From noncommutative Yang-Mills to noncommutative gravity

(through a classical double copy map)

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based on

LJ and Eric Lescano, arXiv:2502.03521

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Background (Double copy 2204.06547)

- Double copy of gauge theory to gravity
  - ⇝ factorization of closed string/graviton amplitudes into open string/gauge theory amplitudes Kawai, Lewellen, Tye '86
  - ⇝ Based on colour–kinematics duality Bern, Carrasco, Johansson '08, '10...

$$\mathcal{A}_n^{L/R} = \sum_I \frac{c_I n_I^{L/R}}{D_I} \to \sum_I \frac{n_I^L n_I^R}{D_I} = M_n^{L\otimes R}$$

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- Classical double copy relating solutions Monteiro, O'Connell, White '14...
  $\rightsquigarrow$ relation between actions?
  $\rightsquigarrow$ left–right factorization in gravity?

$$h_{\mu\nu} \to A_\mu \bar{A}_{\bar{\nu}} + \text{some (nonlinear) field redefinition}$$

  $\rightsquigarrow$ factorization comes for free in double field theory Hohm '11

$$h_{\mu\nu} \to e_{\mu\nu} = h_{\mu\nu} + b_{\mu\nu} \to e_{\mu\bar{\nu}}$$

Background (Double field theory 1305.1907; 1306.2643; 1309.2977)

- Originally constructed to describe the massless sector of closed string theory on toroidal backgrounds Hull, Zwiebach '09; Duff '90; Tseytlin '90; Siegel '93
  $\rightsquigarrow$ fields g, B, d depend on doubled coordinates conjugate to p^{μ} and w_{μ}
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- Both approaches have a well-defined limit to (super)gravity.

Plan

1. Double field theory as double copy of Yang–Mills.
2. Noncommutative DFT as double copy of noncommutative YM.
3. Noncommutative corrections to general relativity.
4. Outlook.

DFT as double copy of YM

Diaz-Jaramillo, Hohm, Plefka '21

Idea: Off-shell (!) double copy from *perturbative* YM to *perturbative* DFT

$$YM^{(2,3)}(A) \to DFT^{(2,3)}(e) \to Sugra^{(2,3)}(h, b, \phi)$$

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Start with Yang–Mills action

$$S_{\rm YM} = -\frac{1}{4}\int d^Dx\, \kappa_{ab} F_{\mu\nu}{}^a F^{\mu\nu b},$$

with

$$F_{\mu\nu}{}^a = 2\partial_{[\mu}A_{\nu]}{}^a + f^a{}_{bc}A_\mu{}^b A_\nu{}^c,$$

for Minkowski metric $\eta_{\mu\nu} = \mathrm{diag}(-,+,+,+)$ and Cartan–Killing metric κ_{ab}.

DFT as double copy of YM

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Expand to quadratic order in A and pass to momentum space:

$$S_{\text{YM}}^{(2)} = -\frac{1}{2}\int d^D k\, \kappa_{ab} k^2 \Pi^{\mu\nu}(k) A_\mu{}^a(-k) A_\nu{}^b(k),$$

with

$$\Pi^{\mu\nu}(k) = \eta^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}, \quad \Pi^{\mu\nu} k_\nu = 0, \quad \Pi^{\mu\nu}\Pi_{\nu\rho} = \Pi^\mu{}_\nu.$$

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Double copy prescription: replace colour index by an additional spacetime index

$$A_\mu{}^a(k) \to e_{\mu\bar{\mu}}(k,\bar{k}), \qquad \kappa_{ab} \to \Pi^{\bar{\mu}\bar{\nu}}(\bar{k}).$$

DFT as double copy of YM

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Quadratic double copy action

$$S^{(2)}_{\rm DC} = -\frac{1}{4}\iint d^D k\, d^D\bar{k}\; k^2 \Pi^{\mu\nu}(k)\bar{\Pi}^{\bar{\mu}\bar{\nu}}(\bar{k})\, e_{\mu\bar{\mu}}(-k,-\bar{k}) e_{\nu\bar{\nu}}(k,\bar{k}).$$

