $\tilde{Q}_s(\mathcal{I}_-^+) = \tilde{Q}_s(\mathcal{I}_+^-)$ Strominger 13
represent the charge conservation equation of a symmetry charge

So far the proof of the subleading theorem is valid only at order $\bar{g}_N g_N$

Can we extend this to all semi non-linear orders $\bar{g}_N g_N^n$?

Pranzetti, Raclariu, LF 23



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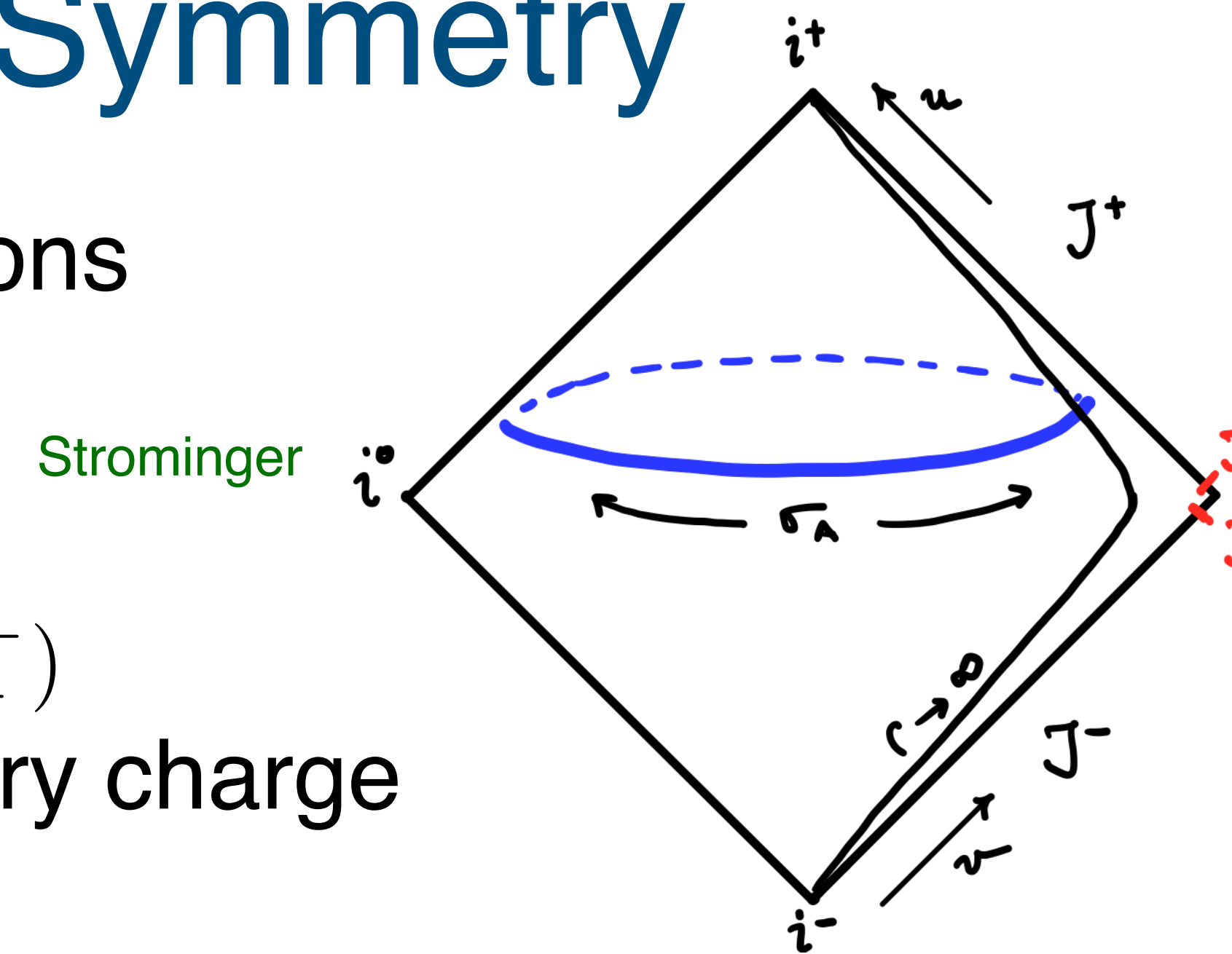
Pranzetti, Raclariu, LF 23

To do this we need to

identify the symmetry algebra ?

understand its (non-linear) action on the gravitational Phase space

Construct the corresponding Noether charge and prove the charge conservation



Noether Charges

Symmetries are linked to conservation laws

$$\partial_u \tilde{Q}_s = D\tilde{Q}_{s-1} + (s+1)C\tilde{Q}_{s-2}$$

We have to show that these leads to charge conservation under non-radiative boundary conditions.

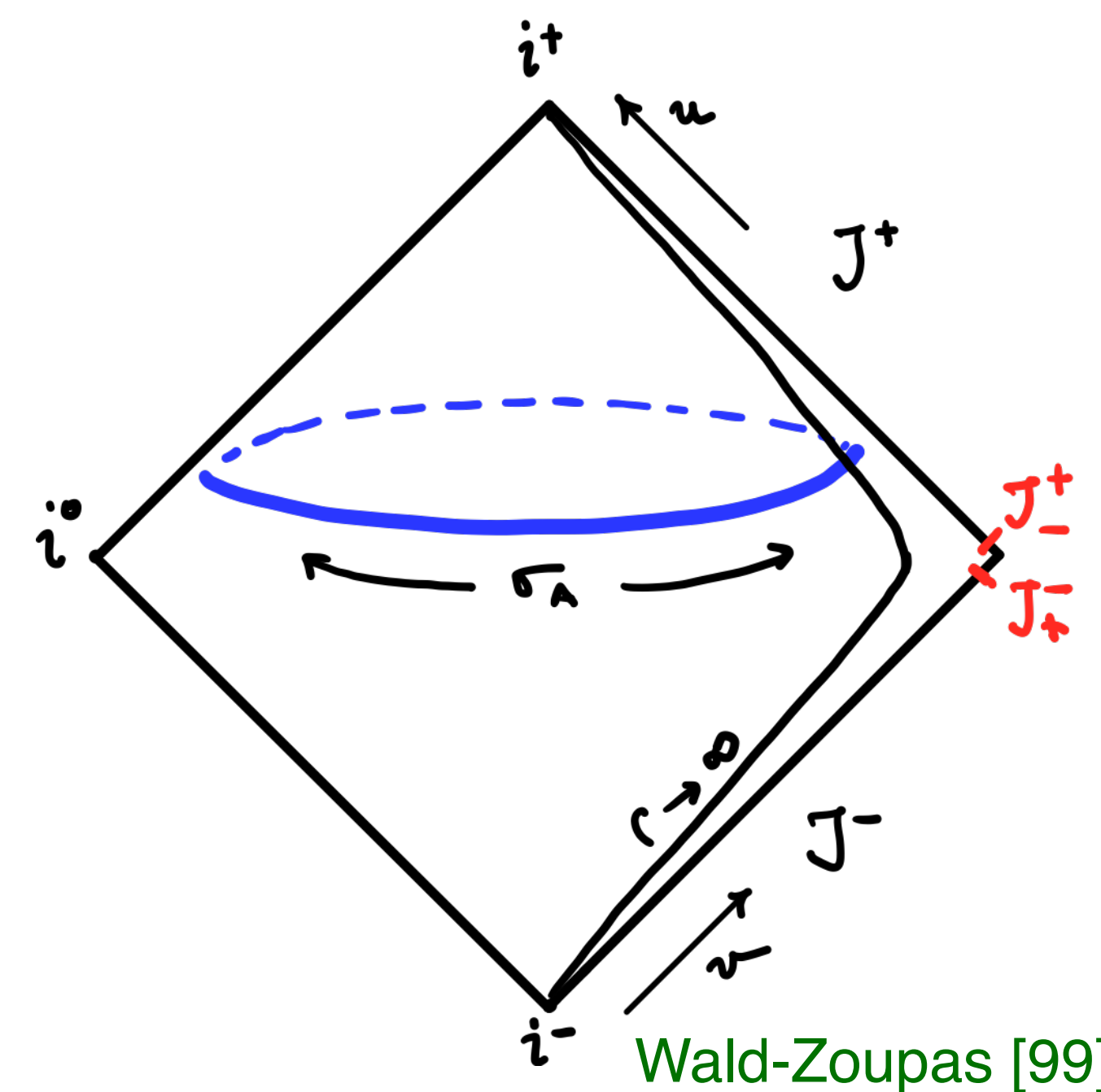
Admissible bdy conditions are Boundary eom defined as $\Theta = 0$

What symplectic potential? $\Theta^{\text{AS}} = \frac{1}{8\pi G} \int_{\mathcal{I}} (\bar{N} \delta C + N \delta \bar{C}) \rightarrow N = \bar{N} = 0$

usual notion of no-radiation but too strong

Change of boundary conditions are achieved through $\Theta \rightarrow \Theta + \delta \ell - d\vartheta$

Key idea: allow complex canonical transformations.



