Emergence of expanding (3+1)-dimensional spacetime in the type IIB matrix model

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Present status of superstring theory

Superstring theory is a promising candidate for a unified theory including quantum gravity

1． Perturbation theory with D-branes including a part of nonperturbative effects

→ numerous (meta-)stable vacua with different spacetime dimensionalities, gauge groups, matter contents and cosmological constants

In particular, 3+1 dimensions are not required, for instance 2+1 and 4+1 are possible

2． Singularity at the beginning of the Universe → Not resolved in perturbation theory

Liu-Moore-Seiberg (2002)

⇒ We need a nonperturbative formulation of superstring theory

Type IIB matrix model

Ishibashi Kawai, Kitazawa, AT (1996)

Proposed as a nonperturbative formulation of superstring theory

$$S = -N\mathrm{Tr}\left(\frac{1}{4}[A_\mu, A_\nu][A^\mu, A^\nu] + \frac{1}{2}\bar{\psi}\Gamma^\mu[A_\mu, \psi]\right)$$

$N \times N$ Hermitian matrices

A_μ : 10D Lorentz vector $(\mu = 0, 1, \ldots, 9)$

ψ : 10D Majorana-Weyl spinor

SO(9,1) symmetry

The action takes the form of the dimensional reduction of 10D N=1 SYM.

Space-time does not exist a priori,
but emerges from the degrees of freedom of matrices.

→ Dimensionality of space-time can be predicted

Crucial properties: 10D N=2 SUSY

$$Q^{(1)} \begin{cases} \delta^{(1)} A_\mu = i\bar{\epsilon}_1 \Gamma_\mu \psi \\ \delta^{(1)} \psi = \frac{i}{2} \Gamma^{\mu\nu} [A_\mu, A_\nu] \epsilon_1 \end{cases}$$

dimensional reduction of
10D N=1 SUSY

$$Q^{(2)} \begin{cases} \delta^{(2)} A_\mu = 0 \\ \delta^{(2)} \psi = \epsilon_2 1_N \end{cases}$$

$$P_\mu \begin{cases} \delta_{\mathrm{T}} A_\mu = c_\mu 1_N \\ \delta_{\mathrm{T}} \psi = 0 \end{cases}$$

$$\begin{cases} \tilde{Q}^{(1)} = Q^{(1)} + Q^{(2)} \\ \tilde{Q}^{(2)} = i(Q^{(1)} - Q^{(2)}) \end{cases} \Longrightarrow [\bar{\epsilon}_1 \tilde{Q}^{(i)}, \bar{\epsilon}_2 \tilde{Q}^{(j)}] = -2\delta^{ij} \bar{\epsilon}_1 \Gamma^\mu \epsilon_2 P_\mu$$

10D N=2 SUSY if P_μ is identified
with momentum, which generates shift of A_μ

The space-time is represented as the eigenvalue distribution of A_μ .

The fact that the model has maximal SUSY suggests strongly that the model includes gravity.

Crucial properties: connection to the world sheet action

Green-Schwarz action of Schild-type for type IIB superstring with κ symmetry fixed

$$S_{\mathrm{S}} = \int d\tau d\sigma \sqrt{-g}\left[\frac{1}{4}\{X_\mu, X_\nu\}\{X^\mu, X^\nu\} - \frac{i}{2}\bar{\Psi}\Gamma^\mu\{X_\mu, \Psi\}\right]$$

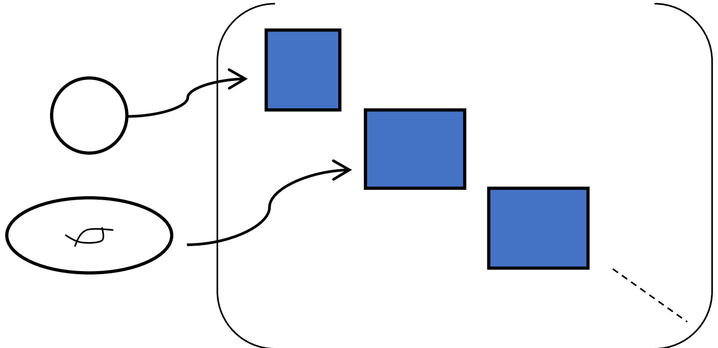
$$\{X, Y\} = \frac{1}{\sqrt{-g}}\left(\frac{\partial X}{\partial \tau}\frac{\partial Y}{\partial \sigma} - \frac{\partial X}{\partial \sigma}\frac{\partial Y}{\partial \tau}\right)$$

matrix regularization

$$\begin{cases} X_\mu(\tau,\sigma) \to A_\mu \\ \Psi(\tau,\sigma) \to \psi \\ \{\ ,\ \} \to \frac{1}{i}[\ ,\ ] \\ \int d\tau d\sigma \to \mathrm{Tr} \end{cases}$$

type IIB matrix model

$$S = -N\mathrm{Tr}\left(\frac{1}{4}[A_\mu, A_\nu][A^\mu, A^\nu] + \frac{1}{2}\bar{\psi}\Gamma^\mu[A_\mu, \psi]\right)$$



multi strings

2nd quantized

Crucial properties　(cont'd)

- Long distance behavior of interaction between D-branes is reproduced.

- Light-cone string field theory for type IIB superstring is reproduced from SD equations for Wilson loops under reasonable assumptions.

Fukuma-Kawai-Kitazawa-AT（1997）

Plan of the present talk

✓ 1. Introduction

2. Lorentzian vs Euclidean

3. How to investigate the model

4. Results of numerical simulations

5. Summary and outlook

Lorentzian vs Euclidean

Euclidean model

$$Z = \int dA \; e^{-S} = \int dA \; \mathrm{Pf}\mathcal{M}_{\mathrm{E}}(A) \; e^{-S_{\mathrm{b}}}$$

connection to worldsheet theory

$$S_{\mathrm{b}} = \frac{N}{4} \sum_{\mu,\nu=0}^{9} \mathrm{Tr}(-[A_\mu, A_\nu]^2)$$: positive semi-definite

SO(10) symmetry

$\mathrm{Pf}\mathcal{M}_{\mathrm{E}}(A)$: complex ⟹ sign problem

The Euclidean model is well-defined without cutoff.

