On Global Embeddings of Assisted Fibre Inflation

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Outline

- **Moduli Stabilization** in Type IIB (**LVS**)
  (*Prerequisite for the reliability and validity of the effective theory*)
- **Fibre Inflation**
- **Global Embedding** in CY compactifications
- **Assisted Fibre Inflation**
- Confronting recent CMB data (**ACT**, **DESI**)

Motivations for Inflation in String Theory

- String theory offers a compelling framework for early-universe inflation.
- "Unifies" Inflation with Quantum Gravity, in the sense that:
- It provides a UV-complete setting where **scalar (moduli) fields** naturally arise from compactification
- Fibre inflation is one of the most compelling string-derived inflation models.
- The inflaton is a Kähler modulus (a "**fibre**" modulus) whose potential is generated by perturbative corrections.

Type IIB Framework

Low energy dynamics of 4D effective SUGRA from type IIB compactified on CY orientifolds captured by:
The superpotential

$$\mathcal{W}_0 = \int \mathbf{G}_3 \wedge \Omega(U_a)$$

and the Kähler potential

$$\mathcal{K}_0 = -\log[-i(S-\bar{S})] - 2\log\mathcal{V} - \log\left[-i\int \Omega\wedge\bar{\Omega}\right]$$

Then, the F-term **scalar potential** is defined:

$$V = e^{\mathcal{K}}\left(\mathcal{K}^{\mathcal{A}\bar{\mathcal{B}}}(D_{\mathcal{A}}\mathcal{W})(D_{\bar{\mathcal{B}}}\overline{\mathcal{W}}) - 3|\mathcal{W}|^2\right)$$

The Superpotential $\mathcal{W}$

The (tree-level) flux-induced superpotential:

$$\mathcal{W}_0 = \int \mathbf{G}_3 \wedge \Omega(U_a)$$

Depends on:

- $\mathbf{G}_3 = F_3 - SH_3$ (field strengths)
- Axio-dilaton modulus $S = C_0 + ie^{-\phi}$
- Complex structure moduli U_a through $\Omega(U_a)$

Stabilizing Complex Structure Moduli and Axiodilaton

Supersymmetric conditions $D_i W = 0$:

1. Axio-dilaton stabilization: $D_S W = 0 \implies \int G_3 \wedge \overline{\Omega} = 0$
2. Complex structure moduli stabilization: $D_{U_\alpha} W = 0 \implies \int G_3 \wedge D_\alpha = 0$

$\Rightarrow$ U_a and S stabilized, (*barring tadpole issues: 2010.10519*),
but **Kähler moduli remain unfixed**, and, despite

their appearance in tree-level **Kähler** potential, the scalar potential is still zero

$$V = e^{\mathcal{K}_0}\left(\sum_{I,J}\mathcal{D}_I\mathcal{W}_0\mathcal{K}_0^{I\bar{J}}\mathcal{D}_{\bar{J}}\mathcal{W}_0 - 3|\mathcal{W}_0|^2\right) \equiv 0$$

due to conditions $D_i W = 0$ and no-scale structure.

Kähler Moduli Stabilization

Kähler moduli can be stabilized via:

- Perturbative corrections to Kähler potential:

$$\mathcal{K}_0 \;\Rightarrow\; \mathcal{K} = -2\log(\mathcal{U}) + \cdots$$

where $\mathcal{U}$ includes $\underline{\alpha'^3}$ and perturbative $\underline{\log(\mathcal{V})}$ corrections :

$$\mathcal{U} = \hat{\mathcal{V}} + \hat{\xi} + \gamma\ln(\hat{\mathcal{V}}),\ \text{where} : \xi, \gamma \propto \chi$$

(see *hep-th/0204254* and *hep-th/1909.10525* respectively)

- Non-perturbative contributions to superpotential:

$$\mathcal{W} = \mathcal{W}_0 + \sum_k \mathcal{A}_k e^{-a_k T_k}$$

Global Embedding & Fibre Inflation

1. **Perturbative LVS** provides a novel way to realise LVS **Fibre Inflation**, *without* implementing *non-perturbative* effects.
2. Then, the simplest realization of Fibre Inflation is based on a:
   K3-fibred CY orientifold with **toroidal-like** volume (*hep-th/2406.01694*):

   $$\mathcal{V} \propto \sqrt{\tau_1 \tau_2 \tau_3} \qquad (1)$$

   τ_α: four-cycle volumes corresponding the CY_3-divisors
3. **Lack of rigid divisors**: in the specific CY_3 this implies that there are no contributions from instantons or gaugino condensation.
4. This prevents the use of standard moduli stabilization schemes like KKLT or LVS