Wald-Zoupas [99]

Harlow-Wu[19]

LF,Pranzetti, Oliveri, Speziale [21]

Odak-Rignon-Bret-Speziale [22]

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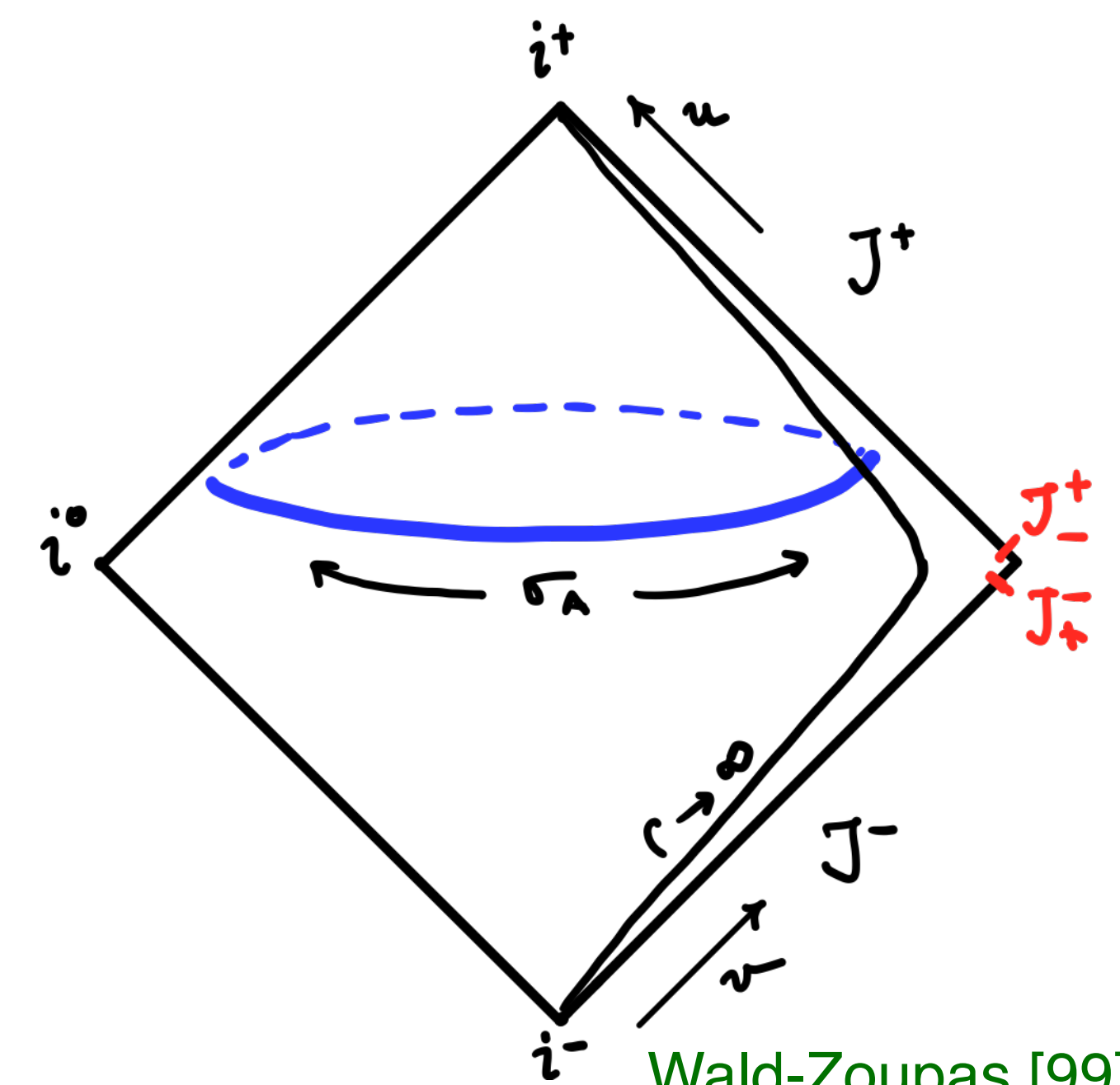
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Admissible bdy conditions are Boundary eom defined as $\Theta = 0$

We chose instead $\Theta^{\text{HAS}} = \frac{1}{4\pi G} \int_{\mathcal{I}} \bar{N} \delta C \rightarrow \bar{N} = 0, N \text{ arbitrary}$

We are therefore looking for a symmetry charge conserved under this holomorphic no-radiation condition



Wald-Zoupas [99]

Harlow-Wu[19]

LF,Pranzetti, Oliveri, Speziale [21]

Speziale [24]

Symmetry Charges

The symmetry parameters are dual to the charges

$$\tau = (\tau_0, \tau_1, \tau_2, \dots)$$

super-translation super-rotation

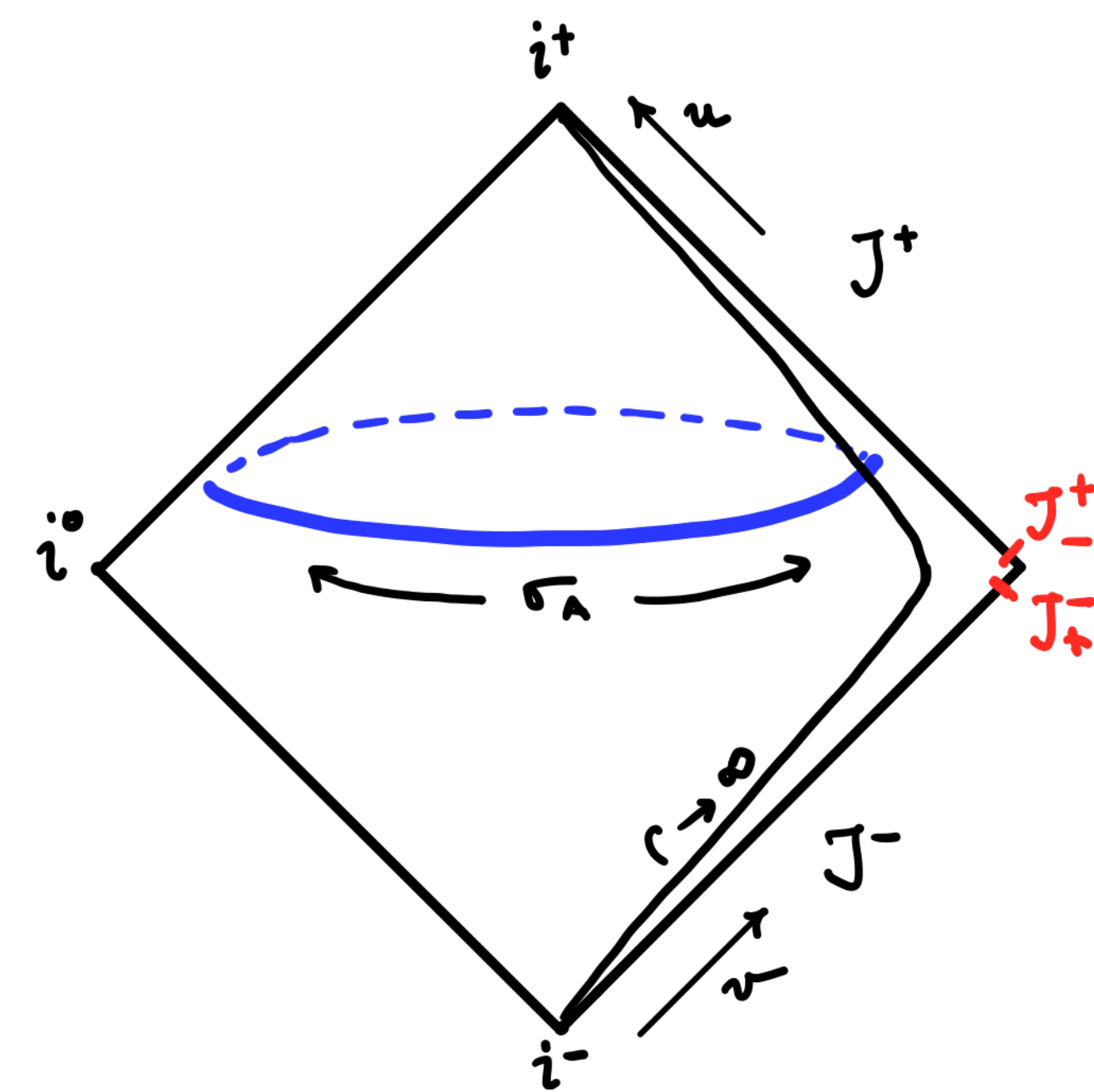
Since $\tilde{Q}_s \in C_{(3,s)}^{\text{Car}} \rightarrow \tau_s \in C_{(-1,-s)}^{\text{Car}}$