Krauth, Nicolai, Staudacher ('98) Austing, Wheater ('01)

Numerical simulations showed SSB of SO(10) to SO(3) due to less fluctuations of the complex phase of Pfaffian for lower dimensions

Nishimura, Vernizzi (2000) Anagnostopoulos, Azuma, Ito, Nishimura, Okubo, Papadoudis (2020)

3D space emerges, but time does not emerge ⟹ study the Lorentzian model

Partition function of Lorentzian model

Kim-Nishimura-AT (2011)

$$S = -N\mathrm{Tr}\left(\frac{1}{4}[A_\mu, A_\nu][A^\mu, A^\nu] + \frac{1}{2}\bar{\psi}\Gamma^\mu[A_\mu, \psi]\right)$$

$$Z = \int dA d\psi \; e^{iS} = \int dA \; \mathrm{Pf}\mathcal{M}(A) \; e^{iS_b}$$

phase factor ⇒ sign problem

polynomial in A_μ , real

connection to worldsheet theory

$$S_b = -\frac{N}{4}\mathrm{Tr}\left([A^\mu, A^\nu][A_\mu, A_\nu]\right)$$

The model is not well-defined as it is

We need a regularization

Partition function of Lorentzian model (cont'd)

$$Z = \int dA d\psi \; e^{iS+iS_m-S_{\rm gf}} \Delta_{\rm FP}$$

We introduce a Lorentz invariant mass term as an infrared regulator

Hatakeyama, Matsumoto, Nishimura, AT, Yosprakob (2020)

$$S_m = -\frac{1}{2} N\gamma \mathrm{Tr}(A_\mu A^\mu) = \frac{1}{2} N\gamma (e^{i\epsilon}\mathrm{Tr}(A_0^2) - e^{-i\epsilon}\sum_{i=1}^{9}\mathrm{Tr}(A_i^2))$$

$\gamma > 0$ $\quad \epsilon \to 0$

Gauge-fixing of Lorentz symmetry ⬅ gauge-volume of Lorentz symmetry is infinite

$$\Delta_{\rm FP} = \det \Omega, \quad \Omega_{ij} = \mathrm{Tr}(A_0)^2\delta_{ij} + \mathrm{Tr}(A_i A_i)$$

Asano, Piensuk, Nishimura, Yamamori (2014)

$$S_{\rm gf} = \frac{\alpha}{2}(\mathrm{Tr}(A_0 Ai))^2$$

In what follows, for simplicity, we perform a Lorentz transformation on the sampled configurations to remove the artifacts caused by the Lorentz boosts, instead of considering the above gauge fixing

Classical solutions

Hatakeyama, Matsumoto, Nishimura, AT, Yosprakob (2020)

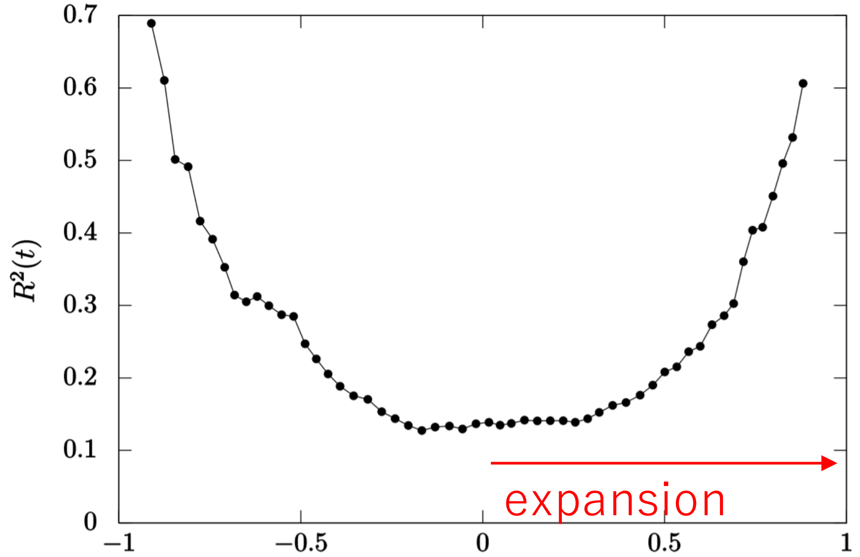
$$Z = \int dA\, e^{i(A^4+\gamma A^2)} \sim \int d\tilde{A}\, e^{i\gamma^2(\tilde{A}^4+\tilde{A}^2)} \qquad A_\mu = \sqrt{\gamma}\tilde{A}_\mu \qquad \gamma^2 \leftrightarrow \frac{1}{\hbar}$$

Classical solutions dominate at large γ .

Classical EOM $\quad [A^\nu,[A_\nu,A_\mu]] = \gamma A_\mu$

$A_\mu = 0$ is always a solution (trivial saddle).

Typical solutions exhibit expanding behavior for $\gamma > 0$ (non-trivial saddles).



But space-time dimensionality is not fixed at the classical level.

Those solutions are hermitian so that they reside on the original contour before deformation (relevant saddles in the Picard-Lefshetz theory, which contribute to summation over saddles). We expect that non-trivial saddles are more dominant than the trivial one due to large entropy when N is large.

How to investigate the model

Complex Langevin method

Parisi (1983), Klauder (1984)

We use the complex Langevin method to overcome the sign problem.

We take the gauge in which A_0 is diagonal.

We make a change of variables to introduce a time-ordering preserving holomorphy

$$\alpha_1 = 0,\ \alpha_2 = e^{\tau_1},\ \alpha_3 = e^{\tau_1} + e^{\tau_2}, \ldots, \alpha_N = \sum_{a=1}^{N-1} e^{\tau_a} \qquad \alpha_1 < \alpha_2 < \ldots < \alpha_N$$

Nishimura, AT (2019)

➡ complexify τ_a and A_i

complex Langevin equation

$$\begin{cases} \dfrac{d\tau_a}{dt_L} = -\dfrac{\partial S_{\rm eff}}{\partial \tau_a} + \eta_a(t_L) \\ \dfrac{d(A_i)_{ab}}{dt_L} = -\dfrac{\partial S_{\rm eff}}{\partial (A_i)_{ba}} + (\eta_i(t_L))_{ab} \end{cases}$$

$$S_{\rm eff} = (\tilde{S}_{\rm b} + \tilde{S}_{\rm m}) - \log {\rm Pf}\mathcal{M}(A) - 2\log\Delta(\alpha) - \sum_{a=1}^{N-1}\tau_a$$

➡ zero eigenvalue of ${\rm Pf}\mathcal{M}(A)$

singular drift problem

t_L : Langevin time ~discretized in practice

η_a, η_i : Gaussian noises

expectation value of holomorphic observables can be calculated by taking samples around sufficiently large t_L

Avoiding the singular drift problem

To avoid the singular drift problem, we add a mass term to the fermionic action.

$$S_f = -\frac{N}{2}\mathrm{Tr}\left(\bar{\psi}\Gamma^{\mu}[A_{\mu}, \psi] + i m_f \bar{\psi}\Gamma^7 \Gamma^{8\dagger}\Gamma^9 \psi\right)$$

$m_f = \infty$: bosonic

$m_f = 0$: SUSY

The effect of fermions is weakened for finite m_f

m_f should be as small as possible

Controlling the quantum fluctuation of bosonic matrices

$$S_m = \frac{1}{2} N\gamma \mathrm{Tr}\left(\mathrm{Tr}(A_0)^2 - \sum_{i=1}^{d} \mathrm{Tr}(A_i)^2 - \xi \sum_{i=d+1}^{9} \mathrm{Tr}(A_i)^2 \right)$$

d , ξ : parameters that can control the quantum fluctuations of bosonic matrices

For large ξ , the bosonic degrees of freedom reduces effectively to $(d+1)$ -dimensional one.

