Non-gravitational signals of

Dark energy under a gauge symmetry

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with Kunio Kaneta, Jiheon Lee, Jaeok Yi

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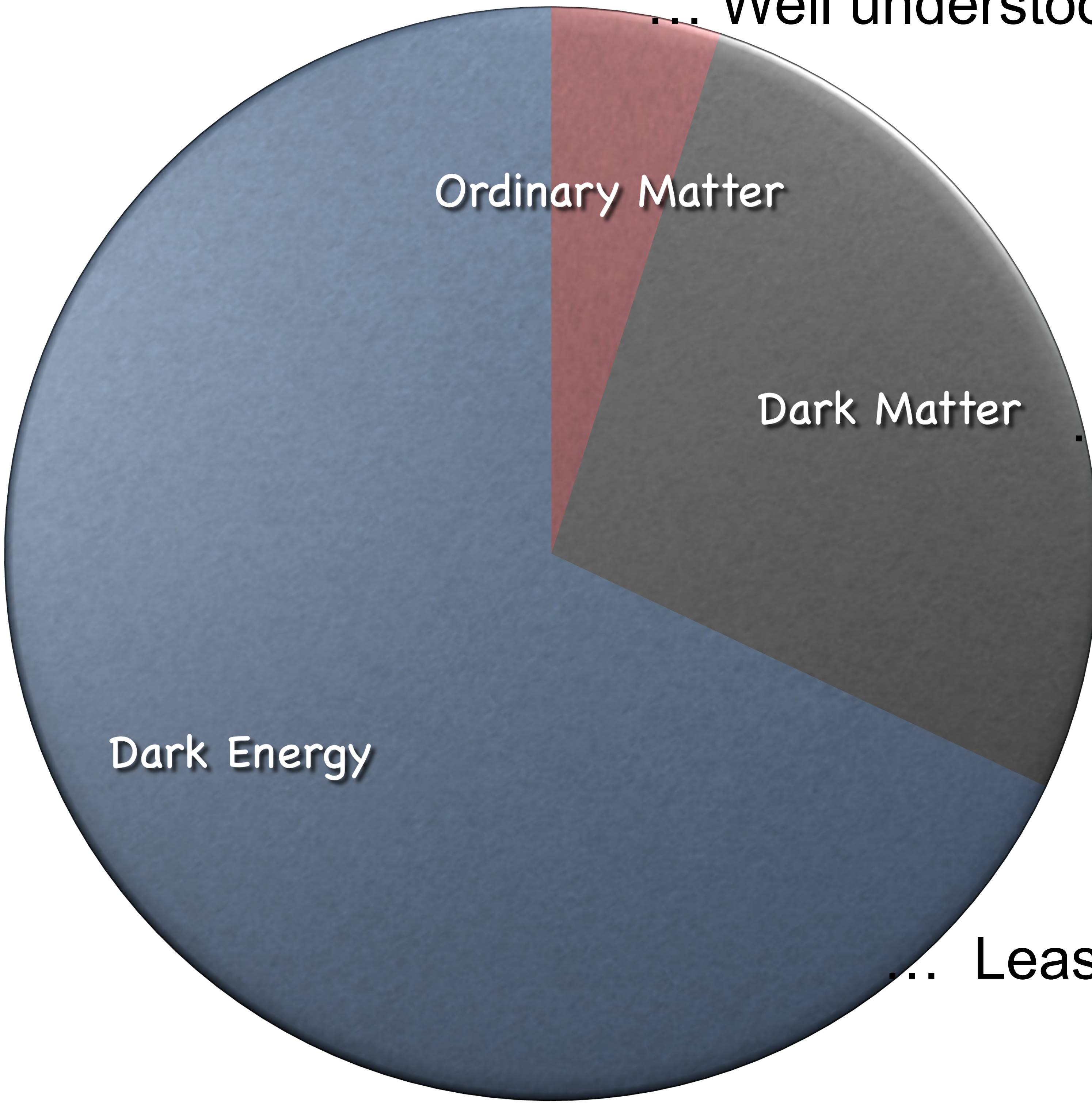
August 31, 2024

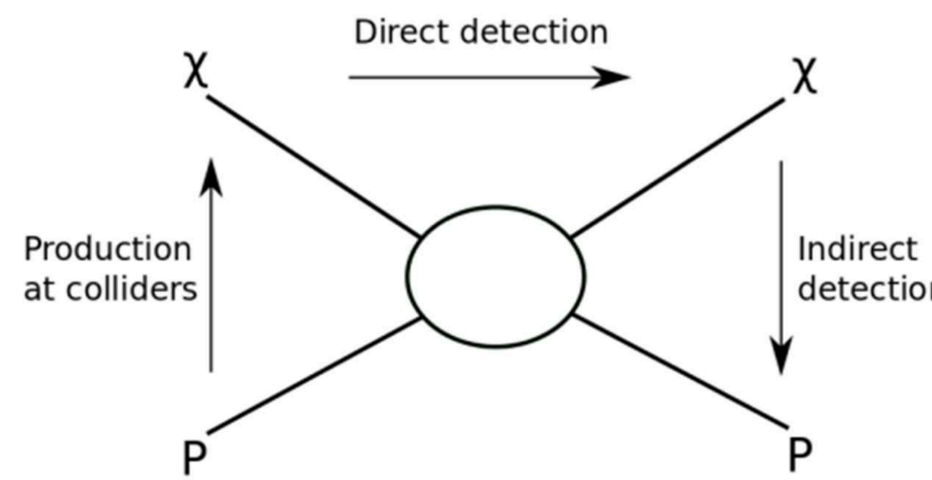
The universe is an enormous direct product of representations of symmetry groups.

- Steven Weinberg -

... Well understood

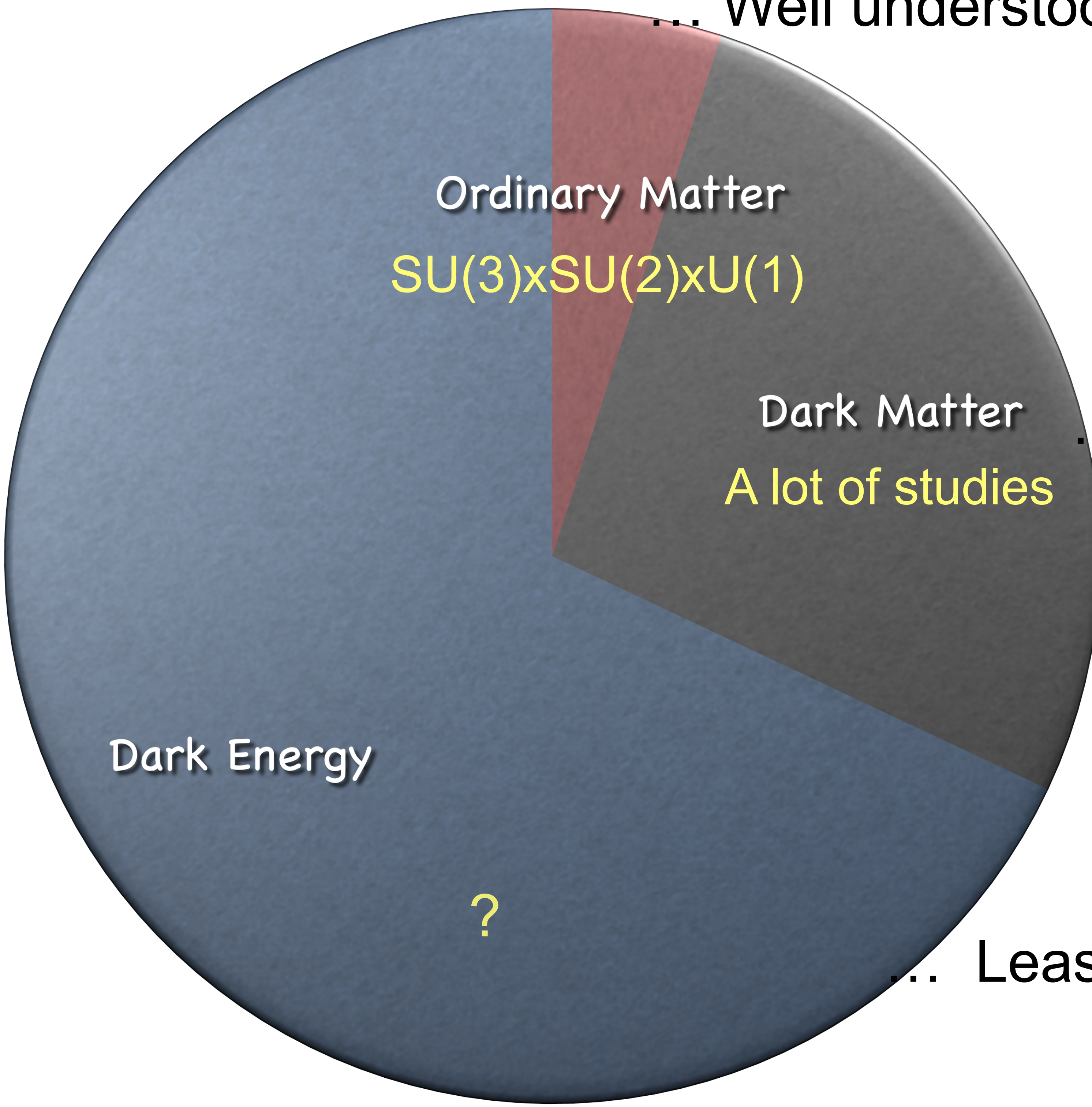


... Global competitions



... Least understood

... Well understood



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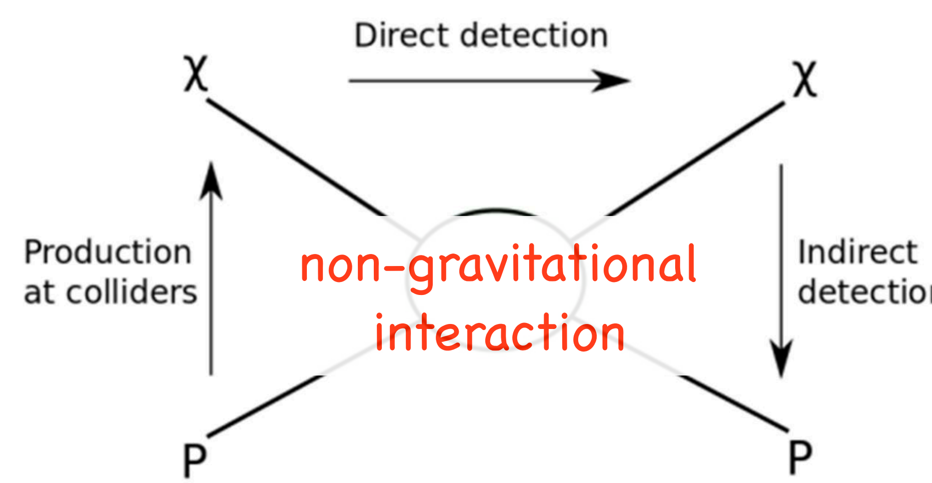

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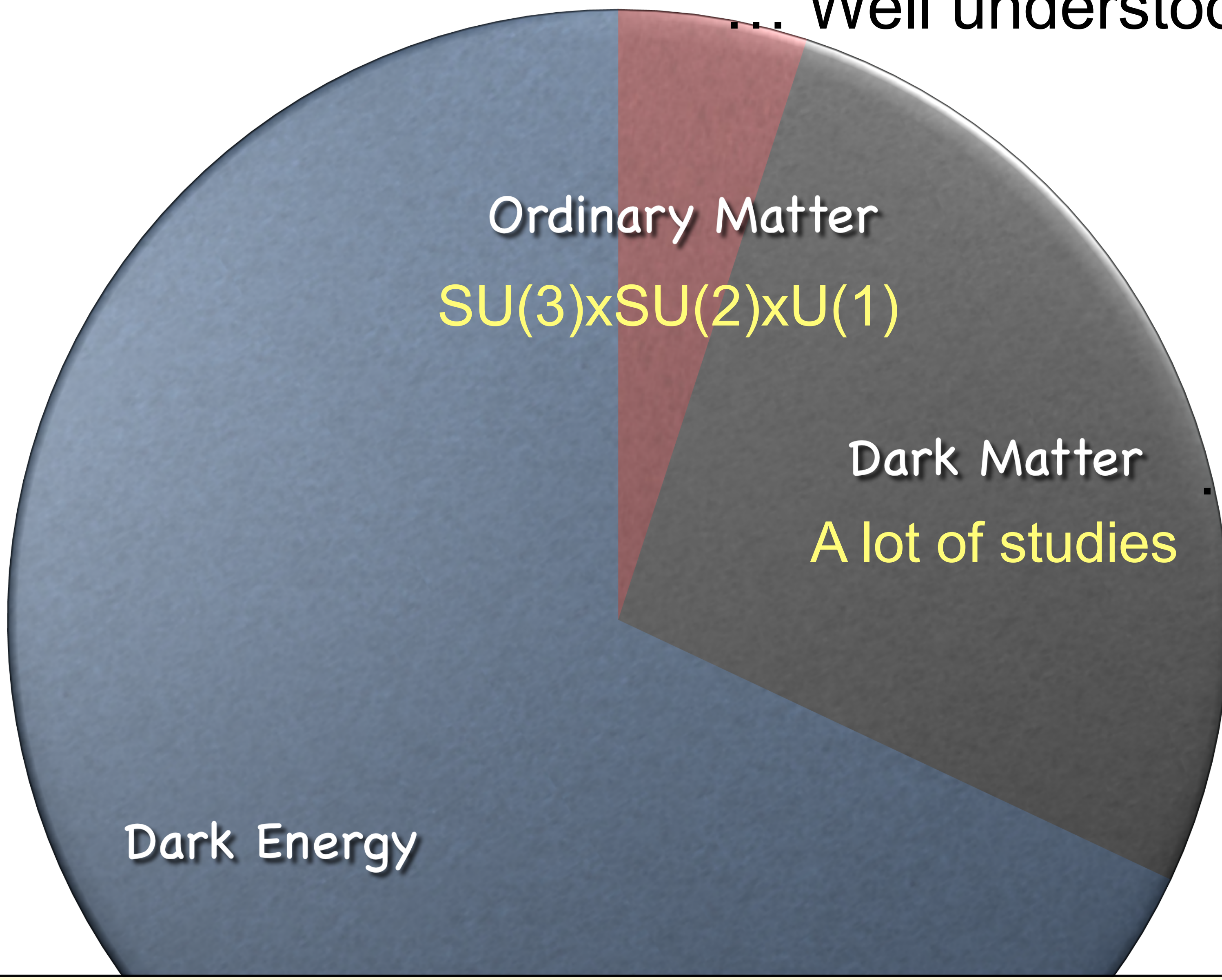

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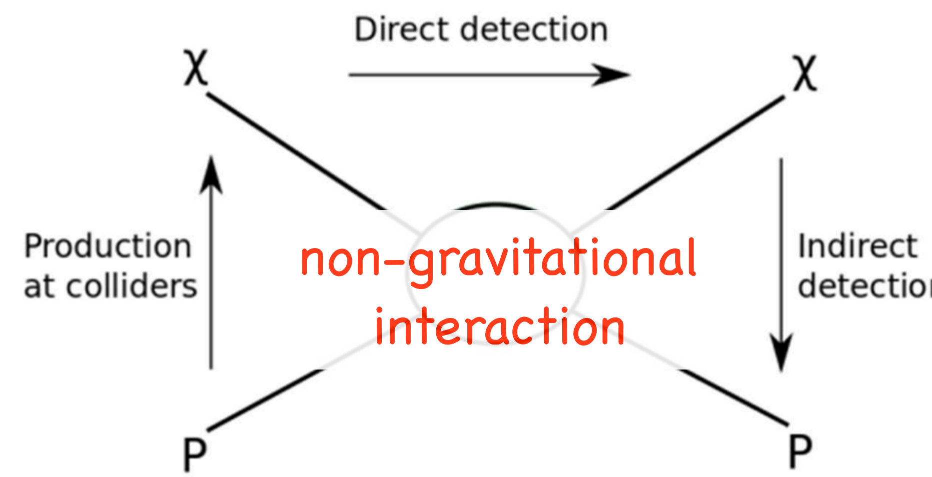


Least understood

Quintessence [Ratra, Peebles (1988)]
: first dark energy field model with a singlet scalar