This allows us to define a **master charge**

$$Q_\tau^u = \sum_{s=0}^{\infty} \int_{S_u} \tilde{Q}_s \tau_s - \frac{1}{4\pi G} \int_{S_u} \tau_0 \bar{N} C$$

This charge is **conserved** under the Holomorphic radiation condition $\bar{N} = 0$ provided τ_s satisfy the dual eom $E_s = 0$

$$E_s = \partial_u \tau_s - D \tau_{s+1} + (s+3) C \tau_{s+2}$$



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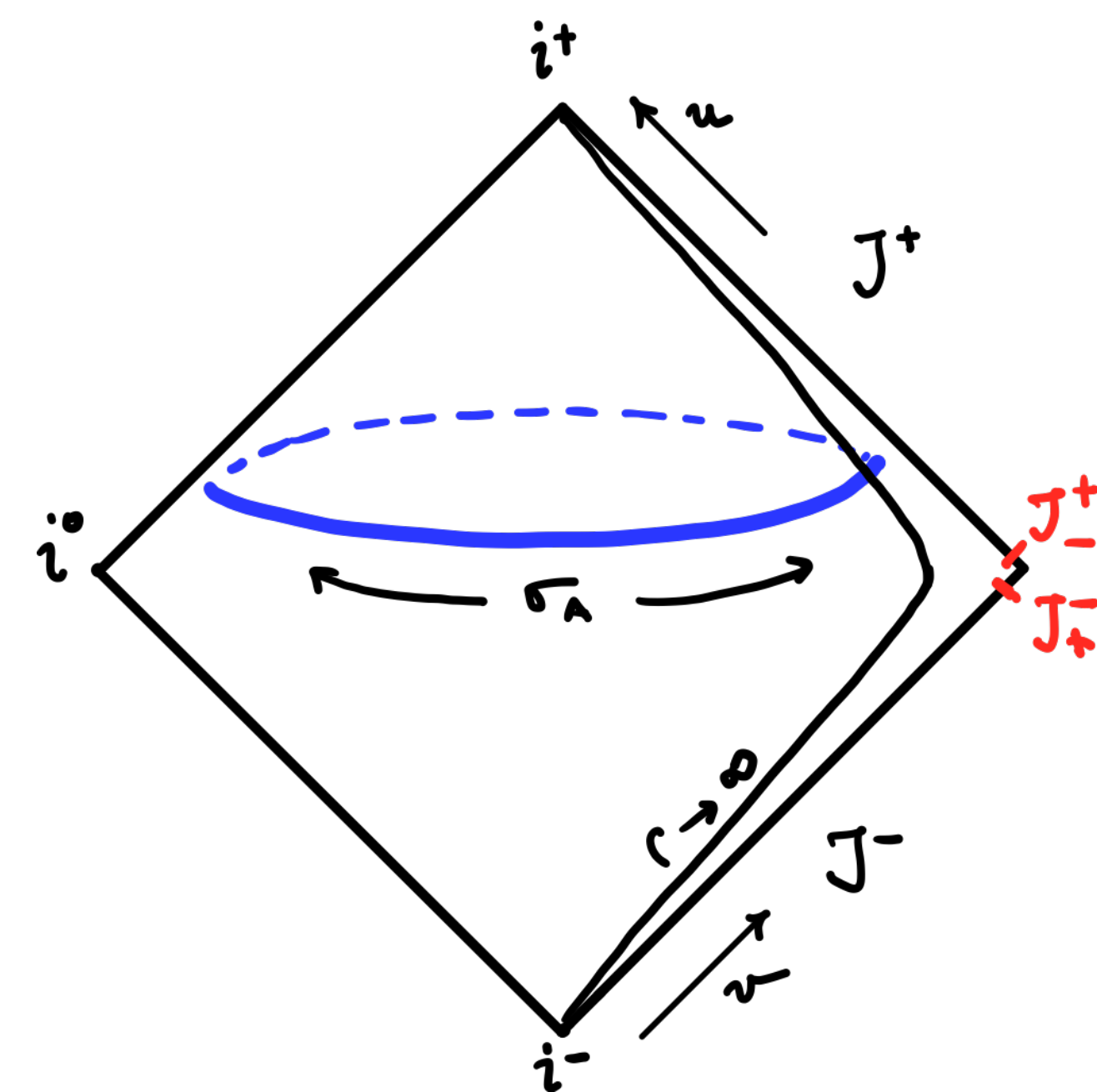
$$\tau = (\tau_0, \tau_1, \tau_2, \dots)$$

satisfy the dual eom $\partial_u \tau_s = D\tau_{s+1} - (s+3)C\tau_{s+2}$

These equation determines τ_s in terms of the celestial parameters T_s
 which appears as initial conditions $T = \tau|_{u=0} \rightarrow \tau(T)$

For instance for BMS

$$\tau_0 = T + \frac{u}{2}DY \quad \tau_1 = \frac{1}{2}Y \quad \tau_2 = 0$$



Symmetry action

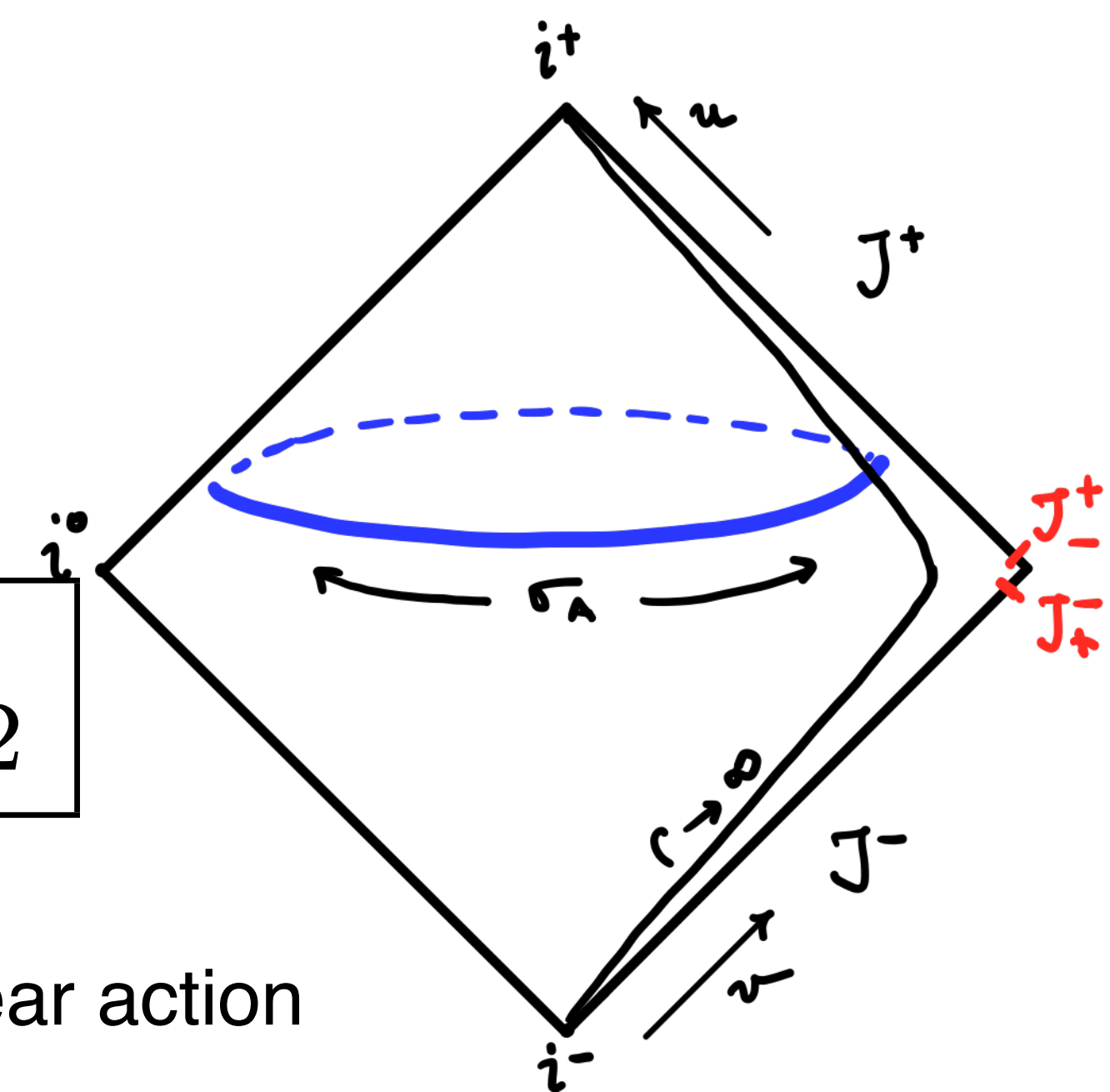
The W symmetry parameters τ acts on the sphere as

