Monopoles and instantons on quantum spaces

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The simplest case z_k, z_k^*, $k = 0, 1$ are the generators of the algebra $\mathscr{A}(S_q^3)$:

$$z_0 z_1 = q^{-1} z_1 z_0 \ , \qquad z_0^* z_1 = q z_1 z_0^* \ , \qquad z_1^* z_0 = q z_0 z_1^*$$
$$z_0^* z_0 = z_0 z_0^* + (1 - q^2) z_1 z_1^* \ , \qquad z_1^* z_1 = z_1 z_1^* \, , \qquad z_1^* z_0^* = q^{-1} z_0^* z_1^* \, ,$$

and sphere relations : $\qquad z_0 z_0^* + z_1 z_1^* = 1 \qquad z_0^* z_0 + q^2 z_1^* z_1 = 1$

An co-action of $U(1)$; the coinvariant elements make up the projective line $\mathscr{A}(\mathbb{C}P_q^1)$ (the standard Podleś sphere)

$$B_0 := z_1^* z_1 \ , \quad B_0 := z_1^* z_0 \ , \quad B_- := z_0^* z_1$$

with $B_0^* = B_0$ and $B_0^* = B_-$.

They satisfy commutation relations

$$B_0 B_0 = q^2 B_0 B_0 \ , \qquad B_0 B_- = q^{-2} B_- B_0 \ , \qquad B_- B_0 = q^4 B_0 B_- + q^2 (1 - q^2) B_0$$

and sphere relations: $B_0 B_- = B_0 (1 - B_0) \, , \qquad B_- B_0 = q^2 B_0 (1 - q^2 B_0)$

The tautological line bundle (the monopole bundle)

via its sections; or the module of equivariant maps; or a projection :

$$p_{(1)} = \begin{pmatrix} z_0^* \\ z_1^* \end{pmatrix} (z_0, z_1) = \begin{pmatrix} 1 - q^2 B_0 & B_- \\ B_+ & B_0 \end{pmatrix} \qquad \longleftrightarrow \qquad \phi = z_0 f + z_1 g$$

for $f, g \in \mathscr{A}(\mathbb{C}P_q^1)$

$$\mathrm{Tr}(p_{(1)}) = 1 + (1 - q^2) B_0$$

The rank of the bundle: $[\varepsilon]([\mathrm{Tr}(p_{(1)}]) = 1 + 0$

The degree of the bundle: $[\varepsilon]([\mathrm{Tr}(p_{(1)}]) = 0 - 1$

The K-theory group $K(\mathbb{C}P_q^1)$ classifies equivalence classes of projection

(i.e. line bundles)

Ring structure on the K-theory group of quantum projective spaces $\mathscr{A}(\mathbb{C}P^n_q)$

Projections $P_N \in \mathrm{Mat}_{\mathrm{d_N}}(\mathscr{A}(\mathbb{C}P^n_q))$, $N = 0, 1, ..., n$

such that $[P_N]$ generate the group $K_0(\mathscr{A}(\mathbb{C}P^n_q))$ (algebraic generators).

Indeed we have $\mathscr{A}(\mathbb{C}P^n_q))$ - bimodules

$$\mathcal{L}_N := P_N(\mathscr{A}(\mathbb{C}P^n_q))^{d_N}$$

(written as right modules) obeying :

$$\mathcal{L}_N \otimes_{\mathscr{A}(\mathbb{C}P^n_q)} \mathcal{L}_M \cong \mathcal{L}_{M+N}$$

Chern numbers coming from pairing with K-homology:

$$\mathrm{ch}_0(\mathcal{L}_N) = 1, \qquad \mathrm{ch}_1(\mathcal{L}_N) = -N$$

$\mathcal{L}_N$ space of sections of a line bundle over $\mathbb{C}P^n_q)$ with winding number $-N$.

Proposition (Arici-Brain-L, 2015)

$$K_0(\mathbb{C}P_q^n)) = \mathbb{Z}[u]/u^{n+1} \simeq \mathbb{Z}^{n+1}$$

with Euler class

$$u = \chi(\mathcal{L}_{-1}) := 1 - [\mathcal{L}_{-1}]$$

Generalise the above to vector bundles with rank higher that 1

So we seek the quantization of spaces whose K-theory in the classical limit is the ring of dual numbers $\mathbb{Z}[t]/(t^2)$.