Global Model

CY_3 hypersurface inside a 4-dimensional toric variety described by data:

Hyp	x_1	x_2	x_3	x_4	x_5	x_6	x_7
4	0	0	1	1	0	0	2
4	0	1	0	0	1	0	2
4	1	0	0	0	0	1	2
	K3	K3	K3	K3	K3	K3	SD

- Columns x_1 to x_7: represent the seven **toric divisors**.
- Row labeled "Hyp": This stands for the **hypersurface**.
- Row $[4, 0, 0, 1, 1, 0, 0, 2]$ defines **degree** of the hypersurface equation.
- CY_3 is defined as the zero-locus of a single polynomial equation

$$P(x_1, \ldots, x_7) = 0$$

The exponents of its monomials satisfy (three) conditions of the form
$0 \cdot \deg(x_1) + 0 \cdot \deg(x_2) + 1 \cdot \deg(x_3) + 1 \cdot \deg(x_4) + 0 \cdot \deg(x_5) + 0 \cdot \deg(x_6) + 2 \cdot \deg(x_7) = 4$

Global Model: Hodge Numbers

The analysis of the divisor topologies (see e.g. Blumenhagen 1104.1187) shows that they can be represented by the following Hodge diamonds:

$$K3 \equiv \begin{array}{ccccc} & & 1 & & \\ & 0 & & 0 & \\ 1 & & 20 & & 1 \\ & 0 & & 0 & \\ & & 1 & & \end{array}, \qquad \mathrm{SD} \equiv \begin{array}{ccccc} & & 1 & & \\ & 0 & & 0 & \\ 27 & & 184 & & 27 \\ & 0 & & 0 & \\ & & 1 & & \end{array}.$$

Global Model: Hodge Numbers

- $h^{1,1} = 3$: number of **Kähler** (odd) moduli, i.e., there are 3 independent ways to deform the Kähler form.

  $$h^{1,1} = \#\text{Torically Invariant Divisors} - \#\text{Linear Relations} - 1 = 7 - 3 - 1 = 3$$

- $h^{2,1} = 115$: number of **complex structure** moduli, control the "shape" of the defining polynomial equation, deforming the complex structure of the manifold.

  $$h^{2,1} = -1 + \sum_{\text{primary } D} \left(h^{2,0}(D) + h^{1,0}(D)\right) + \text{deformation families of curves}$$

- **Euler Number**: $\chi = 2(h^{1,1} - h^{2,1}) = -224$

Divisor Topologies and Volume Form

- First 6 toric divisors: K3 surfaces, 7th divisor: Special Deformation (SD)
- Calculation of intersection numbers between divisors, requires the Stanley-Reisner ideal given by:

$$\mathrm{SR} = \{x_1 x_6,\ x_2 x_5,\ x_3 x_4 x_7\}.$$

  It lists combinations of coordinates that **cannot vanish simultaneously**
- $x_1 x_6$: Divisors D_1 and D_6 do not intersect.
  There is no point on the CY where both $x_1 = 0$ and $x_6 = 0$.
- $x_2 x_5$: Similarly, divisors D_2 and D_5 do not intersect.
- $x_3 x_4 x_7$: D_3, D_4, and D_7 do not have a common intersection.
  At most, two of them can vanish at the same point.

