Top-Down Aspects of Modular Flavor Symmetry

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Outline

- The problem of flavor in the Standard Model
- Traditional flavor symmetries
- Modular flavor symmetries
- Eclectic Flavor Group
- Local Flavor Unification
- Origin of hierarchies for masses and mixing angles from non-universal modular flavor symmetries
- Lessons from top-down model building
- Connecting top-down versus bottom-up constructions?

Importance of localized structures in extra dimensions

(Work with Baur, Knapp-Perez, Liu, Ratz, Ramos-Sanchez, Trautner, Vaudrevange, 2019-23)

The flavor structure of SM

Most of the parameters of the SM concern the flavor sector

- Quark sector: 6 masses, 3 angles and one phase
- Lepton sector: 6 masses, 3 angles, one phase and additional parameters from Majorana neutrino masses

The pattern of parameters

- Quarks: hierarchical masses und small mixing angles
- Leptons: two large and one small mixing angle, hierarchical mass pattern and extremely small neutrino masses

The Flavor structure of quarks and leptons is very different!

Traditional vs Modular Symmetries

Two types of discrete flavor symmetries are available.
The first type is traditional flavor symmetries

- they are linearly realised
- need flavon fields for symmetry breakdown

Another type of symmetries are modular symmetries

- motivated by string theory dualities (Lauer, Mas, Nilles, 1989)
- suggested recently as flavor symmetries (Feruglio, 2017)
- non-linearly realised (no flavon fields needed)
- Yukawa couplings transform as modular forms

Combine with traditional flavor symmetries to the so-called "eclectic flavor group" (Nilles, Ramos-Sanchez, Vaudrevange, 2020)

Bottom-Up (BU) approach

In the BU-approach one is free to choose the flavor group and the representations of the matter fields.

- Many fits from bottom-up perspective with discrete symmetries ($S_3, A_4, S_4, A_5, \Delta(27), \Delta(54)$ etc.)
- Various choices of representations have been considered, e.g. triplets and nontrivial singlets
- Flavor symmetries are spontaneously broken. This can be done by additional flavon fields or the vacuum expectation value of the modulus
- BU-model building leads to (too) many reasonable fits for various choices of groups and representations

We are still missing a top-down explanation of flavor

String Geometry

Strings are extended objects and this reflects itself in special aspects of geometry (including the presence of winding modes). We have:

- normal symmetries of extra dimensions as observed in quantum field theory – traditional flavor symmetries.
- String duality transformations mix winding and momentum modes and lead to discrete modular or symplectic flavor symmetries
- They combine to a unified picture within the concept of eclectic flavor symmetries

In the following we illustrate this with a simple example

- twisted 2D-torus with localized matter fields
- relevant for compactifications with elliptic fibrations

Questions for Top-Down approach

Given the large number of BU-constructions we look for restrictions from TD-considerations

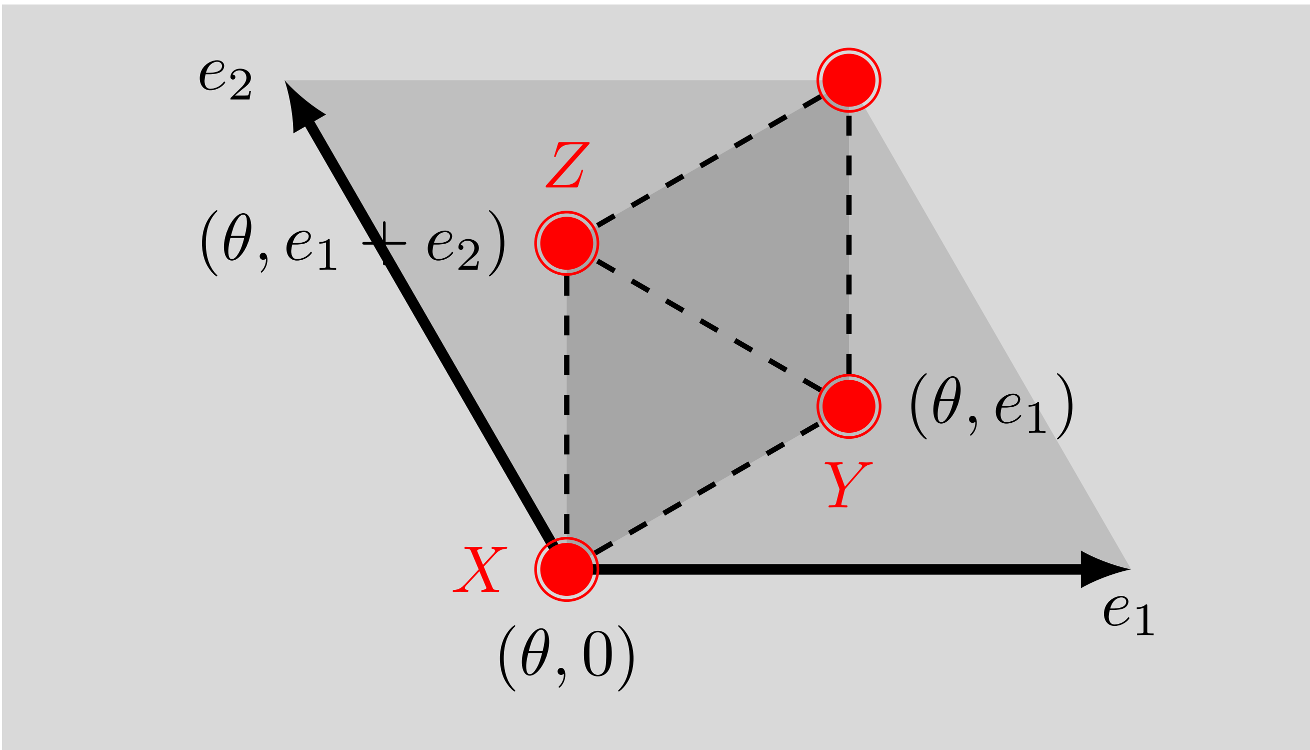
- What are the allowed discrete modular groups?
- Are they accompanied by other symmetries?
- Allowed representations of modular group?
- Restrictions on the modular weights?
- Is there a link between representations and modular weights?
- Information on moduli stabilization and the appearance of hierarchies for masses and mixing angles?

In the following we want to classify the cases of twisted 2-dimensional tori and answer these questions.

Traditional Flavor Symmetries

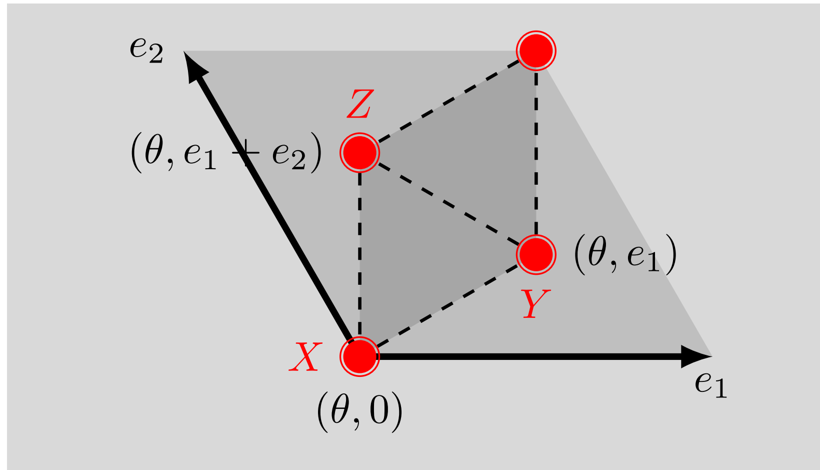
In string theory discrete symmetries can arise form geometry and string selection rules.

