G-structures for AdS_2 solutions

Niall T. Macpherson

University of Oviedo

September 7, 2024

- A. Legramandi, A. Passias, NTM, arXiv:2309.01714 [hep-th], JHEP 06 (2024) 056

Quantum Gravity, Strings and the Swampland,

Corfu Summer Institute, 2024

General idea

- Talk aims:

  - 1: Review G-structure conditions for $\mathcal{N} = 1$ AdS_2 solutions of type II supergravity.

  - 2: Make their utility clear with some interesting examples.

- We begin with some motivation and definitions

Why AdS_2?

There are many reasons to study SUSY AdS_2 $d = 10, 11$ supergravity.

- A principle one is AdS/CFT:

- Know that AdS_{k+1} solutions are dual to CFT_k that live on the AdS boundary.
  - Best understood avatars of AdS/CFT are SUSY with string theory embedding.
  - Low energy/curvature limit is supergravity in $d = 10, 11$
  - $\sim$ strong coupling limit of CFT_k.
- AdS_2 is dual to SCQM, interesting in its own right.
  - Recent proposals for quiver/AdS_2 pairs. [Lozano-Nunez-Ramirez-Speziali]
- Also appears in several higher dim AdS/CFT contexts.
  - Wrapped brane scenarios dual to CFT_d compactified on Σ_{d-1}.
  - Janus/Hades like solutions with higher dim AdS asympotics dual to interfaces.
  - Holographic description of Wilson loops in higher dimensional CFTs.

Why AdS_2?

Another very interesting application for AdS_2 is black holes:

- Famously the near horizon limit of $d = 4$ extremal RN is $AdS_2 \times S^2$.
  - The AdS_2 factor appears to be universal for extremal BHs.
  - What else appears depends on d, symmetries, ang-momentum and asymptotics.
- Near horizon limit of all known BH geometries are solutions to EOM.
  - Constructing near horizons provides stepping stone to full BH geometry.
  - Expect AdS_2/SQCM to be of value to study of BHs.
- Bekenstein–Hawking entropy only require near horizon to compute.
  - embedding AdS_2 into string theory allows micro-state counting [Strominger-Vafa].
- For $\mathcal{N} = 2$ AdS_4 BHs AdS/CFT provides microscopic description of entropy.
  - Computed through extreamisation of topologically twisted index [Benini-Hristov-Zaffaroni]
  - Can compare to AdS_2 computation.
  - CFT side implies likely many more BH geometries than currently known.

Why G-structures and what are they?

Why G-structures?

- SUSY for classical solution requires a Killing spinor. *i.e* $d = 11$ supergravity

$$\nabla_M \epsilon + \frac{1}{288} G_{ABCD} \left(\Gamma_M^{ABCD} - 8\delta_M^A \Gamma^{BCD} \right) \epsilon = 0.$$

  - Not on equal footing with the bosonic fields (g, G), which satisfy geometric conditions.

  - Need a solution in hand to check if its SUSY.

  - Can be rather hard to work with.

- G-structure methods resolved these issues by making SUSY preservation geometric.

But what are G-structures?

- A G-structure is a property a supersymmetric manifold posses.

  - "G" is for group and "structure" is a generalisation of holonomy.

- Simplest solutions of supergravity have only a non trivial metric.

$$\Rightarrow R_{MN} = 0$$

An example

- Of much interest to string-pheno were SUSY $\text{Mink}_4 \times \text{M}_6$.

  - M_6 supports covariantly constant spinor $\nabla_a \eta_+ = 0$.

  - Forming bi-linears it is possible to show equivalence to

$$dJ_2 = 0, \qquad d\Omega_3 = 0.$$

  - Manifolds admitting such forms are CY_3 with SU(3)-holonomy.

- SU(3)-structure manifolds are more general and allow "torsion classes" W_a.

$$dJ_2 = \frac{3}{2}\text{Im}(\overline{W}_1 \Omega) + W_3 + W_4 \wedge J_2, \quad d\Omega_3 = W_1 J_2 \wedge J_2 + W_2 \wedge J_2 + \overline{W}_5 \wedge \Omega_3.$$

  - Necessary for solutions with more bosonic fields turned on.

  - W_a determine properties of M_6, Complex, Kahler, half-flat....

- There exist different G-structures in different dimensions or with more spinors turned on.