Divisor Topologies and Volume Form

- Basis of smooth divisors $\{D_1, D_2, D_3\}$ & respective dual basis $\{\hat{D}_1, \hat{D}_2, \hat{D}_3\}$ implies the intersection polynomial

$$I_3 = 2\,\hat{D}_1\,\hat{D}_2\,\hat{D}_3.$$

- Only non-zero intersection number is:

$$k_{123} = 2$$

- Kähler form:

$$J = t^1\hat{D}_1 + t^2\hat{D}_2 + t^3\hat{D}_3$$

- Volume:

$$\mathcal{V} = \frac{1}{3!}\int J \wedge J \wedge J = \frac{1}{6}\,\kappa_{ijk}t^i t^j t^k = 2t^1 t^2 t^3 = \frac{\sqrt{\tau_1\tau_2\tau_3}}{\sqrt{2}}$$

- Kähler cone conditions:

$$t^1 > 0, t^2 > 0, t^3 > 0$$

Induced Contributions to Scalar Potential

Divisor intersection analysis reveals:

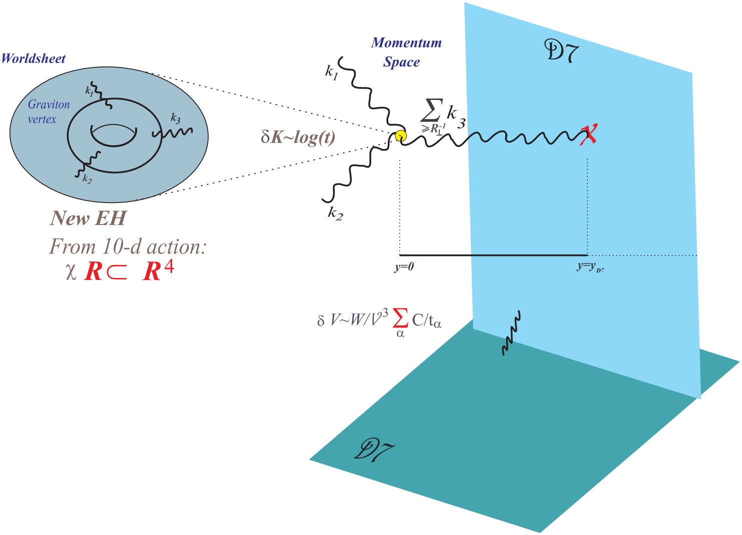
- D7-brane intersections on rigid two-tori $\mathbb{T}^2$ induce **winding-type corrections** (see e.g. hep-th/ 0708.1873):

$$V_{g_s}^{\mathrm{W}} = -\frac{\kappa |W|^2}{\mathcal{V}^3} \sum_a \frac{C_a^w}{t^a}$$

- There are no isolated D7-brane stacks;
  O7-planes are present but there are no O3-planes
- Due to absence of $O3$-planes $\Rightarrow$ $\overline{D3}$-brane uplifting is not favoured.
- Absence of parallel stacks of D7-branes $\Rightarrow$
  no KK-type string-loop corrections to Kähler potential
- K3 divisors have $\Pi = \int c_2 \wedge D_a = 24$, $(c_2 \neq 0)$, hence non-trivial higher derivative effects, i.e., F^4-corrections: (see e.g. 1505.03092, and refs therein)

$$V_{\mathrm{F}^4} = -\frac{\lambda\, \kappa^2\, |W_0|^4}{g_s^{3/2} \mathcal{V}^4}\, 24\, \left(t^1 + t^2 + t^3\right)$$

Contributing terms: schematic depiction



Effective Scalar Potential

- Perturbative corrections to Kähler potential $-2\log\left(\hat{\mathcal{V}}+\hat{\xi}+\gamma\ln(\hat{\mathcal{V}})\right)$ give:

$$\delta V_{pert} \approx \frac{\mathcal{C}_1}{\mathcal{V}^3}\left(\hat{\xi}-4\hat{\eta}+2\hat{\eta}\ln\mathcal{V}\right)$$

All contributions -including uplift- are summarized below:

$$\begin{aligned} V_{\rm eff} \approx\; & V_{\rm up} + \frac{\mathcal{C}_1}{\mathcal{V}^3}\left(\hat{\xi}-4\hat{\eta}+2\hat{\eta}\ln\mathcal{V}\right) \\ & + \frac{\mathcal{C}_2}{\mathcal{V}^4}\left(\mathcal{C}_{w_1}\tau_1+\mathcal{C}_{w_2}\tau_2+\mathcal{C}_{w_3}\tau_3+\frac{\mathcal{C}_{w_4}\tau_1\tau_2}{2(\tau_1+\tau_2)}+\frac{\mathcal{C}_{w_5}\tau_2\tau_3}{2(\tau_2+\tau_3)}+\frac{\mathcal{C}_{w_6}\tau_3\tau_1}{2(\tau_3+\tau_1)}\right) \\ & + \frac{\mathcal{C}_3}{\mathcal{V}^3}\left(\frac{1}{\tau_1}+\frac{1}{\tau_2}+\frac{1}{\tau_3}\right) \end{aligned}$$

where $\mathcal{C}_{w_\alpha}$ are complex-structure moduli dependent quantities.