As an example we consider the orbifold T_2/Z_3



Discrete symmetry $\Delta(54)$

- untwisted and twisted fields
- S_3 symmetry from interchange of fixed points
- $Z_3 \times Z_3$ symmetry from string theory selection rules



- $\Delta(54)$ as multiplicative closure of S_3 and $Z_3 \times Z_3$
- $\Delta(54)$ – a non-abelian subgroup of $SU(3)$
- e.g. flavor symmetry for three families of quarks (as triplets of $\Delta(54)$)

String dualities

Consider a particle on a circle with radius R

- discrete spectrum of momentum modes (KK-modes)
- density of spectrum is governed by m/R (m integer)
- heavy modes decouple for $R \to 0$

Now consider a string

- KK modes as before m/R
- Strings can wind around circle
- spectrum of winding modes governed by nR
- massless modes for $R \to 0$

T-duality

This interplay of momentum and winding modes is the origin of T-duality where one simultaneously interchanges

- momentum $\rightarrow$ winding
- $R \rightarrow 1/R$

This transformation maps a theory to its T-dual theory.

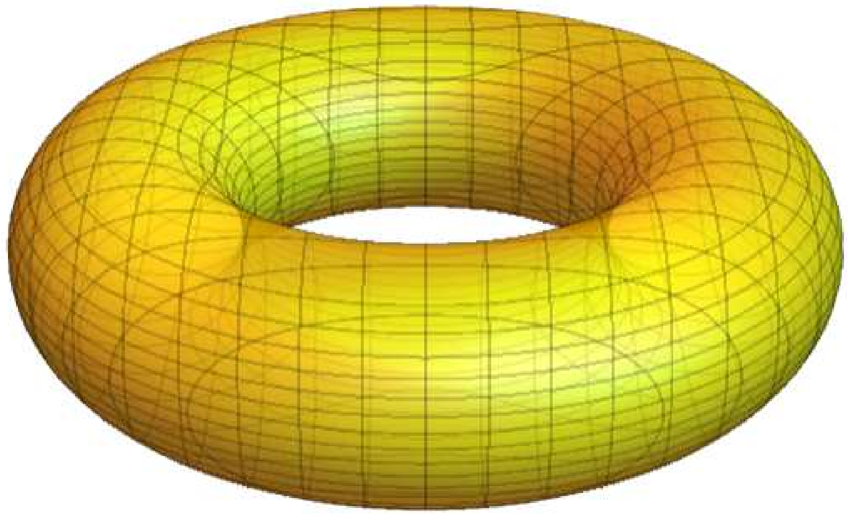
- self-dual point is $R^2 = \alpha' = 1/M_{\mathrm{string}}^2$

If the string scale M_{string} is large, the low energy effective theory describes the momentum states and the winding states are heavy.

Can T-duality play a relevant role for flavor symmetry?

Torus compactification

Strings can wind around several cycles



Complex modulus M (in complex upper half plane)

Modular Transformations

Modular transformations (dualities) exchange windings and momenta and act nontrivially on the moduli of the torus.

In $D = 2$ these transformations are connected to the group $SL(2, Z)$ acting on Kähler and complex structure moduli.

The group $SL(2, Z)$ is generated by two elements

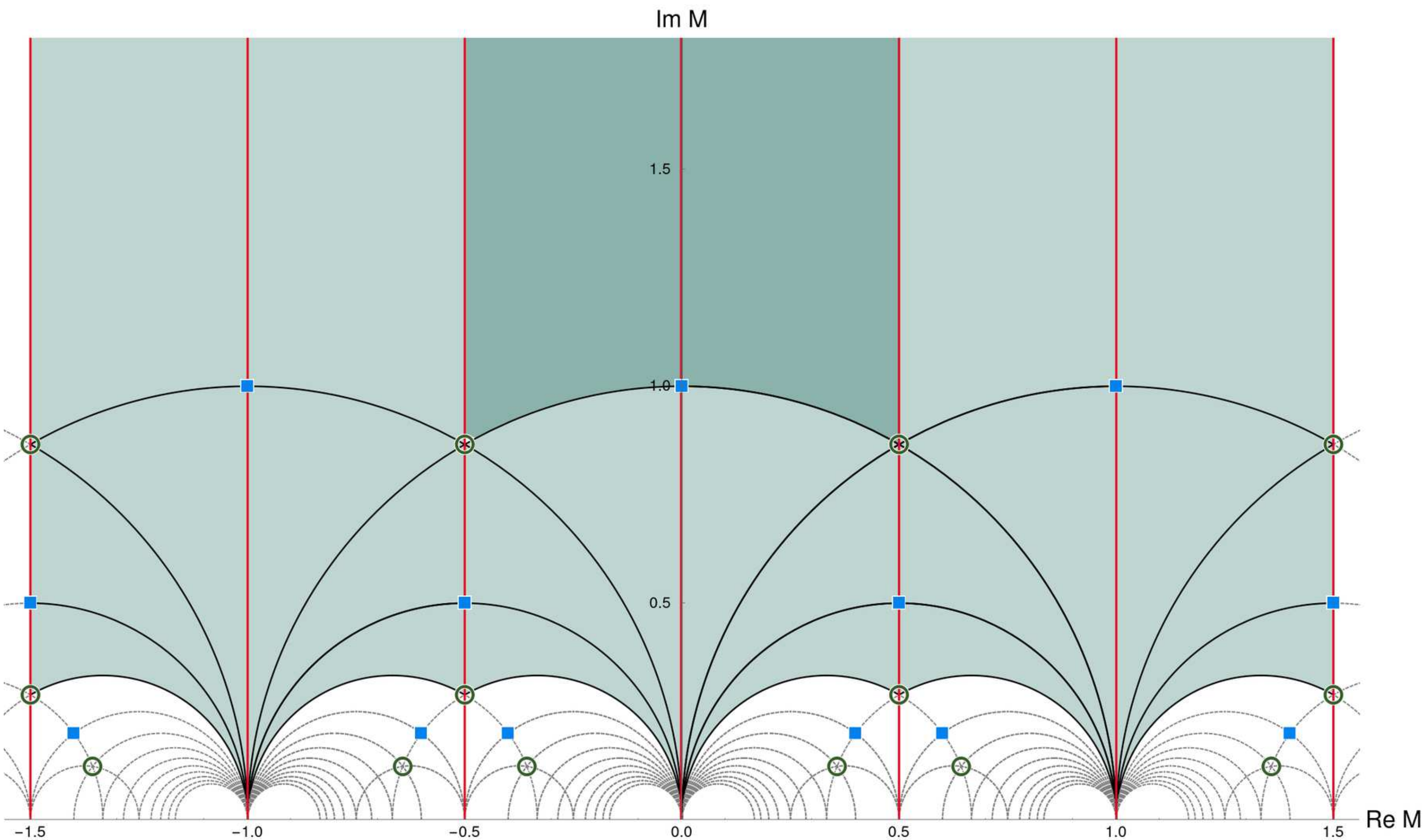
$$\mathrm{S}\ ,\ \mathrm{T}: \quad \text{with}\ \ \mathrm{S}^4 = 1\ \ \text{and}\ \ \mathrm{S}^2 = (\mathrm{ST})^3$$

A modulus M transforms as

$$\mathrm{S}: \quad M \to -\frac{1}{M} \quad \text{and} \quad \mathrm{T}: M \to M + 1$$

Further transformations might include $M \to -\overline{M}$ and mirror symmetry between Kähler and complex structure moduli.