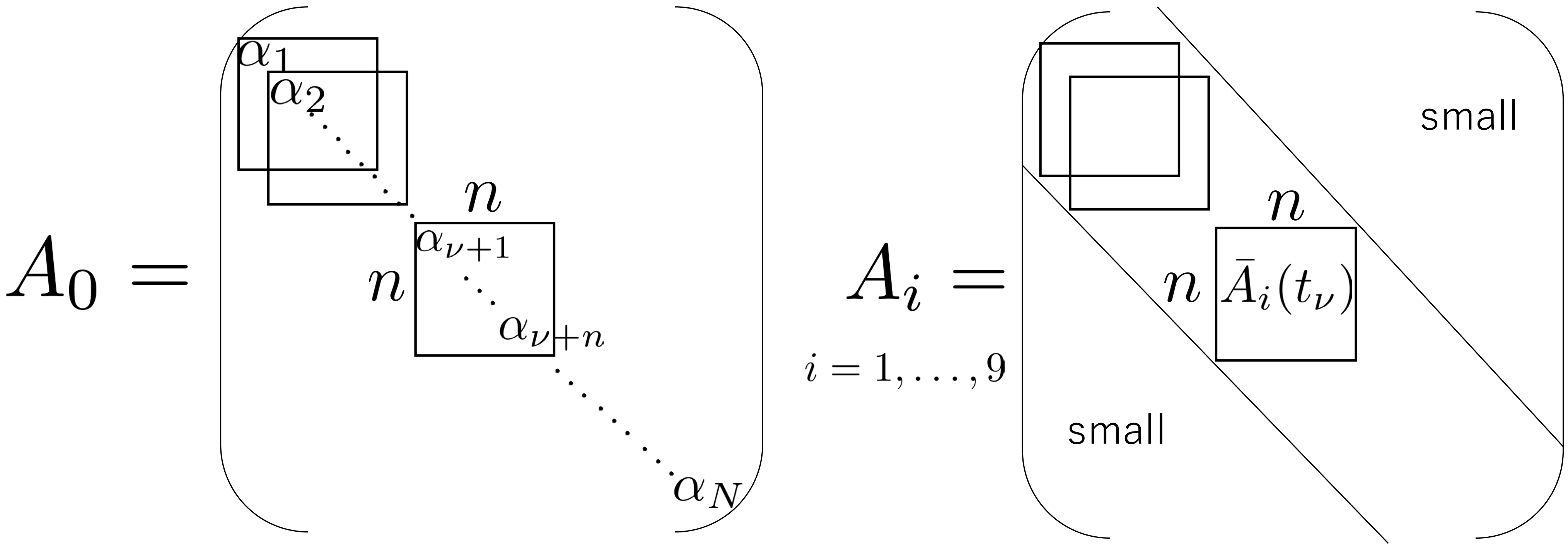
By choosing d and ξ appropriately, we can expect to realize a situation which is close to one where SUSY is respected.

Eventually, we want to take $m_f \to 0,\ \xi \to 1,\ N \to \infty,\ \gamma \to 0$ ➡ target theory

Extracting the time evolution

Kim-Nishimura-AT (2011)

We take the gauge in which A_0 is diagonal.



The state of the universe at time t_ν

definition of time

$$\bar{\alpha}_k = \frac{1}{n}\sum_{i=1}^{n} \alpha_{k+i} \in \mathbb{C}, \ t_\rho = \sum_{k=1}^{\rho} |\bar{\alpha}_{k+1} - \bar{\alpha}_k|$$

A_i has band-diagonal structure, which is nontrivial dynamical property.

locality of time is guaranteed.

~ emergence of time evolution

Removing the effect of Lorentz boost

We choose a Lorentz frame by minimizing $\mathcal{T} = \mathrm{Tr}\left(A_0^\dagger A_0\right)$ w.r.t. Lorentz transformations on each sampled configuration

We perform the (1+1)-dimensional Lorentz transformation

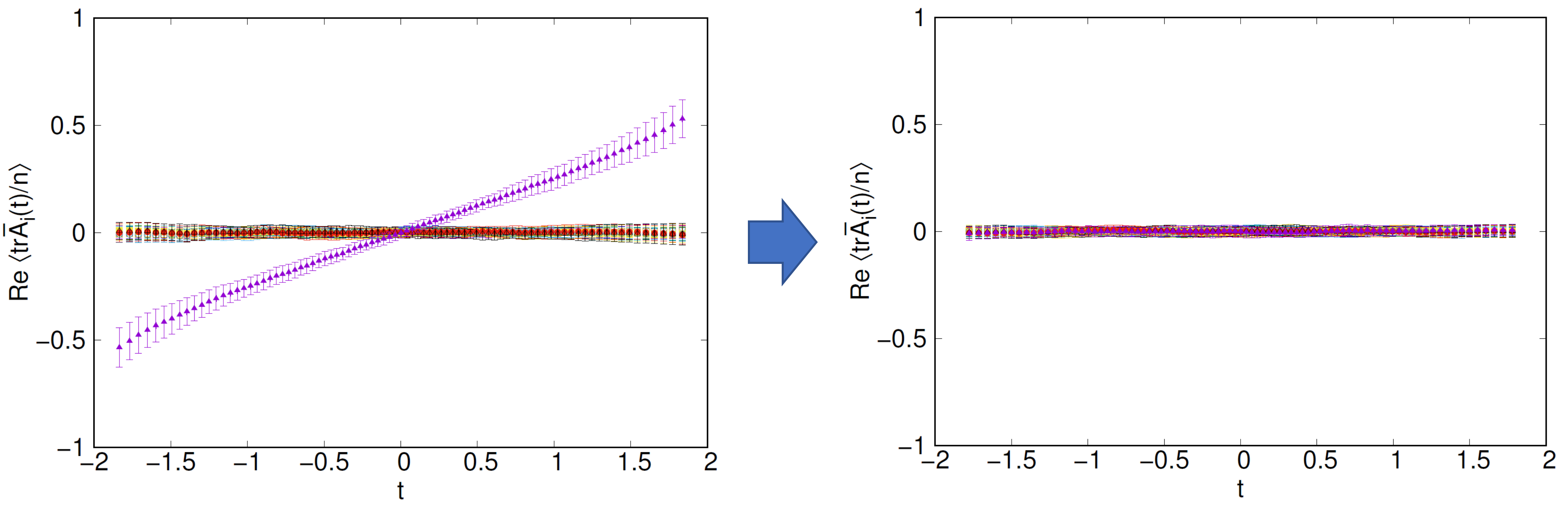
$$\begin{pmatrix} A_0' \\ A_i' \end{pmatrix} = \begin{pmatrix} \cosh\sigma & \sinh\sigma \\ \sinh\sigma & \cosh\sigma \end{pmatrix} \begin{pmatrix} A_0 \\ A_i \end{pmatrix} \qquad i = 1, \ldots, 9$$

iteratively in such a way that $\mathcal{T}$ is minimized w.r.t. σ at each step

Removing the effect of Lorentz boost（cont'd)