  - General $\mathcal{N} = 1$ $\text{Mink}_4 \times \text{M}_6$ can have SU(3) or SU(2)-structure [Graña-Minasian-Petrini-Tomasiello].

- In most cases bosonic fields expressed in terms of G-structure forms.

  - Reduces need to make ansatze.

Talk outline

- **$\mathcal{N}=1$ AdS$_2$ in type II supergravity.**

  - G-structure conditions for AdS$_2$.

- **Applications.**

  - $\mathcal{N}=1$ solutions with foliations involving weak G$_2$ manifolds

  - Small $\mathcal{N}=4$ solutions on with AdS$_2\times$S^2 foliated over CY$_2\times\Sigma_2$.

- **Some comments on $d=11$ case.**

- **Conclusions.**

$\mathcal{N} = 1$ AdS$_2$ in type II supergravity

[A. Legramandi, A. Passias, NTM]

- In general AdS$_2$ solutions of type II supergravity decompose as ($\pm$ for IIA/IIB)

$$ds^2 = e^{2A}ds^2(\text{AdS}_2)+ds^2(\text{M}_8),\qquad F_\pm = e^{2A}f_\pm + \text{vol}(\text{AdS}_2)\wedge\star_8\lambda(f_\pm),$$

$$H = e^{2A}\text{vol}(\text{AdS}_2)\wedge H_1 + H_3,\qquad \Phi=\Phi(\text{M}_8).$$

  - task is derive geometric constraints on internal fields: $(A,\Phi,f_\pm,H_{1,3},ds^2(\text{M}_8))$.

- AdS$_2$ (inverse radius m) supports Majorana-Weyl (MW) Killing spinors $\zeta_\pm$:

$$\nabla_\mu\zeta_\pm = \frac{m}{2}\gamma_\mu\zeta_\mp,\qquad \zeta^c_\pm=\zeta_\pm$$

- $d=10$ MW spinors decompose in terms of $d=8$ MW spinors

$$\epsilon_1=\zeta_+\otimes\chi^1_+ + \zeta_-\otimes\chi^1_-,\qquad \epsilon_2=\zeta_+\otimes\chi^2_\mp+\zeta_-\otimes\chi^2_\pm,$$

  - All of $\chi^{1,2}_\pm$ must be non zero, or $m=0$.

- $\mathcal{N}=1$ means there are two independent real supercharges.
  - Two from $\zeta_\pm$ and one from $\chi^{1,2}_\pm$.

- KSE of type II then imply conditions on $\chi^{1,2}_\pm$. **Can** use to derive geometric conditions.

$\mathcal{N} = 1$ AdS$_2$ in type II: Work smart not hard

- G-structure conditions for general $\mathcal{N} = 1$ type II solutions already exist [Tomasiello]

  - depend on bi-linears:

$$K^{1,2}_M := \frac{1}{32}\overline{\epsilon^{1,2}}\Gamma_M\epsilon^{1,2}, \qquad \not{\Psi}_\pm := \epsilon_1 \otimes \overline{\epsilon_2} = \frac{1}{32}\sum_{n=0}^{10}\frac{1}{n!}\overline{\epsilon^2}\Gamma_{M_n\ldots M_1}\epsilon^1\Gamma^{M_1\ldots M_n}$$

  that imply a poly-form $\Psi_\pm$ and two one forms $2K = K^1 + K^2,\ 2\tilde{K} = K^1 - K^2$.

- Supersymmetry entirely equivalent to the following differential constraints

$$\nabla_{(N}K_{M)} = 0, \qquad d\tilde{K} = \iota_K H, \qquad (d - H\wedge)(e^{-\Phi}\Psi_\pm) = -\frac{1}{32}(\tilde{K}\wedge + \iota_K)F$$

  - As well as some algebraic "Pairing" constraints - complicated, will omit details.

  - Fix metric and other Bosonic fields.

  - Killing vector $K^M\partial_M$ can be time-like/null.

- For AdS$_2\times$M$_8$, $\Psi_\pm$ decompose in terms of forms on AdS$_2$ and M$_8$

  - Can factor out AdS$_2$ data to arrive at geometric conditions on M$_8$ $\Rightarrow$ SUSY.

$\mathcal{N}=1$ AdS$_2$ in type II: Say no to AdS$_3$!