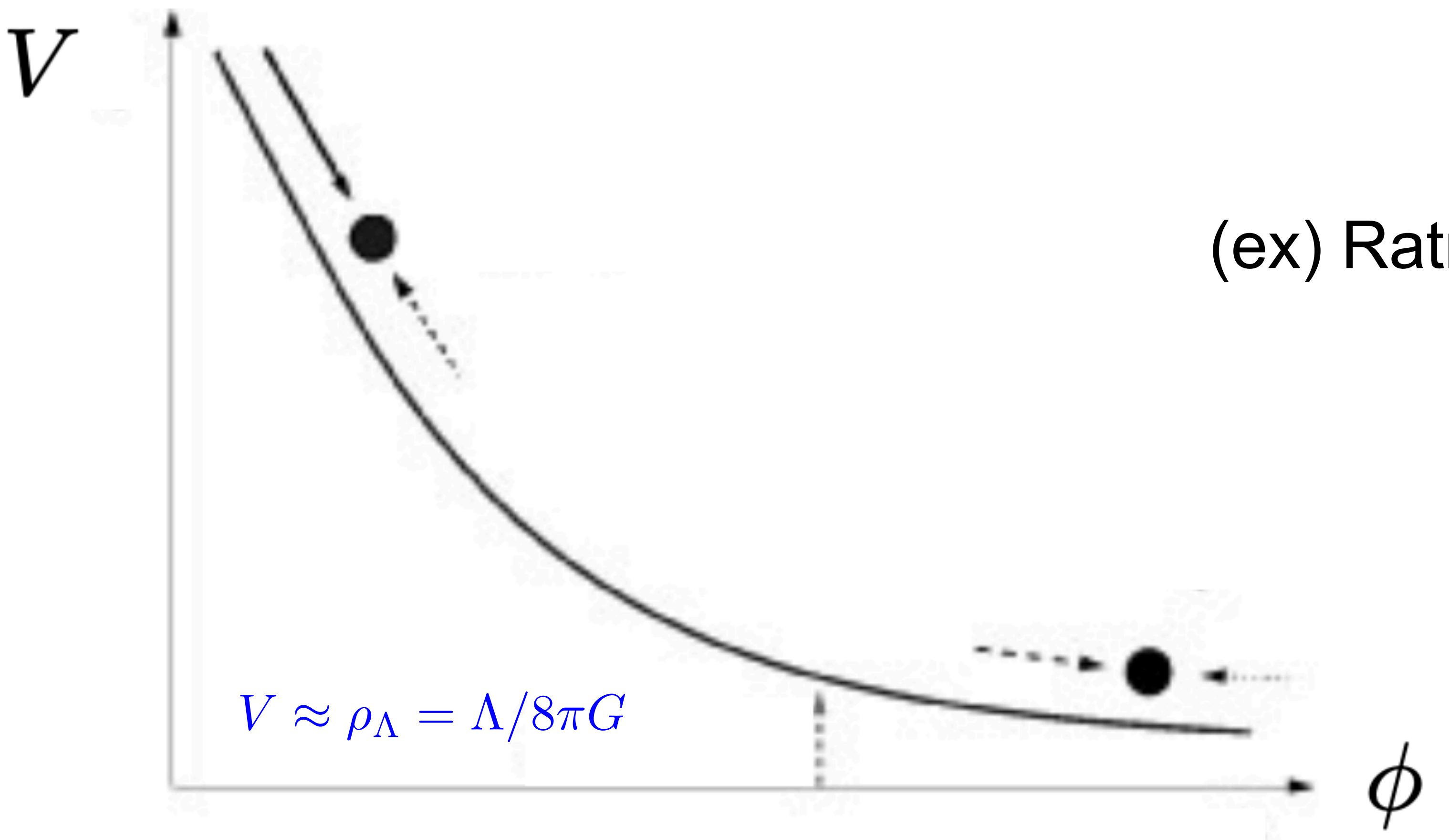
Gauged Quintessence [Kaneta, LEE, Lee, Yi (2023)]
: first gauge symmetry model in
the quintessence scalar field

Outline of this talk

1. Quintessence at a glance
2. Gauged quintessence
3. Misalignment mechanism for the vector boson production
4. Evolution of the universe
5. Remarks on the Hubble tension
6. Non-gravitational signals

Quintessence at a glance

Quintessence



(ex) Ratra-Peebles inverse power potential

$$V(\phi) = \frac{M^{\alpha+4}}{|\phi|^\alpha}$$

Quintessence

- Proposed by Ratra and Peebles (1988).
- Dynamic dark energy model with a scalar field (ϕ).
- A scalar rolls down a potential slowly in the late universe.
- Its potential energy is identified as the dark energy.
- Tracking behavior: The ϕ initial value does not really matter. Only the potential determines the present time value of ϕ and its equation of state w_ϕ (addressing the cosmological coincidence problem) [Steinhardt, Wang, Zlatev (1999)].

Quintessence

$$S = \int d^4x \, \sqrt{-g} \Big[\frac{1}{2} m_{Pl}^2 R - \frac{1}{2} (\partial_\mu \phi)^2 - V(\phi) \Big]$$

$g_{\mu\nu} = \mathrm{Diag}\{-1, a(t)^2, a(t)^2, a(t)^2\}$

$$m_\phi^2 = \frac{\partial^2 V}{\partial \phi^2}$$

(m_ϕ decreases for a Ratra-Peebles potential.)

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V}{\partial \phi} = 0$$

(equation of motion)

$H \equiv \frac{\dot{a}}{a}$ (Hubble parameter)

$$w \equiv \frac{p}{\rho} = \frac{\frac{1}{2}\dot{\phi}^2 - V}{\frac{1}{2}\dot{\phi}^2 + V}$$

(equation of state)

$\rho \propto a^{-3(1+w)}$

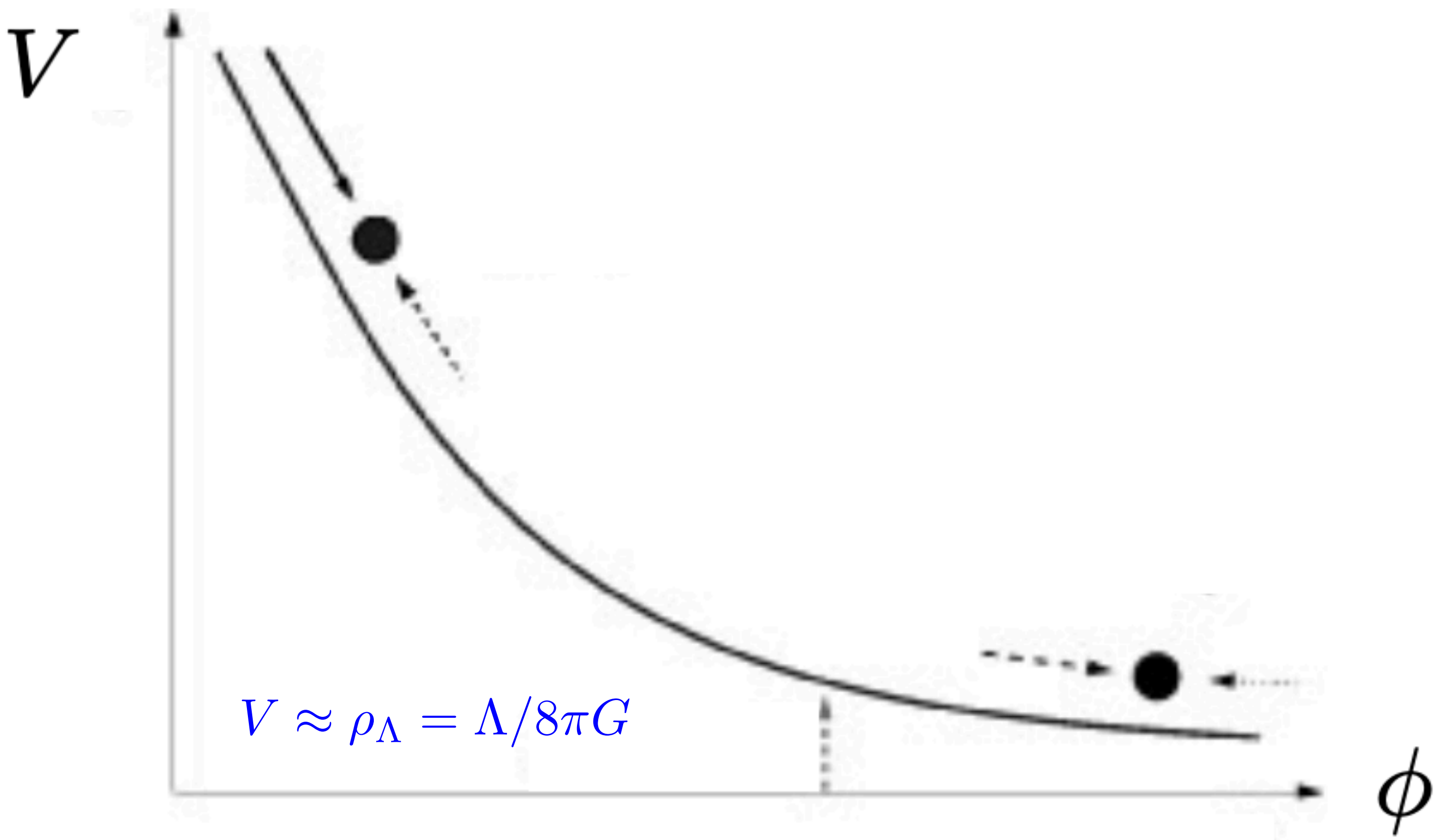
$$= -1 + \frac{\dot{\phi}^2}{V} + \cdots$$

for $\dot{\phi}^2 \ll V(\phi)$ (slow-roll)

$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(1+3w)\rho$

($w < -1/3$ for the accelerated expansion. $w = -1$ for $\mathbf{\Lambda}$)

Quintessence



Conditions for the quintessence dark energy

$V \sim 10^{-123} M_{Pl}^4 \sim 3 \times 10^{-47}\,\mathrm{GeV}^4$ (present dark energy density)

$m_\phi \lesssim H_0 \sim 10^{-42}\,\mathrm{GeV}$ (slow-roll)

Gauged quintessence

Gauged Quintessence

We introduce a dark U(1) gauge symmetry to the quintessence scalar.

$$\Phi = \frac{1}{\sqrt{2}}\phi\, e^{i\eta}$$: complex scalar under the $U(1)_{dark}$ gauge symmetry

$(\phi:\ \text{quintessence scalar})$

$(\eta:\ \text{longitudinal component of the dark gauge boson } X)$

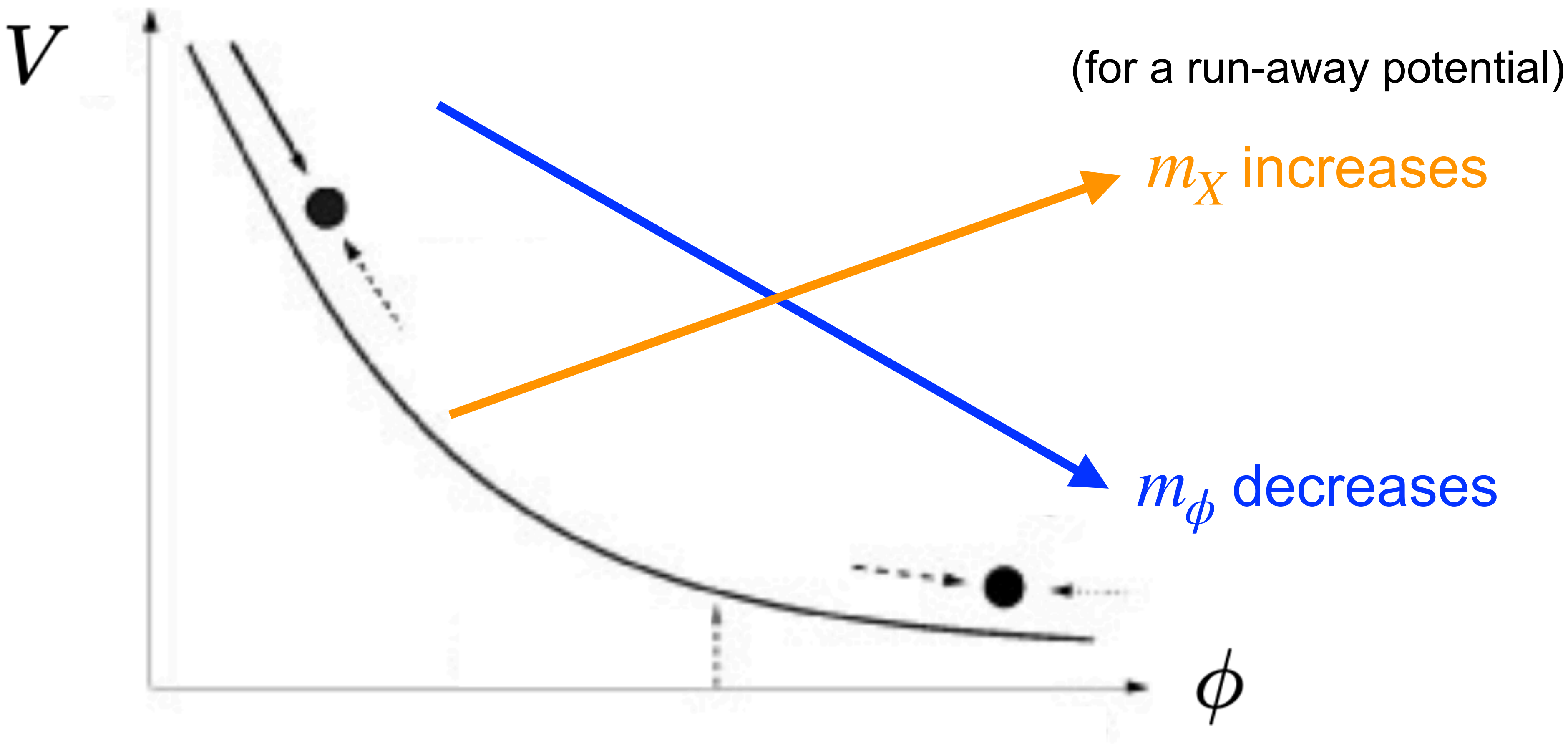
$$S = \int d^4x\,\sqrt{-g}\Big[\frac{1}{2}m_{Pl}^2 R - |D_\mu\Phi|^2 - V_0(\Phi) - \frac{1}{4}\mathbb{X}_{\mu\nu}\mathbb{X}^{\mu\nu}\Big] \qquad D_\mu \equiv \partial_\mu + i g_X \mathbb{X}_\mu$$

$$= \int d^4x\,\sqrt{-g}\Big[\frac{1}{2}m_{Pl}^2 R - \frac{1}{2}(\partial_\mu\phi)^2 - V_0(\phi) - \frac{1}{4}X_{\mu\nu}X^{\mu\nu} - \underbrace{\frac{1}{2}g_X^2\phi^2 X_\mu X^\mu}_{V_{\rm gauge}}\Big]$$

$$\left(\text{in unitary gauge}:\ \eta = 0, \quad X_\mu = \mathbb{X}_\mu + \frac{1}{g_X}\partial_\mu\eta\right)$$

$$m_\phi^2|_0 = \frac{\partial^2 V_0}{\partial\phi^2} + \frac{\partial^2 V_{\rm gauge}}{\partial\phi^2}\,, \quad m_X^2|_0 = g_X^2\phi^2$$ (tree-level masses)

Masses vary over cosmic evolution



$$V_0(\phi) = \frac{M^{\alpha+4}}{|\phi|^\alpha}$$

As the quintessence ϕ rolls down the potential, both m_ϕ and m_X change over cosmic evolution.

$$m_\phi^2|_0 = \frac{\partial^2 V_0}{\partial \phi^2} + \frac{\partial^2 V_{\rm gauge}}{\partial \phi^2}\,, \quad m_X^2|_0 = g_X^2 \phi^2$$

(tree-level masses)

Boltzmann equations for mass-varying ϕ and X

(Neglecting the collision term)

$$\dot{\rho}_\phi + 3H(\rho_\phi + p_\phi) = -\frac{\dot{m}_X}{m_X}(\rho_X - 3p_X)$$

