Corfu Summer Institute September 07 2024

Fibre Inflation in Large Volume Compactifications*

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(arXiv: 2406.0169; 2405.06738(JCAP)).

PRELUDE

▲ **String Theory** provides a robust framework to study Physics Phenomena in a vast range of Energies varying from Planck to minuscule scales!

▲ Early (80s) phenomenological explorations focus mostly on model building of GUTs and SM (*still an active research area*)

▲ Remarkably, the ensuing years, the implications of String Theory for cosmology have been proved equally important!

▲ In fact, in the study of effective field theory models, vital Physics issues near the Planck scale must be addressed!

▲ An important issue is that, in compactifications **large numbers** of massless scalar (**moduli**) fields appear, which must be **stabilised**!

▲ Then, under the right conditions, such fields can solve important problems in cosmology.

In this talk I will discuss :

▲ **Cosmological Inflation** in Type IIB compactifications in the context of **Large Volume Scenarios** (**LVS**) (hep-th/0502058)

▲ One of the most attractive inflationary models that can be realised in **LVS** is **Fibre Inflation**

▲ In this context, two basic approaches will be analysed: Non Perturbative & Perturbative

▲ The role of Kähler Cone Constraints will be examined in the above two approaches

▲ The merits and demerits of these scenarios will be discussed.

TYPE IIB SETUP

The main elements to be used (moduli, fluxes ...)

$(\mathbf{NS}_+, \mathbf{NS}_+)$: graviton, dilaton and 2-form Kalb-Ramond-field:

$$g_{\mu\nu},\ \phi,\ B_{\mu\nu} \to B_2$$

$(\mathbf{R}_-, \mathbf{R}_-)$: scalar, 2- and 4-index fields (*p-form potentials*)

$$\mathbf{C_0}, C_{\mu\nu}, C_{\kappa\lambda\mu\nu} \to C_p,\ p = 0, 2, 4$$

▲ $C_0,\ \phi\ \to$ **axion-dilaton** *modulus:*

$$S = C_0 + i\, e^{-\phi} \equiv C_0 + \frac{i}{g_s}$$

▲ *Field strengths/magnetic fluxes:*

$$F_p := d\, C_{p-1},\ H_3 := d\, B_2,\ \Rightarrow \mathbf{G_3} := F_3 - S H_3$$

▲ *Holomorphic* $(3,0)$*-form :* $\Omega(U_a)$

▲ Let U^i complex-structure (CS) and T_α Kähler moduli ($T_s = c_s - i\tau_s$)

▲ Kähler form J expressed in terms of 2-cycle t^k, i.e., $J = J(t^k)$ is expanded in harmonic forms through definition of the basis

$$\hat{D}_k,\ k = 1, 2, \ldots, h^{1,1}$$

$$J = \sum_{k=1}^{h^{1,1}} t^k \hat{D}_k, \tag{1}$$

Volume of internal space

$$\mathcal{V} = \frac{1}{3!} \int_{CY} J \wedge J \wedge J = \frac{1}{6} k_{ijk} t^i t^j t^k \tag{2}$$

▲ Low energy dynamics of 4D effective SUGRA from type IIB compactified on CY orientifolds can be captured by a holomorphic superpotential W, and a real Kähler potential K

$$\mathcal{W}_0 = \int \mathbf{G_3} \wedge \Omega(U_a) \tag{3}$$

$$\mathcal{K}_0 = -\log[-i(S-\bar{S})] - 2\log\mathcal{V} - \log[-i\int \Omega\wedge\bar{\Omega}] \tag{4}$$

▲ The F-term contributions to the scalar potential of 4D $\mathcal{N}=1$ from the type IIB encoded in

$$V = e^{\mathcal{K}}(K^{\mathcal{A}\overline{\mathcal{B}}}(D_{\mathcal{A}}W)(D_{\overline{\mathcal{B}}}\overline{W}) - 3|W|^2)$$

,

★ $\mathcal{A}$) Non-Perturbative Moduli Stabilisation

Moduli stabilisation in 4D type IIB effective supergravity models follows a **two-step procedure**.