Fundamental Domain



Three fixed points at M$= i$, $\omega = \exp(2\pi i/3)$ and $i\infty$

Modular Functions

String dualities give important constraints on the action of the theory via the modular group $SL(2,Z)$:

$$\gamma: \quad M \rightarrow \frac{aM+b}{cM+d}$$

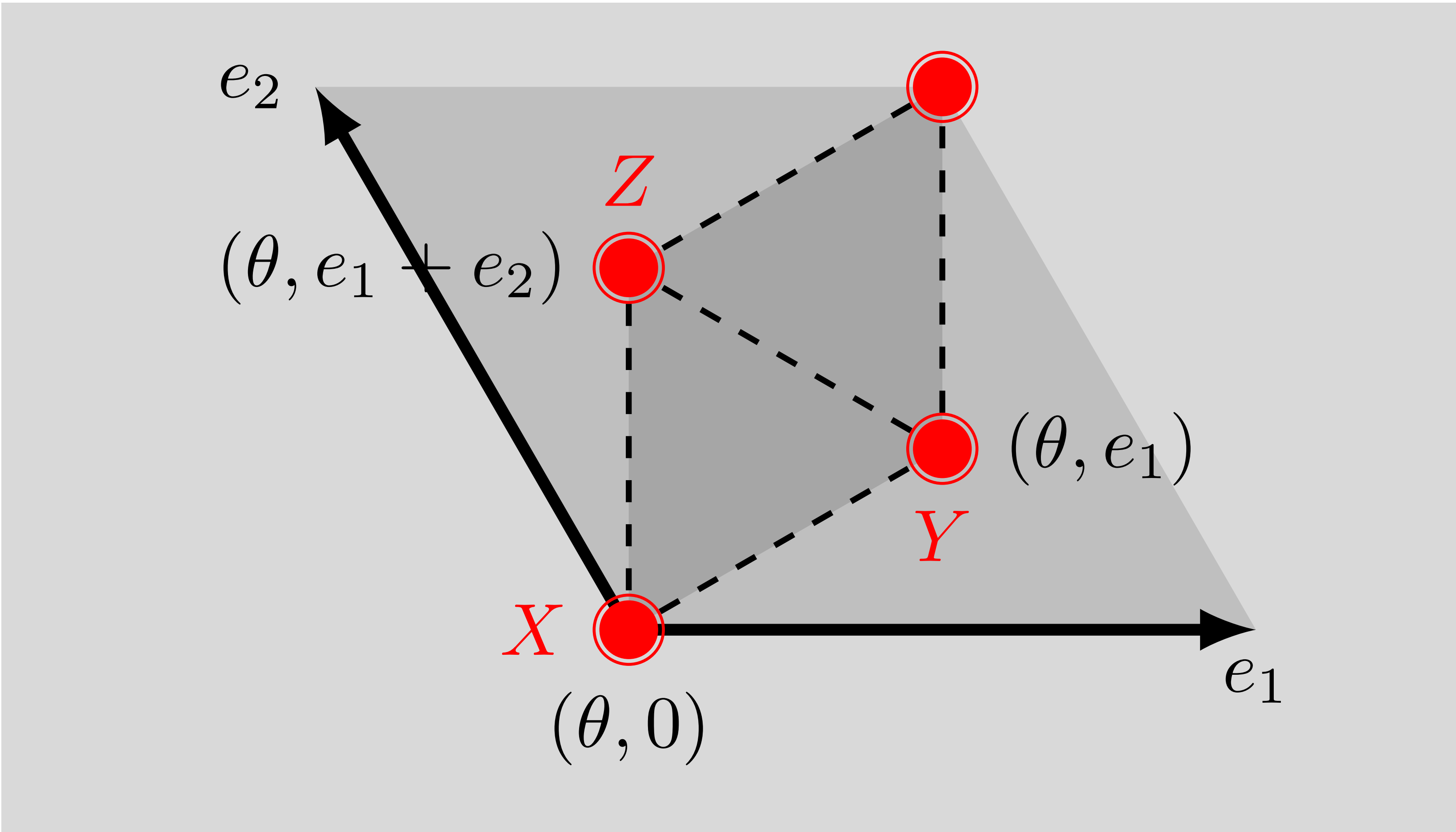
with $ad-bc=1$ and integer a,b,c,d.

Matter fields transform as representations $\rho(\gamma)$ and modular functions of weight k

$$\gamma: \quad \phi \rightarrow (cM+d)^k \rho(\gamma)\phi \; .$$

Yukawa-couplings transform as modular functions as well.
$G = K + \log|W|^2$ must be invariant under T-duality

Towards Modular Flavor Symmetry



Modular flavor symmetry

On the T_2/Z_3 orbifold some of the moduli are frozen,

- lattice vectors e_1 and e_2 have the same length
- angle is 120 degrees

Modular transformations form a subgroup of $SL(2, Z)$

- $\Gamma(3) = SL(2, 3Z)$ as a mod(3) subgroup of $SL(2, Z)$
- discrete modular flavor group $\Gamma'_3 = SL(2, Z)/\Gamma(3)$
- the discrete modular group is $\Gamma'_3 = T' \sim SL(2, 3)$ (which acts nontrivially on twisted fields); the double cover of $\Gamma_3 \sim A_4$ (which acts only on the modulus).
- the CP transformation $M \to -\overline{M}$ completes the picture.

Full discrete modular group is $GL(2, 3)$.