$$\delta_{\tau} C = \tau_0 N - D^2 \tau_0 + 2\tau_1 DC + 3CD\tau_1 - 3C^2\tau_2$$

$\nearrow$
super-translation

$\uparrow$
super-rotation on $C_{(1,2)}^{\text{Car}}$

$\uparrow$
Spin2 non-linear action



infinite dimensional and non-linear generalization of the BMS action.

When $s \geq 2$ τ_s depends on the shear: non-linearity of the action

Symmetry action

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Three remarkable and non-trivial facts: The action closes

$$[\delta_{\tau}, \delta_{\tau'}]C = -\delta_{[[\tau, \tau']] }C$$

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The symmetry preserves the asymptotic eom $\tilde{E}_s = \partial_u \tilde{Q}_s - D\tilde{Q}_{s-1} - (s+1)C\tilde{Q}_{s-2}$

$$\delta_\tau \tilde{E}_s = \sum_{n \geq s} L_s^n(\tau) \tilde{E}_n$$

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The symmetry is canonical $\delta_\tau C = \{Q_\tau, C\}$ $Q_\tau = I_{\delta_\tau} \Theta^{\text{HAS}}$

Symmetry Charges

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We have a compact expression for the charges assuming that $Q_\tau^{i+} = 0$

$$\hat{Q}_\tau = \int_{\mathcal{I}} \bar{N} \delta_\tau C$$

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$$\hat{Q}_{\tau} = \int_{\mathcal{J}} \left(\tau_0 (-D^2 \bar{N} + \bar{N} N) + \tau_1 (\bar{N} DC - 3D(\bar{N} C)) + \tau_2 \bar{N} C^2 \right)$$

Solving the dual eom provides the explicit charge expressions

Symmetry Bracket

The symmetry bracket is

$$[[\tau, \tau']]_s = [\tau, \tau']_s + \delta'_\tau \tau_s - \delta_\tau \tau'_s$$

where

$$[\tau, \tau']_s = \sum_n (n+1) (\tau_n D \tau'_{s+1-n} - \tau'_n D \tau_{s+1-n}) - (s+3) C (\tau_0 \tau'_{s+2} - \tau'_0 \tau_{s+2})$$

flat space contribution ↑

↗ deformation to accomodate any SD background

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It satisfies Jacobi identity

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Three non trivial properties: it preserves the dual eom $E_s = \partial_u \tau_s - D\tau_{s+1} + (s+3)C\tau_{s+2}$

$$E_s([[\tau, \tau']]) = 0 \quad \text{when} \quad E_s(\tau) = E_s(\tau') = 0$$

It satisfies Jacobi identity

The Charges provides a canonical representation of the algebra

$$\{Q_\tau, Q_{\tau'}\} = Q_{[[\tau, \tau']]}$$

Symmetry Bracket

The symmetry bracket is

$$[[\tau, \tau']]_s = [\tau, \tau']_s + \delta'_\tau \tau_s - \delta_\tau \tau'_s$$

where

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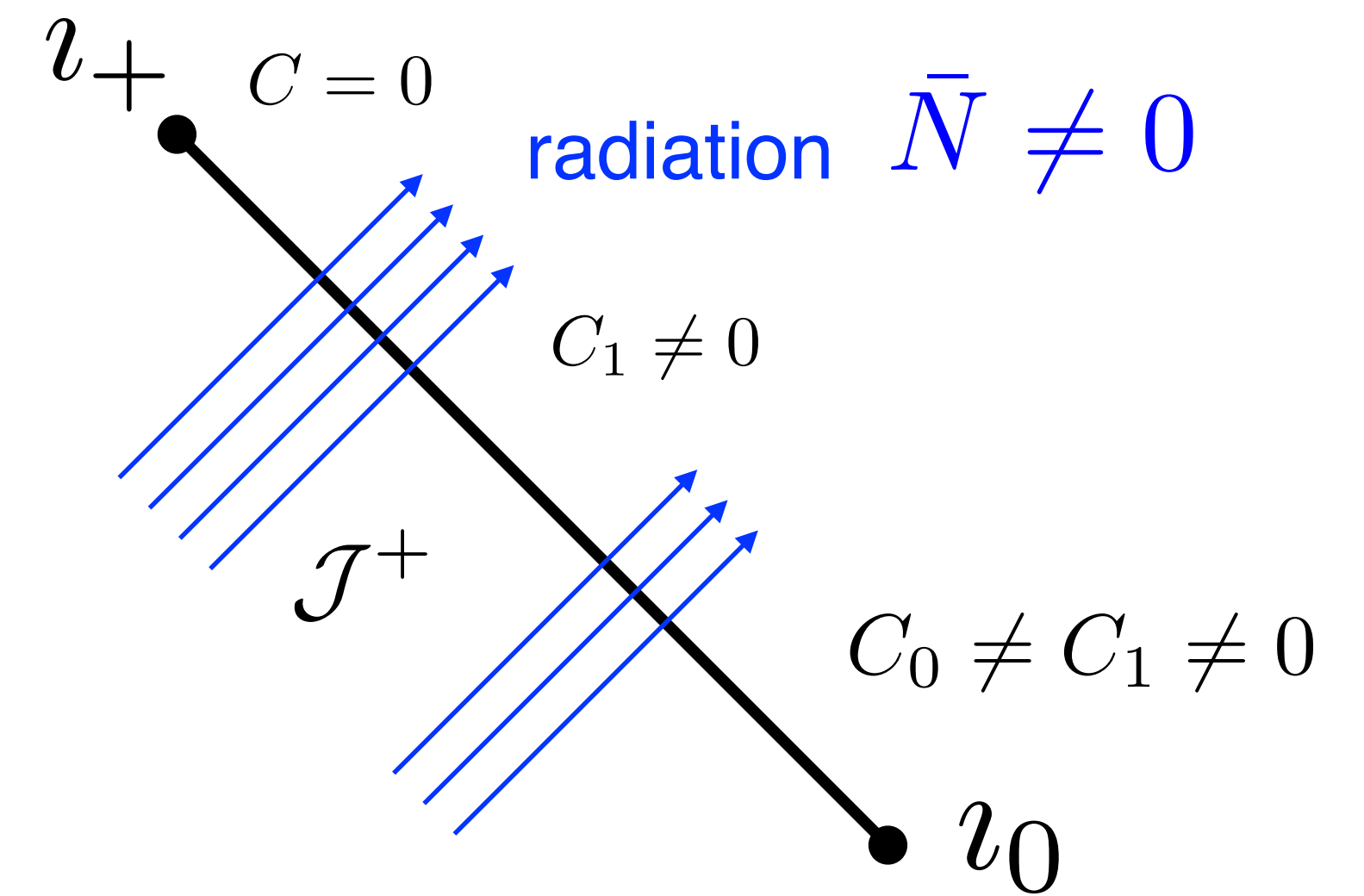
Puzzle: Symmetries are about algebra not algebroids !

From algebroid to algebra

Algebroid are the algebraic expression of **dynamical symmetries when radiation** is present.