▲ First, one fixes the CS moduli U^i and the axio-dilaton S by the leading order $W_0 \equiv W_{\rm flux}$ induced by the 3-form fluxes (F_3, H_3)

▲ No-scale structure protects the Kähler moduli T_α which remain flat.

At a second step, they can be stabilised via non-perturbative corrections arising from the whole series of α' and string-loop (g_s) corrections:

$$W = W_0 + W_{\rm np}(S, T_\alpha),$$
$$K = K_{\rm cs} - \ln\left[-i\,(S - \bar{S})\right] - 2\ln\mathcal{Y}, \tag{5}$$

where generally $\mathcal{Y}$ function of $\mathcal{V}$, α' and string-loop corrections.

★ $\mathcal{B}$) FIBRE INFLATION (FI)

FI models are built in the context of IIB orientifold flux compactifications (0808.0691, . . . , 1709.01518)

The generic geometric set up includes $D3/D7$ branes and $O(3)/O(7)$ planes

▲ The internal (CY) volume is of the generic form

$$\mathcal{V} = f_{\frac{3}{2}}(\tau_i) - \sum_{j=1}^{N_s} \lambda_j \tau_j^{3/2} \tag{6}$$

- τ_i: "large" divisors $i = 1, 2, \ldots N_l$.
- τ_j: "small" blow-up rigid divisors $j = 1, 2, \ldots N_s$.
- $N_l + N_s = h^{1,1}$.
- $f_{\frac{3}{2}}$: degree $\frac{3}{2}$ homogeneous function of τ_i

Leading ${\alpha'}^3$ corrections in the Kähler potential:

$$\xi \;=\; -\frac{\zeta(3)}{4(2\pi)^3}\chi = \hat{\xi}\left(\frac{S-\bar{S}}{2i}\right)^{-3/2} \equiv \hat{\xi}\; g_s^{3/2}\;.$$

The α' correction is incorporated into the Kähler potential through the shift:

$$\hat{\mathcal{V}} \to \mathcal{U} = \hat{\mathcal{V}} + \frac{\hat{\xi}}{2} \equiv \hat{\mathcal{V}} + \frac{\xi}{2}\left(\frac{S-\bar{S}}{2i}\right)^{3/2} \equiv \hat{\mathcal{V}} + \frac{\xi}{2}\frac{1}{{g_s}^{3/2}}\;.$$

Then, the α' corrected Kähler potential acquires the form:

$$\mathcal{K}_{\alpha'} \;=\; -\log(-i(S-\bar{S})) - 2\log(\mathcal{U}) - \log(-i\int \Omega\wedge\bar{\Omega}), \quad (7)$$

The GVW superpotential $\mathcal{W}_0$ given by

$$\mathcal{W}_0 = \int G_3 \wedge \Omega(z_a)\,, \tag{8}$$

is corrected by non-perturbative contributions.

▲ NP contributions can be generated by divisors which are stable under perturbations and have fixed complex structures, i.e., **rigid** ones, such as del Pezzo (dP) divisors. Thus, generically

$$\mathcal{W} = \mathcal{W}_0 + \sum_k \mathcal{A}_k e^{-a_k T_k} \tag{9}$$

which are generated by D-brane instantons and gaugino condensation.

The coefficients $\mathcal{A}_k$ may depend on complex structure moduli, but in most cases they are considered constants.

Procedure and Conditions

Recall that:

$$\mathcal{V} = f_{\frac{3}{2}}(\tau_i) - \sum_{j=1}^{N_s} \lambda_j {\tau_j}^{3/2}$$

Step 1: Overall Volume $\mathcal{V}$ and volumes of N_s small blow-up divisors τ_j are stabilised by α'^3 corrections in K and NP-contributions in W.

$N_l - 1 \equiv h^{1,1} - N_s - 1$ directions remain flat.

$\Rightarrow$ natural inflaton candidates

Step 2: Subleading $\mathcal{O}(g_s)$ corrections due to KK exchange and winding modes fix the remaining d.o.f.

The potential for these moduli is flatter and thus suitable for slow roll inflation.