Eclectic Flavor Groups

We have thus two types of flavor groups

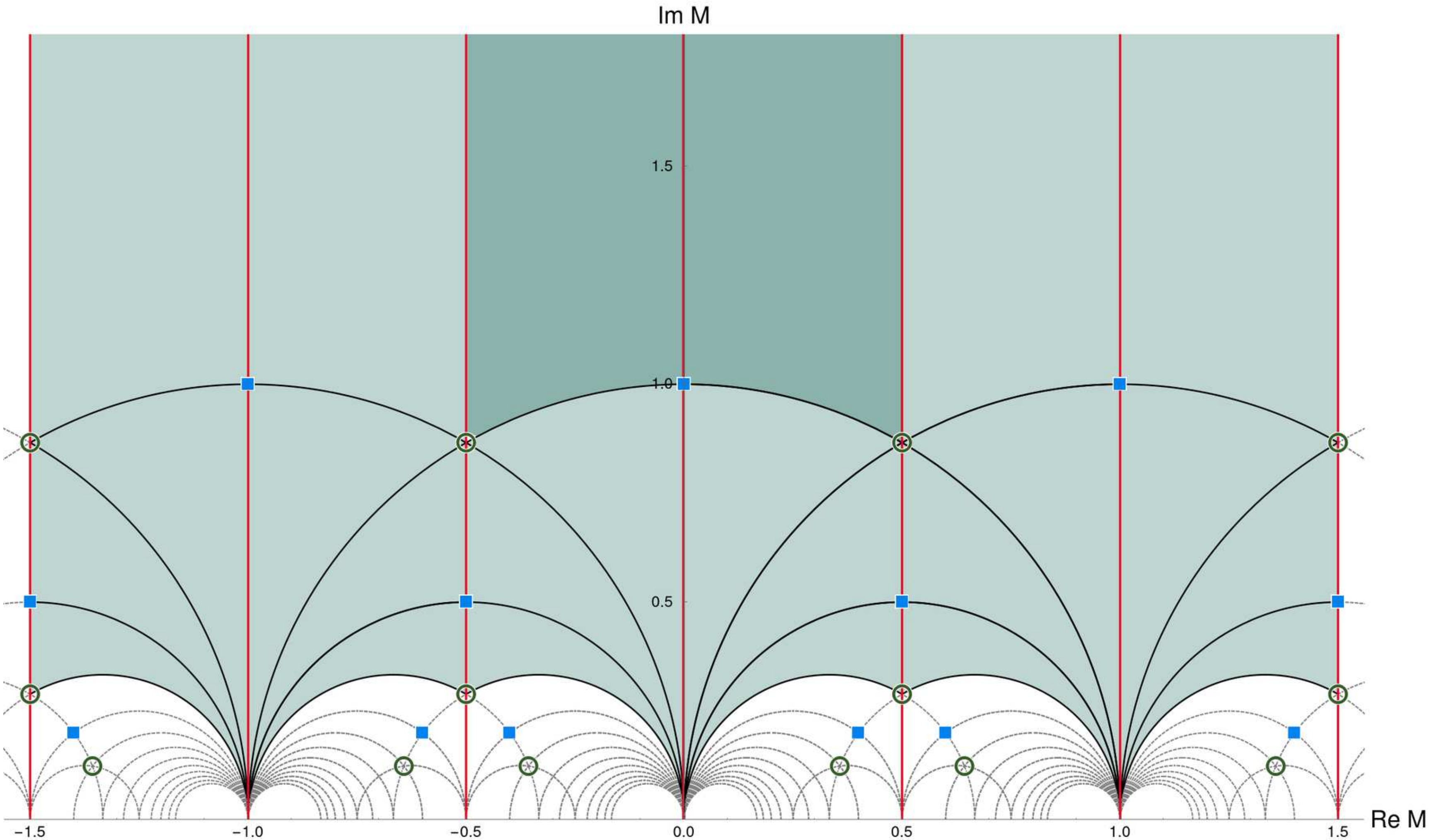
- the traditional flavor group that is universal in moduli space (here $\Delta(54)$)
- the modular flavor group that transforms the moduli nontrivially (here T')

The eclectic flavor group is defined as the multiplicative closure of these groups. Here we obtain for T_2/Z_3

- $\Omega(1) = SG[648, 533]$ from $\Delta(54)$ and $T' = SL(2, 3)$
- $SG[1296, 2891]$ from $\Delta(54)$ and $GL(2, 3)$ including CP

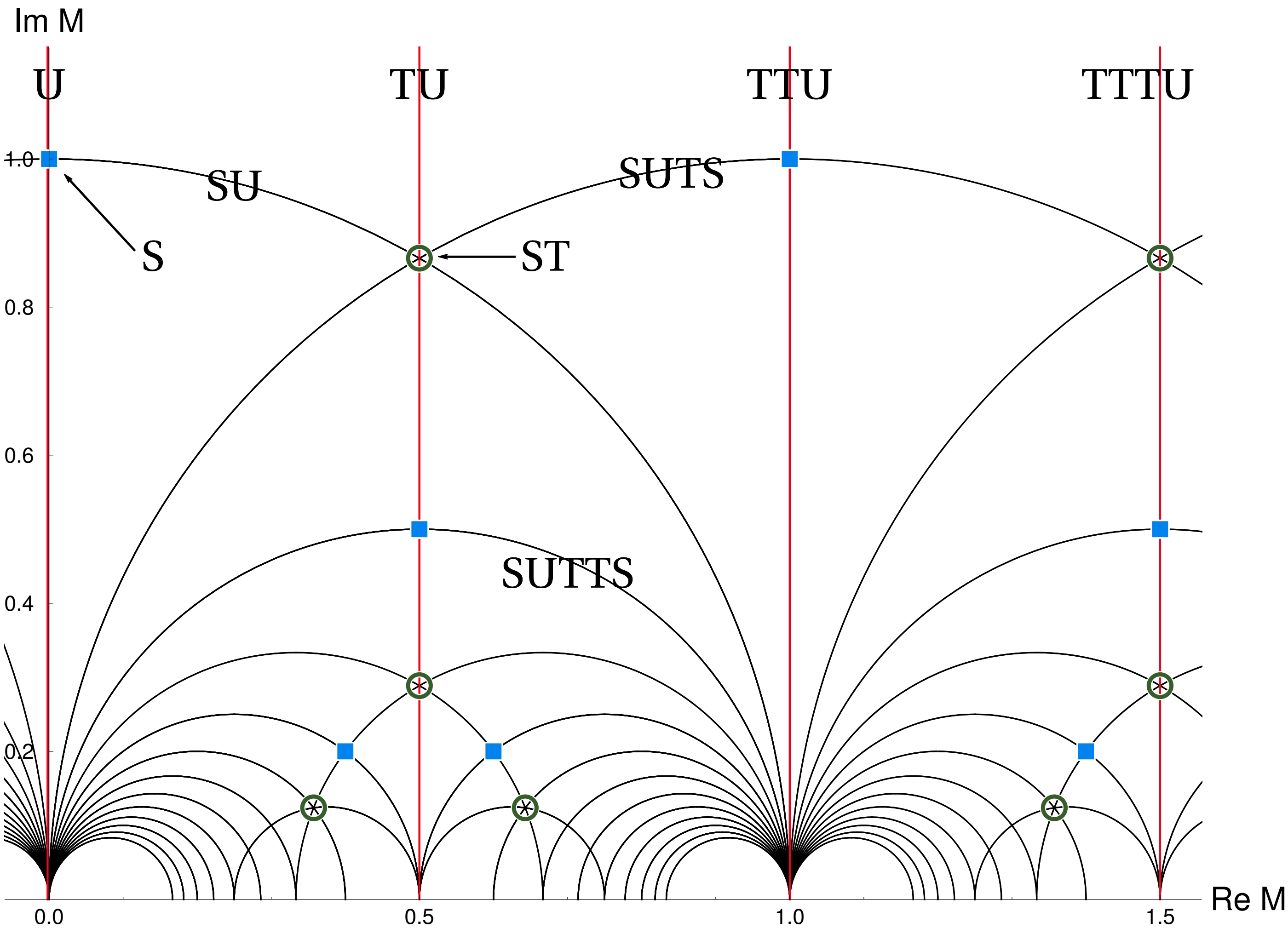
The eclectic group is the largest possible flavor group for the given system, but it is not necessarily linearly realized.