But **true symmetries** expressed through **algebras** only exist at **non-radiative** cuts

Due to the memory effect different non-radiative cuts carry different values of the shear



One needs to identify a symmetry algebra $W_C(S)$ the **wedge algebra** that depends on the value of C at non-radiative cuts

Wedge algebra

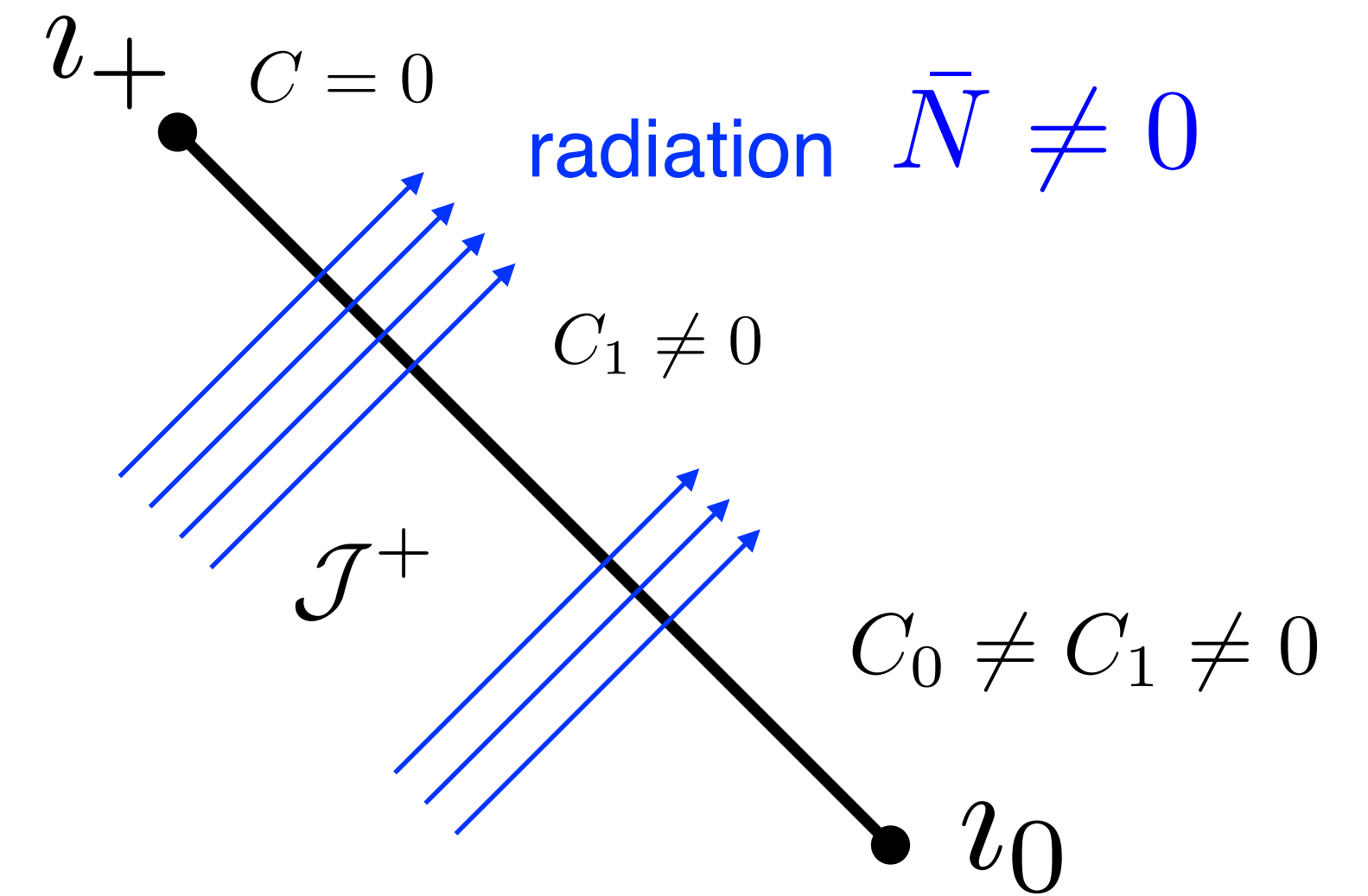
The wedge algebra $W_C(S)$

is defined as the symmetry algebra that preserves C

$$\delta_\tau C = 0$$

for $\tau \in W_C(S)$ C is a field independent parameter

$$W_0(S) = \{T \in C_{(-1, -s)}(S) | D^{s+2} T_s = 0\}$$



Wedge algebra

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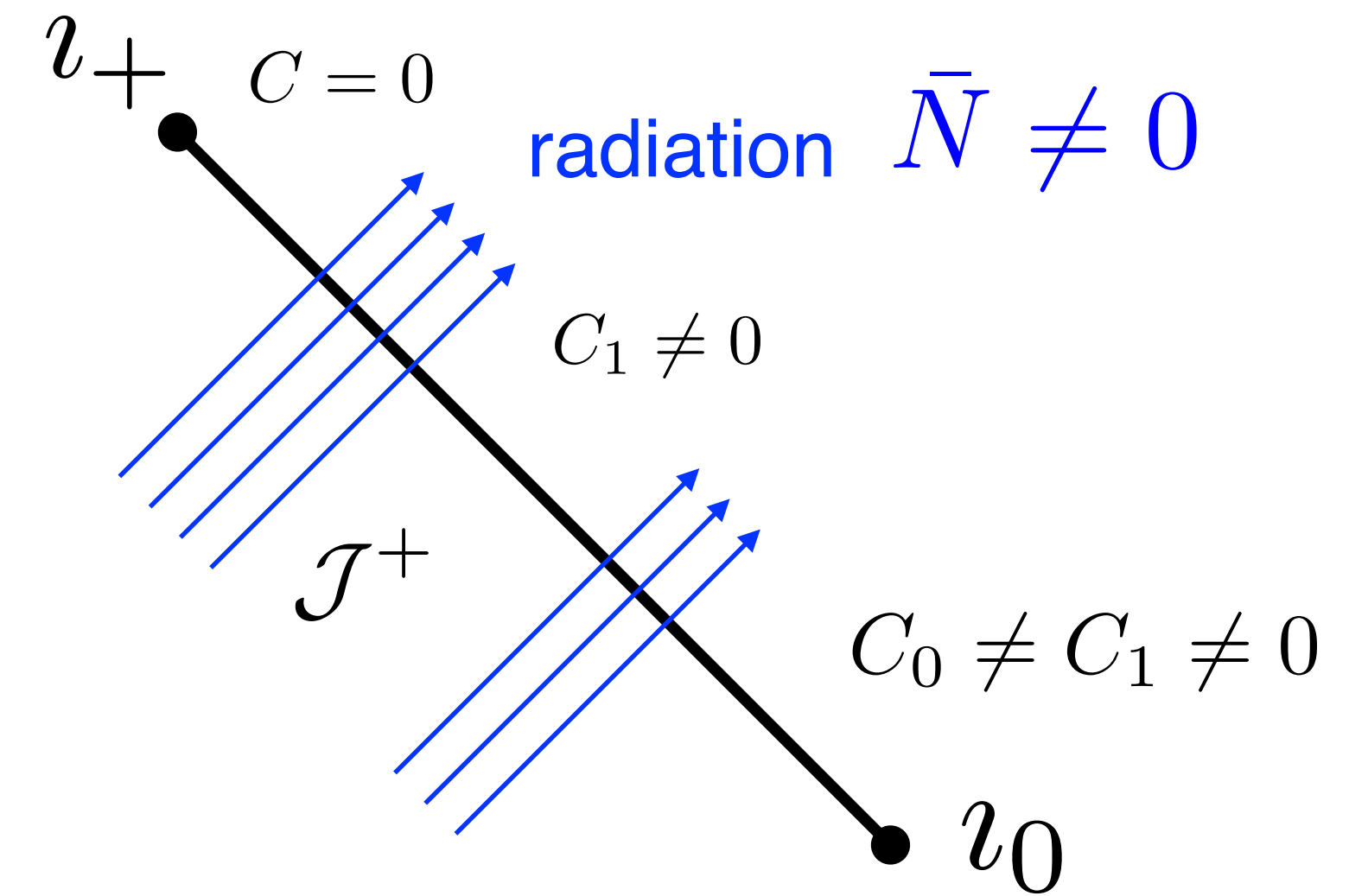
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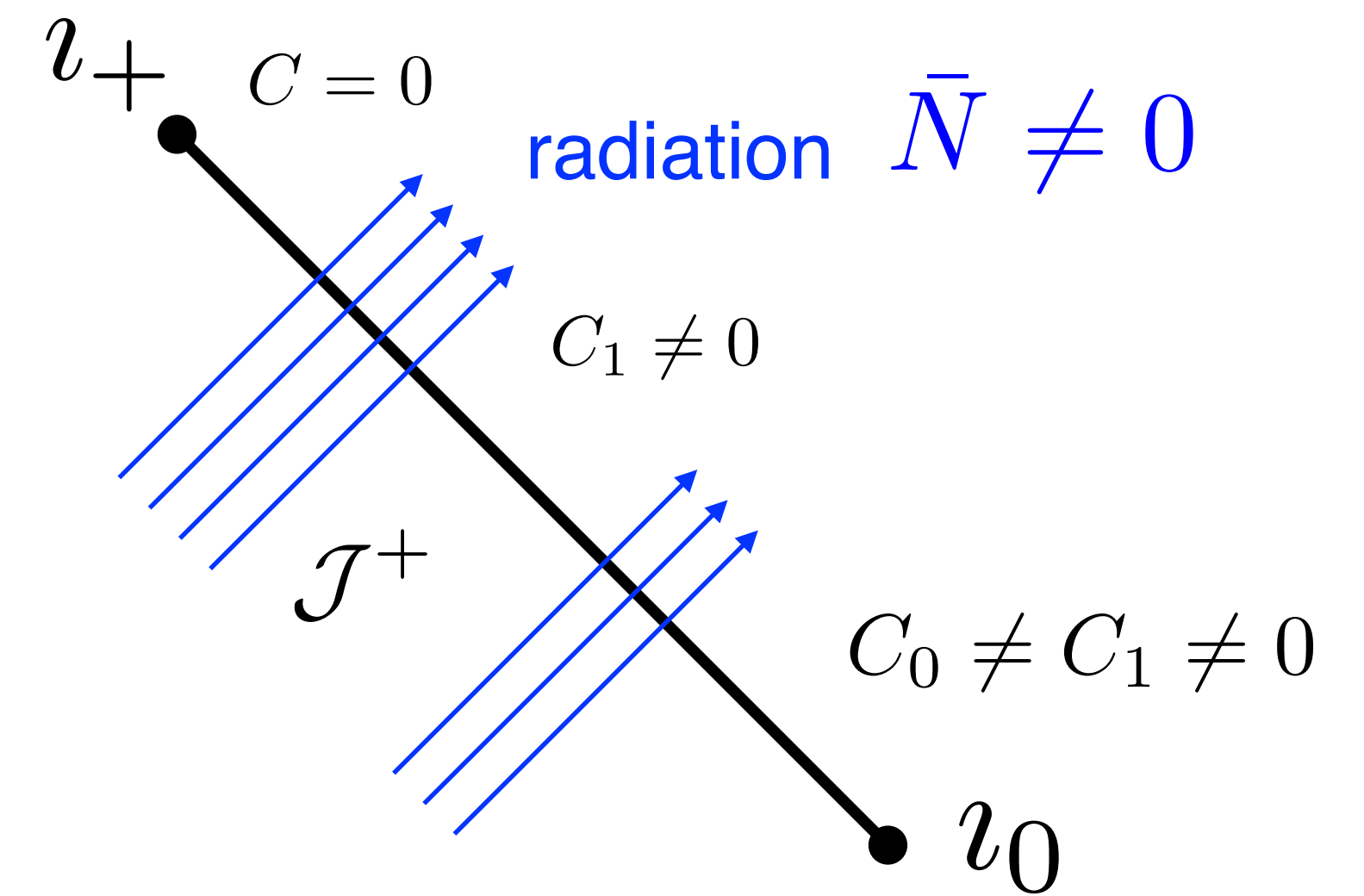
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$$W_0(S_0) = SL^+(2) \ltimes \mathbb{C}^4 = \text{Poinc}^+$$