A simple model with $h^{1,1} = 3$ (see e.g. 1801.05434)

In suitable divisor basis $\hat{D}_b, \hat{D}_f, \hat{D}_s$ with D_s 'diagonal' (i.e. only $k_{sss} \neq 0$, while $k_{ijs} = 0, \forall i \neq s \neq j$), the internal volume is:

$$\mathcal{V} = \lambda_1 \tau_b \sqrt{\tau_f} - \lambda_j {\tau_s}^{3/2}$$

▲ Assuming only α'^3 corrections and

$$W = W_0 + A_s e^{-i a_s T_s}, \; T_s = c_s - i\tau_s$$

where c_s is the C_4 axion.

▲ The scalar potential admits Large Volume minimum if:
1)$\chi < 0$, which implies $h^{1,1} < h^{2,2}$ and $\xi > 0$.
2) The D_s divisor supports NP-effects

▲ This minimal case $h^{1,1} = 3$ leaves only one flat direction τ_f.

String Loop Effects (hep-th/0507131,...,0704.0737)

Subleading string-loop effects known as KK and winding types generate new $V_{g_s}^{KK} + V_{g_s}^{W}$ subleading potential terms for τ_f.

Scalar potential to leading order in minimal FI model:

$$V_{\rm LVS} \approx \frac{|W_0|^2}{\mathcal{V}^2} \left(\frac{\beta_1}{{\tau_f}^2} - \frac{\beta_2}{\mathcal{V}\sqrt{\tau_f}} + \frac{\beta_3 \tau_f}{\mathcal{V}^2} \right) + V_{up}$$

$\beta_{1,2,3}$ positive constants, functions of (W_0, ξ, A_s, k_{sss}) and V_{up} uplift term required to achieve dS minimum.

Kähler Cone Constraints

The Kähler moduli space must be such so that ensures a positive definite Kähler form:

$$\int_{C_i} J > 0$$

This Kähler Cone Condition (KCC) concerns all topologically non-trivial effective curves C_i in the internal manifold (*Mori Cone*).

Thus, while at leading order τ_f remains flat, fixing of $\mathcal{V}$ and τ_s puts bound on field range of τ_f.

KCC translates to constraints of the form

$$\sum_\beta n_{\alpha\beta} t^\beta \ > \ 0, \quad n_{\alpha\beta} \in \mathbf{Z}$$

For $h^{1,1} = 3 \ \Rightarrow n_s t^s + n_b t^b + n_f t^f > 0$ which implies

$$\frac{n_f}{\tau_f} \left(\mathcal{V} + \lambda_s \tau_s^{3/2} \right) + 2\sqrt{2}\, \lambda_b \, n_b \sqrt{\tau_f} > 3\lambda_s \, n_s \sqrt{\tau_s}. \tag{10}$$

In the case of exceptional divisor, $\exists$ diagonal basis where the KCC condition becomes

$$t^s < 0 \leftrightarrow \tau_s > 0$$

Generically, for admissible $\{n_b, n_f, n_s\}$ sets, there are corresponding upper bounds. For example, in a typical model one finds

$$6 < \tau_f < 208$$

For the canonical field $\varphi \sim \sqrt{2}/3 \log(\tau_f)$, these bounds imply:

$$\varphi \lesssim 2.5$$

Notice however, that for a successful slow roll we need

$$\varphi \sim \mathcal{O}(10) M_{Pl}$$

PERTURBATIVE FIBRE INFLATION

The *perturbative LVS* (*Antoniadis et al 1909.10525*) provides a new way to realise LVS inflation, and in particular Fibre Inflation, without implementing non-perturbative effects.

▲ Hence use of **rigid** exceptional divisors can be circumvented, and Kähler Cone Conditions do not put strong bound on the inflaton field's range.

We will demonstrate this feature by considering a compact connected manifold with smooth geometry, more concretely a K3-fibred CY orientifold with toroidal-like volume.