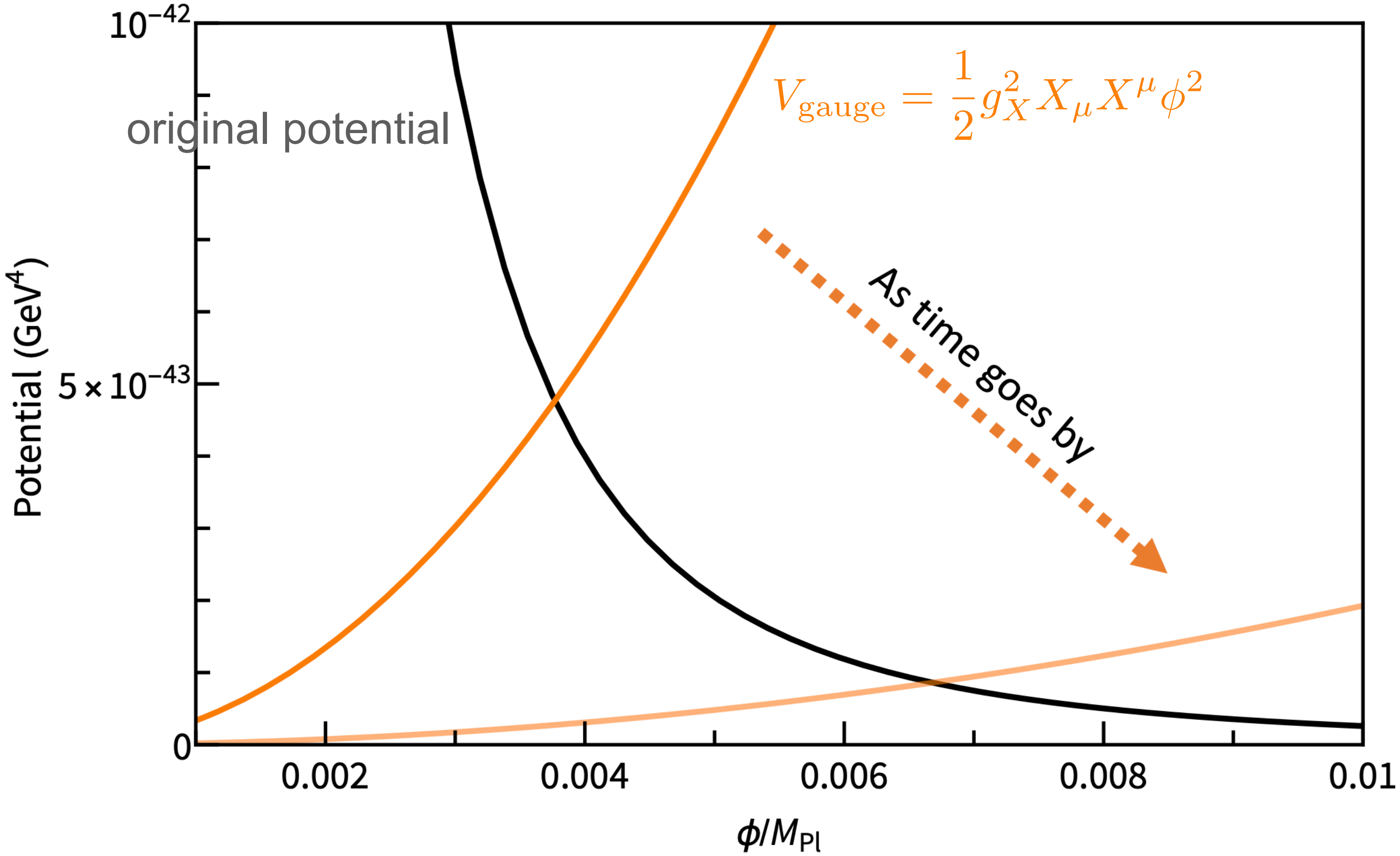
$$\dot{\rho}_X + 3H(\rho_X + p_X) = \frac{\dot{m}_X}{m_X}(\rho_X - 3p_X)$$

The energy flow between the quintessence scalar and the dark gauge boson is proportional to the $\dot{m}_X$.

$$\dot{m}_X > 0 : \text{ Energy flows from } \phi \text{ to } X$$

$$\dot{m}_X < 0 : \text{ Energy flows from } X \text{ to } \phi$$

Potential modified by the gauge symmetry



$$V = V_0 + \frac{1}{2} g_X^2 X_\mu X^\mu \phi^2$$

Combined, they provide interesting phenomenology of the universe.

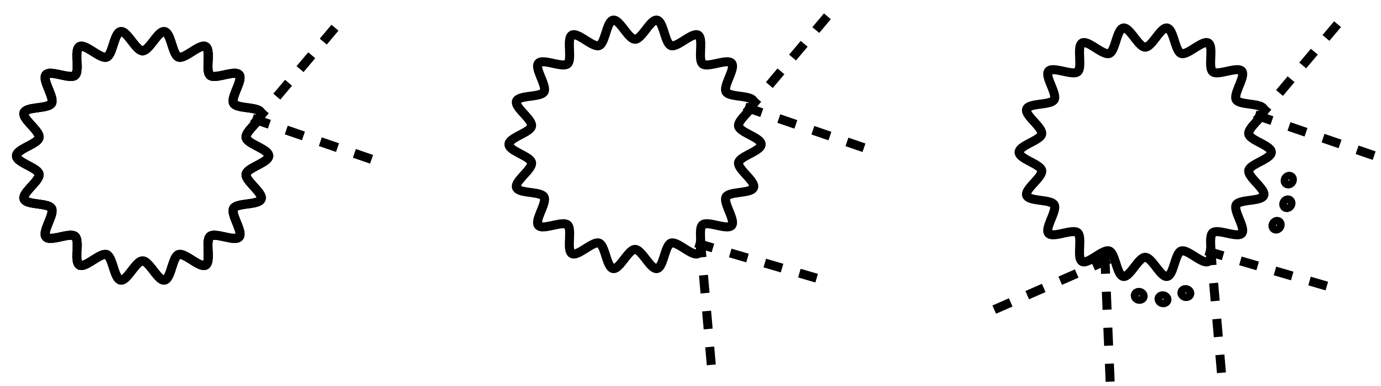
Quantum corrections in the gauged quintessence

1-loop effective potential in the gauged quintessence model

$$V_{\rm eff} = V_0 + \frac{1}{2} g_X^2 X_\mu X^\mu \phi^2 + \frac{\Lambda^2}{32\pi^2} V_0'' + \frac{(V_0'')^2}{64\pi^2}\left(\ln \frac{V_0''}{\Lambda^2} - \frac{3}{2}\right) + \frac{3(m_{\rm X}^2|_0)^2}{64\pi^2}\left(\ln \frac{m_{\rm X}^2|_0}{\Lambda^2} - \frac{5}{6}\right)$$

1-loop correction of the quintessence

Additional 1-loop correction due to the X-boson



$m_X^2|_0 = (g_X \phi)^2$

Quantum corrections in the gauged quintessence

1-loop effective potential in the gauged quintessence model

$$V_{\text{eff}} = V_0 + \frac{1}{2} g_X^2 X_\mu X^\mu \phi^2 + \frac{\Lambda^2}{32\pi^2} V_0'' + \frac{(V_0'')^2}{64\pi^2} \left(\ln \frac{V_0''}{\Lambda^2} - \frac{3}{2} \right) + \frac{3(m_X^2|_0)^2}{64\pi^2} \left(\ln \frac{m_X^2|_0}{\Lambda^2} - \frac{5}{6} \right)$$

$$m_\phi^2 = \frac{\partial^2 V_{\text{eff}}}{\partial \phi^2} = V_0'' + g_X^2 X_\mu X^\mu + \frac{\Lambda^2}{32\pi^2} V_0'''' + \frac{V_0'' V_0''''}{32\pi^2} \left(\ln \frac{V_0''}{\Lambda^2} - 1 \right) + \frac{9 g_X^2 m_X^2|_0}{16\pi^2} \left(\ln \frac{m_X^2|_0}{\Lambda^2} + \frac{1}{3} \right)$$

independent of potential V_0

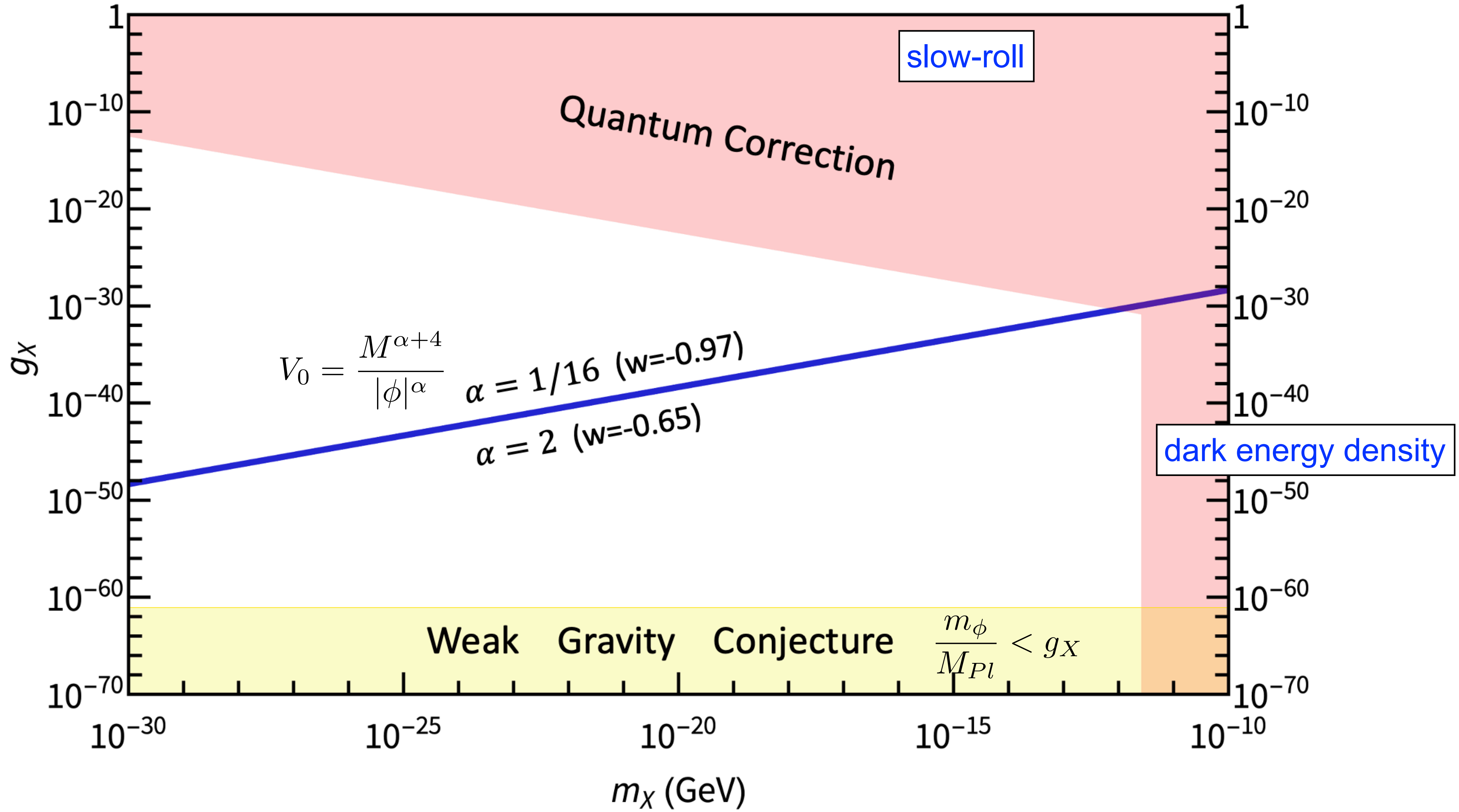
Conditions for the quintessence dark energy

$V \sim 10^{-123} M_{Pl}^4 \sim 3 \times 10^{-47}\,\text{GeV}^4$ (present dark energy density)

$m_\phi \lesssim H_0 \sim 10^{-42}\,\text{GeV}$ (slow-roll)

$m_X^2|_0 = (g_X \phi)^2$

Potential-independent constraints (at present universe)



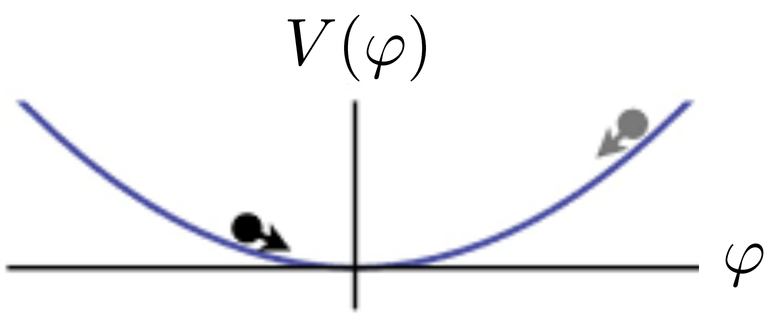
(Blue band: Ratra-Peebles potential case)

Misalignment mechanism
for the vector boson production

Misalignment mechanism for coherent scalar oscillation

[Preskill, Wise, Wilczek (1983)] [Abbott, Sikivie (1983)] [Dine, Fischler (1983)]

Misalignment mechanism is a popular production mechanism of a coherent scalar field (such as QCD axion DM).

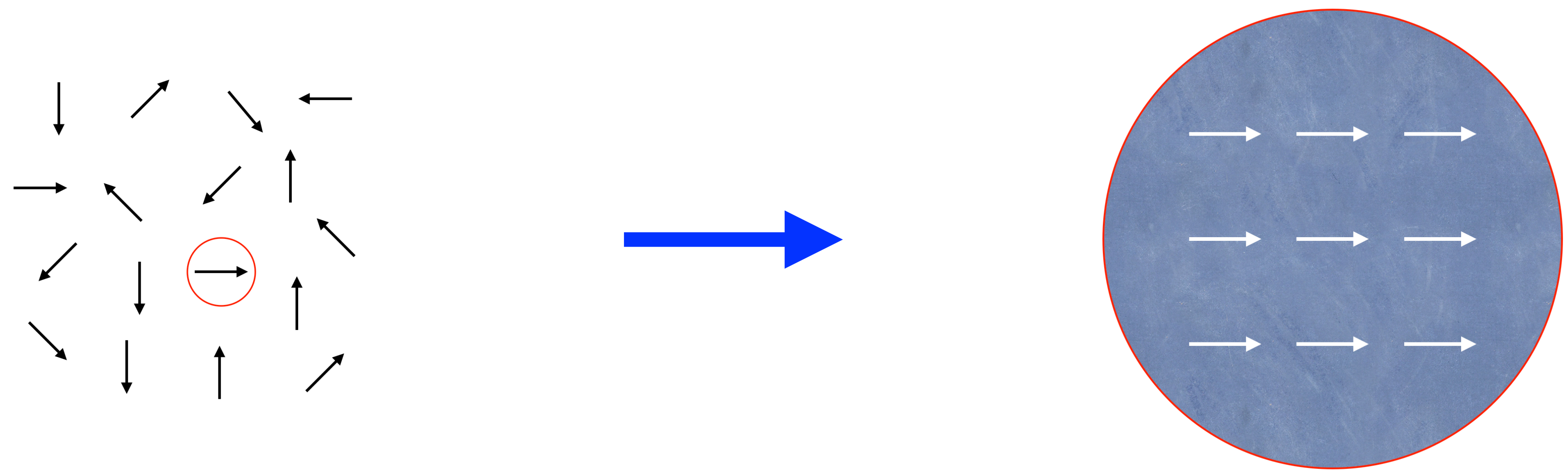


$$\ddot{\varphi} + 3H\dot{\varphi} + m_\varphi^2 \varphi = 0 \qquad \rho_\varphi = \frac{1}{2}\dot{\varphi}^2 + \frac{1}{2}m_\varphi^2\varphi^2$$

Also, look at Sreemanti Chakraborti's talk (Wed).

(i) Inflation makes φ spatially homogeneous: $\varphi(t, \vec{x}) = \varphi(t)$.
(ii) Initially, Hubble friction is large ($H > m_\varphi$), which makes φ frozen and ρ_φ constant.
(iii) When Hubble friction decreases sufficiently ($H \lesssim m_\varphi$), a coherent φ oscillation begins around the potential minimum.
(iv) The oscillator has $p_\varphi = 0$, behaving as non-relativistic matter ($\rho_\varphi \propto a^{-3}$); φ is a CDM despite of lightness. (QCD axion DM: $m_a \sim 10^{-6} - 10^{-2}$ eV)