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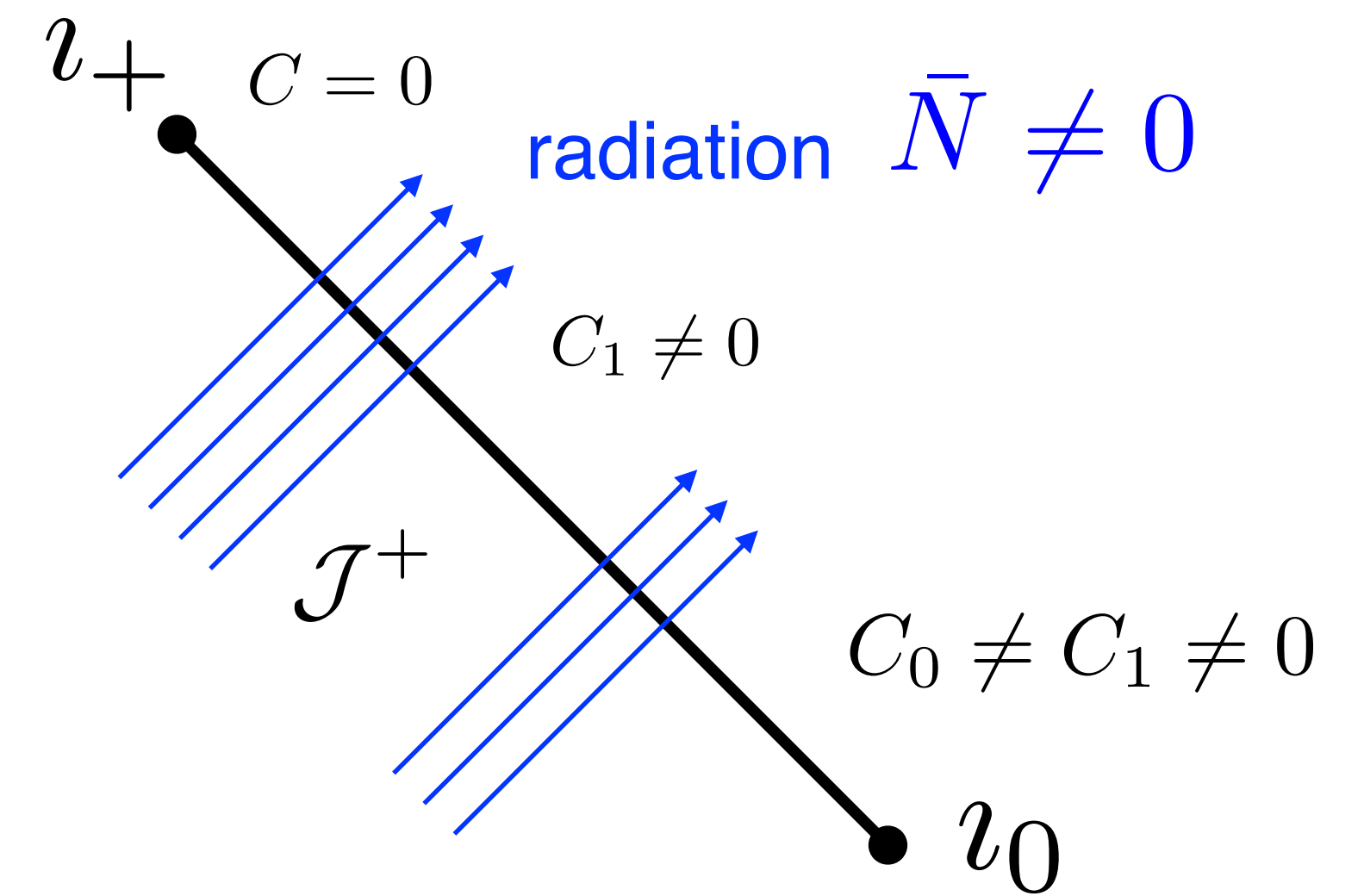
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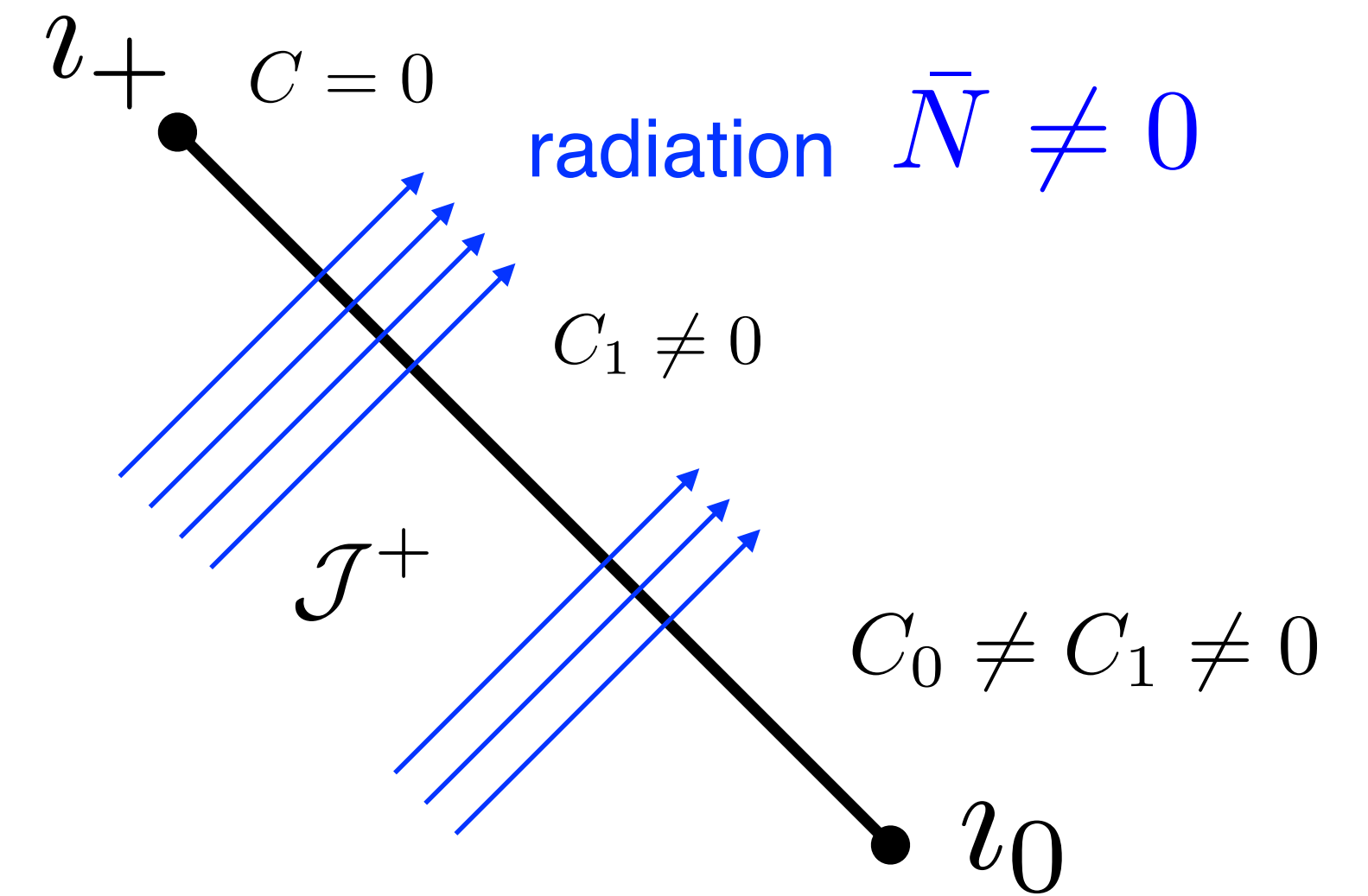
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$$W_0(S_n) \quad \text{Krichever-Novikov like generalisation}$$