Global model:

We consider a CY_3 with $h^{1,1} = 3$ (polytope Id: 249 in the CY database of KS/hep-th 0002240)

It is described by the following toric data:

Hyp	x_1	x_2	x_3	x_4	x_5	x_6	x_7
4	0	0	1	1	0	0	2
4	0	1	0	0	1	0	2
4	1	0	0	0	0	1	2
	$K3$	$K3$	$K3$	$K3$	$K3$	$K3$	SD

Hodge numbers $(h^{2,1}, h^{1,1}) = (115, 3)$,

Euler number $\chi = -224$.

Stanley-Reisner ideal: $\mathrm{SR} = \{x_1 x_6,\ x_2 x_5,\ x_3 x_4 x_7\}$

Analysis of the divisor topologies shows:

- The first 6 toric divisors are K3 surfaces
- The 7^{th} one is described by Hodge numbers $\{h^{0,0} = 1, h^{1,0} = 0, h^{2,0} = 27, h^{1,1} = 184\}$.
- In the divisor basis $\{\hat{D}_1, \hat{D}_2, \hat{D}_3\}$, the Kähler form is

$$J = t^1 \hat{D}_1 + t^2 \hat{D}_2 + t^3 \hat{D}_3$$

- The only non-zero intersection is $k_{123} = 2$ leading to

$$\mathcal{V} = 2\, t^1\, t^2\, t^3 = \frac{1}{\sqrt{2}} \sqrt{\tau_1\, \tau_2\, \tau_3}$$

- The Kähler cone conditions are:

$$\text{KCC:} \qquad t^1 > 0, \quad t^2 > 0, \quad t^3 > 0. \tag{11}$$

Subleading corrections

The divisor intersection analysis shows

▲ All the three $D7$-brane stacks intersect at $\mathbb{T}^2$

▲ There are no non-intersection $D7$-brane stacks and the $O7$-planes along without $O3$-planes present as well.

Therefore

▲ The model does not induce KK-type string-loop corrections to the Kähler potential.

▲ Absence of $O3$-planes $\Rightarrow$ $\overline{D3}$ uplifting is not directly applicable

▲ Because $D7$-brane stacks intersect on non-shrinkable two-torii $\exists$ string-loop effects of the winding-type $V^{\rm W}_{g_s} = -\frac{\kappa|W|^2}{\mathcal{V}^3}\sum_a \frac{C^w_a}{t^a}$

▲ $K3$ basis divisor implies non-zero second Chern number $\Rightarrow$ $\exists$ higher derivative $V_{\rm F^4} \propto \Pi_\alpha t^\alpha$ corrections

All contributions give rise to the following scalar potential:

$$V_{\rm eff} \approx V_{\rm up} + \frac{\mathcal{C}_1}{\mathcal{V}^3}\left(\hat{\xi} - 4\,\hat{\eta} + 2\,\hat{\eta}\,\ln\mathcal{V}\right) \tag{12}$$

$$+\frac{\mathcal{C}_2}{\mathcal{V}^4}\left(\mathcal{C}_{w_1}\tau_1 + \mathcal{C}_{w_2}\tau_2 + \mathcal{C}_{w_3}\tau_3 + \frac{\mathcal{C}_{w_4}\,\tau_1\tau_2}{2(\tau_1+\tau_2)}\right. \tag{13}$$

$$\left. + \frac{\mathcal{C}_{w_5}\,\tau_2\tau_3}{2(\tau_2+\tau_3)} + \frac{\mathcal{C}_{w_6}\,\tau_3\tau_1}{2(\tau_3+\tau_1)}\right) + \frac{\mathcal{C}_3}{\mathcal{V}^3}\left(\frac{1}{\tau_1}+\frac{1}{\tau_2}+\frac{1}{\tau_3}\right) \tag{14}$$

where

$$\begin{aligned} \mathcal{C}_1 &= \frac{3}{4}\,\kappa\,|W_0|^2 = \frac{3}{4}\mathcal{C}_2, \\ \mathcal{C}_3 &= -24\,\lambda\,\kappa^2\,|W_0|^4/g_s^{3/2} \end{aligned} \tag{15}$$

Part (12) fixes the volume $\mathcal{V}$ (*Antoniadis,Chen, GKL 2018*).
Parts (13) and (14) fix one more modulus. Then:
$V_{\rm eff}$ depends on one modulus, $V_{\rm eff} = V(\tau_3)$ which drives inflation