Classically, $t = r - [E]$ is the Euler characteristic (in K-theory) of the vector bundle E with rank r. One can rephrase the relation $t^2 = (r - [E])^2 = 0$ as

$$r^2 - 2r[E] + [E \otimes E] = 0.$$

This result can be dualized and then generalized, in a suitable way, to noncommutative spaces.

For a compact quantum space, sufficient conditions for a morphism of abelian groups $K_0 \to \mathbb{Z}[t]/(t^2)$ compatible with the tensor product of bimodules.

Applications include the standard Podleś sphere S^2_q and a quantum 4-sphere S^4_q coming from quantum symplectic groups. The K-theory is generated by the Euler class of the monopole or instanton bundle respectively.

Explicit formulas for the projections of vector bundles on the sphere S^4_q associated to the principal $SU_q(2)$-bundle $S^7_q \to S^4_q$ via irreducible corepresentations of $SU_q(2)$, and compute their characteristic classes.

Characteristic classes of noncommutative vector bundles

A unital *-algebra $\mathscr{B}$, and its C*-enveloping algebra B

B is a ‘compact quantum space’, and $\mathscr{B}$ describes some additional structure on this quantum space. In addition, an inclusion of unital *-algebras

$$\mathscr{B} \subseteq \mathscr{A},$$

where $\mathscr{A}$ describes some auxiliary noncommutative space

Interested in classes in $K_0(B)$ that are represented by finitely generated projective right $\mathscr{B}$-modules (“noncommutative vector bundles”).

Specialize to right modules that “trivialize” over $\mathscr{A}$ (that is, their “pullback” to the total space is trivial), and denote by $\mathrm{Vect}_{\mathscr{A}}(\mathscr{B})$ their class.

We construct an isomorphism

$$\mathrm{ch} : K_0(B) \to \mathbb{Z}[t]/(t^2)$$

of abelian groups that is “multiplicative”, in the sense that it is compatible with the tensor product in $\mathrm{Vect}_{\mathscr{A}}(\mathscr{B})$.

A first result:

Proposition:
With mild assumptions, if $K_0(B) \cong \mathbb{Z}^2$ and there is $\mathscr{E} \in \mathsf{Vect}_{\mathscr{A}}(\mathscr{B})$ with $\mathsf{ch}_1(\mathscr{E}) = \pm 1$, then $(1, [\mathscr{E}])$ is a basis of $K_0(B)$ and

$$r^2 - 2r[\mathscr{E}] + [\mathscr{E} \otimes_{\mathscr{B}} \mathscr{E}] = 0, \qquad r := \mathsf{ch}_0(\mathscr{E}).$$

This identity is the analogue of the classical statement $t^2 = 0$ for $t := r - [E]$.

The construction of $\mathbb{Z}$-linear maps $K_0(B) \to \mathbb{Z}$ induced by an even Fredholm module. We focus on 1-summable Fredholm modules.

1-summable even Fredholm modules

Let B be a unital C*-algebra, $\pi_1, \pi_2 : B \to \mathcal{B}(\mathcal{H})$ be two bounded *-representations on a Hilbert space $\mathcal{H}$ and $F, \gamma \in \mathcal{B}(\mathcal{H} \otimes \mathbb{C}^2)$ the operators

$$F = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \gamma = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$

where 1 is the identity on $\mathcal{H}$.

Let π be the representation of B on $\mathcal{H} \otimes \mathbb{C}^2$ direct sum of π_1 and π_2. Note:

$$\gamma F[F, \pi(b)] = \begin{pmatrix} \pi_1(b) - \pi_2(b) & 0 \\ 0 & \pi_1(b) - \pi_2(b) \end{pmatrix}.$$