Local Flavor Unification



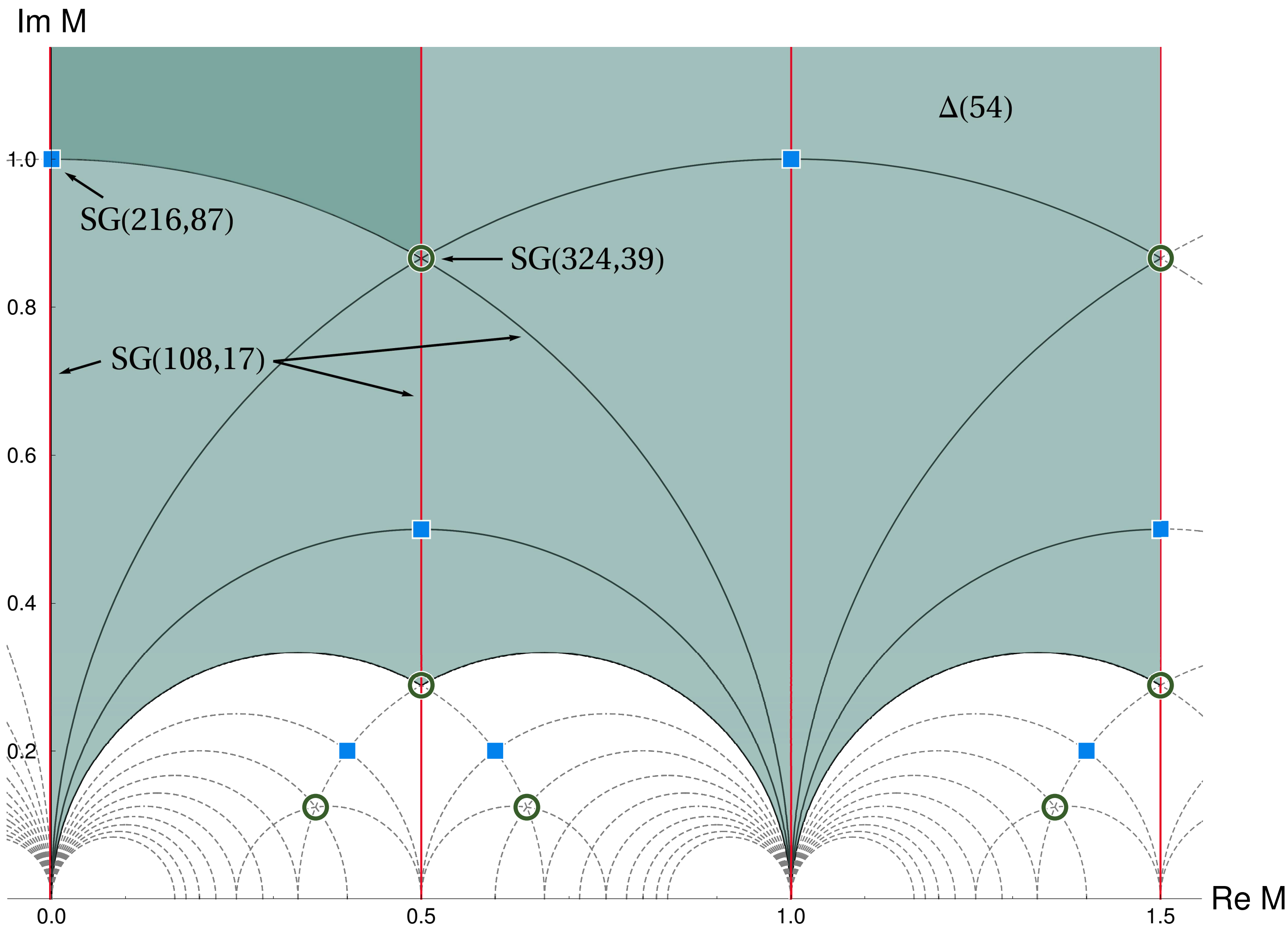
Moduli space of $\Gamma(3) = SL(2, 3Z)$

Fixed lines and points



$$\mathrm{S}: M \to -\frac{1}{M}, \quad \mathrm{T}: M \to M+1 \quad \text{and} \quad \mathrm{U}: M \to -\overline{M}$$

Moduli space of flavour groups



"Local Flavor Unification"

Unification of Flavor and CP

Summary of predictions of the string picture:

- traditional flavor symmetries (universal in moduli space)
- modular flavor symmetries and CP are non-universal in moduli space

They unify in the eclectic picture of flavor symmetry.
You cannot just have one without the other.

The non-universality in moduli space leads to

- enhanced symmetries at specific points in moduli space
- hierarchical structures of masses, mixing angles and phases in vicinity of fixed points or lines
- potentially different pictures for quarks and leptons

Classification

- Modular symmetry $SL(2, Z)$ is intrinsically related to the 2-torus T^2 with two moduli T and U
- Chiral fermions require a twist Z_k of T^2, embedded in 6-dimensiponal compact space.
- relevant are Z_k for $k = 2, 3, 4, 6$
- for $k = 3, 4, 6$ the T-modulus is fixed to allow for the twist
- higher k lead to larger modular symmetries Γ_k at the expense of smaller traditional flavor symmetries
- In the Z_3 we have $\Omega(1) = SG[648, 533]$ from $\Delta(54)$ and $T' = SL(2, 3)$
- embedding in 6-dimensional space gives additional R-symmetries

Z_2 orbifold: two moduli

Here the twist does not constrain the moduli T and U

- and we have the full $SL(2,Z)_T \times SL(2,Z)_U$.
- The discrete modular group is $\Gamma_2 \times \Gamma_2 \times Z_2$,
- where $\Gamma_2 = S_3$ and
- Z_2 interchanges T and U (known as mirror symmetry).
- The traditional flavor group is the product of $(D_8 \times D_8)/Z_2$ and a Z_4 R-symmetry.

This leads to an

- eclectic group with 2304 elements (excluding CP)
- or 4608 elements (including CP)

with a rich pattern of local flavor group enhancements.

Z_4 and Z_6

The 2-dimensional Z_3 orbifold gives already a promising result: $\Delta(54)$ and T'. What about the others?

- Z_6 gives modular group $\Gamma_2 \times \Gamma_3' = S_3 \times T' = [144, 128]$
- traditional flavor group is abelian (one fixed point)
- Z_6 eclectic group is $[144, 128] \times Z_{36}^R$ $(5184$ elements)
- Z_4 gives only the dihedral modular group $2D_3 = [12, 1]$ at the level of massless string states (and not $\Gamma_4' = S_4'$)
- traditional flavor symmetry is $[64, 185]$ (including the R-symmetry) and Z_4 eclectic group is $[384, 5614]$

Interpretation of quark- and lepton-multiplets is less clear than in the Z_3 case (Baur, Nilles, Ramos-Sanchez, Trautner, Vaudrevange, 2024)

Questions for Top-Down approach

The TD-approach was supposed to answer the following questions.

- What are the allowed discrete modular groups?
- Are they accompanied by other symmetries?
- Allowed representations of modular group?
- Restrictions on the modular weights?
- Is there a link between representations and modular weights?
- Information on moduli stabilization and the appearance of hierarchies for masses and mixing angles?

In the class of models based on twisted 2-dimensional tori we obtain severe restrictions from the TD-approach.