Inflationary dynamics:

Define the canonically normalised fields,

$$\varphi^\alpha = \frac{1}{\sqrt{2}} \ln \tau_\alpha, \ \alpha \in \{1,2,3\}, \ \text{so that}$$

$$\mathcal{V} \propto e^{\frac{1}{\sqrt{2}}(\varphi^1+\varphi^2+\varphi^3)}$$

The scalar potential takes the form

$$V = \mathcal{C}_0 \left(\mathcal{C}_{\text{up}} + \mathcal{R}_0 e^{-\gamma\phi} - e^{-\frac{\gamma}{2}\phi} + \mathcal{R}_1 e^{\frac{\gamma}{2}\phi} + \mathcal{R}_2 e^{\gamma\phi} \right), \tag{16}$$

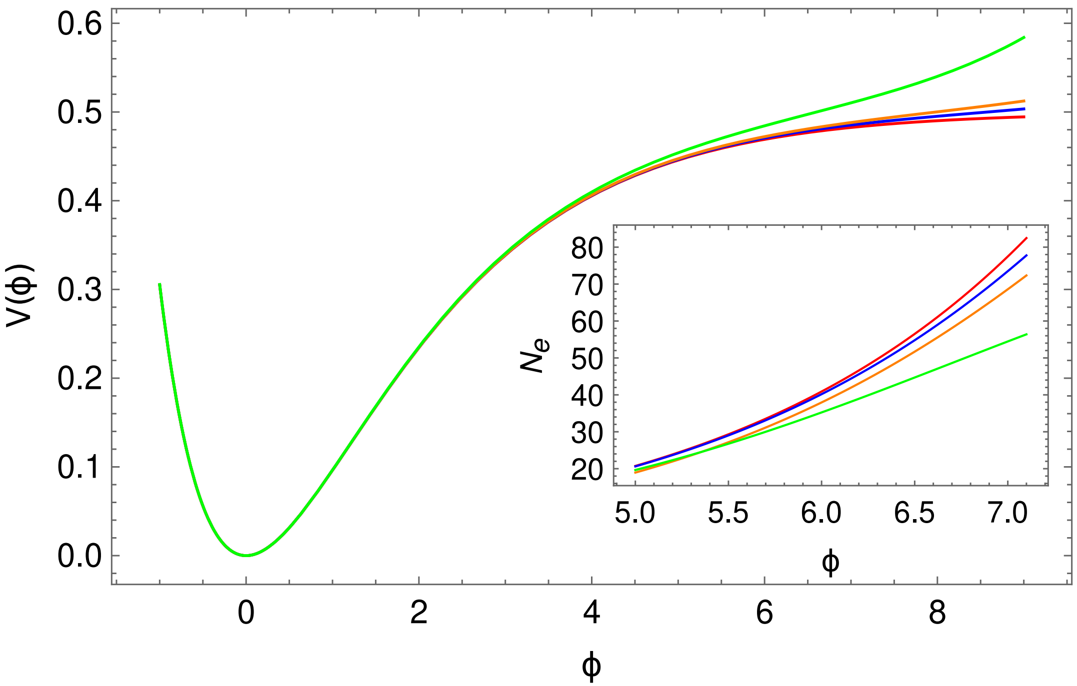
$\gamma = \frac{\sqrt{2}}{3}$, $\varphi = \langle \varphi \rangle + \phi$, and $C_{up} = 1 - \mathcal{R}_0 - \mathcal{R}_1 - \mathcal{R}_2$
is the required up-lift for dS vacuum.
Notice $\overline{D3}$ up-lift not possible due to absence of $O(3)$-planes
Nevertheless, D7-brane or T-uplift (1512.04558) can be implemented.