If the difference $\pi_1(b) - \pi_2(b)$ is of trace class on $\mathcal{H}$ for all b in a dense unital *-subalgebra $\mathscr{B}$ of B, then $(B, \mathcal{H} \otimes \mathbb{C}^2, F, \gamma)$ is a 1-summable even Fredholm module. It defines a $\mathbb{Z}$-module map

$$\varphi : K_0(B) \to \mathbb{Z}.$$

For $[p]$ the K-theory class of a projection $p \in M_N(\mathscr{B})$, this is given by

$$\varphi([p]) = \tfrac{1}{2}\,\mathsf{Tr}_{\mathcal{H}\otimes\mathbb{C}^2}(\gamma F[F,\pi(\mathsf{Tr}\,p)]) = \mathsf{Tr}_{\mathcal{H}}\big(\pi_1(\mathsf{Tr}\,p) - \pi_2(\mathsf{Tr}\,p)\big)\ ,$$

where $\mathsf{Tr}\,p = \sum_{i=1}^N p_i^i \in \mathscr{B}$ and $\mathsf{Tr}_{\mathcal{H}}$ (resp. $\mathsf{Tr}_{\mathcal{H}\otimes\mathbb{C}^2}$) is the trace of operators on the Hilbert space $\mathcal{H}$ (resp. $\mathcal{H}\otimes\mathbb{C}^2$).

Note:

(i) $\varphi([p])$ only depends on the K-theory class of p;

(ii) it is an integer being the index of a Fredholm operator

Thus: $\qquad \varphi \in K_0(B)^\vee$.

(In general, in this way one constructs a $\mathbb{Z}$-module map $K^0(B) \to K_0(B)^\vee$ which needs not to be injective or surjective)

Proposition

Assume that there is a character $\varepsilon : \mathscr{A} \to \mathbb{C}$ and a bounded unital *-representation $\pi : \mathscr{A} \to \mathcal{B}(\mathcal{H})$ such that

$$\pi(b) - \varepsilon(b) \in \mathcal{L}^1(\mathcal{H}) \ , \quad \forall\, b \in \mathscr{B}.$$

Then:

(i) There is a homomorphism of unital semirings

$$\mathsf{ch} : \big(\mathsf{Vect}_{\mathscr{A}}(\mathscr{B}), \oplus, \otimes\big) \to \mathbb{Z}[t]/(t^2), \qquad \mathsf{ch}(\mathscr{E}) = \mathsf{ch}_0(\mathscr{E}) + \mathsf{ch}_1(\mathscr{E})t,$$

given, for any $\mathscr{E} \cong p\mathscr{B}^N$, by the formulas

$$\mathsf{ch}_0(\mathscr{E}) := \varepsilon(\mathsf{Tr}\, p), \qquad \mathsf{ch}_1(\mathscr{E}) = \mathsf{Tr}_{\mathcal{H}}\big(\pi(\mathsf{Tr}\, p) - \varepsilon(\mathsf{Tr}\, p)\big).$$

(ii) The underlying morphism of abelian semigroups is the composition of (**??**) with a homomorphism $[\mathsf{ch}] : K_0(B) \to \mathbb{Z}[t]/(t^2)$.

(iii) If (u, v) is a trivializing pair for $\mathscr{E}$, then the number of column of u is $r = \mathsf{ch}_0(\mathscr{E})$ and thus depends only on the K-theory class of $\mathscr{E}$.

Note

The maps $[\mathsf{ch}_0]$ and $[\mathsf{ch}_1]$ are the index maps of the Fredholm modules associated with the pair or representations $(\varepsilon, 0)$ on $\mathbb{C}$ and (π, ε) on $\mathcal{H}$.

Here $\varepsilon(b)$ acts on $\mathcal{H}$ as a scalar multiple of the identity.

The representations ε and π extend from $\mathscr{B}$ to the C*-enveloping algebra B, due to the universal property of the latter.

The map ch is compatible with the tensor product of elements of $\mathsf{Vect}_{\mathscr{A}}(\mathscr{B})$, which in components means that

$$\begin{aligned} \mathsf{ch}_0(\mathscr{E} \otimes_{\mathscr{B}} \widetilde{\mathscr{E}}) &= \mathsf{ch}_0(\mathscr{E})\mathsf{ch}_0(\widetilde{\mathscr{E}}) \ , \\ \mathsf{ch}_1(\mathscr{E} \otimes_{\mathscr{B}} \widetilde{\mathscr{E}}) &= \mathsf{ch}_0(\mathscr{E})\mathsf{ch}_1(\widetilde{\mathscr{E}}) + \mathsf{ch}_1(\mathscr{E})\mathsf{ch}_0(\widetilde{\mathscr{E}}) \ , \end{aligned} \tag{1}$$

for all $\mathscr{E}, \widetilde{\mathscr{E}} \in \mathsf{Vect}_{\mathscr{A}}(\mathscr{B})$.