Misalignment mechanism for coherent vector oscillation



vector fields in random direction
(before the inflation)

vector fields in one direction in visible universe
(during/after inflation)

$$\partial_\mu X^{\mu\nu} + 3HX^{0\nu} - m_X^2 X^\nu = 0$$

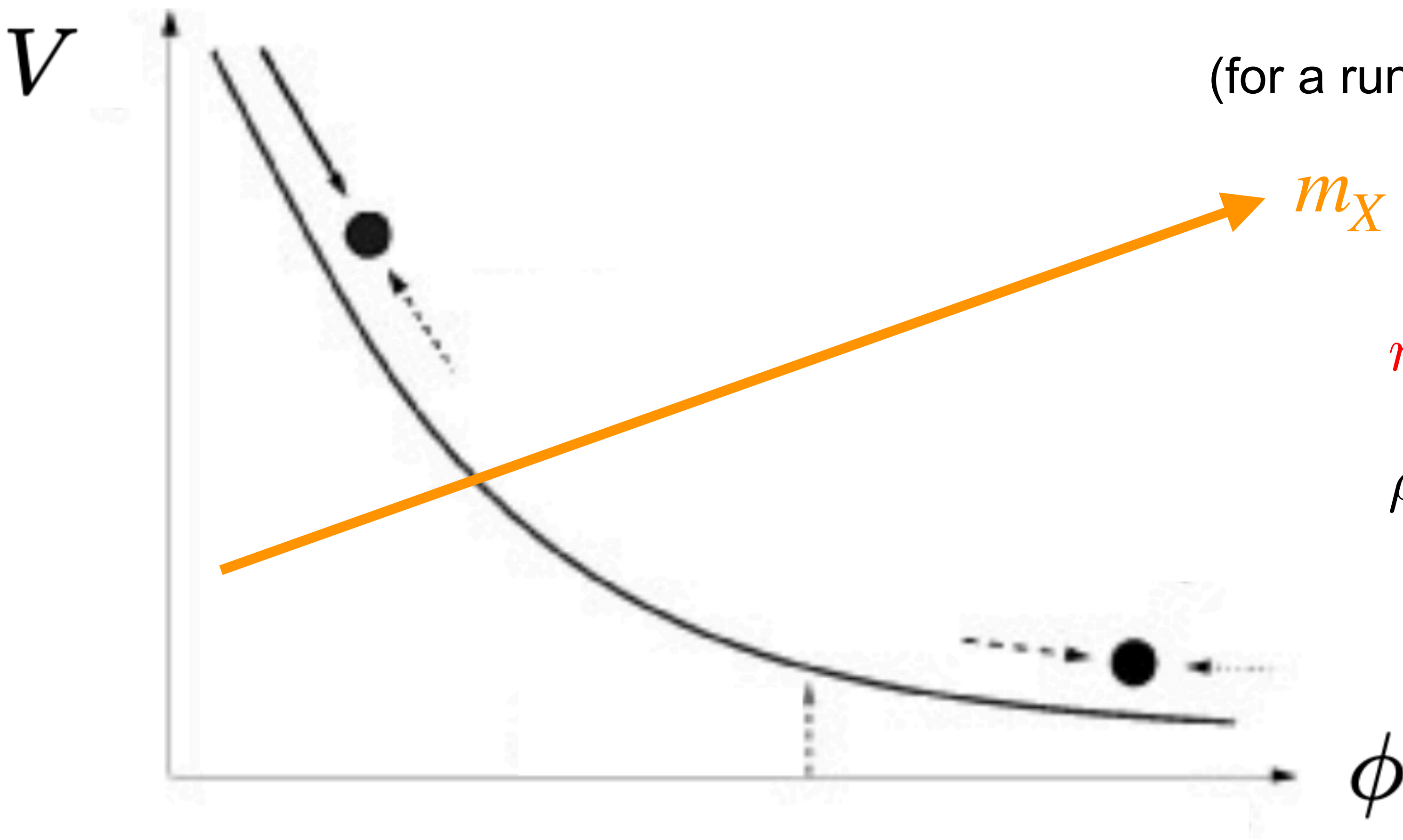
zero mode (spatially homogeneous): $X_\mu(t,\vec{x}) = X_\mu(t) = (X_0(t), \vec{X}(t))$

$$X_0 = 0 \qquad \ddot{X} + H\dot{X} + m_X^2 X = 0 \qquad \rho_X = \boxed{\frac{1}{2a^2}}\left(\dot{X}^2 + m_X^2 X^2\right)$$

Unlike the scalar case, the ρ_X is highly suppressed by the scale factor, and it is hard to retain the ρ_X through the inflation. (Typical inflation e-folding is 60.)

Naive misalignment does not really work for a sizable vector boson production.

Vector misalignment in the gauged quintessence model



(for a run-away potential)

m_X increases

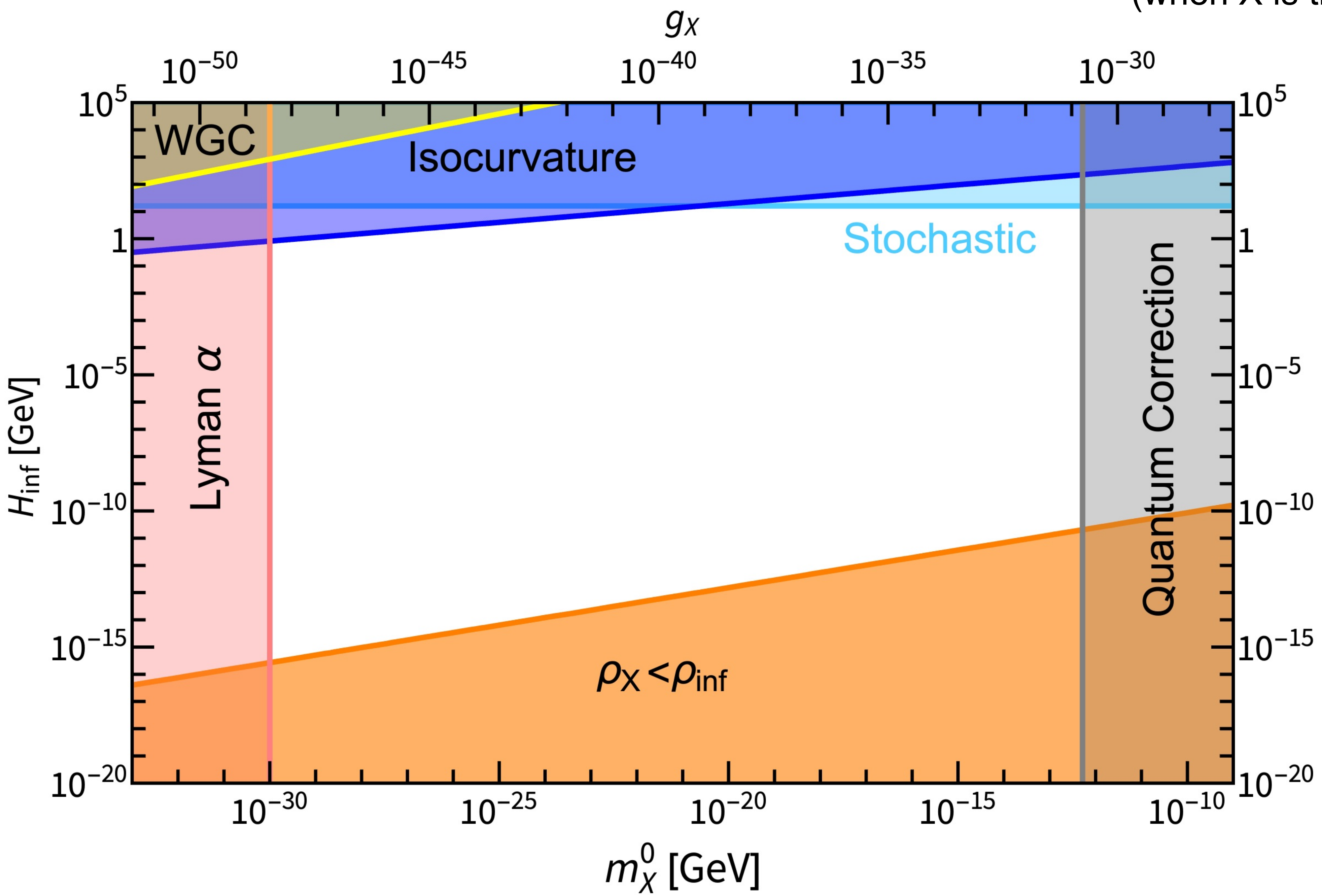
$$m_X = g_X \phi$$

$$\rho_X = \frac{1}{2a^2}\left(\dot{X}^2 + m_X^2 X^2\right)$$

As ϕ increases by many orders of magnitude,
m_X may increase by many orders of magnitude too
overcoming the suppression by the scale factor.

Vector misalignment in the gauged quintessence model

(when X is the whole DM)



Misalignment mechanism with a mass-varying vector boson may work to produce a sizable vector boson energy density.

Evolution of the universe

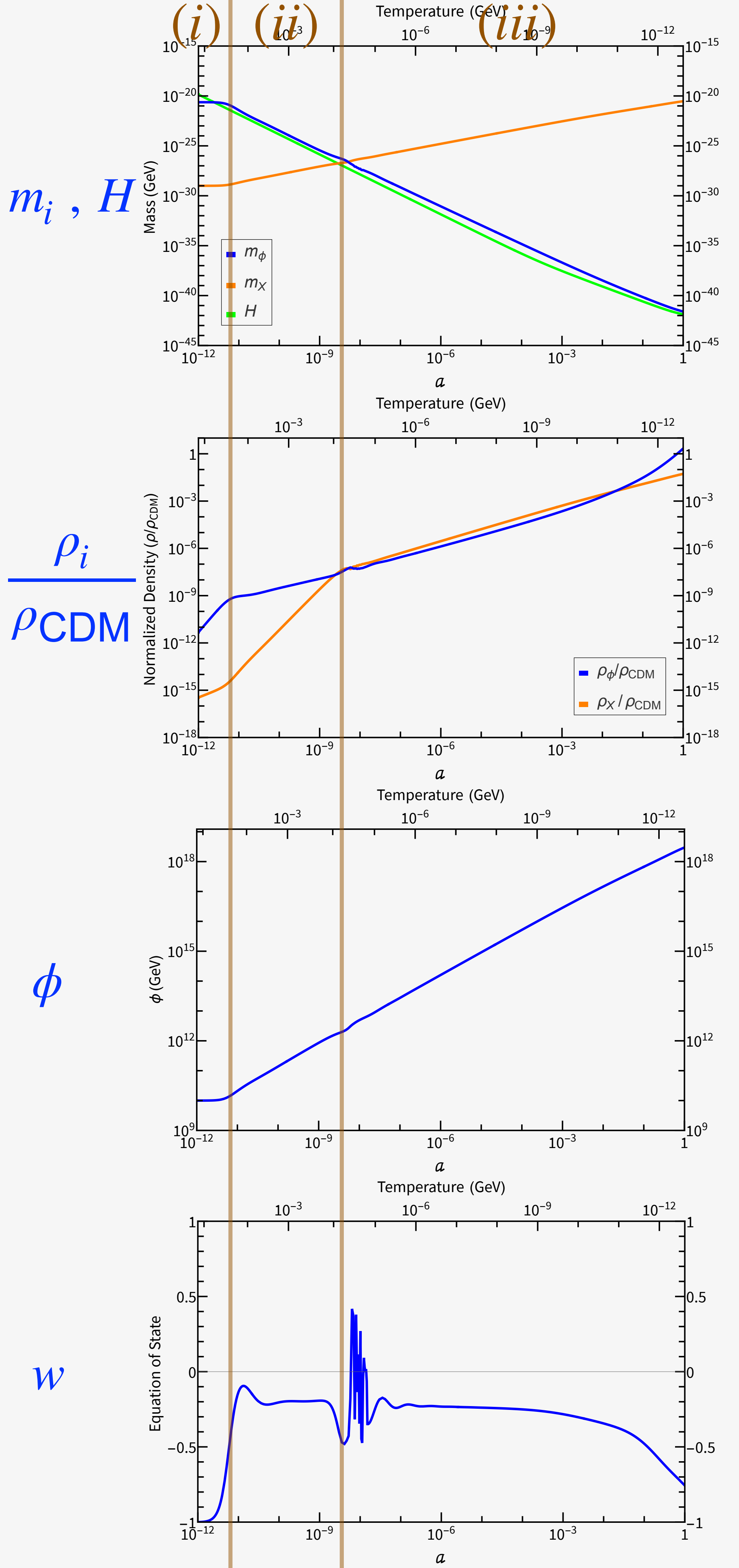


Illustration of how things evolve over the cosmic time.

X may have a sizable relic density, but we assume it is a subdominant DM (less than 10% relic density of the dominant CDM).

The dynamics of ϕ and X change drastically when the hierarchy among m_ϕ, m_X, H change over time.

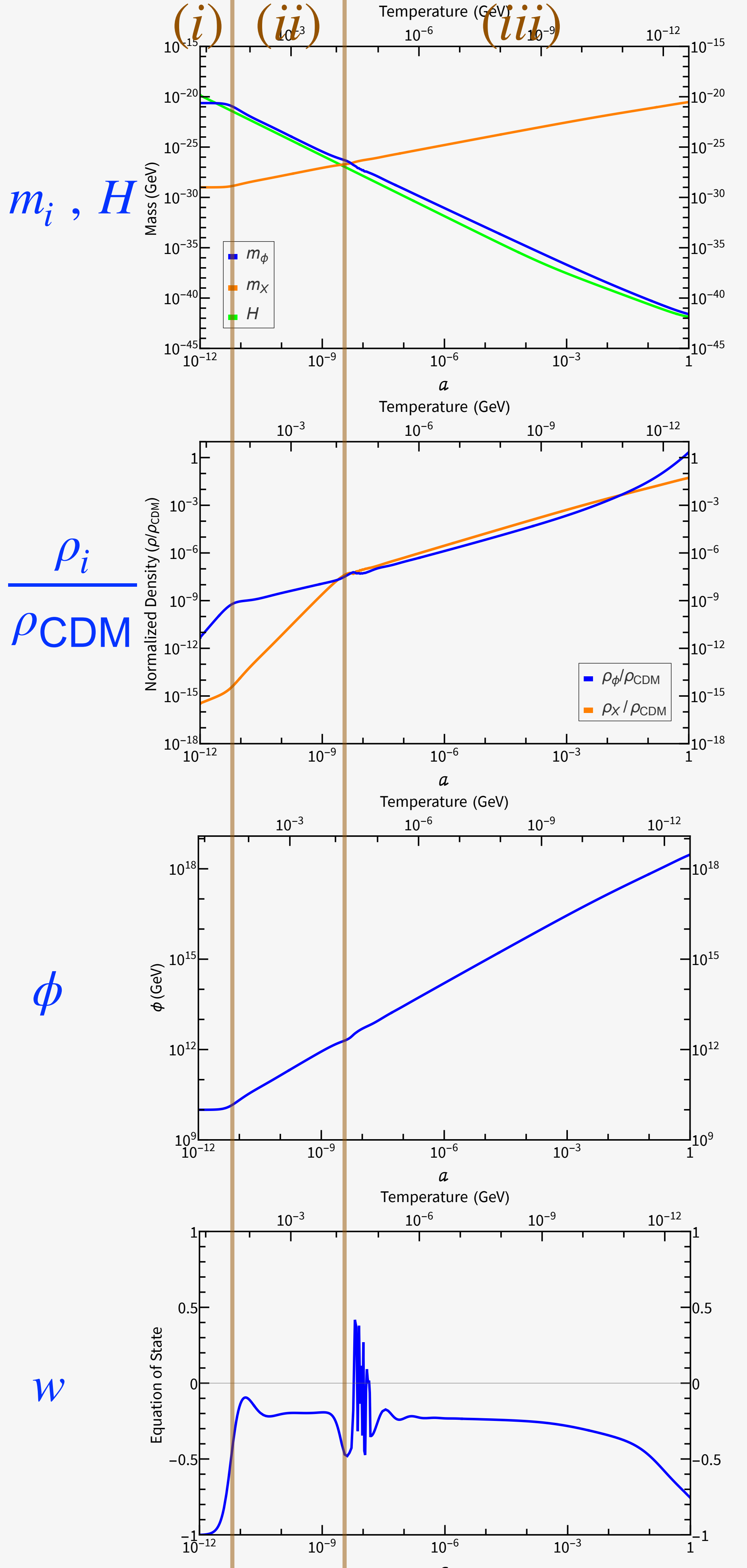
$(i)\ H > m_\phi,\ m_X \quad (ii)\ m_\phi > H > m_X \quad (iii)\ m_X,\ m_\phi > H$

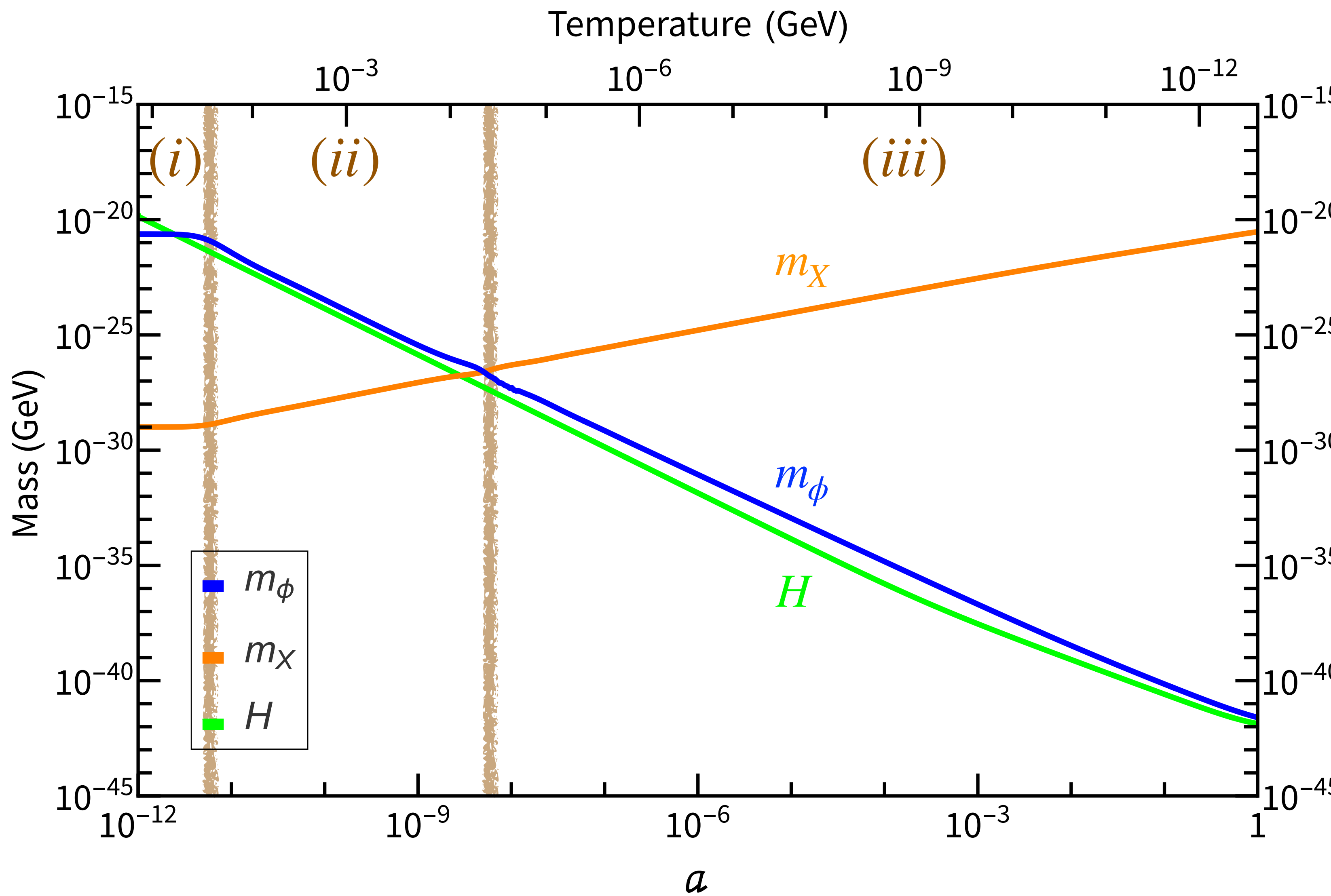
Equations of motion for ϕ and X (coupled via $V_{\rm gauge}$)