Slow Roll Inflation: Parameters

To study inflationary dynamics, we must examine the appropriate conditions for slow-roll inflation, which are determined by the set of slow-roll parameters

$$\epsilon_V,\ \eta_V,\ \xi_V^{(2)}$$

These are defined in terms of the scalar potential and its derivatives:

$$\epsilon_V \equiv \frac{1}{2}\left(\frac{V_\phi}{V}\right)^2 \ll 1$$

$$\eta_V \equiv \frac{V_{\phi\phi}}{V} \ll 1$$

$$\xi_V^{(2)} \equiv \frac{V_\phi V_{\phi\phi\phi}}{V^2} \ll 1$$

Assisted Fibre Inflation

In Assisted Inflation (AI):

- Several scalar fields contribute to inflationary dynamics
- Each one of them having its own potential energy
- Combined effect leads to flatter effective scalar potential
- AI: natural in string theory due to **multiple moduli** fields
- Reduces individual field excursions compared to single-field case

Multi-field Inflationary Dynamics

Using e-folding number N as time coordinate ($dN = Hdt$):

$$\frac{d^2\Phi^a}{dN^2} + \Gamma^a_{bc}\frac{d\Phi^b}{dN}\frac{d\Phi^c}{dN} + \left(3 + \frac{1}{H}\frac{dH}{dN}\right)\frac{d\Phi^a}{dN} + \frac{\mathcal{G}^{ab}\partial_b V}{H^2} = 0$$

Friedmann equation:

$$H^2 = \frac{1}{3}\left(V(\Phi^a) + \frac{1}{2}H^2\mathcal{G}_{ab}\frac{d\Phi^a}{dN}\frac{d\Phi^b}{dN}\right)$$

Observables

The best way to test the validity of an inflationary model is to examine predictions for the following two observables:

- Spectral index n_s: This enters in the definition of the **primordial power spectrum** $\mathcal{P}_R(k) = A_s \left(\frac{k}{k_s}\right)^{n_s - 1}$
- Tensor-to-scalar ratio r, describing the ratio of the tensor perturbations to the scalar ones.

• Exploring the parameter space to meet current data:
1.) Planck:
$n_s = 0.9651 \pm 0.0044, \ \alpha_s = dn_s/d\ln k = -0.0041 \pm 0.0067, \ r < 0.036$
2.) Planck+ACT+DESI:

$$n_s = 0.9743 \pm 0.0034, \quad \alpha_s = dn_s/d\ln k = 0.0062 \pm 0.0052$$

• In case 2:
i) n_s differs from Planck by $\sim 2\sigma$ (disfavours Starobinsky and Higgs models),
ii) negative values of α_s are disfavoured

Single Field Slow Roll Inflation

It's useful to start with single field scenario which reveals the merits of the present approach
For canonically normalized fields $\varphi = \frac{1}{\sqrt{2}} \ln \tau$:

$$V \simeq \mathcal{C}_0 \left(\mathcal{R}_{\rm LVS} + \mathcal{R}_0 e^{-2\gamma\phi} - e^{-\gamma\phi} + \mathcal{R}_1 e^{\gamma\phi} + \mathcal{R}_2 e^{2\gamma\phi} \right)$$

with $\gamma = 1/\sqrt{3}$.

- First three terms: Starobinsky-type (determine minimum)
- Last two terms: create steepening and bending of V_{eff}