Corollary
Under the assumptions as before, for every $\mathscr{E} \in \mathsf{Vect}_{\mathscr{A}}(\mathscr{B})$ and every positive integer k, for the module

$$\mathscr{E}^{\otimes_{\mathscr{B}} k} := \underbrace{\mathscr{E} \otimes_{\mathscr{B}} \mathscr{E} \otimes_{\mathscr{B}} \ldots \otimes_{\mathscr{B}} \mathscr{E}}_{k \text{ times}}$$

one has

$$\mathsf{ch}_0(\mathscr{E}^{\otimes_{\mathscr{B}} k}) = \mathsf{ch}_0(\mathscr{E})^k \,, \qquad \mathsf{ch}_1(\mathscr{E}^{\otimes_{\mathscr{B}} k}) = k \, \mathsf{ch}_0(\mathscr{E})^{k-1} \mathsf{ch}_1(\mathscr{E}) \,.$$

Proposition
Under the assumptions as before, if $K_0(B) \cong \mathbb{Z}^2$ and there exists $\mathscr{E} \in \mathsf{Vect}_{\mathscr{A}}(\mathscr{B})$ with $\mathsf{ch}_1(\mathscr{E}) = \pm 1$, then $(1, [\mathscr{E}])$ is a basis of $K_0(B)$ and

$$r^2 - 2r[\mathscr{E}] + [\mathscr{E} \otimes_{\mathscr{B}} \mathscr{E}] = 0,$$

where $r := \mathsf{ch}_0(\mathscr{E})$.

This identity is the analogue of the classical statement $t^2 = 0$ for $t := r - [E]$.

Modules associated to corepresentations

In the examples we are interested in, the inclusion $\mathscr{B} \subseteq \mathscr{A}$ is a Hopf-Galois

H is a Hopf algebra with bijective antipode,

$\mathscr{A}$ a right H-comodule algebra with coaction δ_R

$\mathscr{B} := \mathscr{A}^{\mathrm{co}H}$ the subalgebra of $\mathscr{A}$ of coinvariant elements, and assume that $\mathscr{B} \subset \mathscr{A}$ is a faithfully flat Hopf-Galois extension.

Associated bundles, via left H comodule (co-representations) trivialize when pulled back to the total space of a noncommutative principal bundle

Vector bundles on a quantum 4-sphere

Let $0 < q < 1$. The *quantum symplectic 7-sphere* is the noncommutative space dual the unital *-algebra $\mathcal{O}(S_q^7)$ generated by elements $\{x_i, x_i^*\}_{i=1,\dots,4}$ with commutation relations

$$x_1 x_2 = q x_2 x_1 \qquad \dots$$
$$\vdots$$

and sphere relations:

$$x_1 x_1^* + x_2 x_2^* + x_3 x_3^* + x_4 x_4^* = 1,$$
$$q^8 x_1^* x_1 + q^6 x_2^* x_2 + q^2 x_3^* x_3 + x_4^* x_4 = 1.$$

This algebra was studied in LPR 2006 as a quantum homogeneous space of the quantum symplectic group $\mathcal{O}(Sp_q(2))$.

A character $\varepsilon : \mathcal{O}(S^7_q) \to \mathbb{C}$ is given on generators by

$$\varepsilon(x_i) = 0 \quad \text{for } i \neq 4, \qquad \varepsilon(x_4) = 1 \, .$$

It is the restriction to $\mathcal{O}(S^7_q)$ of the counit of the quantum group $\mathcal{O}(Sp_q(2))$.

The quotient of $\mathcal{O}(S^7_q)$ by the ideal generated by x_1 is the unital *-algebra $\mathcal{O}(S^5_q)$ of a quantum 5-sphere DL2021.