The action is invariant under

$$\delta e_{\mu\bar{\nu}} = k_\mu \bar{\lambda}_{\bar{\nu}} + \bar{k}_{\bar{\nu}} \lambda_\mu$$

imposing level matching

$$\bar{k}^2 = k^2.$$

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$$S_{\rm DC}^{(2)} = -\frac{1}{4}\int_{k,\bar{k}}\Big(k^2 e^{\mu\bar{\nu}}e_{\mu\bar{\nu}} - k^\mu k^\rho e_{\mu\bar{\nu}}e^{\bar{\nu}}{}_\rho - \bar{k}^{\bar{\nu}}\bar{k}^{\bar{\rho}}e_{\mu\bar{\nu}}e^\mu{}_{\bar{\rho}} + \tfrac{1}{k^2}k^\mu k^\rho\bar{k}^{\bar{\nu}}\bar{k}^{\bar{\rho}}e_{\mu\bar{\nu}}e_{\rho\bar{\rho}}\Big)$$

Additionally, introduce an auxiliary scalar field $\phi(k,\bar{k})$:

$$S_{\rm DC}^{(2)} = -\frac{1}{4}\int_{k,\bar{k}}\Big(k^2 e^{\mu\bar{\nu}}e_{\mu\bar{\nu}} - k^\mu k^\rho e_{\mu\bar{\nu}}e^{\bar{\nu}}{}_\rho - \bar{k}^{\bar{\nu}}\bar{k}^{\bar{\rho}}e_{\mu\bar{\nu}}e^\mu{}_{\bar{\rho}} - k^2\phi^2 + 2\phi k^\mu\bar{k}^{\bar{\nu}}e_{\mu\bar{\nu}}\Big)\,.$$

DFT as double copy of YM

Diaz-Jaramillo, Hohm, Plefka '21

Fourier transform to local action in doubled position space

$$S^{(2)}_{\text{DFT}} = \frac{1}{4}\int d^Dx\, d^D\bar{x}\, \left(e^{\mu\bar{\nu}}\Box e_{\mu\bar{\nu}} + \partial^\mu e_{\mu\bar{\nu}}\partial^\rho e_\rho{}^{\bar{\nu}} + \bar{\partial}^{\bar{\nu}} e_{\mu\bar{\nu}}\bar{\partial}^{\bar{\sigma}} e^\mu{}_{\bar{\sigma}} - \phi\Box\phi + 2\phi\partial^\mu\bar{\partial}^{\bar{\nu}} e_{\mu\bar{\nu}}\right),$$

which reproduces the standard quadratic DFT action Hull,Zwiebach '09.

$\rightsquigarrow$ linearized gravity recovered for $x = \bar{x}$ with $e_{\mu\nu} = h_{\mu\nu}$, and $h := h^\mu{}_\mu = \phi$.

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Contribution cubic in A

$$S_3 = -\frac{i}{6(2\pi)^{D/2}}\int_{k_i} \delta\left(k_1 + k_2 + k_3\right) f_{abc}\pi^{(0)\mu\nu\rho} A_\mu{}^a(k_1) A_\nu{}^b(k_2) A_\rho{}^c(k_3)\,,$$

with

$$\pi^{(0)\mu\nu\rho}(k_1, k_2, k_3) = \eta^{\mu\nu}(k_1 - k_2)^\rho + \eta^{\nu\rho}(k_2 - k_3)^\mu + \eta^{\rho\mu}(k_3 - k_1)^\nu\,.$$