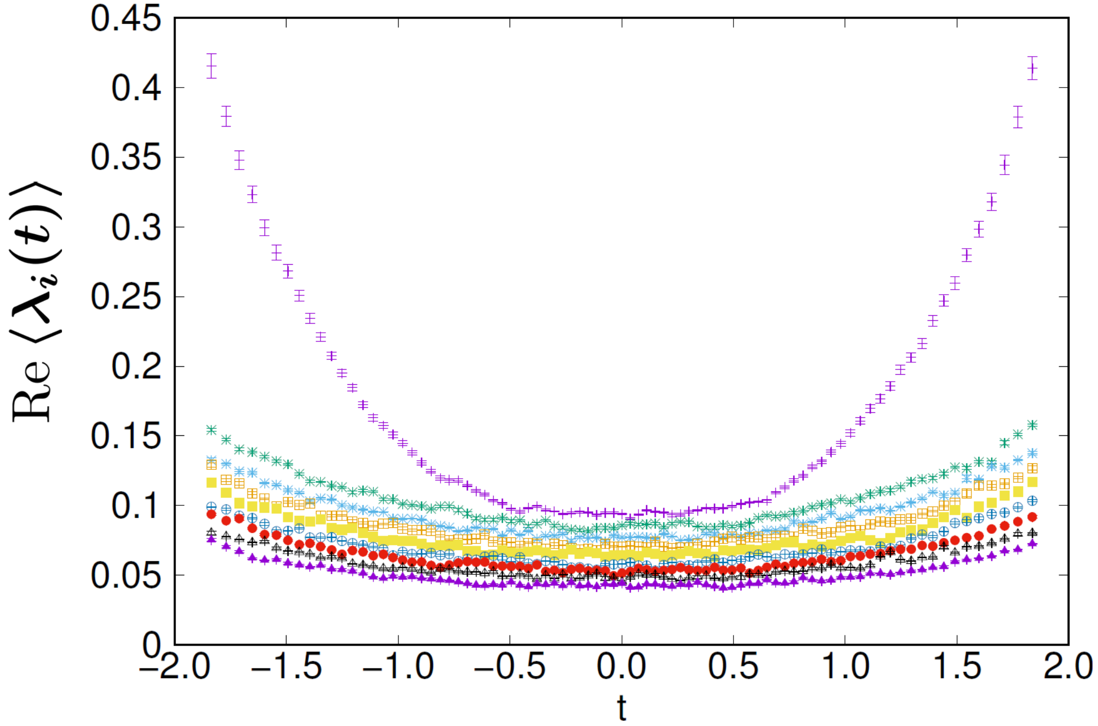
"center of mass" at each time $\frac{1}{n}\mathrm{tr}(\bar{A}_i(t))$ $i = 1, \ldots, 9$

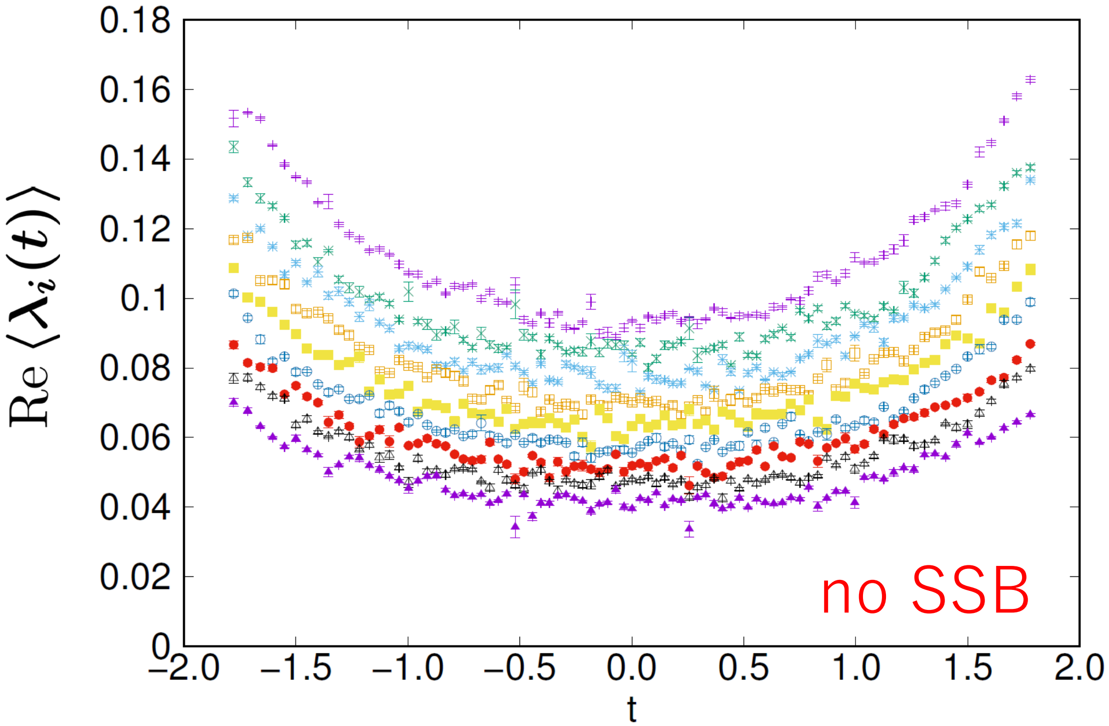
bosonic model $N = 96,\ \gamma = 4$



Removing the effect of Lorentz boost (cont'd)

bosonic model $N = 96,\ \gamma = 4$





$\lambda_i(t)$ $(i = 1, \ldots, 9)$: eigenvalues of $T_{ij}(t) = \frac{1}{n} \mathrm{tr}(\bar{A}_i(t) \bar{A}_j(t))$

analog of moment of inertia tensor

SUSY (effects of fermions) is important to obtain (3+1)-dimensional spacetime

Results of numerical simulations

Set-up

We include fermions and put $d = 5$

$$S_m = \frac{1}{2} N\gamma \mathrm{Tr}\left(\mathrm{Tr}(A_0)^2 - \sum_{i=1}^{d} \mathrm{Tr}(A_i)^2 - \xi \sum_{i=d+1}^{9} \mathrm{Tr}(A_i)^2 \right)$$