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V_0}{\partial \phi} + g_X^2 X_\mu X^\mu \phi = 0$$

$$\partial_\mu X^{\mu\nu} + 3HX^{0\nu} - g_X^2 \phi^2 X^\nu = 0$$

Benchmark parameters: $\alpha = 1$, $M = 2.2 \times 10^{-6}$ GeV, $g_X = 10^{-39}$
$\dot{X} = 0$, $\dot{\phi} = 0$ (at $a = 10^{-12}$)

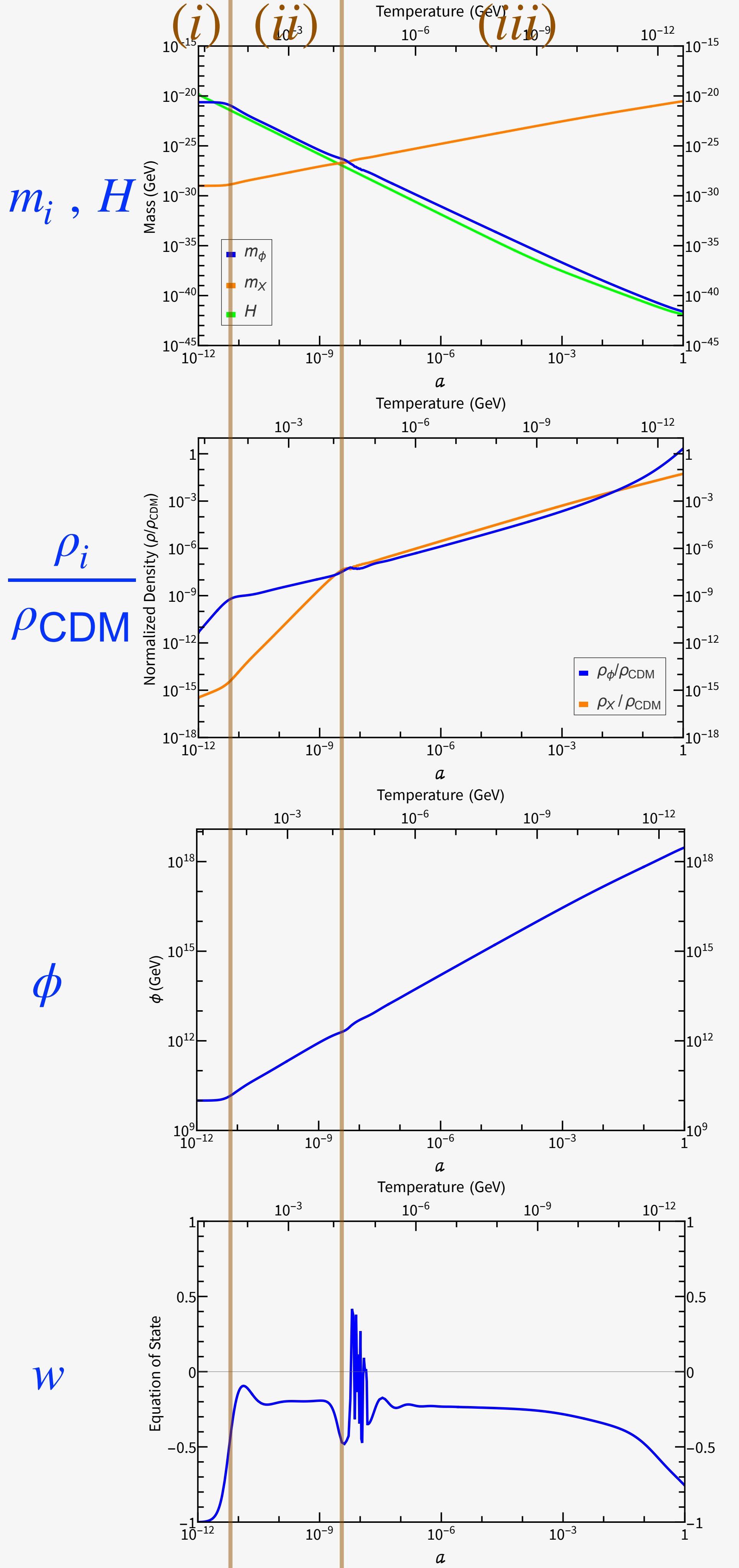


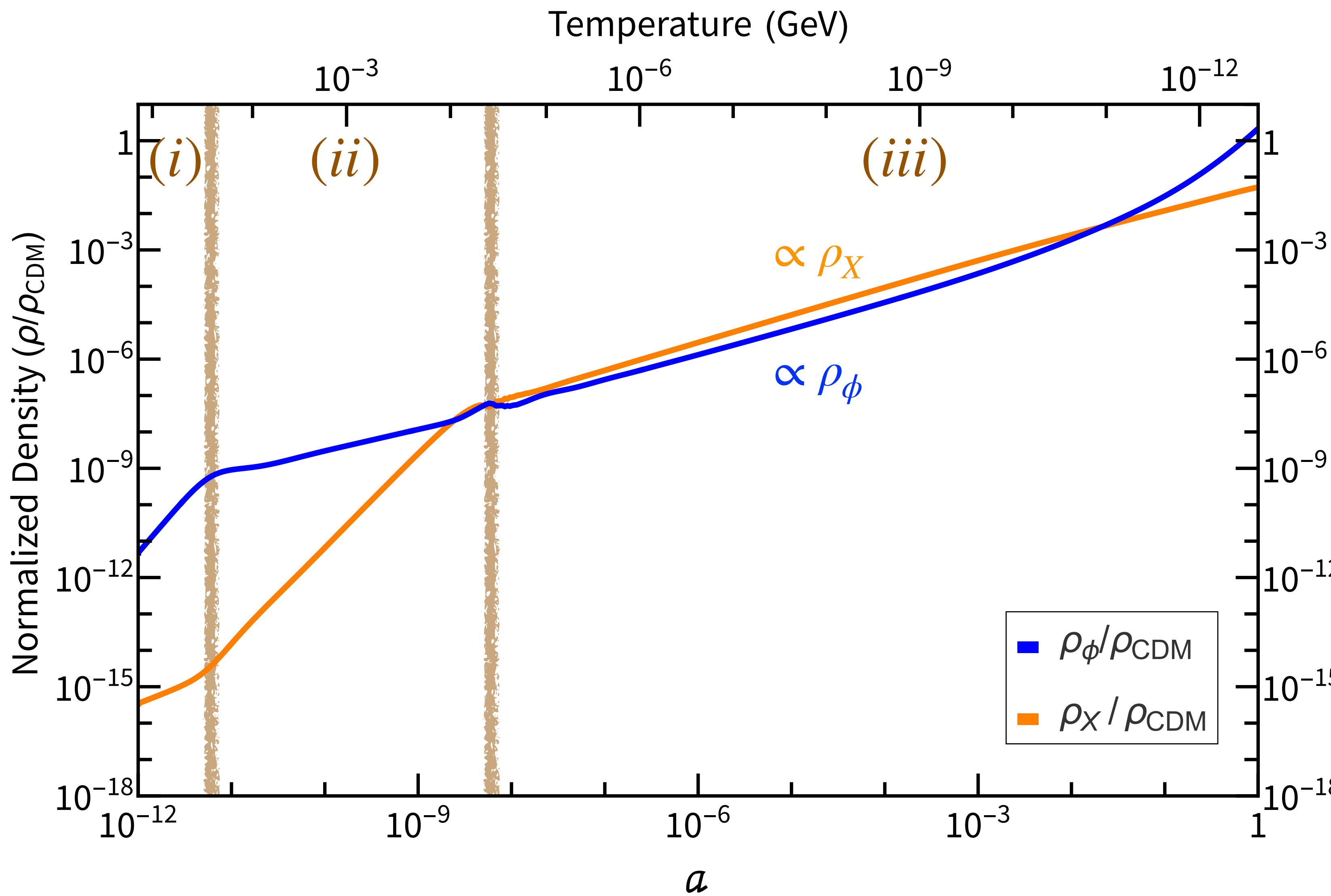


(i) $H > m_\phi, m_X$: Both ϕ and X are frozen by Hubble friction.

(ii) $m_\phi > H > m_X$: X is frozen, but ϕ rolls down potential.

(iii) $m_X, m_\phi > H$: X is in coherent oscillations (DM), ϕ rolls.





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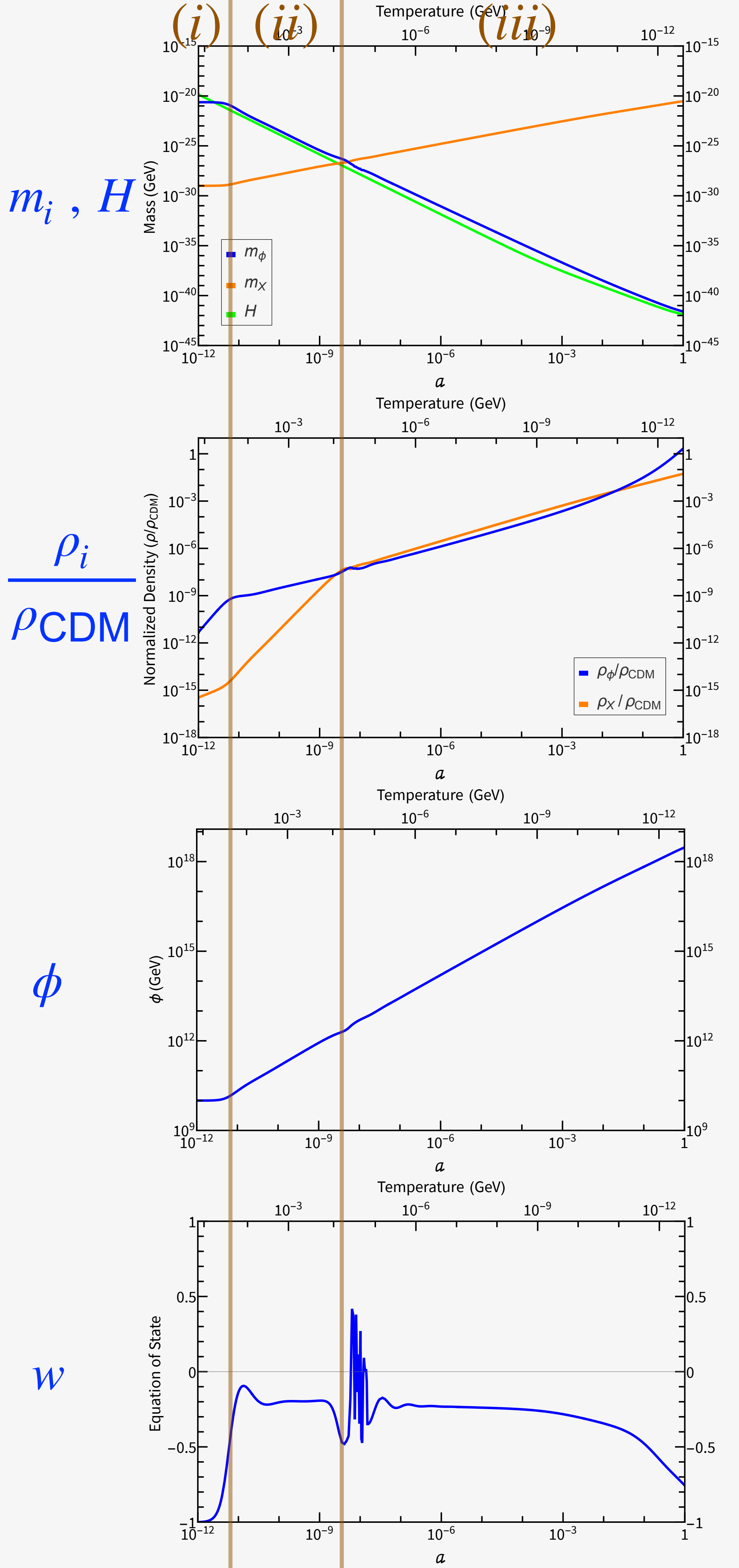