- The condition $\nabla_{(N}K_{M)}=0$ is actually very powerful, one finds ($\chi^{1,2}:=\chi^{1,2}_{+}+\chi^{1,2}_{-}$)

$$K=\frac{1}{64}\bigg(e^{A}(|\chi^1|^2+|\chi^2|^2)v_1+e^{A}(\chi^{1\dagger}\hat{\gamma}\chi^1\mp\chi^{2\dagger}\hat{\gamma}\chi^2)u_1\bigg)-\frac{1}{32}f_0k$$

where in particular the AdS$_2$ n-forms (v_1,v_1,f_0) obey

$$\nabla_{(\mu}(v_1)_{\nu)}=0,\qquad \nabla_{(\mu}(u_1)_{\nu)}=-mf_0g^{\text{AdS}_2}_{\mu\nu}.$$

- Actually imposing that $K^M\partial_M$ is Killing means that

$$\nabla_{(a}k_{b)}=0,\qquad \mathcal{L}_kA+\frac{m}{2}e^{-A}(\chi^{1\dagger}\hat{\gamma}\chi^1\mp\chi^{2\dagger}\hat{\gamma}\chi_2)=0,$$
$$d(e^{-A}(|\chi^1|^2+|\chi^2|^2))=0,\qquad d(e^{-A}(\chi^{1\dagger}\hat{\gamma}\chi^1\mp\chi^{2\dagger}\hat{\gamma}\chi^2))+2me^{-2A}k=0.$$

- So either $k=0$ or $k^a\partial_a$ is Killing w.r.t M$_8$ but not e^{2A}.

  - If Killing can take $k^a\partial_a=\partial_\rho$ wlog

$$\Rightarrow\quad e^{2A}ds^2(\text{AdS}_2)+ds^2(\text{M}_8)=e^{2A_7}\overbrace{\Big[m^2\cosh^2\rho\, ds^2(\text{AdS}_2)+d\rho^2\Big]}^{\text{AdS}_3}+ds^2(\text{M}_7)$$

  - So one has AdS$_3$ unless $k_a:=\frac{1}{2}(\chi^{1\dagger}\gamma_a\chi^1\mp\chi^{2\dagger}\gamma_a\chi^2)=0$ and $\chi^{1\dagger}\hat{\gamma}\chi^1\mp\chi^{2\dagger}\hat{\gamma}\chi^2=0$.
  - So $K^M\partial_M$ is time-like for true AdS$_2$ solutions.

$\mathcal{N}=1$ AdS_2 in type II: Necessary and sufficient condtions

- Proceeding in kind with the rest of the $d=10$ geometric SUSY constraints one finds:
- Conditions for no AdS_3

$$(\chi^{1\dagger}\gamma_a\chi^1 \mp \chi^{2\dagger}\gamma_a\chi^2)=0,\qquad \chi^{1\dagger}\hat{\gamma}\chi^1 \mp \chi^{2\dagger}\hat{\gamma}\chi^2=0,\qquad |\chi_1|^2=|\chi_2|^2$$

- Conditions for SUSY phrased in terms of following:

$$e^A\cos\beta := \chi^{1\dagger}\hat{\gamma}\chi^1,\qquad e^A\sin\beta V := \chi^{1\dagger}\gamma_a\chi^1 e^a,\qquad e^A := |\chi^1|^2$$

$$\psi := \chi^1\otimes\chi^{2\dagger},\qquad \hat{\psi} := \hat{\gamma}\chi^1\otimes\chi^{2\dagger}.$$

- $\mathcal{N}=1$ SUSY is equivalent to imposing

$$e^{2A}H_1 = me^A\sin\beta V - d(e^{2A}\cos\beta),\qquad d(e^A\sin\beta V)=0,$$

$$d_{H_3}(e^{-\Phi}\psi_\pm) = \pm\frac{1}{16}e^A\sin\beta V\wedge f_\pm,$$

$$d_{H_3}(e^{A-\Phi}\hat{\psi}_\mp) - me^{-\Phi}\psi_\pm = \mp\frac{1}{16}e^{2A}(\star_8\lambda f_\pm + \cos\beta f_\pm),$$

$$(\psi_\pm, f_\pm)_8 = \pm\frac{1}{4}e^{-\Phi}\left(m - \frac{1}{2}e^A\sin\beta\iota_V H_1\right)\text{vol}(\text{M}_8).$$

- final condition comes from Pairing constraints, proving equivalence to this is not easy.