Important questions

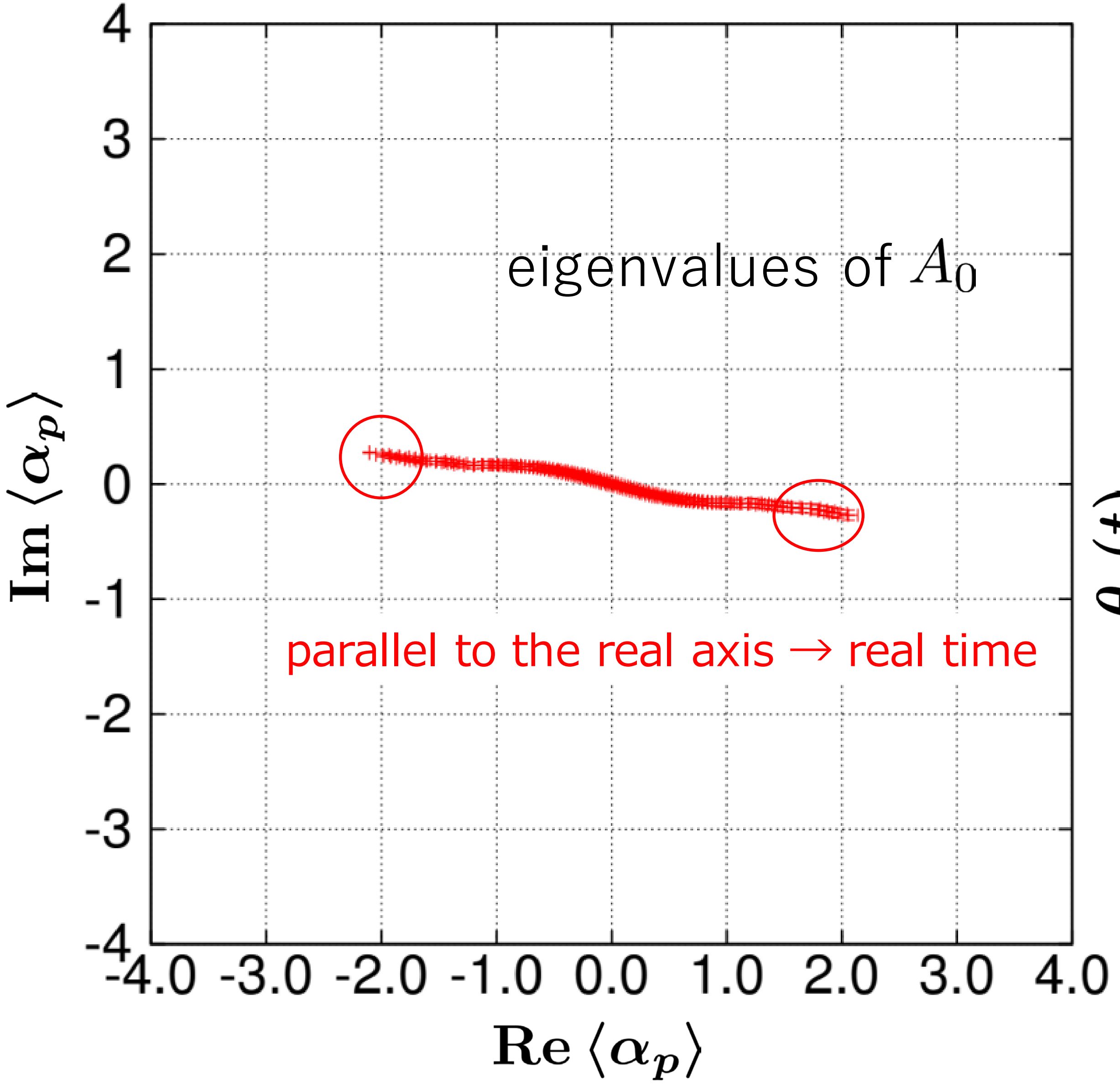
Is real spacetime obtained?

Spacetime dimensionality?

← weight e^{iS} is complex

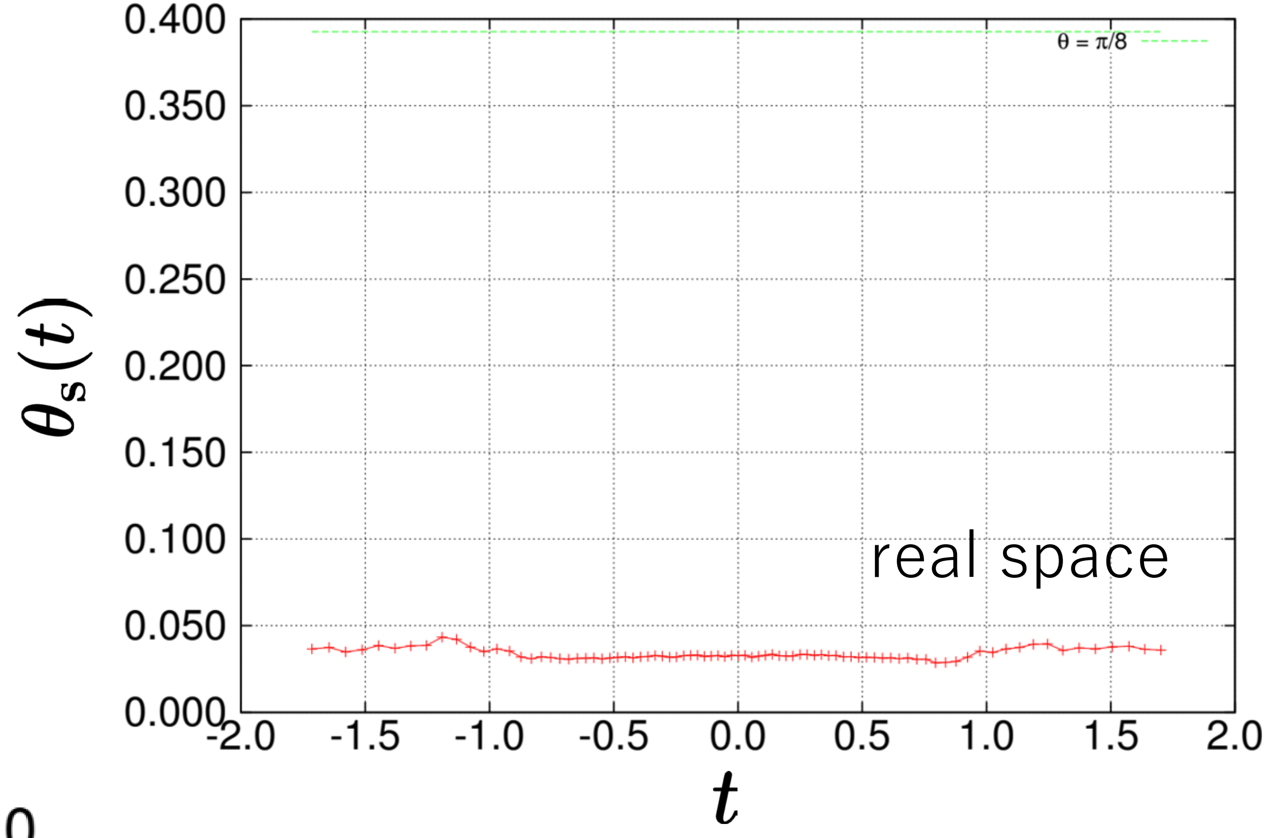
Emergence of real spacetime

$N = 96,\ n = 12,\ \gamma = 4,\ m_{\mathrm{f}} = 3.5,\ \xi = 16$



complex phase of space

$$\left\langle \sum_{i=1}^{9} \frac{1}{n} \mathrm{tr} \left(\bar{A}_i(t) \right)^2 \right\rangle \sim e^{2i\theta_{\mathrm{s}}(t)}$$