Three Field Inflationary Dynamics

Scalar potential in real field basis $\Phi^a = \{\mathcal{V}, t^2, t^3\}$:

$$
\begin{aligned}
V(\mathcal{V}, t^2, t^3) = & \frac{\mathcal{C}_{\rm up}}{\mathcal{V}^p} + \frac{\mathcal{C}_1}{\mathcal{V}^3}\left(\hat{\xi} + 2\hat{\eta}\ln\mathcal{V} - 8\hat{\eta} + 2\hat{\sigma}\right) \\
& - \frac{\mathcal{C}_2}{\mathcal{V}^3}\left(2\mathcal{C}_{w_1}\frac{t^2 t^3}{\mathcal{V}} + \frac{\mathcal{C}_{w_2}}{t^2} + \frac{\mathcal{C}_{w_3}}{t^3} + \frac{\mathcal{C}_{w_4} t^2 t^3}{\mathcal{V} + 2(t^2)^2 t^3}\right. \\
& \left. + \frac{\mathcal{C}_{w_5}}{2(t^2 + t^3)} + \frac{\mathcal{C}_{w_6} t^2 t^3}{\mathcal{V} + 2t^2 (t^3)^2}\right) \\
& + \frac{\mathcal{C}_3}{\mathcal{V}^3}\left(\frac{1}{2t^2 t^3} + \frac{t^2}{\mathcal{V}} + \frac{t^3}{\mathcal{V}}\right) + \cdots
\end{aligned}
$$

- Potential has exchange symmetry $t_2 \leftrightarrow t_3$.
- Uplift cases: $p = 2$ (D-term), $p = 8/3$ (T-brane), $p = 4/3$ ($\overline{D3}$-brane)

Benchmark Models

Standard Cosmological Observables

$$
\begin{aligned}
&p = 2, \quad \chi(\mathrm{CY}) = -224, \quad \eta_0 = 2, \quad \sigma_0 = 0, \quad g_s = 0.3, \\
&|W_0| = 5.6, \quad \mathcal{C}_{w_1} = 0.0008, \quad \mathcal{C}_{w_2} = -0.0008, \quad \mathcal{C}_{w_3} = -0.0008, \\
&\mathcal{C}_{w_4} = -0.02, \quad \mathcal{C}_{w_5} = 0.4, \quad \mathcal{C}_{w_6} = -0.02, \quad \lambda = -0.0001;
\end{aligned} \tag{2}
$$

$$
\begin{aligned}
&\mathcal{C}_{\mathrm{up}} = 5.38229 \cdot 10^{-3} \quad \langle \mathcal{V} \rangle = 1067, \quad \langle t^2 \rangle = 0.698, \quad \langle t^3 \rangle = 0.698, \\
&\langle \phi^1 \rangle = 5.97586, \quad \langle \phi^2 \rangle = -3.00448, \quad \langle \phi^3 \rangle = -3.00448,
\end{aligned}
$$

$$
\begin{aligned}
&\mathcal{V}^* = 1335, \quad (t^2)^* = 22.349, \quad (t^3)^* = 22.349, \\
&\phi^{1*} = 6.1589, \quad \phi^{2*} = 1.5, \quad \phi^{3*} = 1.5, \quad \Delta\phi = 5.88, \quad N = 54.5, \\
&P_s^* = 2.10 \cdot 10^{-9}, \quad \mathbf{n_s^* = 0.966}, \quad \boldsymbol{\alpha_s^* = -6.30 \cdot 10^{-4}}, \quad r^* = 6.46 \cdot 10^{-3}.
\end{aligned}
$$

Benchmark Models: Results

ACTivated Cosmological Observables

$$\mathbf{p} = \mathbf{2}, \quad \chi(\mathrm{CY}) = -224, \quad \eta_0 = 2, \quad \sigma_0 = 0, \quad g_s = 0.298,$$
$$|W_0| = 5, \quad \mathcal{C}_{w_1} = 0.001, \quad \mathcal{C}_{w_2} = -0.0008, \quad \mathcal{C}_{w_3} = -0.0008,$$
$$\mathcal{C}_{w_4} = -0.1, \quad \mathcal{C}_{w_5} = 0.33, \quad \mathcal{C}_{w_6} = -0.1, \quad \lambda = -0.00017; \tag{3}$$

$$\mathcal{C}_{\mathrm{up}} = 5.38229 \cdot 10^{-3} \quad \langle \mathcal{V} \rangle = 1129, \quad \langle t^2 \rangle = 1.14437, \quad \langle t^3 \rangle = 1.14437,$$
$$\langle \phi^1 \rangle = 6.02224, \quad \langle \phi^2 \rangle = -2.42149, \quad \langle \phi^3 \rangle = -2.42149,$$