$\mathcal{N}=1$ AdS_2 in type II: What G-structure?

- No AdS_3 conditions are restrictions on allowed $\chi^{1,2}_\pm$ and so $(\psi,\hat{\psi}) \Rightarrow$ G-structure.

  - Need only: Unit norm $\chi_\pm$, 1-form U s.t $\iota_V U = 0$ and phase $e^{i\alpha}$ to span $\chi^{1,2}_\pm$.

  - $\chi_\pm \Rightarrow d=8$ G_2-structure which is broken to SU(3)-structure by U.

- We define $d=7$ bi-linears orthogonal to V in terms of SU(3)-structure bi-linears

$$\psi^{(7)}_\pm = \frac{1}{2}\bigg(\psi^{\text{SU(3)}}_\pm + i\psi^{\text{SU(3)}}_\mp \wedge U\bigg), \qquad \psi^{\text{SU(3)}}_+ = \frac{1}{8}e^{i\alpha}e^{-iJ_2}, \qquad \psi^{\text{SU(3)}}_- = \frac{1}{8}\Omega_3,$$

  In terms of which we have

$$\psi_\pm = e^A\text{Re}\Big[\psi^{(7)}_\pm + \cos\beta\psi^{(7)}_\mp \wedge V\Big], \qquad \psi_\mp = e^A\sin\beta V \wedge \text{Re}\Big[\psi^{(7)}_\mp\Big],$$
$$\hat{\psi}_\pm = e^A\text{Re}\Big[\psi^{(7)}_\pm \wedge V + \cos\beta\psi^{(7)}_\pm\Big], \qquad \hat{\psi}_\mp = \pm e^A\sin\beta\text{Re}\Big[\psi^{(7)}_\mp\Big].$$

- M_8 supports SU(3)-structure generically.

  - enhanced to G_2-structure when $e^{i\alpha}=i$, then

$$\Phi_3 = -(J_2\wedge U + \text{Re}\Omega_3), \qquad \star_7\Phi_3 = \frac{1}{2}J_2\wedge J_2 - U\wedge\text{Im}\Omega_3.$$

$\mathcal{N}=1$ AdS$_2$ in type II: Summery

- AdS$_2$ solutions in type II supergravity

$$ds^2 = e^{2A} ds^2(\mathrm{AdS}_2) + ds^2(\mathrm{M}_8), \qquad F_\pm = e^{2A} f_\pm + \mathrm{vol}(\mathrm{AdS}_2) \wedge \star_8 \lambda(f_\pm),$$

$$H = e^{2A}\mathrm{vol}(\mathrm{AdS}_2) \wedge H_1 + H_3, \qquad \Phi = \Phi(\mathrm{M}_8).$$

- Preserve $\mathcal{N}=1$ supersymmetry when following hold

$$e^{2A}H_1 = me^A \sin\beta V - d(e^{2A}\cos\beta), \qquad d(e^A \sin\beta V) = 0,$$

$$d_{H_3}(e^{-\Phi}\psi_\pm) = \pm\frac{1}{16}e^A \sin\beta V \wedge f_\pm,$$

$$d_{H_3}(e^{A-\Phi}\hat{\psi}_\mp) - me^{-\Phi}\psi_\pm = \mp\frac{1}{16}e^{2A}(\star_8\lambda f_\pm + \cos\beta f_\pm),$$

$$(\psi_\pm, f_\pm)_8 = \pm\frac{1}{4}e^{-\Phi}\left(m - \frac{1}{2}e^A \sin\beta \iota_V H_1\right)\mathrm{vol}(\mathrm{M}_8).$$

- $(\psi, \hat{\psi})$ expressed in terms of SU(3)-structure.
- Totally geometric conditions. No spinors any more!

- SUSY implies that one has a solutions when

$$dH_3 = 0, \qquad \iota_V(d_{H_3} f_\pm) = 0, \qquad \cos\beta\left[d(e^{-2\Phi}\star_8 H_1) + \frac{1}{2}(f_\pm, f_\pm)_8\right] = 0.$$

- follows from integrability proof for time-like $K^M\partial_M$ [Legramandi-Martucci-Tomasiello]

Applications

[A. Legramandi, A. Passias, NTM]

Applications: $\mathcal{N}=1$ with foliation involving weak G_2 manifold.