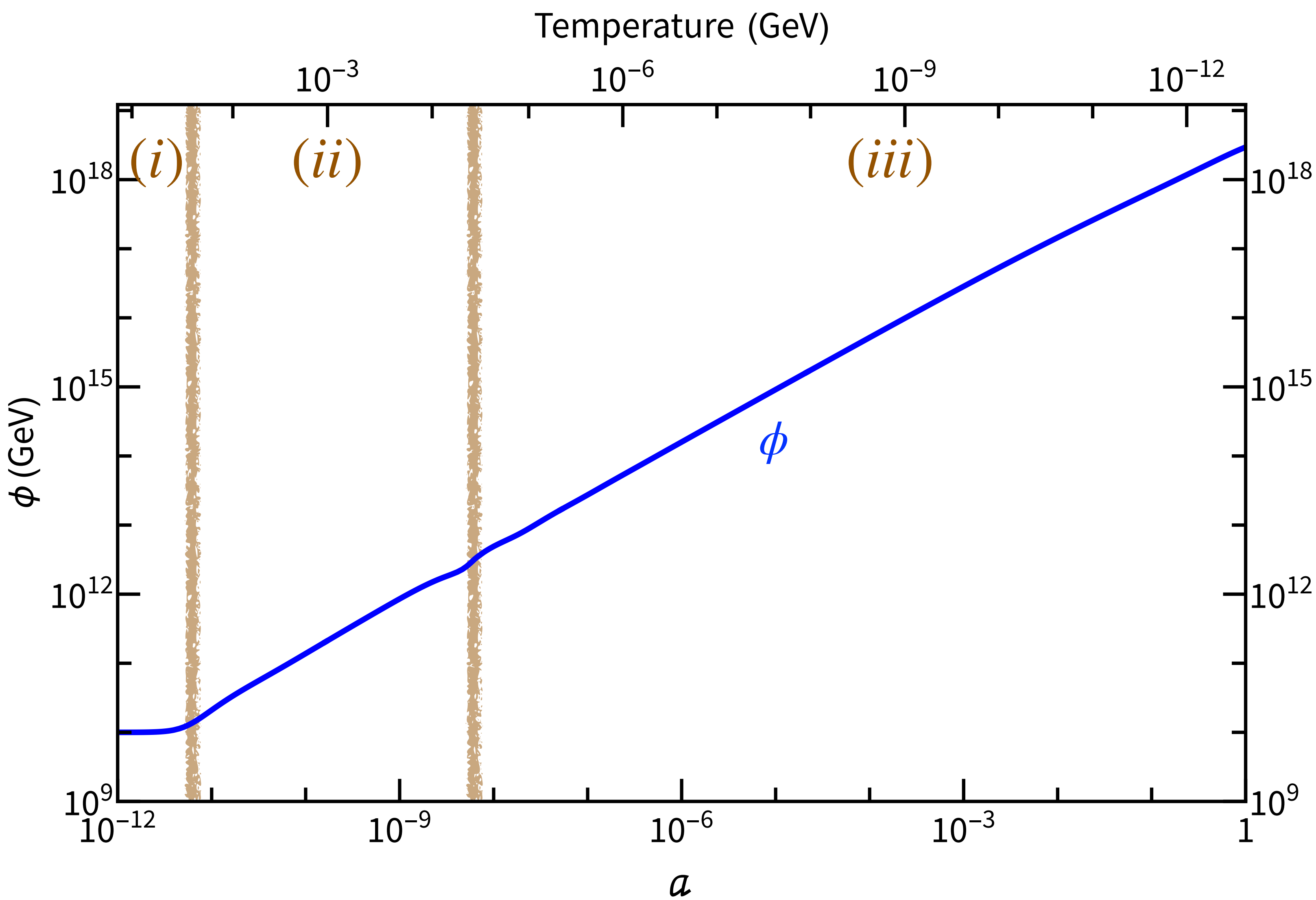

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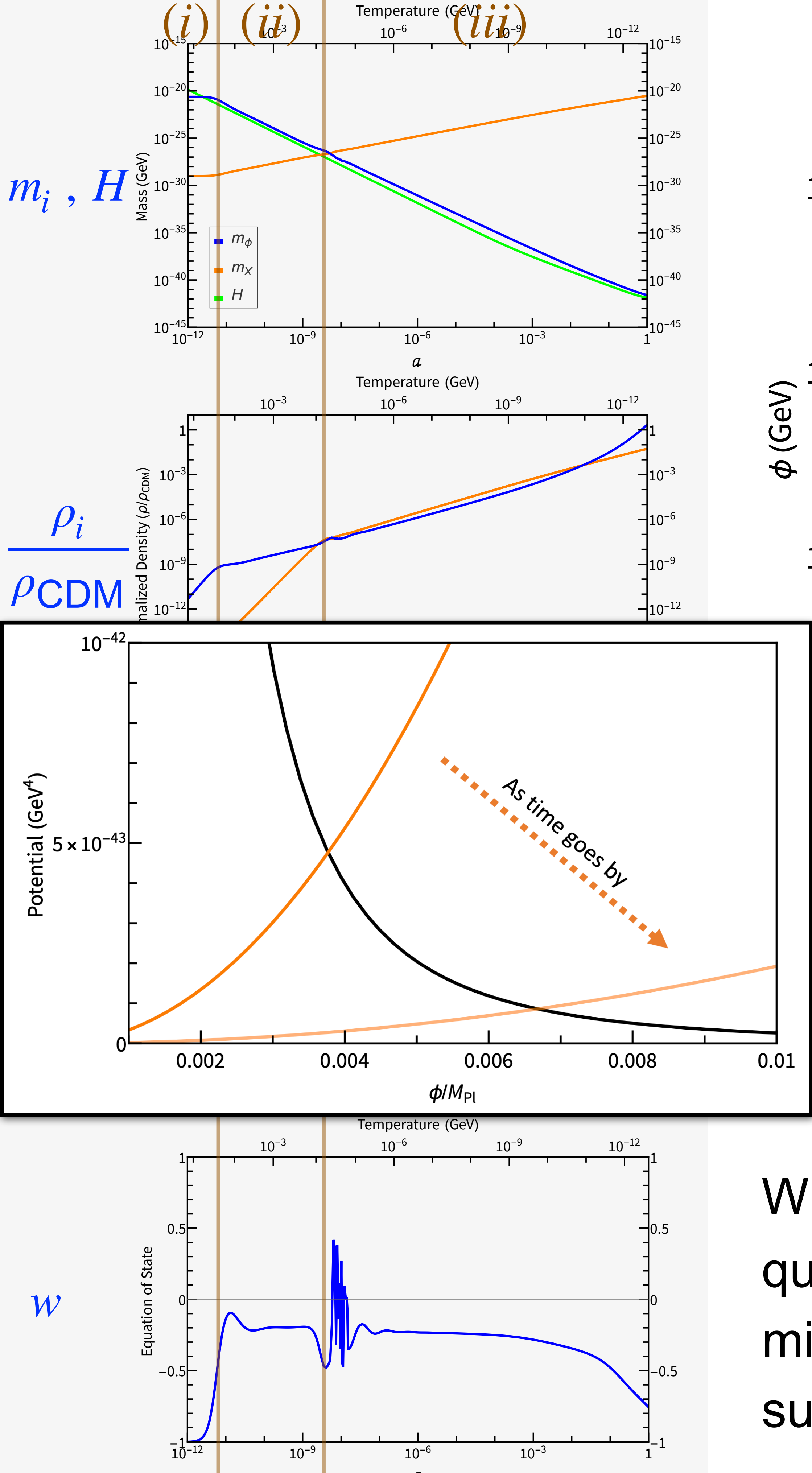
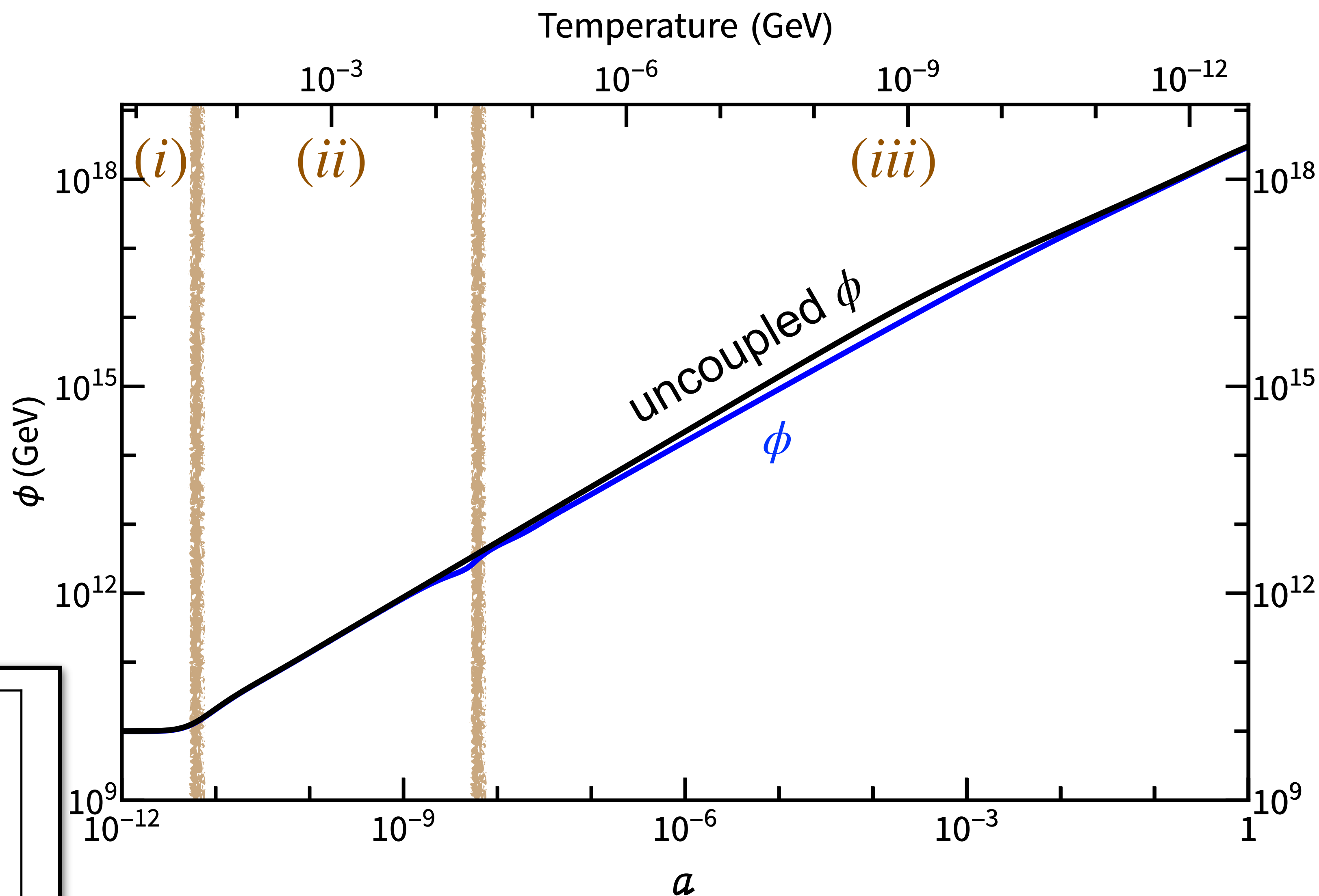
When $V_{\rm gauge} \sim \rho_X$ becomes sizable, it affects the quintessence dynamics. ϕ oscillates around the minimum of the potential until $V_{\rm gauge}$ becomes subdominant. ϕ gets back to the tracking solution.





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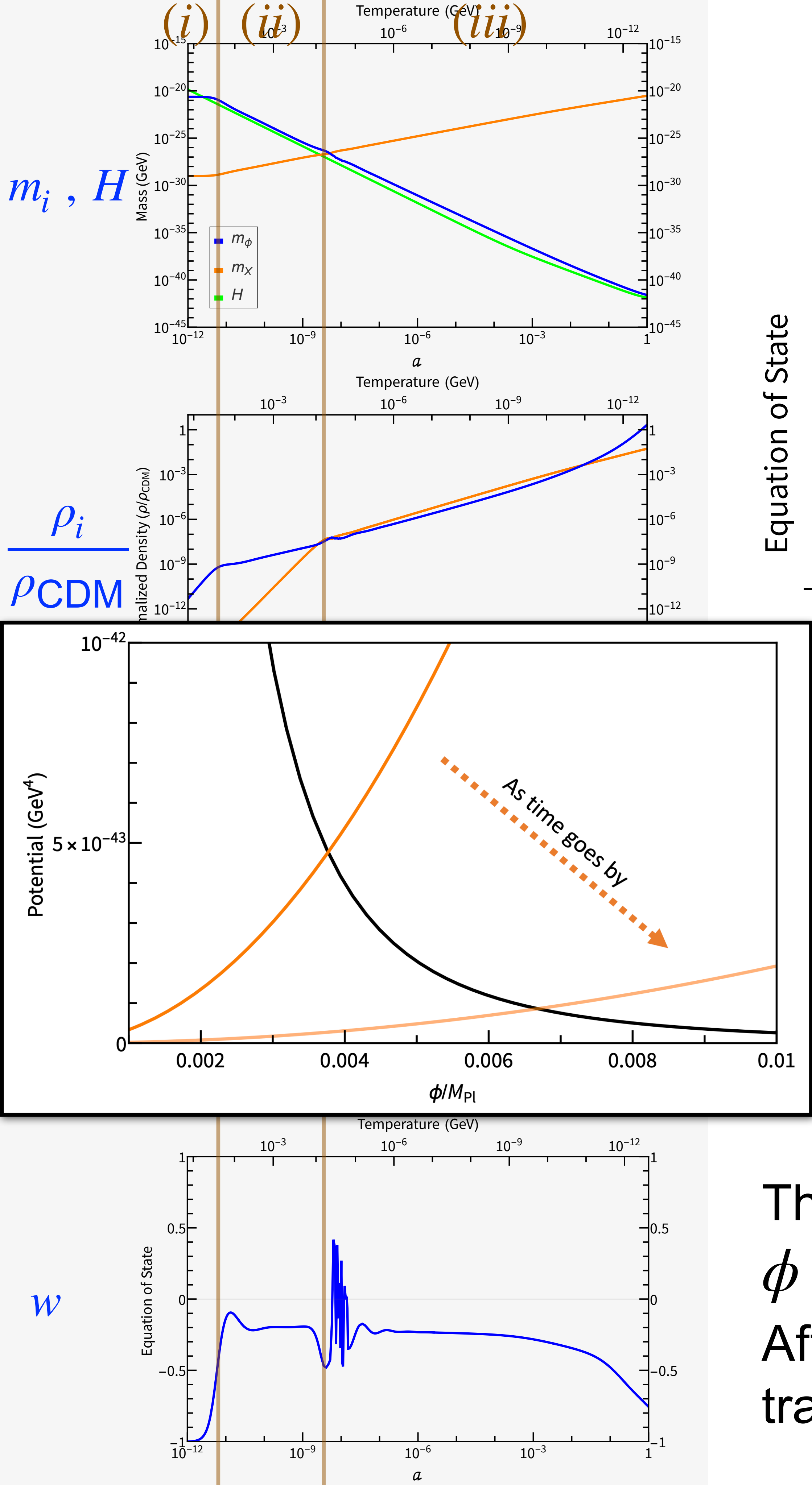

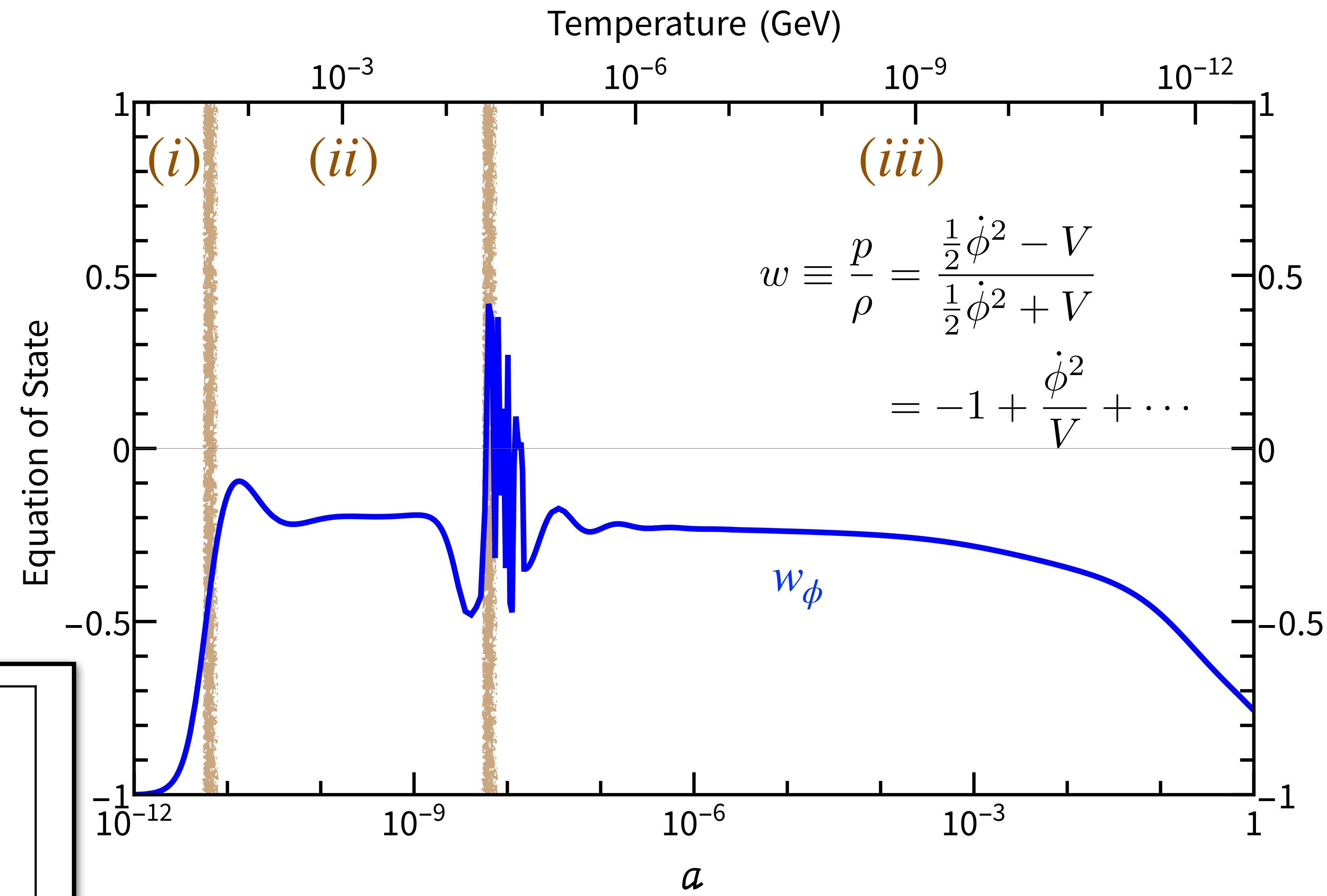

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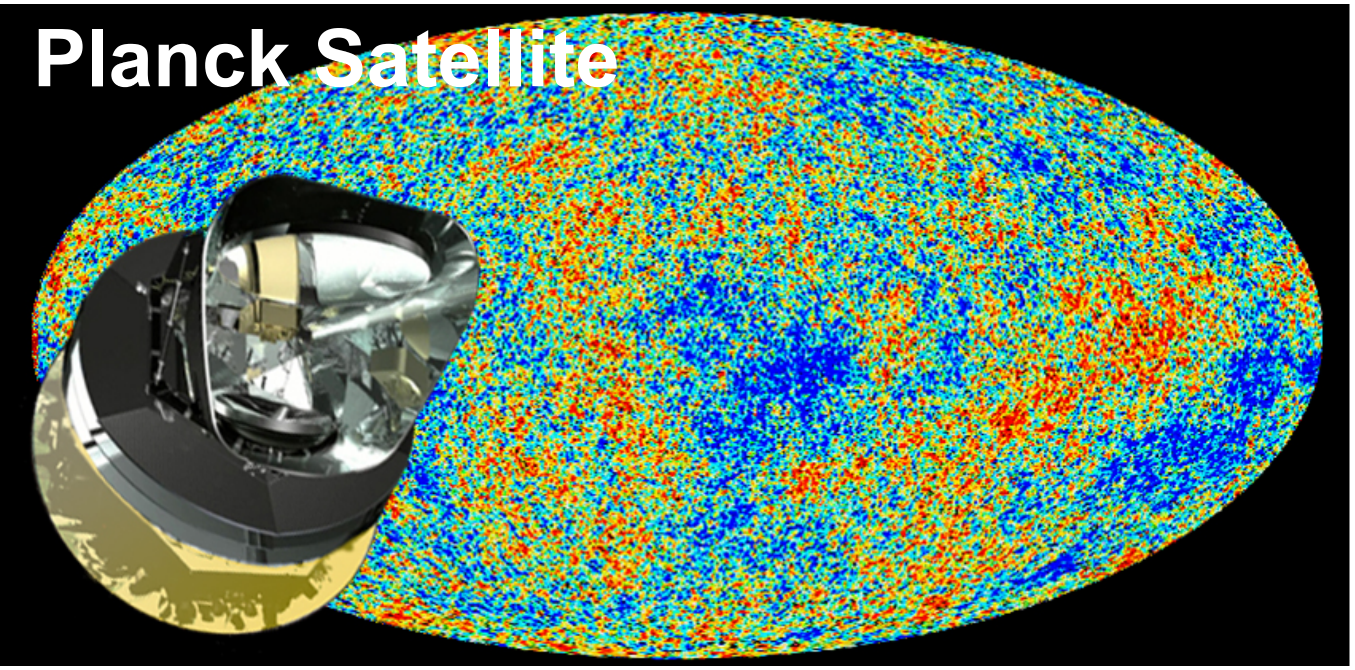
) $m_\phi > H > m_X$: X is frozen, but ϕ rolls down potential.

i) $m_X, m_\phi > H$: X is in coherent oscillations (DM), ϕ rolls.