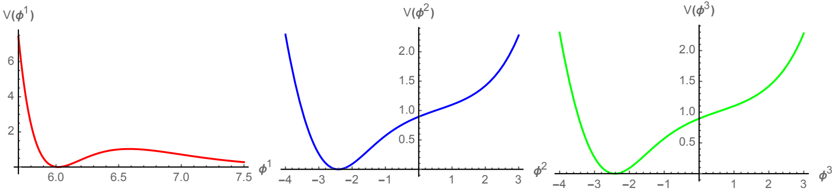
$$\mathcal{V}^* = 1258, \quad (t^2)^* = 25.798, \quad (t^3)^* = 25.798,$$
$$\phi^{1*} = 6.1105, \quad \phi^{2*} = 1.35, \quad \phi^{3*} = 1.35, \quad \Delta\phi = 5.36, \quad N = 55.6,$$
$$P_s^* = 2.12 \cdot 10^{-9}, \quad \mathbf{n_s^*} = \mathbf{0.976}, \quad \alpha_s^* = -\mathbf{5.808} \cdot \mathbf{10^{-4}}, \quad r^* = 2.71 \cdot 10^{-3}.$$

$$\mathbf{p} = \mathbf{8/3}, \quad \chi(\mathrm{CY}) = -224, \quad \eta_0 = 6, \quad \sigma_0 = -4, \quad g_s = 0.295, \tag{4}$$
$$|W_0| = 5, \quad \mathcal{C}_{w_1} = 0.001, \quad \mathcal{C}_{w_2} = -0.0008, \quad \mathcal{C}_{w_3} = -0.0008,$$
$$\mathcal{C}_{w_4} = -0.1, \quad \mathcal{C}_{w_5} = 0.33, \quad \mathcal{C}_{w_6} = -0.1, \quad \lambda = -0.00017;$$

$$\mathcal{C}_{\mathrm{up}} = 5.32455, \quad \langle \mathcal{V} \rangle = 1123.23, \quad \langle t^2 \rangle = 1.14996, \quad \langle t^3 \rangle = 1.14996,$$
$$\langle \phi^1 \rangle = 6.01802, \quad \langle \phi^2 \rangle = -2.41341, \quad \langle \phi^3 \rangle = -2.41341,$$

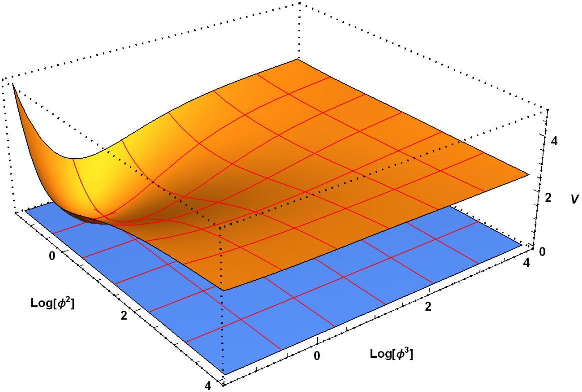
$$\mathcal{V}^* = 1258, \quad (t^2)^* = 25.8, \quad (t^3)^* = 25.8,$$
$$\phi^{1*} = 6.11069, \quad \phi^{2*} = 1.35, \quad \phi^{3*} = 1.35, \quad \Delta\phi = 5.32, \quad N = 55.5,$$
$$P_s^* = 2.095 \cdot 10^{-9}, \quad \mathbf{n_s^*} = \mathbf{0.9763}, \quad \alpha_s^* = -\mathbf{5.763 \cdot 10^{-4}}, \quad r^* = 2.73 \cdot 10^{-3}.$$

$V_{\rm eff}(\phi^i)$ Plots



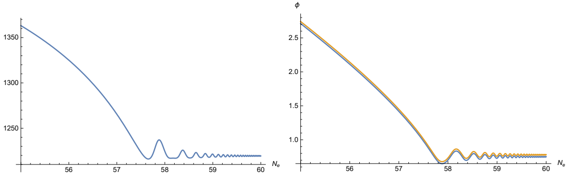
One dimensional plot of $V \cdot 10^{10}$ while keeping the other two moduli at their minimum. ϕ^2 and ϕ^3: identical evolution (reflecting CY symmetry)

Potential Symmetry



For fixed volume $\mathcal{V}$, the potential $V(\phi_2, \phi_3)$ exhibits flat direction along diagonal suitable for slow-roll inflation.