- Family of massive IIA $\mathcal{N}=8$ solutions exists, foliations of $AdS_2\times S^7$ over interval [Dibitetto-Passias].

  - S^7 supports weak G_2-structure: Exists $\tilde{\Phi}_3$ s.t

$$d\tilde{\Phi}_3 = 4\star_7\tilde{\Phi}_3$$

  - Many other weak G_2-manifolds: Compact examples with G_2 cone singularities

$$ds^2(M_7) = d\alpha^2 + \sin^2\alpha ds^2(B_6), \qquad B_6 = (S^6,\, S^3\times S^3,\, \mathbb{CP}^3,\, \mathbb{F}^3).$$

- What about also allowing fluxes to depend on $(\tilde{\Phi}_3, \star_7\tilde{\Phi}_3)$?

- We assume ansatz such that weak G_2-structure is respected

$$ds^2 = e^{2C}ds^2(M_7) + e^{2k}d\rho^2, \qquad H_3 = 0$$
$$f_+ = F_0 + e^k p\tilde{\Phi}_3\wedge d\rho + g\star_7\tilde{\Phi}_3 + e^k q\text{vol}(M_7)\wedge d\rho,$$

  - (e^A, e^k, e^C, g, q) functions of ρ only, $\partial_\rho M_7 = 0$.

- We also assume we are in G_2-structure limit with

$$V = e^k d\rho, \qquad \Phi_3 = e^{3C}\tilde{\Phi}_3$$

  - Actually large assumption, not required that $\Phi_3\propto\tilde{\Phi}_3$.

Applications: $\mathcal{N}=1$ with foliation involving weak G_2 manifold.

- Though the G-structure conditions find class of form

$$\frac{ds^2}{L^2}=\sqrt{\frac{h}{h''}}\left[\frac{hh''\sqrt{1-7v}}{8\Delta}ds^2(\text{AdS}_2)+\left(\frac{h''}{8h\sqrt{1-7v}}d\rho^2+\frac{\sqrt{1-7v}}{(v-1)^2}ds^2(\text{M}_7)\right)\right],$$

$$H=\frac{L^2}{8\sqrt{2}}d\left(\frac{hh'(1-7v)}{\Delta}-\rho\right)\wedge\text{vol}(\text{AdS}_2),\qquad e^{-\Phi}=\frac{\sqrt{\Delta}(1-v)^{\frac{7}{2}}}{c_0L^3(1-7v)^{\frac{5}{4}}}\left(\frac{h''}{h}\right)^{\frac{3}{4}},$$

$$\Delta=2hh''-(1-7v)(h')^2,$$

-None of the RR flux terms generically zero.

- One has a solution whenever away from sources

$$\partial_\rho\left(\frac{(1-v)^{\frac{7}{2}}h''}{(1-7v)}\right)=F_0,\quad \partial_\rho\left(\sqrt{v}\partial_\rho\left(\frac{h\sqrt{v}}{\sqrt{1-v}}\right)\right)-\frac{2v^{\frac{3}{4}}}{(1-v)(1-7v)}\partial_\rho\left((1-v)^{\frac{3}{2}}v^{\frac{1}{4}}h'\right)=0,$$

- complicated in general but for $v=v_0$ truncate to

$$h'''=F_0,\quad v_0(1+5v_0)=0,\quad\Rightarrow\quad h=c_1+c_2\rho+c_3\rho^2+\frac{1}{3!}F_0\rho^3\ \text{(Locally)}$$

- $v_0=0$: expected generalisation of $\text{AdS}_2\times\text{S}^7\times\mathcal{I}$ to general weak G_2 manifolds.
- $(1+5v_0)=0$: unexpected solution with $(\tilde{\Phi}_3,\star_7\tilde{\Phi}_3)$ in fluxes.
- F_0 can be piece-wise constant $\Rightarrow$ D8 sources along interval.

- interesting global solutions a la AdS_7 in massive IIA.

Can use $\mathcal{N}=1$ conditions to construct solutions with extended SUSY.

- Must make SU(3)-structure forms charged under R-symmetry.