Answers

So far we have the following answers:

- Discrete modular groups:
  S_3, T' and $2D_3$ and nothing else (no A_4 etc.).
- Are they accompanied by other symmetries?
  Yes. There is a traditional flavor symmetry as well.
- Allowed representations of modular group?
  Very restrictive, e.g. no irreducible triplets so far.
- Restrictions on the modular weights?
  Only very few choices allowed.
- Link between representations and modular weights?
  Yes. Modular weights are fixed by representations.

So far none of the BU-models satisfies the TD-restrictions.

Outlook

This opens up a new arena for flavor model building and connections to bottom-up constructions:

- need more explicit string constructions
- need more BU-models that satisfy the TD rules.
- the concept of local flavor symmetries allows different flavor groups for different sectors of the theory, as for example quarks and leptons
- but it is not only the groups but also the representations of matter fields that are relevant. Not all of the possible representations appear in the massless sector.

There is still a huge gap between "top-down and bottom-up" constructions

Open Questions

So far $\Delta(54) \times T'$ seems to be the favourite model

- numerous bottom-up models with these groups
- successful realistic string models from Z_3 orbifolds
- Z_2, Z_4 and Z_6 alternatives seem less attractive

"Local Flavor Unification" leads to successful fits in the vicinity of fixed points and lines

(Feruglio, 2022-23; Petcov, Tanimoto, 2022; Abe, Higaki, Kawamura, Kobayashi, 2023)

- moduli stabilization favours boundary of moduli space, but AdS-minima (Cvetic, Font, Ibanez, Lüst, Quevedo, 1991)
- uplift moves them slightly away from the boundary

(Knapp-Perez, Liu, Nilles, Ramos-Sanchez, Ratz, 2023)

Summary

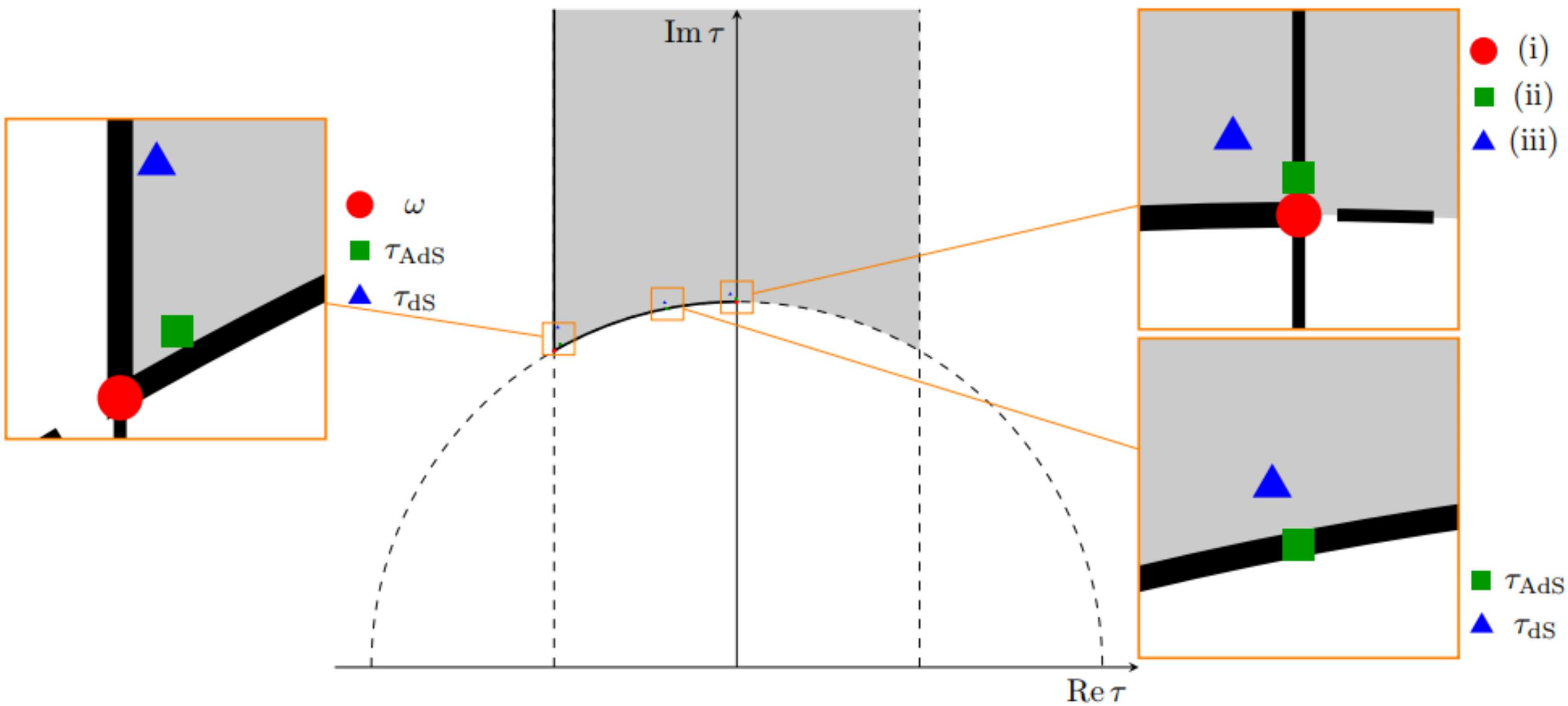
String theory provides the necessary ingredients for flavor:

- traditional flavor group
- discrete modular flavor group
- a natural candidate for CP
- the concept of local flavour unification