The irreducible bounded *-representation of $\mathcal{O}(S^5_q)$ there when pulled back to $\mathcal{O}(S^7_q)$ gives a representation π on $\ell^2(\mathbb{N}^2)$, with canonical orthonormal basis $\big(|k_1, k_2\rangle \big)_{k_1,k_2 \in \mathbb{N}}$, that on generators reads:

$$\begin{aligned} \pi(x_1) &= 0 \\ \pi(x_2)\,|k_1, k_2\rangle &= q^{k_1+2k_2}\,|k_1, k_2\rangle \\ \pi(x_3)\,|k_1, k_2\rangle &= q^{k_1}\sqrt{1-q^{4(k_2+1)}}\,|k_1, k_2+1\rangle \\ \pi(x_4)\,|k_1, k_2\rangle &= \sqrt{1-q^{2(k_1+1)}}\,|k_1+1, k_2\rangle \, . \end{aligned} \tag{2}$$

The instanton bundle

The algebra $\mathcal{O}(S^7_q)$ carries a coaction of $\mathcal{O}(SU_q(2))$, which makes it a faithfully flat Hopf–Galois extension of its subalgebra of coinvariant elements.

$$T_{(1)} := \begin{pmatrix} \alpha & -q\gamma^* \\ \gamma & \alpha^* \end{pmatrix}$$

The generators of $\mathcal{O}(S^7_q)$ arranged in the matrix

$$u := \begin{pmatrix} qx_1 & qx_2 \\ -q^2x_2^* & q^3x_1^* \\ -x_3 & x_4 \\ x_4^* & qx_3^* \end{pmatrix} .$$

The subalgebra of $\mathcal{O}(S^7_q)$ made of coinvariant for the coaction is $\mathcal{O}(S^4_q)$.

Geometrically this is a quantum principal bundle on the quantum 4-sphere S^4_q with structure quantum group $SU_q(2)$ and total space S^7_q.

The $*$-algebra $\mathcal{O}(S_q^4)$ is generated by the entries of the matrix

$$p := uu^* = \begin{pmatrix} q^{-2}y_0 & 0 & y_1 & y_2 \\ 0 & y_0 & q^{-2}y_2^* & -q^2 y_1^t* \\ y_1^* & q^{-2}y_2 & 1-q^{-4}y_0 & 0 \\ y_2^* & -q^2 y_1 & 0 & 1-q^2 y_0 \end{pmatrix} , \tag{3}$$

where

$$y_0 := q^4(x_1x_1^* + x_2x_2^*) \qquad y_1 := -qx_1x_3^* + qx_2x_4^* \qquad y_2 := qx_1x_4 + q^2x_2x_3 \ .$$

The relations among generators are encoded in the equality $p^2 = p$.

The projection p in (3) determines a class $[\mathscr{E}]$ in the K-theory of $\mathcal{O}(S_q^4)$:

Geometrically the projection p describes the quantum vector bundle on the 4-sphere S_q^4 associated with the Hopf–Galois extension $\mathcal{O}(S_q^4) \subset \mathcal{O}(S_q^7)$ via the fundamental corepresentation $T_{(1)}$ of $\mathcal{O}(SU_q(2))$.

The K-theory ring

The C*-enveloping algebra of $\mathcal{O}(S^7_q)$ is isomorphic to the one of the Vaksman-Soibelman quantum 7-sphere $C(S^7_q)$. Its K-theory is $K_0\big(C(S^7_q)\big) \cong \mathbb{Z}$

$K_0\big(C(S^7_q)\big) \cong \mathbb{Z}$ is generated by the class of the unit.

(alway the case for these odd dimensional quantum spheres HL).

We let $C(S^4_q)$ be the C*-enveloping algebra of $\mathcal{O}(S^4_q)$, not the closure in $C(S^7_q)$.

The group $K_0\big(C(S^4_q)\big)$ is isomorphic to $\mathbb{Z}^2$.

When restricted to the subalgebra $\mathcal{O}(S^4_q)$, the character $\varepsilon : \mathcal{O}(S^7_q) \to \mathbb{C}$ reduces to the trivial representation, while π in (2) reads

$$\begin{aligned}
\pi(y_0)\,|k_1,k_2\rangle &= q^{4+2k_1+4k_2}\,|k_1,k_2\rangle \\
\pi(y_1)\,|k_1,k_2\rangle &= q^{k_1+2k_2}\sqrt{1-q^{2k_1}}\,|k_1-1,k_2\rangle \\
\pi(y_2)\,|k_1,k_2\rangle &= q^{2(k_1+k_2+2)}\sqrt{1-q^{4(k_2+1)}}\,|k_1,k_2+1\rangle \ .
\end{aligned}$$

For all $b \in \mathcal{O}(S^4_q)$, the operator $(\pi-\varepsilon)(b)$ is of trace class.