The oscillation in the ϕ equation of state reflects the ϕ oscillation around the minimum of the potential. After $V_{\rm gauge}$ becomes subdominant, it restores tracking solution.

Hubble tension

Modern measurements of the Hubble constant



(early Universe) H_0 ~ 67 km/s/Mpc

(i) By fitting CMB data to the $\boldsymbol{\Lambda}$-CDM model

(precise but model-dependent)

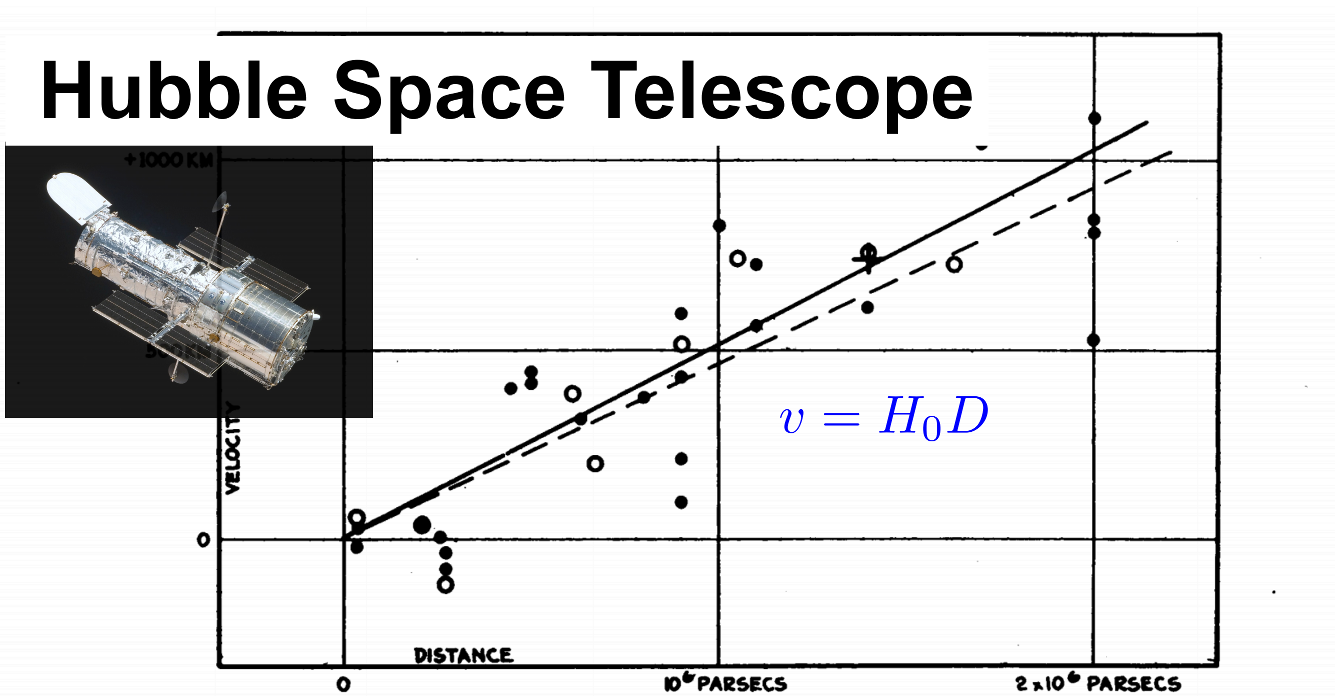


FIGURE 1

(late Universe) H_0 ~ 73 km/s/Mpc

(ii) With the observation of the expansion : standard candles (Cepheid variables + Supernovae) and redshifts

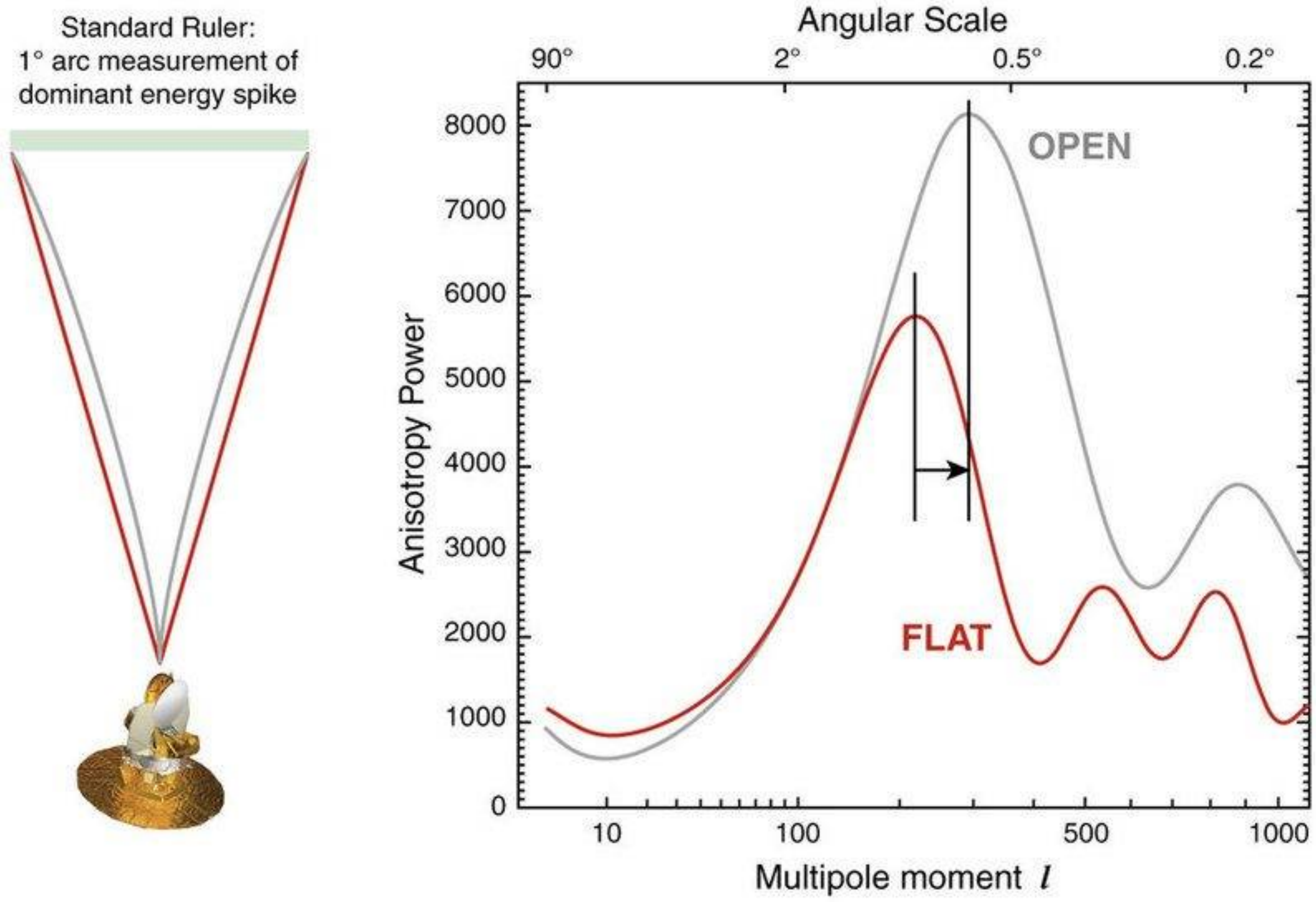
(less precise but model-independent)

Hubble tension: However, look at Pasquale Di Bari's talk (Thu).

significant discrepancy in H_0 values between the early and late universe.

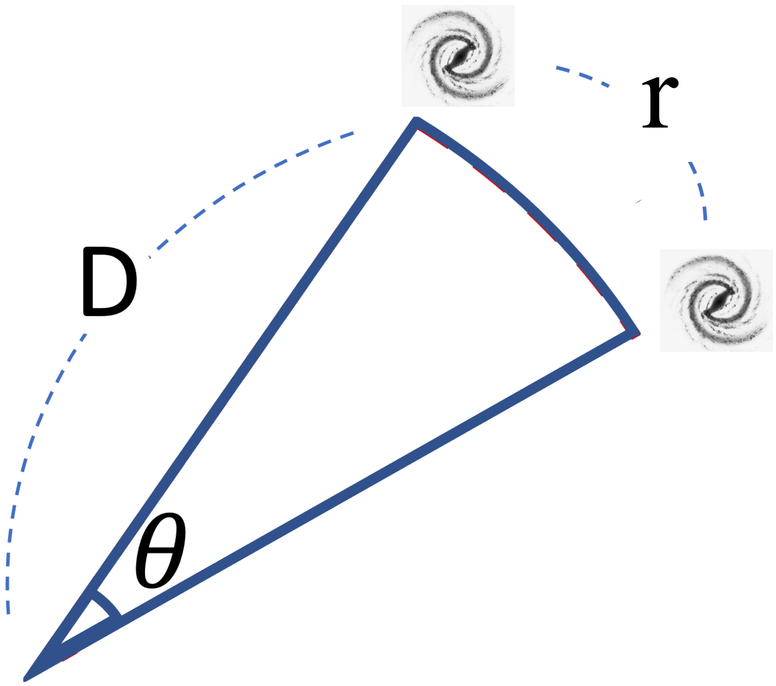
(Potential hint of the new cosmology.)

Sound horizon in CMB



Addressing Hubble tension with a new DE model is constrained by baryon acoustic oscillations.

Baryon Acoustic Oscillations



$$(sound\ horizon):\ r_s = \int_{z_s}^{\infty} dz\, \frac{c_s(z)}{H(z)}$$

$$(distance\ to\ the\ sound\ horizon):\ D(z_s) = \int_0^{z_s} dz\, \frac{c}{H(z)}$$

(observed quantity) (early universe)

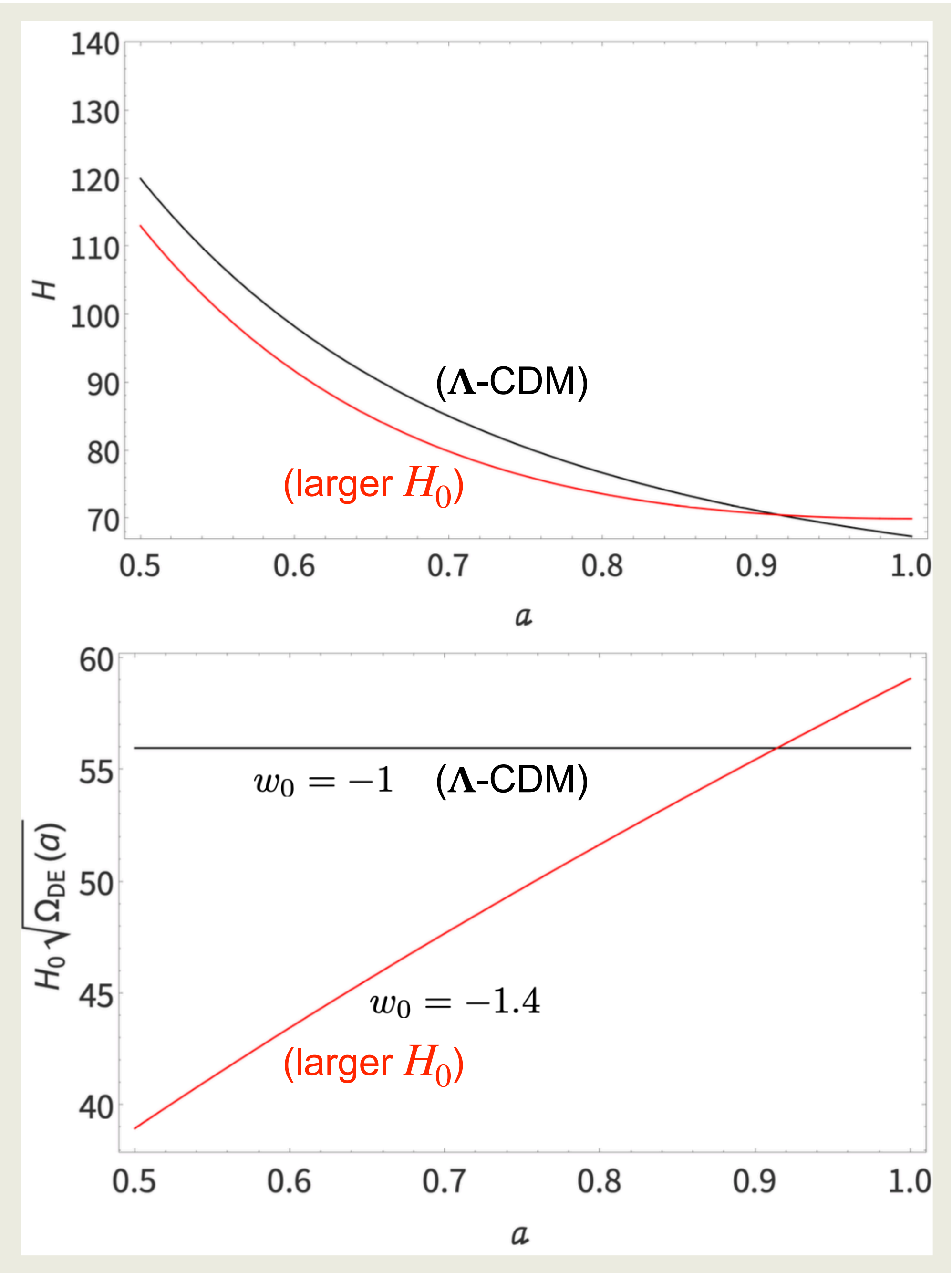
$$\boxed{(angular\ size\ of\ the\ sound\ horizon) = \frac{(sound\ horizon)}{(distance\ to\ the\ sound\ horizon)}}$$

(late universe)

DE became dominant only in the late universe.

Unless we want to bring new physics in the early universe changing the sound horizon (r, early universe physics), the comoving distance to the last scattering (D, late universe physics) should remain intact with a new DE model.

Hubble tension



$$D(z_s) = \int_0^{z_s} dz\,\frac{c}{H(z)}$$

$$H^2 = \left(\frac{\dot a}{a}\right)^2 = H_0^2\left(\Omega_{\rm DE}+\Omega_{\rm matter}\right)$$

$$\rho \propto a^{-3(1+w)}$$

To keep D unchanged, a larger H_0 (resolving Hubble tension) should be compensated by a smaller H <u>in the recent past</u>

: It demands w(DE) < -1 ($\mathbf{\Lambda}$-CDM value).

In the uncoupled quintessence model, w(DE) > -1 (worsening Hubble tension).

If an interacting DE model with $w_{\rm eff}$(DE) < -1 can be found, it may alleviate Hubble tension.