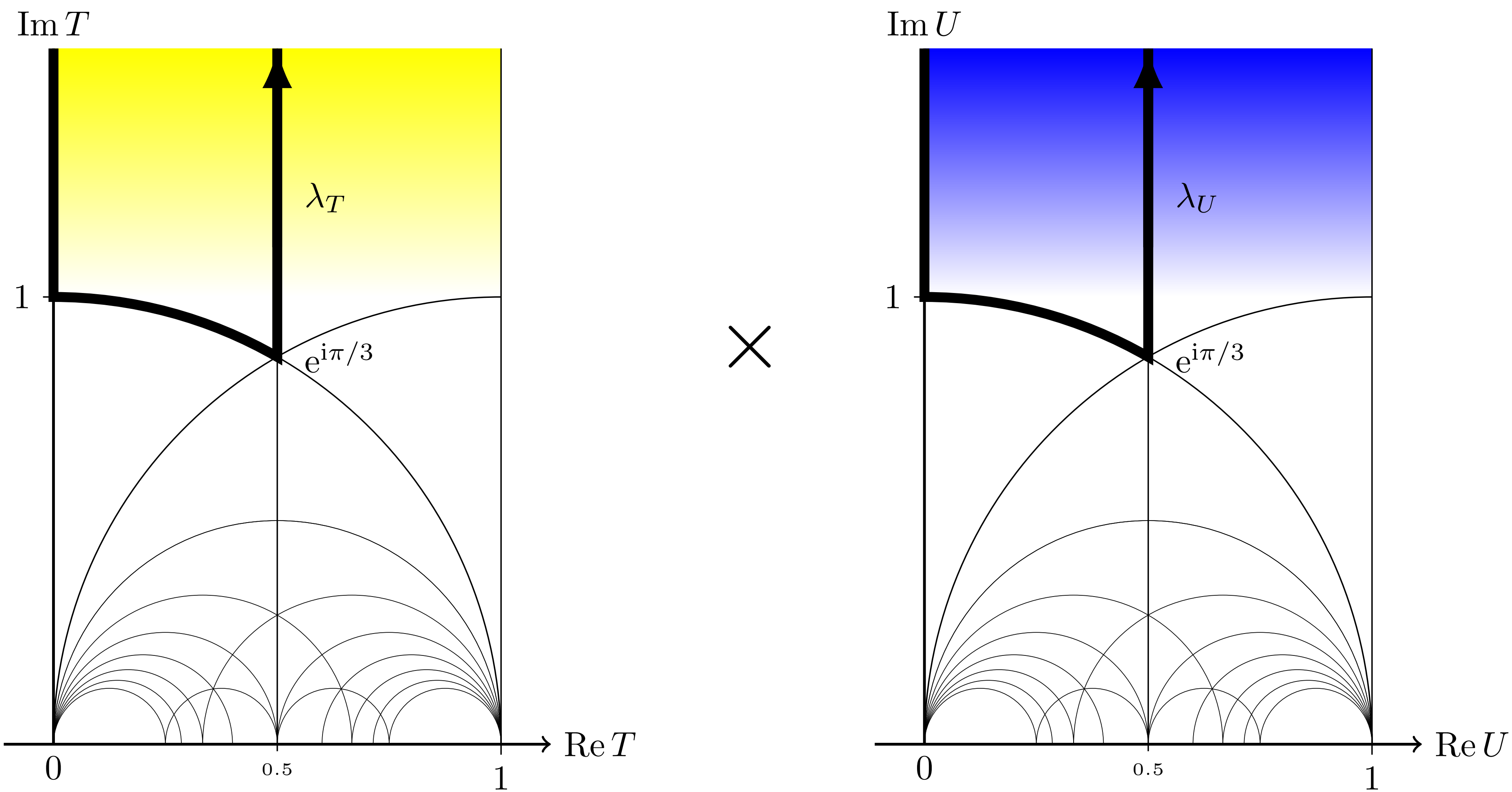
The eclectic flavor group provides the basis:

- this includes a non-universality of flavor symmetry in moduli space
- allows hierarchies of masses and mixing angles