Let $\mathscr{E} = p\,(\mathcal{O}(S^4_q))^4$ be the right $\mathcal{O}(S^4_q)$-module of sections of the vector bundle $p = uu^*$.

Since u has 2 columns, $\mathsf{ch}_0(\mathscr{E}) = \varepsilon(\mathsf{Tr}\, p) = 2$.

Since $(\pi - \varepsilon)$ factors to the quotient $\mathcal{O}(S^4_q)/\mathbb{C}$, one computes

$$\mathsf{ch}_1(\mathscr{E}) = \mathsf{Tr}_{\mathcal{H}}\big(\pi(\mathsf{Tr}\, p) - \varepsilon(\mathsf{Tr}\, p)\big) = \frac{q^2 + q^4 - 1 - q^6}{(1-q^2)(1-q^4)} = -1.$$

The general theory then applies and

$K_0\big(C(S^4_q)\big)$ is a free $\mathbb{Z}$-module generated by [1] and [$\mathscr{E}$], and relation

$$4 - 4[\mathscr{E}] + [\mathscr{E} \otimes_{\mathcal{O}(S^4_q)} \mathscr{E}] = 0. \tag{4}$$

From the general theory we compute the characteristic classes of modules associated to irreducible corepresentations of $H := \mathcal{O}(SU_q(2))$.

These irreducible corepresentations are labelled by $n \in \mathbb{N}$. Let V_n be the vector space underlying the $n+1$ dimensional irreducible corepresentation, and call

$$\mathscr{E}_n := \mathcal{O}(S^7_q) \,\square^H\, V_n.$$

the associated $\mathcal{O}(S^4_q)$-bimodule. In particular, $\mathscr{E}_1 \cong \mathscr{E}$

For every $n \geq 1$,

$$\mathsf{ch}_1([\mathscr{E}_n]) = -\frac{1}{6} n(n+1)(n+2). \tag{5}$$

From the known decomposition $V_1 \otimes V_n \cong V_{n+1} \oplus V_{n-1}$, using (**??**) and (**??**), one gets the bimodule isomorphism

$$\mathscr{E}_1 \otimes_{\mathcal{O}(S^4_q)} \mathscr{E}_n \cong \mathscr{E}_{n+1} \oplus \mathscr{E}_{n-1}. \tag{6}$$

From (1) we get

$$\mathsf{ch}_1([\mathscr{E}_{n+1}]) = \mathsf{ch}_0([\mathscr{E}_1])\mathsf{ch}_1([\mathscr{E}_n]) + \mathsf{ch}_0([\mathscr{E}_n])\mathsf{ch}_1([\mathscr{E}_1]) - \mathsf{ch}_1([\mathscr{E}_{n-1}]).$$

Using $\mathsf{ch}_1([\mathscr{E}_1]) = -1$, and $\mathsf{ch}_0([\mathscr{E}_n]) = \dim(V_n) = n + 1$, we find

$$\mathsf{ch}_1([\mathscr{E}_{n+1}]) = 2\mathsf{ch}_1([\mathscr{E}_n]) - n - 1 - \mathsf{ch}_1([\mathscr{E}_{n-1}]).$$

Formula (5) follows by induction on $n \geq 1$.

A special case of (6) is the isomorphism $\mathscr{E}_1 \otimes_{\mathcal{O}(S^4_q)} \mathscr{E}_1 \cong \mathscr{E}_2 \oplus \mathscr{E}_0$,

The free bimodule $\mathscr{E}_0$ is the analogue of the determinant line bundle of $\mathscr{E}_1$. With an abuse of notations we can denote it by $\mathscr{E}_1 \wedge_{\mathcal{O}(S^4_q)} \mathscr{E}_1$.

Then, the relation (4) can be interpreted as the vanishing of the "square" of the Euler class in K-theory of the instanton bundle on S^4_q given by:

$$\chi(\mathscr{E}_1) = 1 - [\mathscr{E}_1] + [\mathscr{E}_1 \wedge_{\mathcal{O}(S^4_q)} \mathscr{E}_1] = 2 - [\mathscr{E}_1].$$

Again this parallel the classical result.

Projections from corepresentations of $SU_q(2)$

One constructs explicitly a trivializing pairs and then projections describing the vector bundles on the quantum 4-sphere S_q^4 associated to finite-dimensional irreducible corepresentations of the Hopf algebra $\mathcal{O}(SU_q(2))$.

thanks you