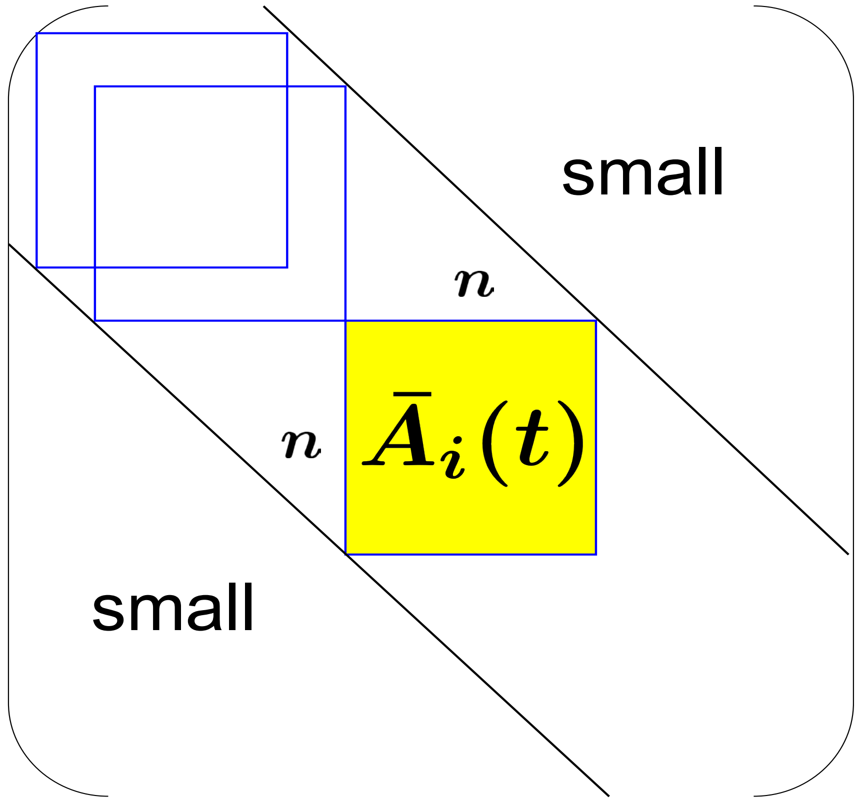


Emergence of real spacetime

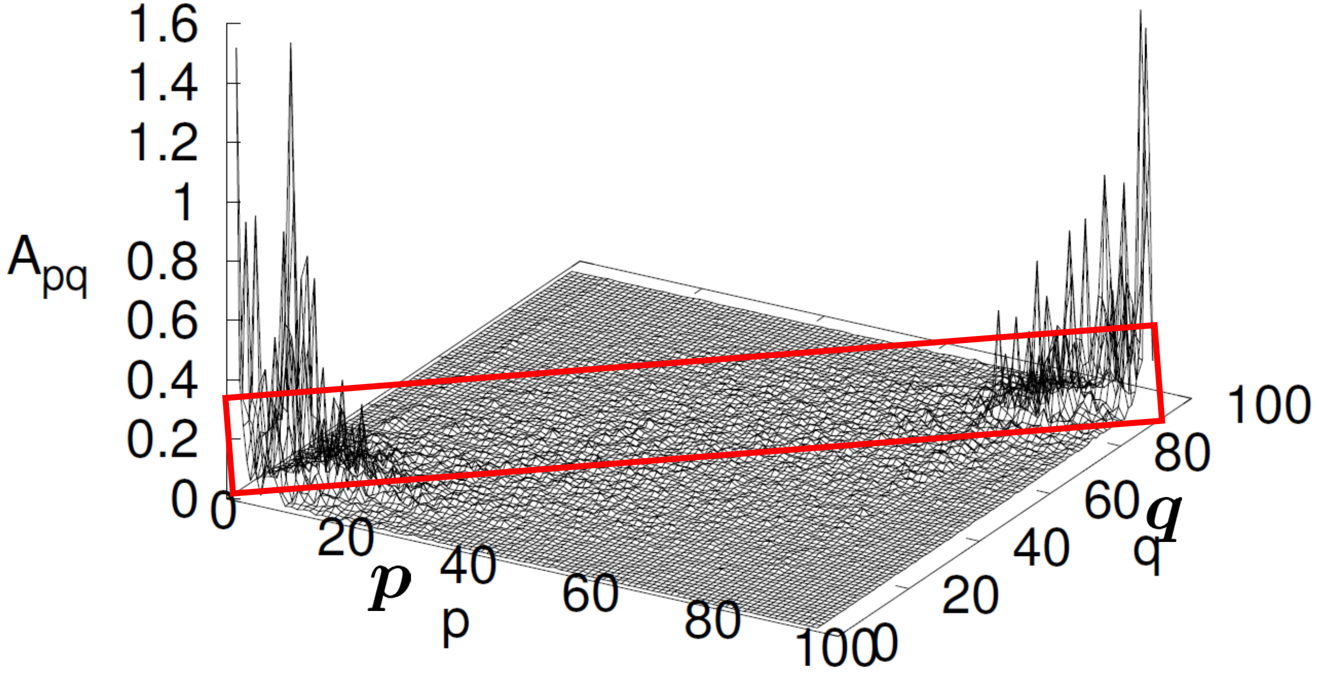
Band-diagonal structure

$$N = 96,\ n = 12,\ \gamma = 4,\ m_{\mathrm{f}} = 3.5,\ \xi = 16$$

$\boldsymbol{A_i} =$



$$\mathcal{A}_{pq} = \frac{1}{9}\sum_{i=1}^{9} \left|(A_i)_{pq}\right|^2 \quad (1 \leq p, q \leq N = 96)$$