Double copy: $f_{abc} \to \bar{\pi}^{(0)\bar{\mu}\bar{\nu}\bar{\rho}}$

DFT as double copy of YM

Diaz-Jaramillo, Hohm, Plefka '21

Cubic action for the double copy of Yang-Mills theory

$$
\begin{aligned}
S^{(3)}_{\rm DFT} \;&=\; \frac{1}{8}\int_{x,\bar{x}} e_{\mu\bar{\mu}}\Big(2\partial^{\mu}e_{\rho\bar{\rho}}\,\bar{\partial}^{\bar{\mu}}e^{\rho\bar{\rho}} - 2\partial^{\mu}e_{\nu\bar{\rho}}\,\bar{\partial}^{\bar{\rho}}e^{\nu\bar{\mu}} \\
&- \; 2\partial^{\rho}e^{\mu\bar{\rho}}\,\bar{\partial}^{\bar{\mu}}e_{\rho\bar{\rho}} + \partial^{\rho}e_{\rho\bar{\rho}}\,\bar{\partial}^{\bar{\rho}}e^{\mu\bar{\mu}} + \bar{\partial}_{\bar{\rho}}e^{\mu\bar{\rho}}\,\partial_{\rho}e^{\rho\bar{\mu}}\Big),
\end{aligned}
$$

which reproduces the cubic (gauged fixed) DFT action Hull, Zwiebach '09.

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$$S^{(3)}_{\rm DFT} = \frac{1}{8}\int_{x,\bar{x}} e_{\mu\bar{\mu}}\Big(2\partial^{\mu}e_{\rho\bar{\rho}}\,\bar{\partial}^{\bar{\mu}}e^{\rho\bar{\rho}} - 2\partial^{\mu}e_{\nu\bar{\rho}}\,\bar{\partial}^{\bar{\rho}}e^{\nu\bar{\mu}} - 2\partial^{\rho}e^{\mu\bar{\rho}}\,\bar{\partial}^{\bar{\mu}}e_{\rho\bar{\rho}} + \partial^{\rho}e_{\rho\bar{\rho}}\,\bar{\partial}^{\bar{\rho}}e^{\mu\bar{\mu}} + \bar{\partial}_{\bar{\rho}}e^{\mu\bar{\rho}}\,\partial_{\rho}e^{\rho\bar{\mu}}\Big),$$

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Limits:

- for $x=\bar{x}$ universal NSNS sector of the low energy limit of string theory

$$e_{\mu\nu} = h_{\mu\nu} + b_{\mu\nu}$$

- for $x=\bar{x}$ and $b_{\mu\nu}=0=\phi$ linearized general relativity

$$e_{\mu\nu} = h_{\mu\nu}$$

Noncommutative Yang–Mills (ncYM)

Consider the noncommutative extension of SU(N) Yang–Mills action

Jurco, Möller, Schraml, Schupp, Wess '01

$$S^{*}_{\mathrm{YM}} = -\frac{1}{4}\mathrm{Tr}\int d^D x\, F^{*}_{\mu\nu} \star F^{*\mu\nu},$$

with the Moyal–Weyl product

$$(f \star g)(x) = \exp\left(\tfrac{i}{2}\theta^{\rho\sigma}\partial_{x^\rho}\partial_{y^\sigma}\right) f(x)g(y)\Big|_{y\to x}.$$

Noncommutative field strength is

$$F^{*}_{\mu\nu} = 2\partial_{[\mu}A^{*}_{\nu]} - i[A^{*}_{\mu}, A^{*}_{\nu}]_{\star},$$

where A^* is a solution of the Seiberg–Witten map Seiberg, Witten '99.
The $SU(N)$ generators T_a satisfy

$$[T_a, T_b] = i f_{ab}{}^{c} T_c\,, \{T_a, T_b\} = \frac{1}{N}\delta_{ab} + d_{ab}{}^{c} T_c\,,$$
$$\mathrm{Tr}(T_a T_b T_c) = \frac{1}{4}(d_{abc} + i f_{abc})\,.$$

ncYM: perturbative corrections

- Quadratic action: no corrections in θ.
- Cubic action: linear θ corrections appear.

