Supersymmetric Poisson sigma models, revisited

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Joint work with **Athanasios Chatzistavrakidis** and **Sylvain Lavau** [2504.13114]

Motivation and synopsis

The Poisson sigma model [Ikeda, 1993] is the most general sigma model of **AKSZ**-type [Alexandrov–Kontsevich–Schwarz–Zaboronsky, 1995] in **two dimensions**, The Poisson sigma model [Ikeda, 1993] is the most general sigma model of **AKSZ**-type [Alexandrov–Kontsevich–Schwarz–Zaboronsky, 1995] in **two dimensions**, and simply reads

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More precisely, one recovers the formula for the **star-product** on $\mathbb{R}^d$ equipped with an *arbitrary* Poisson structure Π, as a **correlation function** on the disc,

$$f \star g = \big\langle f(X)\, g(X) \big\rangle_{\mathrm{PSM}} = f \cdot g + \tfrac{i\hbar}{2}\{f,g\} + \mathcal{O}(\hbar^2) \,,$$

with $f, g \in \mathcal{C}^\infty(\mathbb{R}^d)$ two functions in target space.

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Synopsis

Explore the 'usual' geometric structures encoded by a Poisson supermanifold equipped with a compatible **odd** homological vector field.
As a by-product, we derive a new example of **super-Poisson sigma model** based on the **coadjoint representation up to homotopy** of a Poisson manifold.

Plan of the talk

1. Introduction: Poisson supermanifolds

2. Super-Poisson sigma model and coadjoint ruth

3. Outlook

Introduction: Poisson supermanifolds

Poisson supermanifolds

Consider a supermanifold $\mathcal{M}$ with even coordinates x^μ and odd ones θ^a. It is (non-canonically) diffeomorphic to a **parity-shifted vector bundle**, i.e. $\mathcal{M} \cong \Pi\mathcal{E}$ for $\mathcal{E} \twoheadrightarrow M$ a vector bundle over the *body* M of $\mathcal{M}$ [Batchelor, 1977].

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If its algebra of functions is a Poisson superalgebra, i.e. $\mathcal{C}^\infty(\mathcal{M}) \cong \Gamma(\wedge\mathcal{E}^*)$ is equipped with an $\mathbb{R}$-bilinear **graded-antisymmetric, even**, bracket

$$\{-,-\} : \mathcal{C}^\infty(\mathcal{M}) \wedge \mathcal{C}^\infty(\mathcal{M}) \longrightarrow \mathcal{C}^\infty(\mathcal{M}) \,,$$

obeying the **Leibniz rule** and **Jacobi identity**, then $\big(\mathcal{M}, \{-,-\}\big)$ is called a **Poisson supermanifold**.

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Equivalently, it means that $\mathcal{M}$ is equipped with a Poisson bivector

$$\mathcal{P} \in \Gamma(\wedge^2 T\mathcal{M}) \qquad \text{such that} \qquad \{f,g\} = \mathcal{P}(\mathrm{d}f \wedge \mathrm{d}g) \,, \qquad \forall\, f,g \in \mathcal{C}^\infty(\mathcal{M}) \,,$$

i.e. its components locally takes the form

$$\{x^\mu, x^\nu\} = \mathcal{P}^{\mu\nu}(x,\theta^2) \,, \quad \{x^\mu, \theta^a\} = \theta^b\, \mathcal{P}_b^{\mu a}(x,\theta^2) \,, \quad \{\theta^a, \theta^b\} = \mathcal{P}^{ab}(x,\theta^2) \,,$$

where the notation θ^2 is meant to highlight the fact that the various components appearing above depend on **quadratic combinations** of the odd coordinates—the components are **even**.

Generic case

We can expand the components of the super-Poisson bivector **in powers of θ**,

$$\mathcal{P}^{\mu\nu} = \Pi^{\mu\nu}(x) + \tfrac{1}{2}\theta^a\theta^b\mathcal{P}_{ab}{}^{\mu\nu}(x) + \dots\,,$$
$$\mathcal{P}^{\mu a} = \theta^b\Gamma^{\mu a}_b(x) + \dots\,,$$
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- The components g^{ab} are that of a section $g \in \Gamma(S^2\mathcal{E}^*)$ which **preserved** by the contravariant connection.

Super-Poisson sigma model and coadjoint ruth

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The pair (X^μ, θ^a) originates from coordinates on $\mathcal{M} \cong \mathcal{E}[0,1]$, and (A_μ, χ_a) are their momenta, i.e. $\omega_{T^*\mathcal{M}} = \mathrm{d}X^\mu \wedge \mathrm{d}A_\mu + \mathrm{d}\theta^a \wedge \mathrm{d}\chi_a$.

Action and gauge (super)symmetry

The action principle takes the relatively simple form [Ikeda, 1993]

$$S[X, A, \theta, \chi] = \int_\Sigma A_\mu \wedge \mathrm{d}X^\mu + \chi_a \wedge \mathrm{d}\theta^a + \tfrac{1}{2}\,\mathcal{P}^{\mu\nu}(X, \theta^2)\, A_\mu \wedge A_\nu$$
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and, by virtue of the fact that $\mathcal{P}$ is a **super-Poisson bivector**, is invariant under the gauge transformations

$$\delta_\varepsilon X^\mu = \varepsilon_\nu \mathcal{P}^{\nu\mu} + \varepsilon_a \theta^b \mathcal{P}_b^{\mu a}\,,$$
$$\delta_\varepsilon A_\mu = \mathrm{d}\varepsilon_\mu + A_\nu \partial_\mu \mathcal{P}^{\nu\rho}\varepsilon_\rho + \varepsilon_\nu \chi_a \partial_\mu \mathcal{P}^{\nu|a} + \varepsilon_a A_\nu \partial_\mu \mathcal{P}^{\nu|a} - \varepsilon_a \chi_b \partial_\mu \mathcal{P}^{ba}\,,$$
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where ε_μ and ε_a are **respectively bosonic** and **fermionic** gauge parameters.