Emergence of (3+1)-dimensional expanding spacetime

$N = 96,\ n = 12,\ \gamma = 4,$

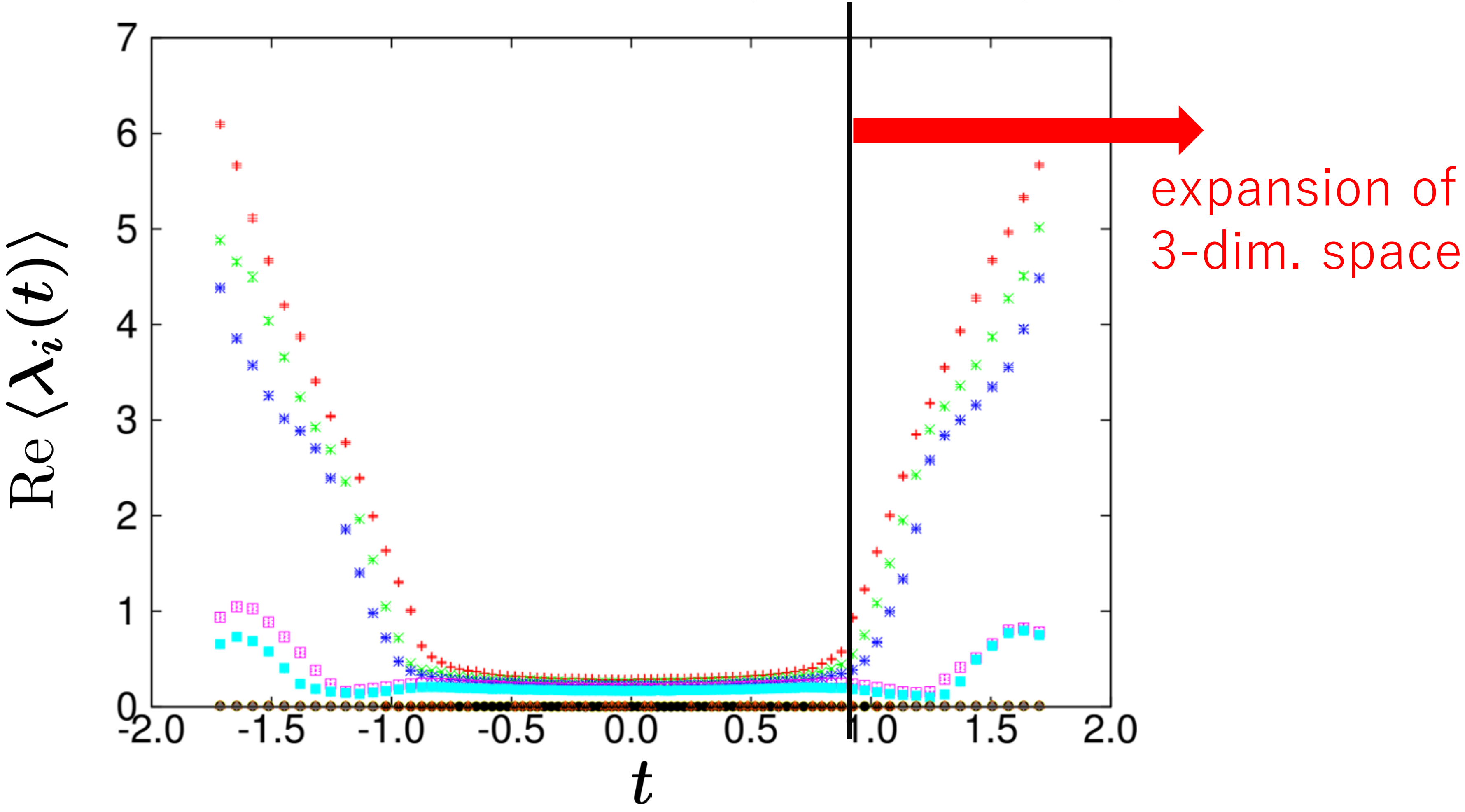
$m_{\rm f} = 3.5,\ \xi = 16$

$\lambda_i(t)\ (i = 1, \ldots, 9)$

eigenvalues of

$$T_{ij}(t) = \frac{1}{n}\mathrm{tr}(\bar{A}_i(t)\bar{A}_j(t))$$

analog of moment of inertia tensor

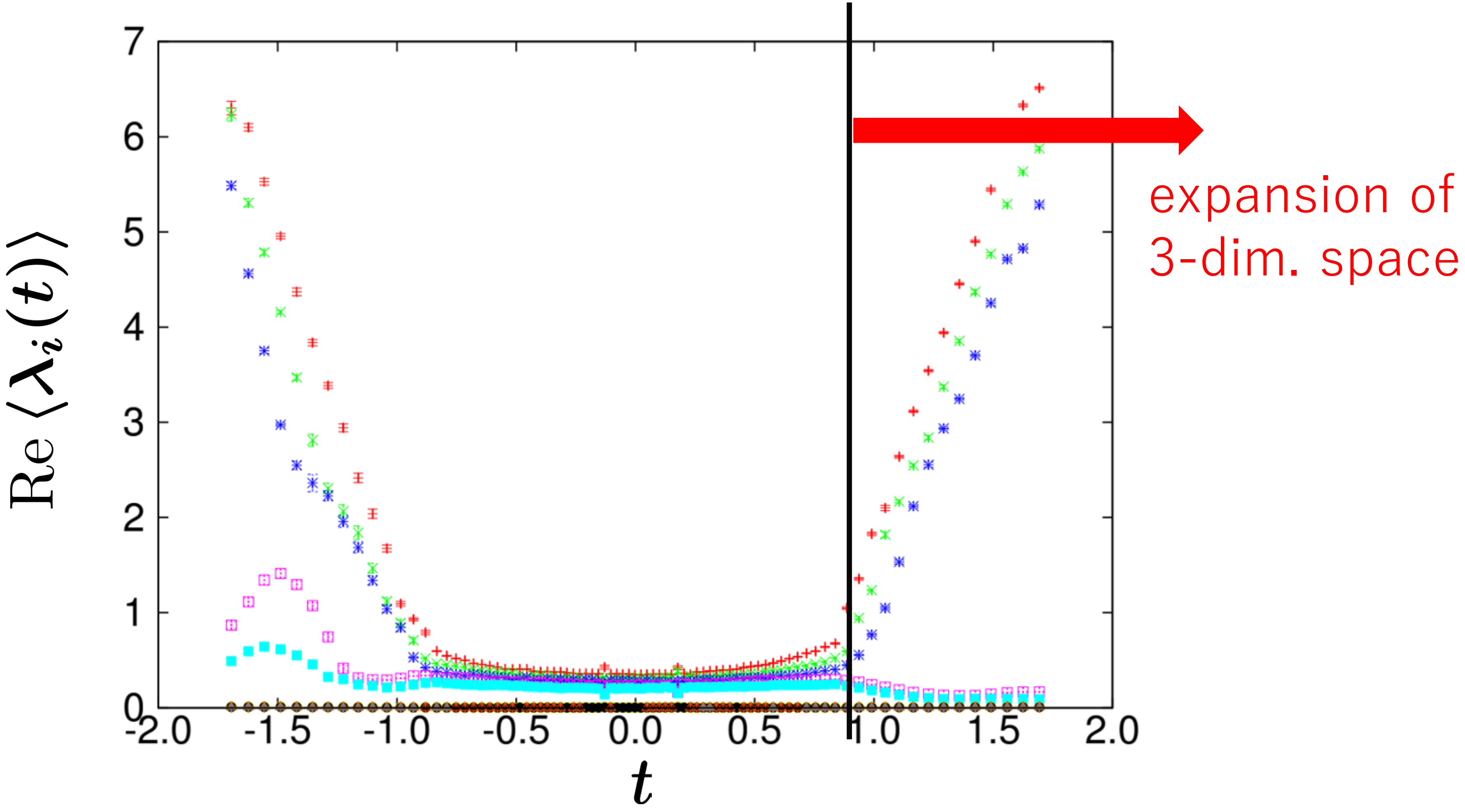


After a critical time, 3 out of 5 directions are expanding (SSB of SO(5) occurs)
→We consider that the effect of fluctuations of fermions and bosons are balanced so thatexpansion of 3-dim. space is realized in a stable way

Emergence of (3+1)-dimensional expanding spacetime

$N = 96,\ n = 12,\ \gamma = 4,$

$m_{\mathrm{f}} = 3.0,\ \xi = 14$



Increasing the effect of fermions by decreasing m_f
leads to increasing the effect of bosons by decreasing ξ

Speculation on mechanism of SSB

$\mathrm{Pf}\mathcal{M}(A_0, A_1, \cdots, A_9) = 0$ if there are only two nonzero A_μ at $m_f = 0$

Krauth, Nicolai, Staudacher (1998)