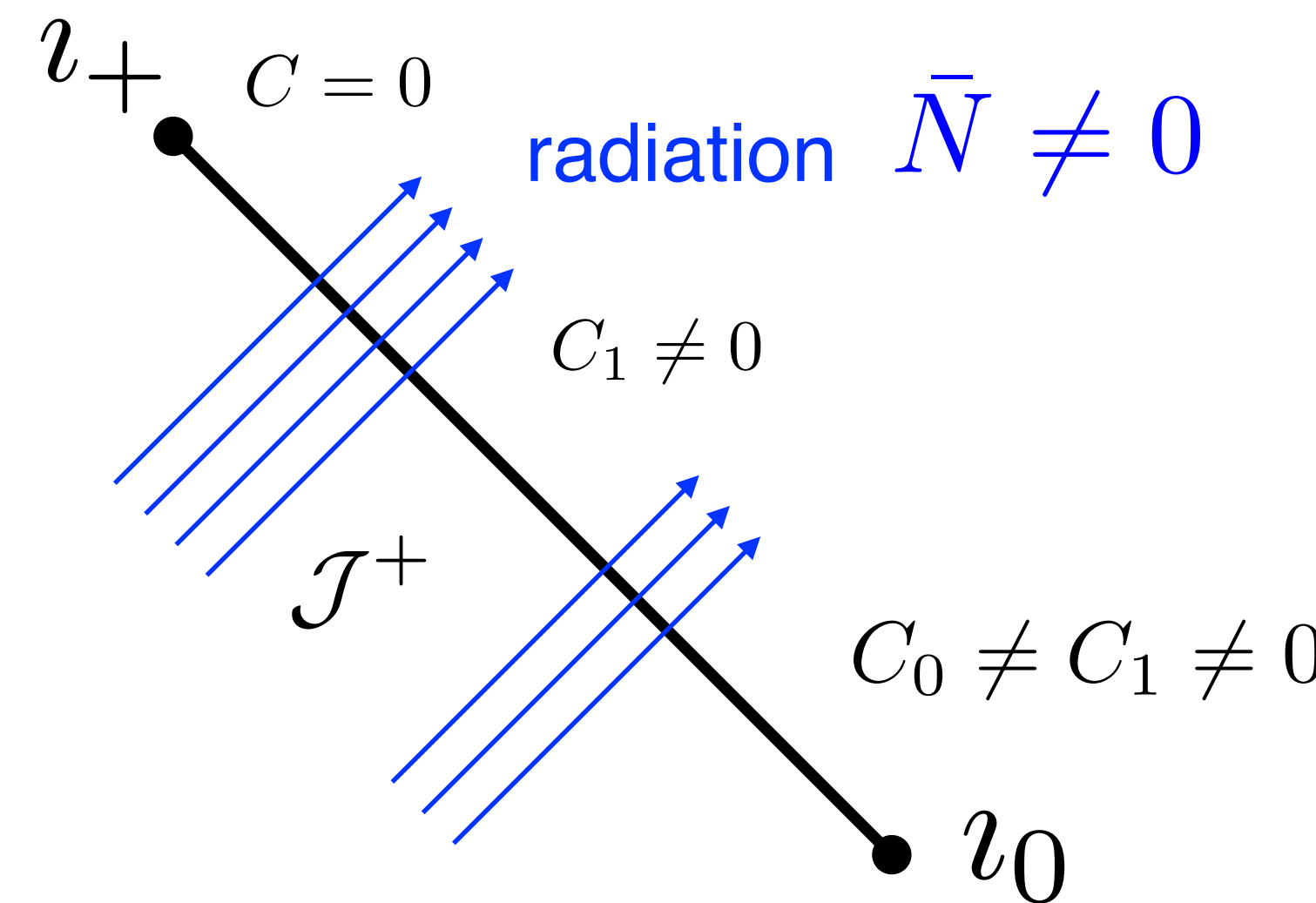
Wedge algebra

the wedge algebra $W_C(S)$ for C is **isomorphic** to $W_0(S)$

$$D^2 T_0 = 2D(CT_1) + CDT_1 - 3C^2 T_2$$

$$D^3 T_1 = 3D^2(CT_2) + 2D(CDT_2) + CD^2 T_2 + \dots$$

$\vdots$



The isomorphism $T^G : W_C(S) \rightarrow W_0(S)$ involves the Goldstone G : $D^2 G = C$

$$T^G(\tau) = P \overrightarrow{\exp} \int_0^1 (\text{ad}_{tG} G)(\tau)$$

From Carrollian to Twistor

Whats the connection with twistor theory?