In particular, the transformations generated by the **fermionic** ones correspond to **local supersymmetry**.

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$$S[X, A, \theta, \chi] = \int_\Sigma A_\mu \wedge \mathrm{d}X^\mu + \tfrac{1}{2}\,\Pi^{\mu\nu}\, A_\mu \wedge A_\nu \\ + \chi_\mu \wedge \nabla\theta^\mu + \tfrac{1}{4}\theta^\mu\theta^\nu\Pi^{\rho(\kappa}\,\overline{R}_{\mu\nu}{}^{\lambda)}{}_\rho\,\chi_\kappa \wedge \chi_\lambda\,,$$

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where

$$\nabla\chi_\mu := \mathrm{d}\theta^\mu + \mathrm{d}X^\kappa\,\Gamma^\mu_{\kappa\lambda}\,\theta^\lambda\,,$$

for ∇ any connection on M, *not necessarily torsion-free*, and $\overline{R}_{\kappa\lambda}{}^\nu{}_\rho$ the components of the curvature of the connection $\overline{\nabla}$ defined by

$$\overline{\nabla}_X Y = \nabla_Y X + [X,Y]\,, \qquad X,Y \in \Gamma(TM)\,, \qquad \text{i.e.} \qquad \overline{\Gamma}^\lambda_{\mu\nu} = \Gamma^\lambda_{\nu\mu}\,,$$

called the **basic connection** [See Thanasis Chatzistavrakidis' talk].

De Rham supersymmetry

This model enjoys an interesting **global supersymmetry**,

$$\delta_S X^\mu = \theta^\mu\,, \qquad \delta_S \theta^\mu = 0\,,$$

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Put differently, this **global supersymmetry** follows from the **compatibility** between the Poisson bracket on $\mathcal{C}^\infty(T[0,1]M) \cong \Omega_M$ and the de Rham differential on M.

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Put differently, this **global supersymmetry** follows from the **compatibility** between the Poisson bracket on $\mathcal{C}^\infty(T[0,1]M) \cong \Omega_M$ and the de Rham differential on M.

Are there more examples, and what is the organising principle behind them?

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- $[0,1]$: The cotangent lift $\mathcal{Q}_S$ of an **odd** homological vector field $q_S\in\Gamma(T\mathcal{M})$ on the **base supermanifold**, i.e.

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  In other words, $(T^*[1,0]\mathcal{M},\mathcal{Q}_S)$ is a $\mathcal{Q}$-bundle [Kotov–Strobl, 2007].

Coadjoint representation up to homotopy

Famously, a **Lie algebroid** $\left(\mathcal{E} \twoheadrightarrow M, \mathcal{E} \xrightarrow{t} TM, [-,-]_{\mathcal{E}}\right)$ is equivalent to a **supermanifold** equipped with an **odd homological vector field** of homogeneity one, $\left(\mathcal{E}[0,1], q_S = \theta^a {t_a}^\mu \frac{\partial}{\partial x^\mu} - \frac{1}{2}\theta^b\theta^c {C_{bc}}^a \frac{\partial}{\partial \theta^a}\right)$ [Vaintrob, 1997], and their **representation up to homotopy** correspond to $\mathcal{Q}$-bundles over $\mathcal{E}[0,1]$ [Abad–Crainic, 2009].

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into a $\mathcal{Q}$-bundle. Concretely, its homological vector field read

$$\begin{aligned}\mathcal{Q}_S = \theta^a t_a{}^\mu \frac{\partial}{\partial x^\mu} - \frac{1}{2}\theta^b\theta^c C_{bc}{}^a \frac{\partial}{\partial \theta^a} \\ + \left(\theta^a \overline{\Gamma}^\nu_{a\mu} a_\nu - \frac{1}{2}\theta^a\theta^b S_{ab\mu}{}^c \chi_c\right)\frac{\partial}{\partial a_\mu} + \left(-t_a{}^\mu a_\mu + \theta^b \overline{\Gamma}^c_{ba}\chi_c\right)\frac{\partial}{\partial \chi_a},\end{aligned}$$

where $\overline{\Gamma}^a_{bc}$ and $\overline{\Gamma}^\nu_{a\mu}$ are the components of the **basic connection**, and $S_{ab\mu}{}^c$ that of the **basic curvature**.

$\mathcal{Q}_S$-invariant super-Poisson structure

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- Another solution that presents itself is the **cotangent Lie algebroid** $\mathcal{E} = T^*M$, in which case the anchor is the Poisson bivector on the base, $t_a{}^{\mu} \rightsquigarrow \Pi^{\alpha\mu}$, so that we can also read the above condition as a definition (modulo the kernel of $\Pi^{\#}$).

This leads to the *'contravariant'* version of the ABST-G model, whose action reads

$$S[X,A,\theta,\chi] = \int_\Sigma A_\mu \wedge \mathrm{d}X^\mu + \chi^\mu \wedge \nabla\theta_\mu + \tfrac{1}{2}\Pi^{\mu\nu} A_\mu \wedge A_\nu - \tfrac{1}{4}\theta_\kappa\theta_\lambda S^{\kappa\lambda}{}_{\mu\nu}\chi^\mu \wedge \chi^\nu\,,$$

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In particular, we see that it generate what could be called a **Poisson supersymetry** (regardless of the regularity of Π).

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Thank you for your attention!