- Will consider case of small $\mathcal{N}=4$ AdS$_2$ solutions

  -SU$(2)_R$ R-symmetry, thus consider decomposing

$$ds^2 = e^{2C} ds^2(\text{S}^2) + ds^2(\text{M}_4) + V^2 + U^2, \qquad H_3 = e^{2C} \tilde{H}_1 \wedge \text{vol}(\text{S}^2) + \tilde{H}_3,$$

- S^2 supports the embedding coords $\mu_{\mathfrak{a}}$, SO(3) triplets

  - Can decompose SU(3)-structure forms in terms of these and SU(2)-structure forms $j_{\mathfrak{a}}$

$$J_2 = e^{2C} \text{vol}(\text{S}^2) - \mu_{\mathfrak{a}} j_{\mathfrak{a}}, \qquad \Omega_3 = e^{C} \left(d\mu_{\mathfrak{a}} \wedge j_{\mathfrak{a}} + i\epsilon_{\mathfrak{abc}} \mu_{\mathfrak{b}} d\mu_{\mathfrak{c}} \wedge j_{\mathfrak{c}} \right),$$

$$j_{\mathfrak{a}} \wedge j_{\mathfrak{b}} = 2\delta_{\mathfrak{ab}} \text{vol}(\text{M}_4)$$

- By insisting that RR sector is SU$(2)_R$ singlet can construct $\mathcal{N}=4$ class:

$$ds^2 = \frac{u}{\sqrt{h_3 h_7}} \left(\frac{1}{\Delta_2} ds^2(\text{AdS}_2) + \frac{1}{\Delta_1} ds^2(\text{S}^2) \right) + \sqrt{\frac{h_3}{h_7}} ds^2(\text{CY}_2) + \frac{\sqrt{h_3 h_7}}{u} (dx_1^2 + dx_2^2) \Big],$$

$$e^{-\Phi} = c_0 \sqrt{\Delta_1 \Delta_2} h_7, \qquad \Delta_1 = 1 + \frac{(\partial_{x_1} u)^2}{h_3 h_7}, \qquad \Delta_2 = 1 - \frac{(\partial_{x_2} u)^2}{h_3 h_7}.$$

  - and all NS and RR fluxes are non trivial, depend on primitive $(1,1)$-forms $X^{(1,1)}_{1,2}$

Applications: Class of small $\mathcal{N} = 4$ solutions

- Functions in the metric have dependence

  - $h_3 = h_3(\text{CY}_2, x_a)$, $h_7 = h_7(x_a)$, $u = u(x_a)$

- Supersymmetry amounts to solving

$$\nabla_2^2 u = 0.$$

- Bianchi identities of fluxes impose

$$d_4 X_1^{(1,1)} = d_4 X_2^{(1,1)} = 0, \quad \partial_{x_2} X_1^{(1,1)} = \partial_{x_1} X_2^{(1,1)}, \quad \partial_{x_1}(h_7^2 X_1^{(1,1)}) = -\partial_{x_2}(h_7^2 X_2^{(1,1)}),$$

$$\nabla_2^2 h_7 = 0, \quad \frac{h_7}{u}\nabla_4^2 h_3 + \nabla_2^2 h_3 + h_7\left((X_1^{(1,1)})^2 + (X_2^{(1,1)})^2\right) = 0,$$

  - Generalised D3-D7 system extended in $\text{AdS}_2 \times \text{S}^2$.
  - $h_3 = h_3(x_a)$ and $X_{1,2}^{(1,1)} = 0$ limit, 3 harmonic functions [Chiodaroli-D'Hoker-Gutperle-Krym]
  - Contains further classes with ∂_{x_1} or ∂_{x_2} isometries [Lozano-Nunez-Ramirez-Speziali]

- Have solution whenever these PDEs are solved.

- Fixing $u = 1$ gives embedding of extremal RN near horizon into IIB.

$$ds^2 = \frac{1}{\sqrt{h_3 h_7}}\left(ds^2(\text{AdS}_2) + ds^2(\text{S}^2)\right) + \sqrt{\frac{h_3}{h_7}}ds^2(\text{CY}_2) + \sqrt{h_3 h_7}(dx_1^2 + dx_2^2)$$

Comments on $d = 11$ case

[J. Hong, NTM, L. A. Pando Zayas]

Comments on $d = 11$ case

- Some time ago provided G-structure conditions for AdS_2 in $d = 11$

$$ds^2 = e^{2\Delta} ds^2(\text{AdS}_2) + ds^2(\text{M}_9), \qquad G = e^{2\Delta}\text{vol}(\text{AdS}_2) \wedge G_2 + G_4.$$

- Necessary geometric conditions for general $\mathcal{N} = 1$ solutions exist

$$d\Xi_2 = \iota_K G, \qquad \nabla_{(M} K_{N)} = 0,$$
$$d\Sigma_5 = \iota_K \star G - \Omega_2 \wedge G, \qquad \star dK = \frac{2}{3}\Xi_2 \wedge \star G - \frac{1}{3}\Sigma_5 \wedge G.$$

  - Simple but only sufficient for time-like K [Gauntlett-Pakis].