Field Evolution



Canonical fields evolution: ϕ^1 (left) and ϕ^2, ϕ^3 (right). Fields ϕ^2, ϕ^3 have identical evolution due to CY symmetry.

Field excursions

- Inflaton moves "diagonally"
- Distance smaller compared to the sum of two individual directions.

For $p = 8/3$ case:

$$\Delta\phi^1 \simeq 0.0926, \quad \Delta\phi^2 = \Delta\phi^3 \simeq 3.763 \qquad \Longrightarrow \qquad \Delta\phi \simeq 5.32, \tag{5}$$

- Underlying exchange symmetry $2 \leftrightarrow 3$.
- In the single- and two-field versions of this model, the total inflaton shift is

$$\Delta\phi \simeq 5.3$$

- Observe the relation (for $n = 2$)

$$\Delta\phi^n \simeq \Delta\phi/\sqrt{n}$$

Conclusions

- Novel multi-field fibre inflation in type IIB pLVS
- Global Embedding in CY_3 with 6 $K3$ and $h^{1,1} = 3, \ h^{2,1} = 115$.
- Volume stabilized at $\mathcal{V} \simeq 10^3$ with $g_s \simeq 0.3, \ W_0 \simeq 5$
- Uplifting mechanisms: D-term ($p = 2$), T-brane ($p = 8/3$)
- Various cases, compatible with Planck/ACT/DESI cosmological data
- Trans-Planckian displacement issues addressed:

  While single-field requires $\Delta\phi \lesssim 6M_p$,
  Assisted inflation reduces individual excursions to $\simeq 3.5M_p$

Conclusions (cont.)

- **Future directions**:
- Extend to $n \geq 2$ inflatons,
- K3 fibrations with $h^{1,1} > 3$ and Swiss-cheese volume:

$$\mathcal{V} = f_{3/2}(\tau_i) - \sum_j \lambda_j \tau_j^{3/2}$$

- Detailed backreaction analysis

Thank You

APPENDIX

Involutions and Tadpole Conditions

- $\forall$ holomorphic involution $\Leftrightarrow$ introduce D3/D7-branes and fluxes
- D7-tadpoles canceled by introducing N_a D7-brane stacks

$$\sum_a N_a\left([D_a]+[D'_a]\right)=8\,[\mathrm{O7}]\,. \tag{6}$$

- D7-branes and O7-planes & contributions from H_3 and F_3 fluxes, D3-branes and O3-planes:

$$N_{\mathrm{D3}}+\frac{N_{\mathrm{flux}}}{2}+N_{\mathrm{gauge}}=\frac{N_{\mathrm{O3}}}{4}+\frac{\chi(\mathrm{O7})}{12}+\sum_a\frac{N_a\left(\chi(D_a)+\chi(D'_a)\right)}{48}\,, \tag{7}$$

- Simple case: D7-tadpoles canceled by placing 4 D7-branes (plus their images) on top of an O7-plane:

$$N_{\mathrm{D3}}+\frac{N_{\mathrm{flux}}}{2}+N_{\mathrm{gauge}}=\frac{N_{\mathrm{O3}}}{4}+\frac{\chi(\mathrm{O7})}{4}\,. \tag{8}$$

- Involution $x_7 \to -x_7 \;\Rightarrow$ better brane setting.
- Only one fixed point set with $\{O7 = D_7\}$ along with no $O3$-planes.
- brane configuration of three stacks of $D7$-branes each wrapping $\{D_1, D_2, D_3\}$:

$$8\,[O_7] = 8\,([D_1] + [D_1']) + 8\,([D_2] + [D_2']) + 8\,([D_3] + [D_3'])\;, \tag{9}$$

along with the $D3$ tadpole cancellation condition being given as

$$N_{\rm D3} + \frac{N_{\rm flux}}{2} + N_{\rm gauge} = 0 + \frac{240}{12} + 8 + 8 + 8 = 44\,, \tag{10}$$

- VEV of W_0 intertwined with $Q_{\rm D3}$ after CS stabilization,

$$2\pi g_s\,|W_0|^2 < Q_{\rm D_3} = 88, \tag{11}$$

- Notice that our models: $|W_0| = 5$, $g_s \approx 0.3$