Linear θ correction in momentum space

$$S_\theta^{(3)} = \frac{i}{6(2\pi)^{\frac{D}{2}}} \int_k \delta(k_1 + k_2 + k_3) d_{abc} \pi^{(1)\mu\nu\rho} A_\mu{}^a(k_1) A_\nu{}^b(k_2) A_\rho{}^c(k_3) \, ,$$

where

$$\pi^{(1)\mu\nu\rho} = \frac{3}{4}\Big(- \theta^{(\mu|\sigma} k_{1\sigma}(k_2^\nu k_3^\rho) + k_{2\tau} k_3^\tau \eta^{|\nu\rho)}) \\ + 2\theta^{(\mu|\sigma} k_{1\tau} k_{2\sigma} k_3^\tau \eta^{|\nu\rho)} - 2\theta^{\tau\sigma} k_{1\tau} k_{2\sigma} k_3^{(\mu} \eta^{\nu\rho)}\Big) \, .$$

Following the YM$\times$YM structure of the double copy prescription we identify

$$d_{abc} \to \frac{ip}{8} \bar{\pi}^{(1)\bar{\mu}\bar{\nu}\bar{\rho}} \, ,$$

with p a free parameter of order 1.

ncYM: perturbative corrections

- Identification introduces a second noncommutativity parameter $\bar{\theta}^{\bar{\mu}\bar{\nu}}$.
- The resulting double copy action is proportional to $\theta\bar{\theta}$.
- Include θ^2 corrections in ncYM Möller '04; Dimitrijević, Radovanović, Simonović '12

$$S^{(3)}_{\theta^2} = \frac{i}{6(2\pi)^{\frac{D}{2}}} \int_k \delta(k_1 + k_2 + k_3) f_{abc} \pi^{(2)\mu\nu\rho} A_\mu{}^a(k_1) A_\nu{}^b(k_2) A_\rho{}^c(k_3) \,,$$

where

$$\pi^{(2)\mu\nu\rho} = \frac{3}{8} \theta^{\sigma\tau} k_{1\sigma} k_{2\tau} \left(\theta^{[\mu\nu} k_{1\kappa} k_2^{\rho]} k_3^\kappa - \theta^{\kappa[\nu} k_{1\kappa} k_2^\rho k_3^{\mu]} \right) .$$

Double copy identification (including the undeformed part)

$$f_{abc} \to \frac{i}{8} (\pi^{(0)\bar{\mu}\bar{\nu}\bar{\rho}} + \pi^{(2)\bar{\mu}\bar{\nu}\bar{\rho}})$$

NC DFT and gravity limit

In momentum space with $K = (k, \bar{k})$:

$$S^{(3)}_{\text{up to }\theta^2}|_{\text{DFT}} = -\frac{1}{48(2\pi)^{\frac{D}{2}}} \int_{k,\bar{k}} \delta(K_1 + K_2 + K_3) \times$$

$$\left(p\bar{\pi}^{(1)}_{\bar{\mu}\bar{\nu}\bar{\rho}} \pi^{(1)\mu\nu\rho} + \bar{\pi}^{(0)}_{\bar{\mu}\bar{\nu}\bar{\rho}} \pi^{(2)\mu\nu\rho} + \bar{\pi}^{(2)}_{\bar{\mu}\bar{\nu}\bar{\rho}} \pi^{(0)\mu\nu\rho} \right) e_\mu{}^{\bar{\mu}}(K_1) e_\nu{}^{\bar{\nu}}(K_2) e_\rho{}^{\bar{\rho}}(K_3)\,.$$

$\rightsquigarrow$ back to coordinate space

To obtain perturbative corrections to linearized GR

- set $D = 4$, solve level matching and strong constraint $x = \bar{x}$,
- set $\theta = \bar{\theta}$, and take $e_{\mu\nu} = h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}$.
- Apply field redefinitions and gauge fixing (we use $\partial^\mu h_{\mu\nu} = 0 = h$).