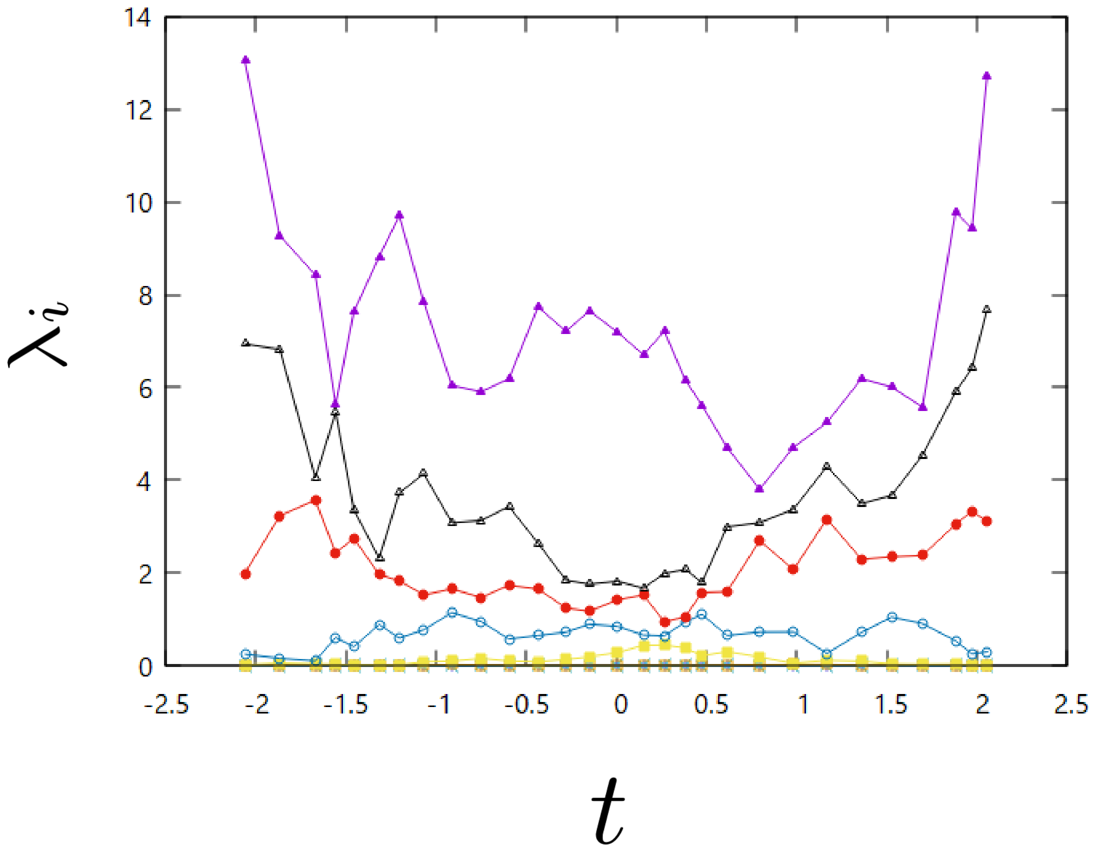
For sufficiently small m_f, it is expected that spacetimes with at least 3 expanding directions are enhanced

Conclusion and outlook

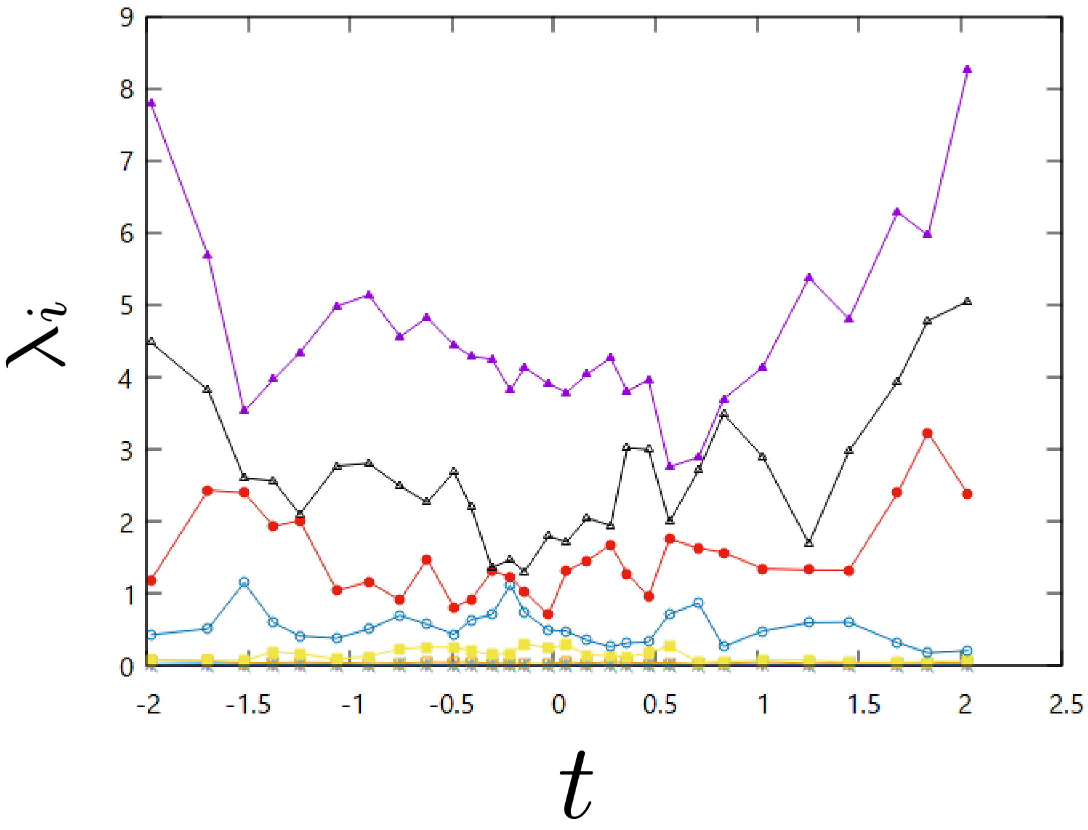
- We performed complex Langevin simulations of the Lorentzian type IIB matrix model
- We introduced a Lorentz invariant mass term for bosons as an infrared regulator
- We introduced a mass term for fermions to avoid the singular drift problem
- We modified the mass term for bosons to balance the effects of fluctuations of bosons and fermions by introducing d and ξ
- We performed a Lorentz transformation on the sampled configurations to remove the artifacts caused by the Lorentz boosts
- We performed simulations with $d=5$ and found that the SO(d) rotational symmetry is spontaneously broken and (3+1)-dimensional expanding spacetime appears at some point in time
- In order to investigate whether the (3+1)–dimensional spacetime emerges in the original model, we need to take the limits of $m_f \to 0,\ \xi \to 1,\ N \to \infty,\ \gamma \to 0$, eventually
- Gauge-fixed calculation

Gauge-fixed calculation preliminary

$N = 32,\ m_f = 3.5,\ \gamma = 4,\ d = 5,\ \xi = 16$



$N = 32,\ m_f = 2.0,\ \gamma = 4,\ d = 5,\ \xi = 10$



(3+1)-dimensional expanding spacetime for smaller m_f and ξ at smaller N