A benchmark model:

$$\mathcal{C}_0 = 5.78 \times 10^{-10}, \;\; \mathcal{R}_1 = 5.00 \times 10^{-5}, \;\; \mathcal{R}_2 = 1.00 \times 10^{-7}$$

which correspond to string parameters:

$$|W_0| = 145, \;\; g_s = 0.3, \;\; \langle \mathcal{V} \rangle = 1.5 \times 10^4$$



Efolds, scalar perturbation amplitude, spectral index:

$$N_e{}^* = 51, \quad P_s = 2.1 \times 10^{-9}, \quad n_s{}^* = 0.966$$

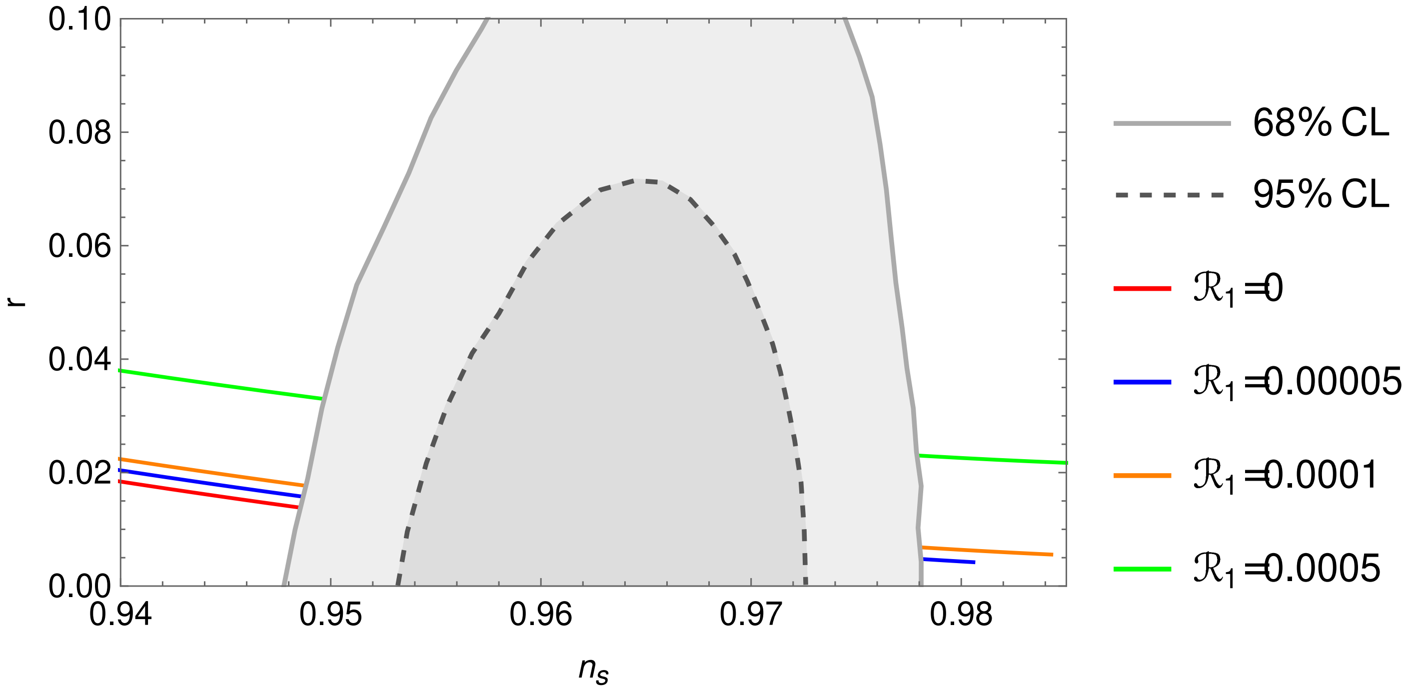


Figure 1: Plot of spectral index n_s vs tensor-to scalar ratio r.

CONCLUSIONS

In this talk, I have presented the two basic directions that we have explored for a fully fledged stringy fibre inflation scenario:

▲ Realisation of Fibre Inflation in Perturbative LVS. (PLVS)

• It was shown that Kähler Cone Conditions are milder and easy to satisfy in PLVS.

• This gives the opportunity to construct a robust string scenario to realise FI

▲▲ Global Embedding within simple CYs having:

• minimal number of Kähler moduli to accommodate inflation

• simple toroidal volume $\mathcal{V} = \sqrt{\tau_1 \tau_2 \tau_3}$.

THANK YOU