Corrections to GR

From the linearized expression:

$$\mathcal{L}_{Riem^3} = \theta^{\mu\nu}\theta^{\rho\sigma}\Big(\frac{p}{144}R^{\lambda\kappa\tau\xi}R_{\lambda\kappa\tau\xi}R_{\mu\nu\rho\sigma} + \frac{5p}{18}R^{\lambda\xi\tau\kappa}R_{\lambda\tau\xi\mu}R_{\kappa\nu\rho\sigma}$$
$$-\frac{2p}{9}R^{\lambda\tau\xi\kappa}R_{\lambda\tau\xi\mu}R_{\kappa\rho\nu\sigma} - \frac{p}{18}R^{\lambda\xi\tau}{}_{\mu}R_{\lambda\rho\tau}{}^{\kappa}R_{\xi\sigma\kappa\nu} + \frac{1}{12}R^{\lambda\xi\tau}{}_{\mu}R_{\lambda}{}^{\kappa}{}_{\tau\rho}R_{\xi\sigma\kappa\nu}$$
$$+\frac{16p+3}{72}R^{\lambda\xi\tau\kappa}R_{\lambda\mu\tau\rho}R_{\xi\nu\kappa\sigma} - \frac{20p-3}{72}R^{\lambda\xi\tau\kappa}R_{\lambda\mu\tau\rho}R_{\xi\sigma\kappa\nu}\Big)\,.$$

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The linearized equations of motion

$$\Box h^{\mu\nu} = -\frac{1}{8}\partial^{\nu}h^{\sigma\lambda}\partial^{\mu}h_{\lambda\sigma} - \frac{1}{4}\partial^{\sigma}h_{\lambda}{}^{\nu}\partial_{\sigma}h^{\lambda\mu} + \frac{1}{4}\partial_{\sigma}h_{\lambda}{}^{\nu}\partial^{\lambda}h^{\mu\sigma}$$
$$-\frac{1}{8}\eta^{\mu\nu}\partial_{\lambda}h^{\sigma\rho}\partial_{\rho}h^{\lambda}{}_{\sigma} + \frac{3}{16}\eta^{\mu\nu}\partial_{\lambda}h^{\rho\sigma}\partial^{\lambda}h_{\rho\sigma} + \frac{1}{2}C^{\mu\nu}(\theta^2)\,,$$

where

$$C^{\mu\nu}(\theta^2) = C_1^{\mu\nu}(\partial^3 h\partial^3 h) + C_2^{\mu\nu}(\partial^4 h\partial^2 h)\,.$$

$\rightsquigarrow \theta^{0i} = 0.$

Summary & Outlook

$$
\begin{array}{rcl}
\mathrm{ncYM}^{(2,3)} \to & \mathrm{DFT}^{(2,3)} + \mathrm{ncDFT}^{(3)} & \\
& \downarrow \qquad\quad \downarrow & \\
& \mathrm{GR}^{(2,3)} + \theta^2 \mathrm{Riem}^3 & .
\end{array}
$$

Summary & Outlook

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- Role of colour–kinematics duality in ncYM?

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- Role of colour–kinematics duality in ncYM?
  Intrinsic mixing of gauge and kinematics!
  Identification

$$
f_{abc} + d_{abc} \rightarrow \frac{i}{8}\left(\bar{\pi}^{(0)\bar{\mu}\bar{\nu}\bar{\rho}} + p\bar{\pi}^{(1)\bar{\mu}\bar{\nu}\bar{\rho}} + \bar{\pi}^{(2)\bar{\mu}\bar{\nu}\bar{\rho}}\right) ,
$$

  from duality? cf. Monteiro '22; Szabo, Trojani '23; Szabo '24

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- Role of colour–kinematics duality in ncYM? cf. Monteiro ’22; Szabo, Trojani ’23; Szabo ’24
- Explicit solutions? eg. Kerr-Schild in DFT: Lee ’18; Lescano, Rodríguez ’20
- Comparison with existing results? cf. Marija’s talk: Bežanić, Dimitrijević Ćirić, Nikolić, Radovanović ’25