- Assuming this for $\text{AdS}_2 \Rightarrow \text{SU}(4)$-structure.
  - Provided conditions for such solutions [J. Hong, NTM, L. A. Pando Zayas].

- But now realise $\nabla_{(M} K_{N)} = 0$ here also implies "no AdS_3" conditions.
  - Other structures possible, but solutions are AdS_3 *i.e* SU(4) is general.

- Assumed round AdS_2 - in extremal Kerr-Newman near horizon, has U(1) fibered over it.
  - Some embeddings into $d = 11$ known [Couzens-Marcus-Stemerdink-Heisteeg].
  - In general such solutions will lift from IIA limit of $d = 10$ AdS_2 conditions.

Conclusions

- **Have provided G-structure conditions for $\mathcal{N}=1$ AdS$_2$ in $d=10$**

  -Also classified solutions in terms of torsion classes (too long for here).

  - Applications for AdS/CFT and black holes.

- **Have provided some new AdS$_2$ examples**:

  - $\mathcal{N}=1$ solutions with weak G$_2$-manifolds governed by $h'''=F_0$

  - Broad class of small $\mathcal{N}=4$ solutions on AdS$_2\times$S$^2\times$ CY$_2\times\Sigma_2$.

- **Many other interesting applications**:

  - $\mathcal{N}=8$ solutions should be fruitful: $\mathcal{N}=(8,0)$ AdS$_3$ suggests interesting Janus type solutions in massive IIA exist.

  - Wrapped Brane scenarios dual to compactified CFTs.

  - $\mathcal{N}=2$ AdS$_2$ solutions compatible with AdS$_4$ BH near-horizons.

  - Holographic duals to $\mathcal{I}$ extreamisation.

- **Interesting to generalise to type II solutions with U(1) fibered over AdS$_2$**

  - Needed for some extremal near horizons solutions, *i.e* Kerr-Newman.

Thank you

$d = 11$ AdS_2 SU(4)-structure conditions

- Recall AdS_2 in $d = 11$ decomposable as

$$ds^2 = e^{2\Delta} ds^2(\mathrm{AdS}_2) + ds^2(\mathrm{M}_9), \qquad G = e^{2\Delta}\mathrm{vol}(\mathrm{AdS}_2) \wedge G_2 + G_4.$$

- For SU(4)-structure to happen M_9 must support a chiral Dirac spinor χ
- SU(4)-structure forms defined as

$$e^{\Delta} V = \chi^{\dagger}\gamma_a \chi e^a, \qquad e^{\Delta} J_2 = -\frac{i}{2}\chi^{\dagger}\gamma_{ab}\chi e^{ab}, \qquad e^{\Delta}\Omega_4 = \frac{1}{4!}\chi^{c\dagger}\gamma_{abcd}\chi e^{abcd},$$

- Conditions on SU(4)-structure forms

$$\begin{aligned}
&d(e^{\Delta} J_2) = 0,\\
&d(e^{2\Delta} V) + e^{\Delta} J_2 + e^{2\Delta} G_2 = 0,\\
&d(e^{\Delta} V \wedge \mathrm{Im}\Omega_4) - e^{\Delta} J_2 \wedge G_4 = 0,\\
&d(e^{2\Delta}\mathrm{Re}\Omega_4) - e^{\Delta} V \wedge \mathrm{Im}\Omega_4 + e^{2\Delta}(\star_9 G_4 - V \wedge G_4) = 0,\\
&\star_9 \big(2V \wedge \star_9 G_2 + \mathrm{Re}\Omega \wedge G_4\big) + 6 d\Delta = 0\\
&J_2 \wedge J_2 \wedge G_4 = 0,\\
&e^{\Delta}\big(2J_2 \wedge \star_9 G_2 - V \wedge \mathrm{Im}\Omega_4 \wedge G_4\big) = 6\mathrm{Vol}(\mathrm{M}_9).
\end{aligned}$$