Back-up Slides

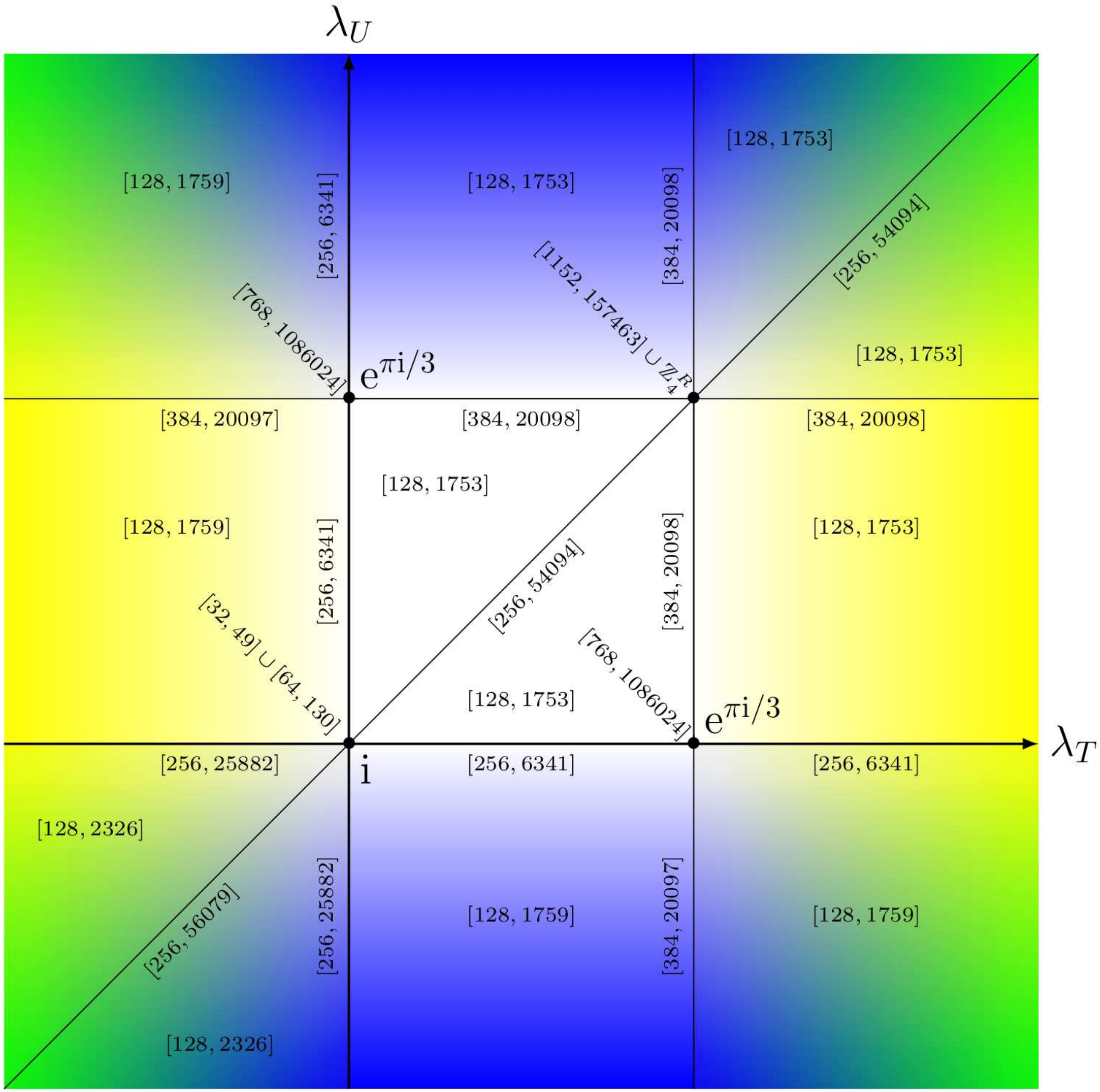


Z_2-orbifold

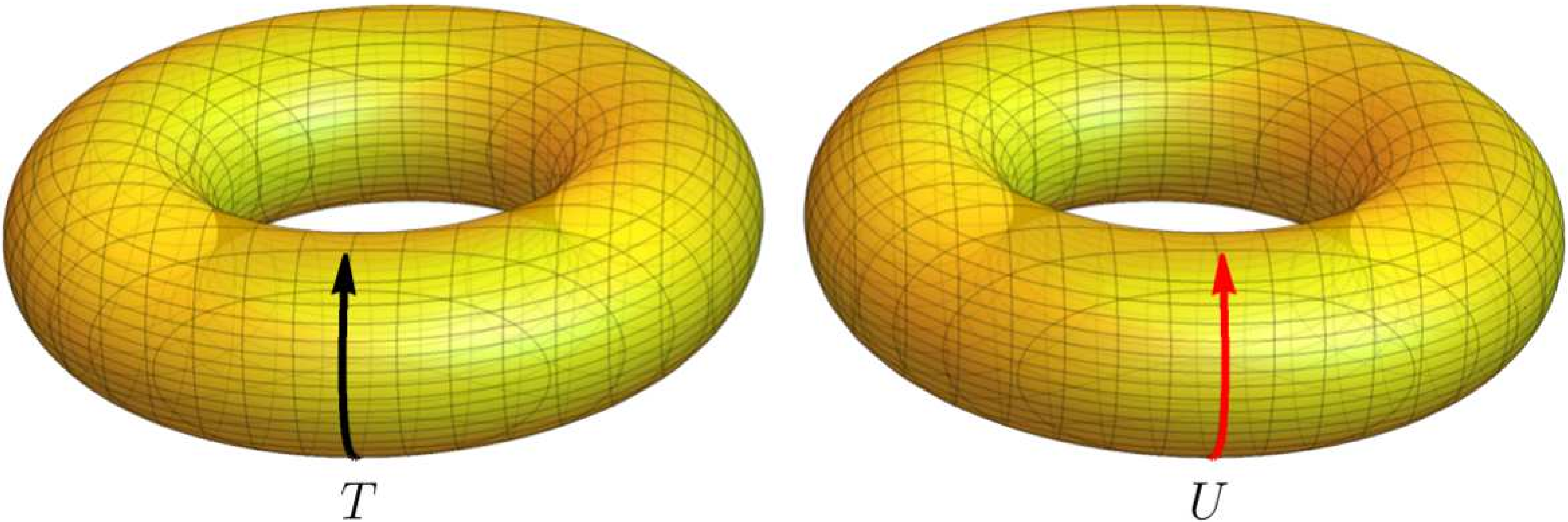


Here we have two unconstrained moduli: T and U

Enhancement

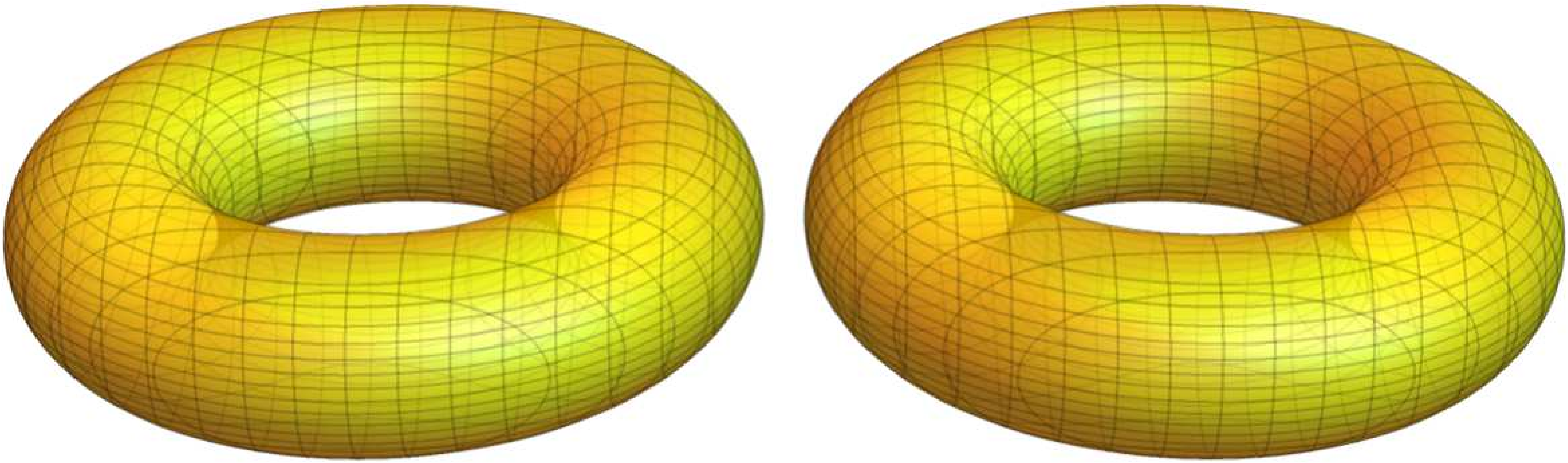


Auxiliary Surface: Double Torus



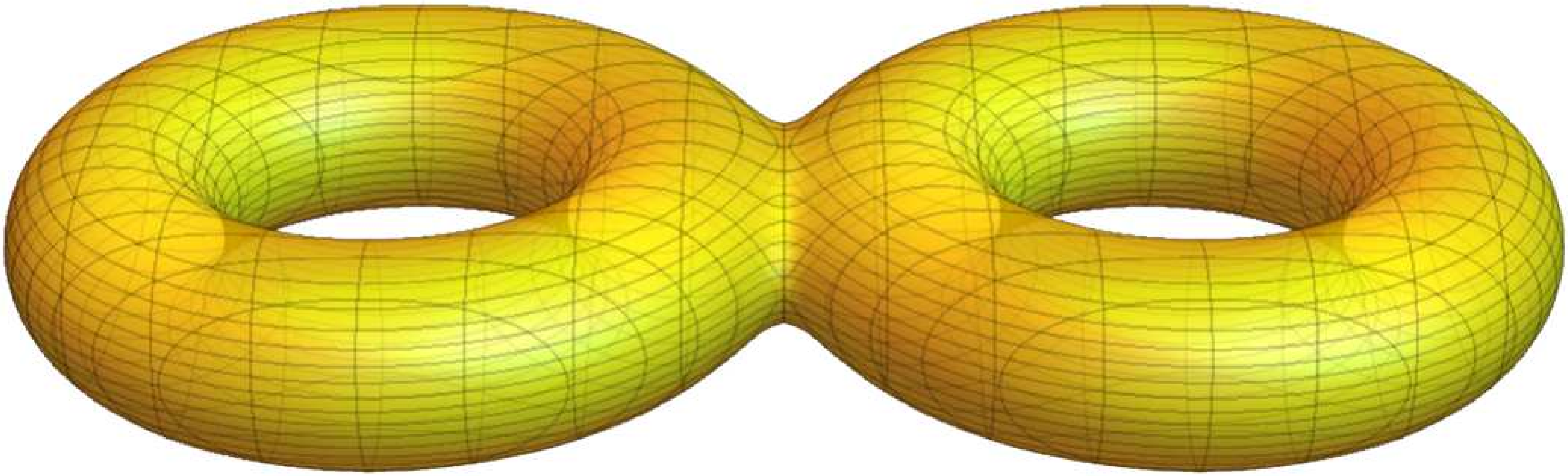
Auxiliary surface for two moduli: $SL(2,Z)_T \times SL(2,Z)_U$

Riemann surface of genus 2



Auxiliary surface for two moduli: T and U

Riemann Surface of Genus 2



Auxiliary surface with three moduli: T + U + Wilson line

Siegel Modular Forms

This leads to a generalization of the modular group to larger groups $Sp(2g, Z)$ characterized through Riemann surfaces of higher genus g:

- for $g = 2$ the Siegel modular group $Sp(4, Z)$
- includes $SL(2, Z)_{U,T}$ and describes three moduli.
- Fundamental domain (6 points, 5 lines, 2 surfaces)
- Orbifold twists are connected to fixed loci in fundamental domain
- Discrete modular group $\Gamma_{g,k}$ ($\Gamma_{1,k} = \Gamma_k$)
- $\Gamma_{2,2} = S_6$ includes $S_3 \times S_3$ and mirror symmetry
- $\Gamma_{2,3}$ has already 51840 elements