Twistor space provide a fibration of $\mathcal{I}$ complexified

$$\mathbb{PT} \rightarrow \mathcal{I}_{\mathbb{C}}$$

$$(\mu^{\alpha}, \bar{\lambda}_{\dot{\alpha}}) \rightarrow (u = [\mu\lambda], z, \bar{z})$$

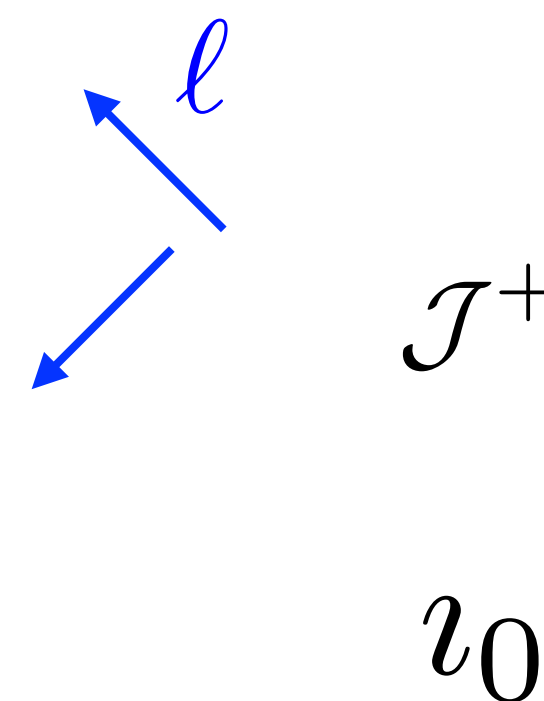
Remember that $u \in C_{(-1,0)}^{\text{Car}}$

$$\lambda \propto (1, z)$$

What does the fiber coordinate represents ?

$$q = [\mu\check{\lambda}] \in C_{(0,1)}^{\text{Car}}$$

$$[\lambda\check{\lambda}] = 1.$$



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Adamo-Newman 10

The fiber coordinate $q = [\mu\check{\lambda}] \in C_{(0,1)}^{\text{Car}} \quad [\lambda\check{\lambda}] = 1.$

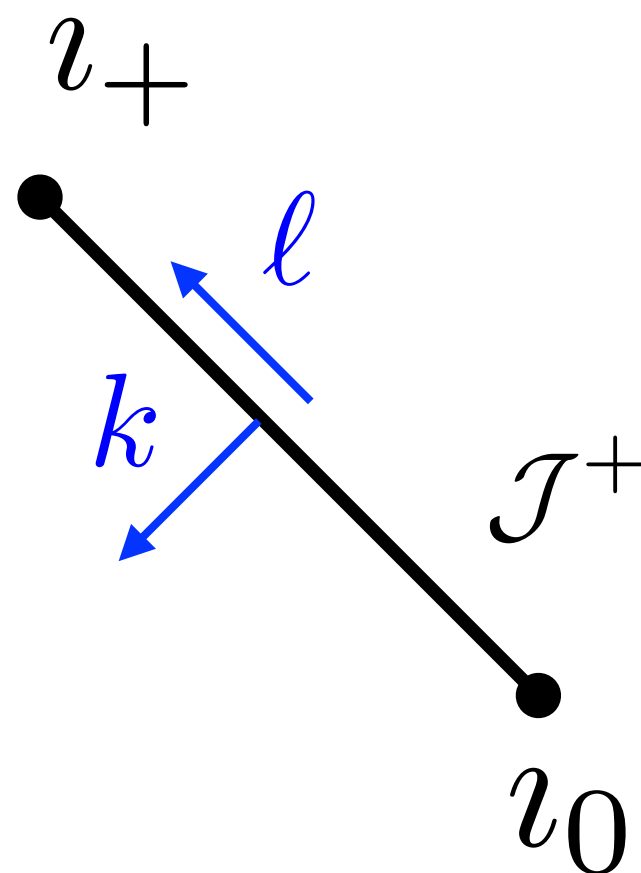
represents the angle at which a congruence of null geodesic intersect $\mathcal{I}$

Carrollian complex structure on $\mathcal{I}$: $\ell^a q_{ab} = 0 \quad q_{ab} = m_{(a} \bar{m}_{b)}$

A null geodesic congruence is characterised by a null vector k_q

Such that $k_q \cdot \ell = 1 \quad k_q \cdot m = q$

$m_q = m - q\ell$ holomorphic frame transverse to k and ℓ



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The total space isomorphic to twistor space is the space of null rays reach scri at a cut u and at an angle q

$$\lambda \propto (1, z)$$

Adamo-Newman 10

It is the total space of the bundle $\mathcal{N} = O(1, -1) \oplus \mathcal{I} \rightarrow \mathcal{I}$

Like scri is the total space of $\mathcal{I} = O(1, 1) \oplus \mathbb{CP}_1 \rightarrow \mathbb{CP}_1$

From Carrollian to twistor

The symmetry parameters can be converted into a function of $q \in C_{(0,1)}^{\text{Car}}$

$$\tau = (\tau_0, \tau_1, \tau_2, \dots) \rightarrow \hat{\tau}(q) = \sum_{s=0}^{\infty} \tau_s q^{s+1}$$

The dual eom can be simply written as

$$q\partial_u\tau - D\tau + C\tau = 0$$

It is equivalent to the twistor equation $\partial h + \{h, \tau\} = 0$

For the twistor potential $h = (\partial_u^{-1}C)D\lambda \in \Omega^{(1,0)}(\mathbb{PT}, O(2))$.

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The W bracket on τ can be recasted as a Poisson bracket on-shell of the dual eom

$$[\tau, \tau']_s = \sum_{n=0}^{s+1} (n+1) (\tau_n D \tau'_{s+1-n} - \tau'_n D \tau_{s+1-n}) - (s+3) C (\tau_0 \tau'_{s+2} - \tau'_0 \tau_{s+2})$$

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$$\sum_s [\tau, \tau']_s q^{s+1} = \{\hat{\tau}, \hat{\tau}'\} + \text{dual EOM}$$

where

$$\boxed{\{\hat{\tau}, \hat{\tau}'\} = \partial_q \tau \partial_u \tau' - \partial_q \tau' \partial_u \tau}$$

is the **twistor Poisson bracket**.

Non-linearity

We have seen that $\tilde{Q}_s = \tilde{Q}_s^S + \tilde{Q}_s^H + \tilde{Q}_s^{SH}$ for $s \geq 2$

The super-Hard contributions requires an extension of the amplitude that includes collinear external states and extension of the subsubleading theorems

Can we obtain these SH contribution from amplitudes?

Narayanan, Jorstad, To appear

We have seen that $E_3 = 0$ is valid in Einstein gravity

→ One expect sub^3 leading theorem to be valid. Amplitude proof?

Quantum

What is the quantization of the W-algebra ?

Preliminary investigation of the quantum commutator algebra shows that there exist potential Lie algebra anomalies

$$[\hat{Q}_\tau, \hat{Q}_{\tau'}] = Q_{[\tau, \tau']} + A_{(\tau, \tau')}$$

where $A_{(\tau, \tau')}$ depends only on C not on $\bar{N}$

First investigation suggests that $A=0$ for the wedge.

Lie algebra anomaly versus VOA anomalies?

Anomalies in SD gravity due to all + one loop amplitudes.

Costello-Paquette 23

Blitteston 24

Conclusion

We have shown that there exists a semi-perturbative sector of GR $\bar{g}_N g_N^n$ which is integrable and carry the representation of an infinite dimensional symmetry algebra $Wc(S)$ with bracket

$$[\tau, \tau']_s = \sum_{n=0}^{s+1} (n+1) (\tau_n D \tau'_{s+1-n} - \tau'_n D \tau_{s+1-n}) - (s+3) C (\tau_0 \tau'_{s+2} - \tau'_0 \tau_{s+2})$$

This is non-linear deformation of $LW_{1+\infty}$

And corresponds to the quantization of SD gravity

Costello-Paquette 23
Blitteston 24

$s=2$ and $s=3$ are exact symmetry of Einstein gravity. If no quantum anomaly are present

Can we use this semi-perturbative sector as a basis for a new form of perturbative quantization in the same way we use free theory as an asymptotic basis?

Can we use this basis to dress asymptotic states and adress the IR problem.

Self-dual Polarization

If instead of

$$\Theta^{\text{HAS}} = \frac{1}{4\pi G} \int_{\mathcal{J}} \bar{N} \delta C \quad \rightarrow \quad \bar{N} = 0, N$$

Self-dual Polarization

we introduce the potential $h \in C_{(0,2)}^{\text{Car}}$ such that $\partial_u h = C$

And work with $\Theta^{\text{SD}} = \frac{1}{4\pi G} \int_{\mathcal{I}} \dot{N} \delta h$

The no-radiation condition is $\dot{N} = 0$, N arbitrary

The symmetry parameters are $\tau = (\tau_{-1}, \tau_0, \tau_1, \tau_2, \dots)$

subject to

$$E_s = 0, s \geq -1$$

The master charge is $Q_{\tau}^u = \sum_{s=-1}^{\infty} \int_{S_u} \tilde{Q}_s \tau_s$

The mass is the covariant mass not the boundary mass

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The global symmetry algebra: $W_0(S_0) =$ central extension of Poinc^+

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