[Valentino, Melchiorri, Mina (2017)]
[Lee, Lee, Colgain, Sheikh-Jabbari, Thakur (2022)]

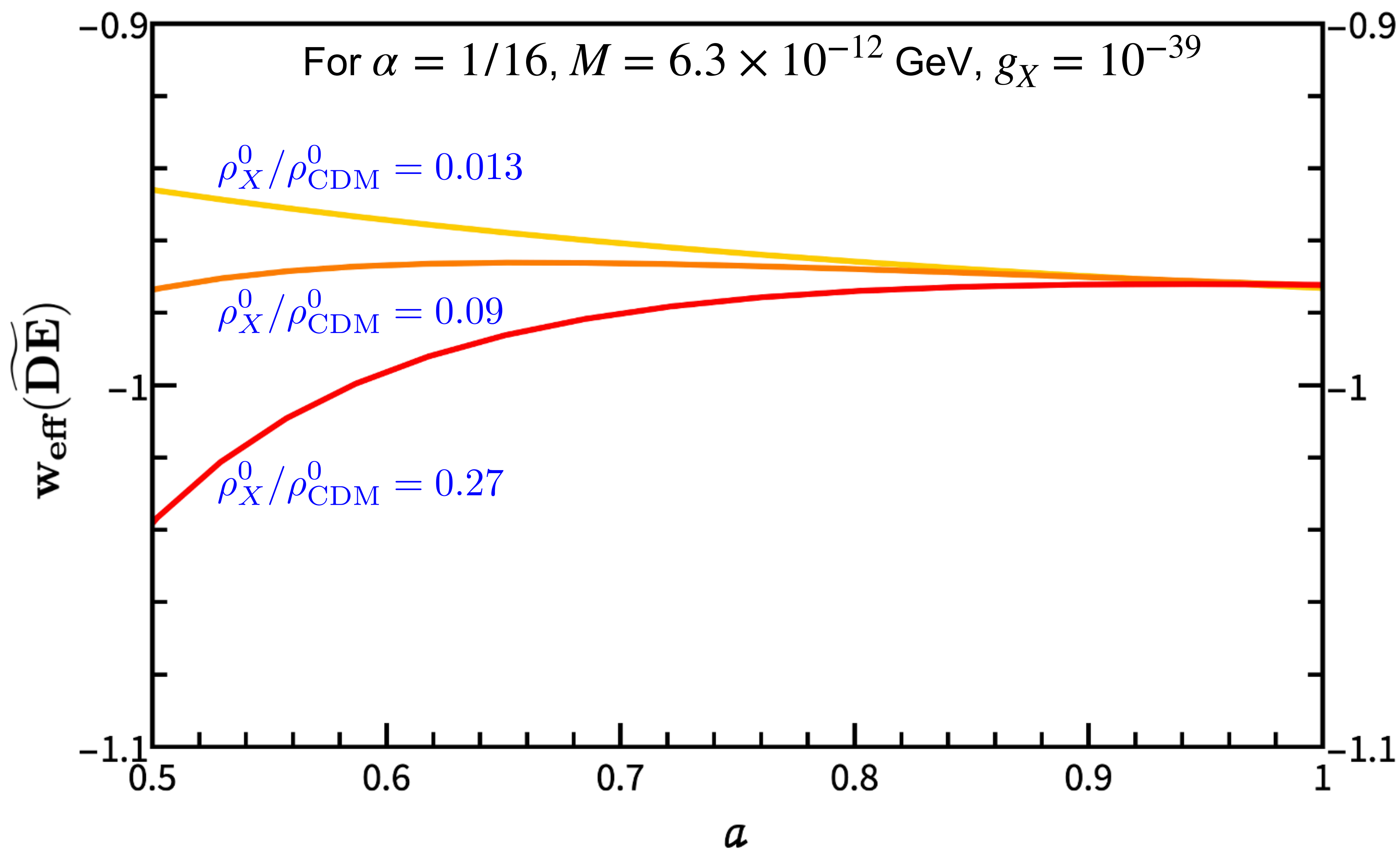
(quintessence) $$w \equiv \frac{p}{\rho} = \frac{\frac12\dot\phi^2 - V}{\frac12\dot\phi^2 + V} = -1 + \frac{\dot\phi^2}{V} + \cdots$$

Does the gauged quintessence show $w_{\rm eff}$(DE) < -1?

w for the effective DE density in the gauged quintessence

$$w_{\rm eff}(\widetilde{DE}) = -1 + \frac{1}{\rho_{\widetilde{DE}}}\left((1+w_0)\rho_\phi + \left(\frac{m_X}{m_X^0} - 1\right)\frac{\rho_X^0}{a^3}\right)$$

$$\rho_{\widetilde{DE}} = \rho_\phi + \rho_X - \rho_X^0 a^{-3}$$



For $\dot{m}_X > 0$, $w_{\rm eff}(\widetilde{DE})$ is lower than the uncoupled quintessence.

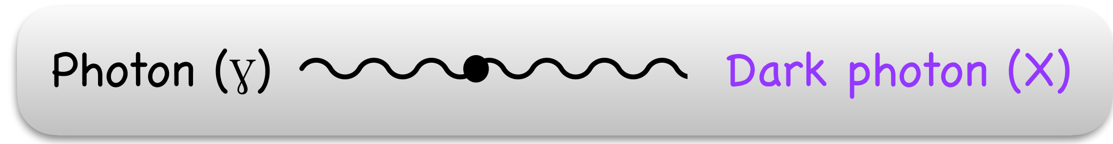
It can be even lower than the Λ-CDM (w = -1).

Possibility of alleviating the Hubble tension. (It requires numerical fitting study.)

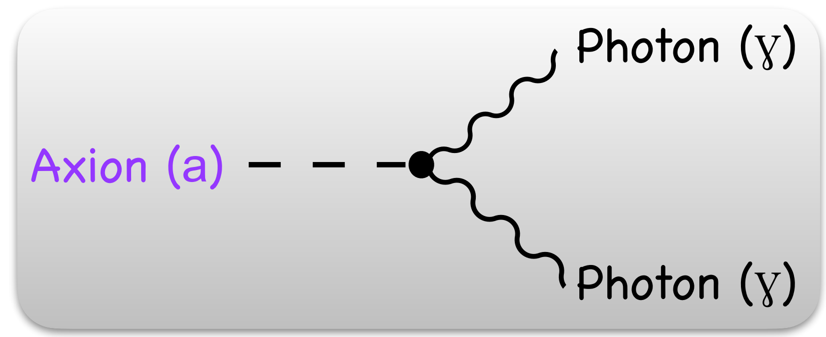
Non-gravitational signals

Using the portal concept

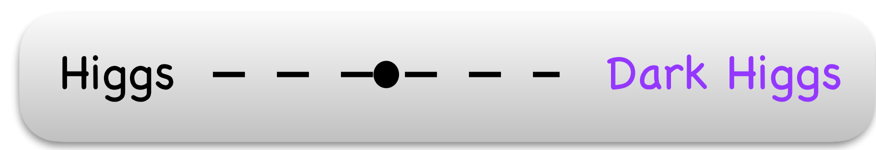
(i) Vector Portal $\frac{\varepsilon}{2} F_{\mu\nu} X^{\mu\nu}$



(ii) Axion Portal $\frac{G_{a\gamma\gamma}}{4} a F_{\mu\nu} \tilde{F}^{\mu\nu}$

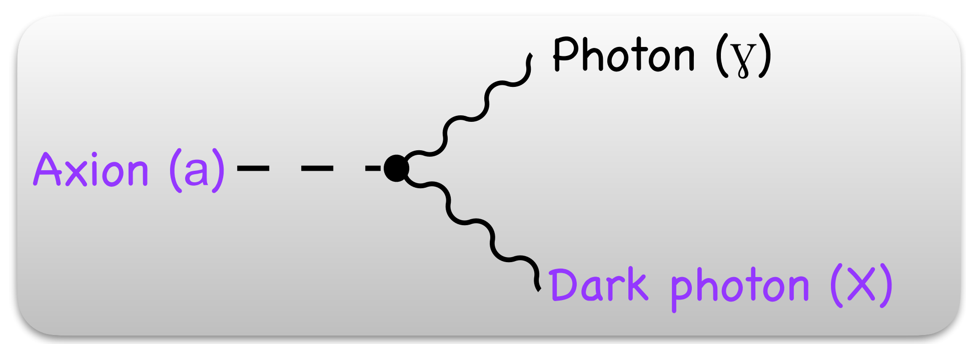


(iii) Higgs Portal $\kappa \, |S|^2 |H|^2 + \mu \, S |H|^2$



(iv) Neutrino Portal $y \, LHN$

(v) Dark Axion Portal $\frac{G_{a\gamma X}}{4} F_{\mu\nu} \tilde{X}^{\mu\nu}$



How can we have non-gravitational signals of the gauged quintessence model?
The portal is a popular concept in studying the dark matter physics.
We adopt it to study the dark energy sector (specifically, the vector portal).

Production of the dark gauge boson

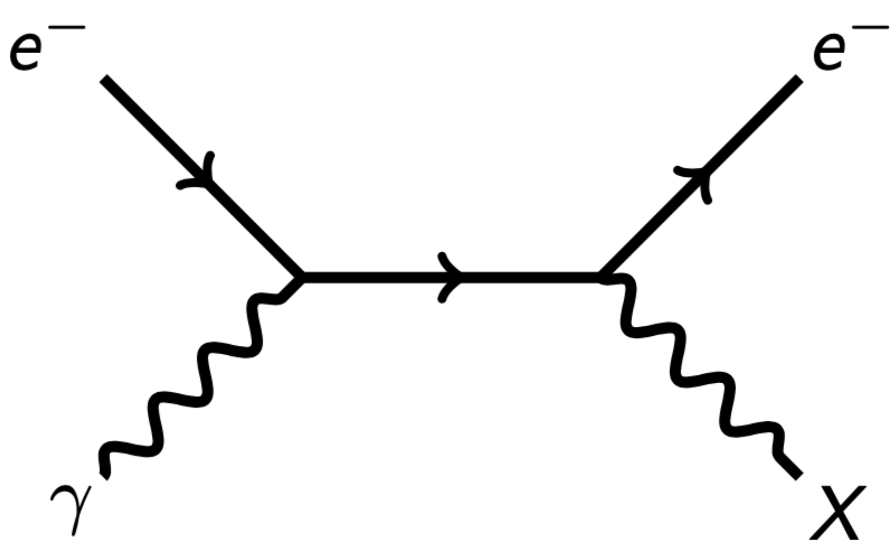
$$\frac{\varepsilon}{2} F_{\mu\nu} X^{\mu\nu}$$

We take the vector portal
for the dark gauge boson production & decay.

Effective kinetic mixing of the photon and dark photon in thermal bath:

$$|\bar{\varepsilon}|^2 = \varepsilon^2 \frac{m_X^4}{(m_X^2 - m_\gamma^2)^2 + (\omega D)^2}$$

Redondo (2008)

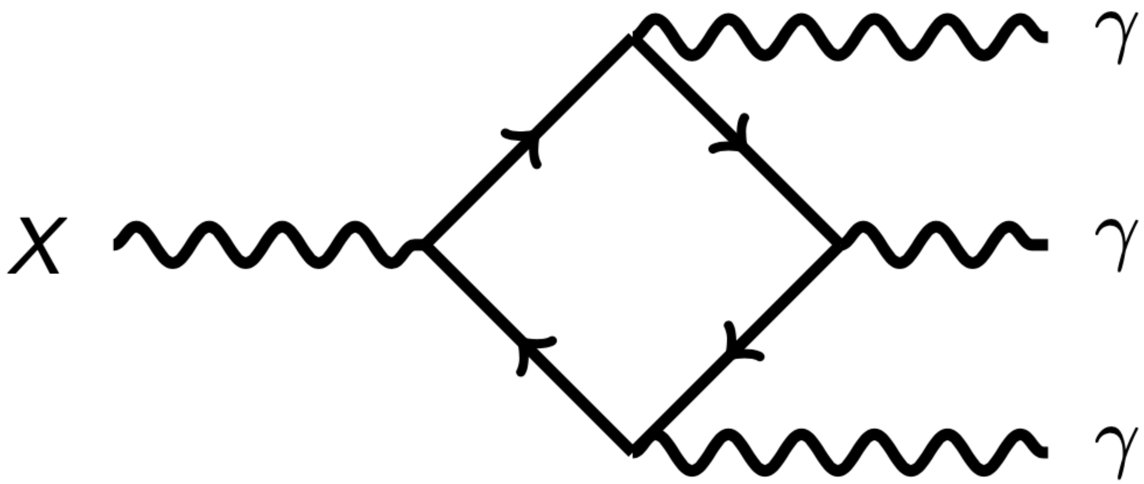


$$\sigma \propto |\bar{\varepsilon}|^2$$

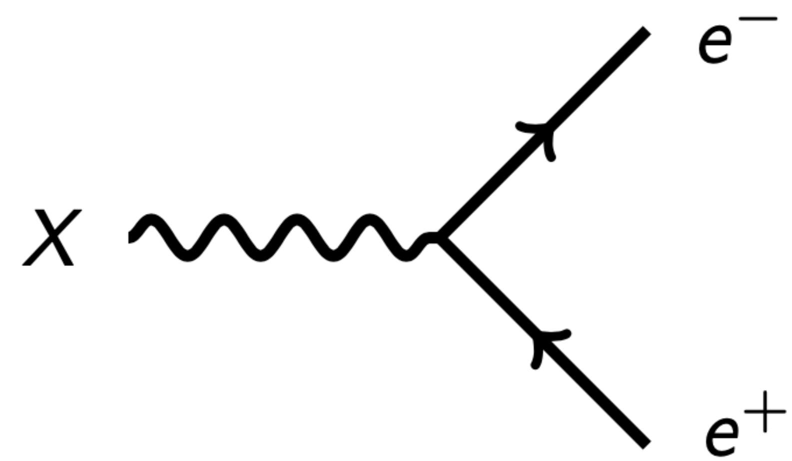
Decay of the dark gauge boson

Time Flow $\longrightarrow$

Early Universe $(m_X < 2m_e)$



Late Universe $(m_X > 2m_e)$



$$\Gamma_{\gamma\gamma\gamma} = \frac{17\alpha_{\rm em}^4 \varepsilon^2}{11664000\pi^3} \frac{m_X^9}{m_e^8}$$

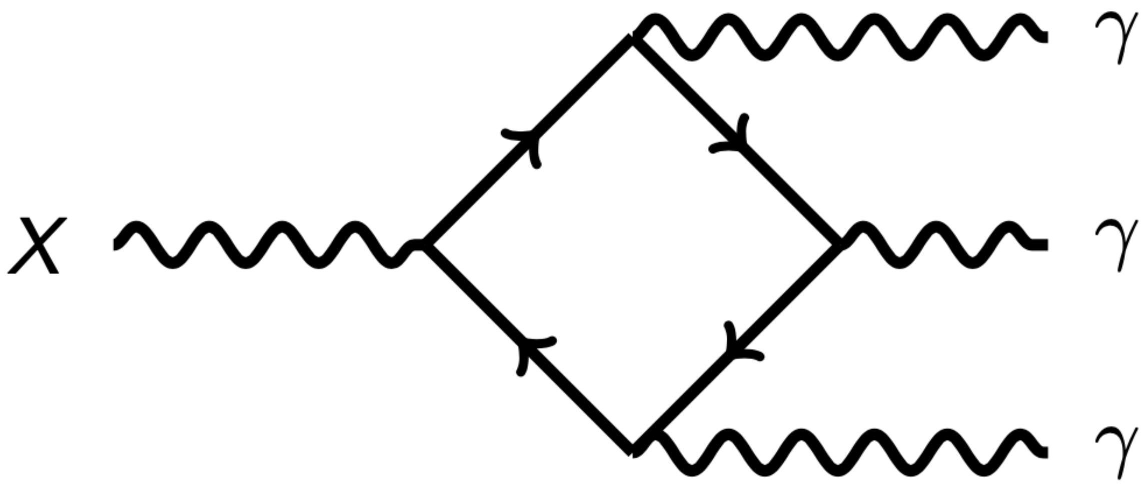
$$\Gamma_{e^+e^-} = \frac{\alpha_{\rm em}\varepsilon^2 m_X}{3} \sqrt{1 - \left(\frac{2m_e}{m_X}\right)^2} \left(1 + \frac{2m_e^2}{m_X^2}\right)$$

The typical (mass-fixed) dark photon has only one dominant decay mode (either $\gamma\gamma\gamma$ or e^+e^-) depending on the dark photon mass.
The mass-varying dark photon's dominant decay mode may change over cosmic time. We may have both signals ($\gamma\gamma\gamma$ and e^+e^-).

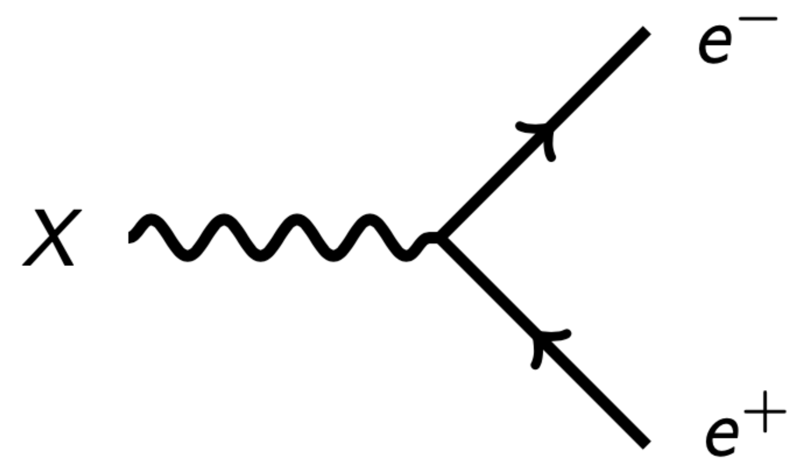
Decay of the dark gauge boson

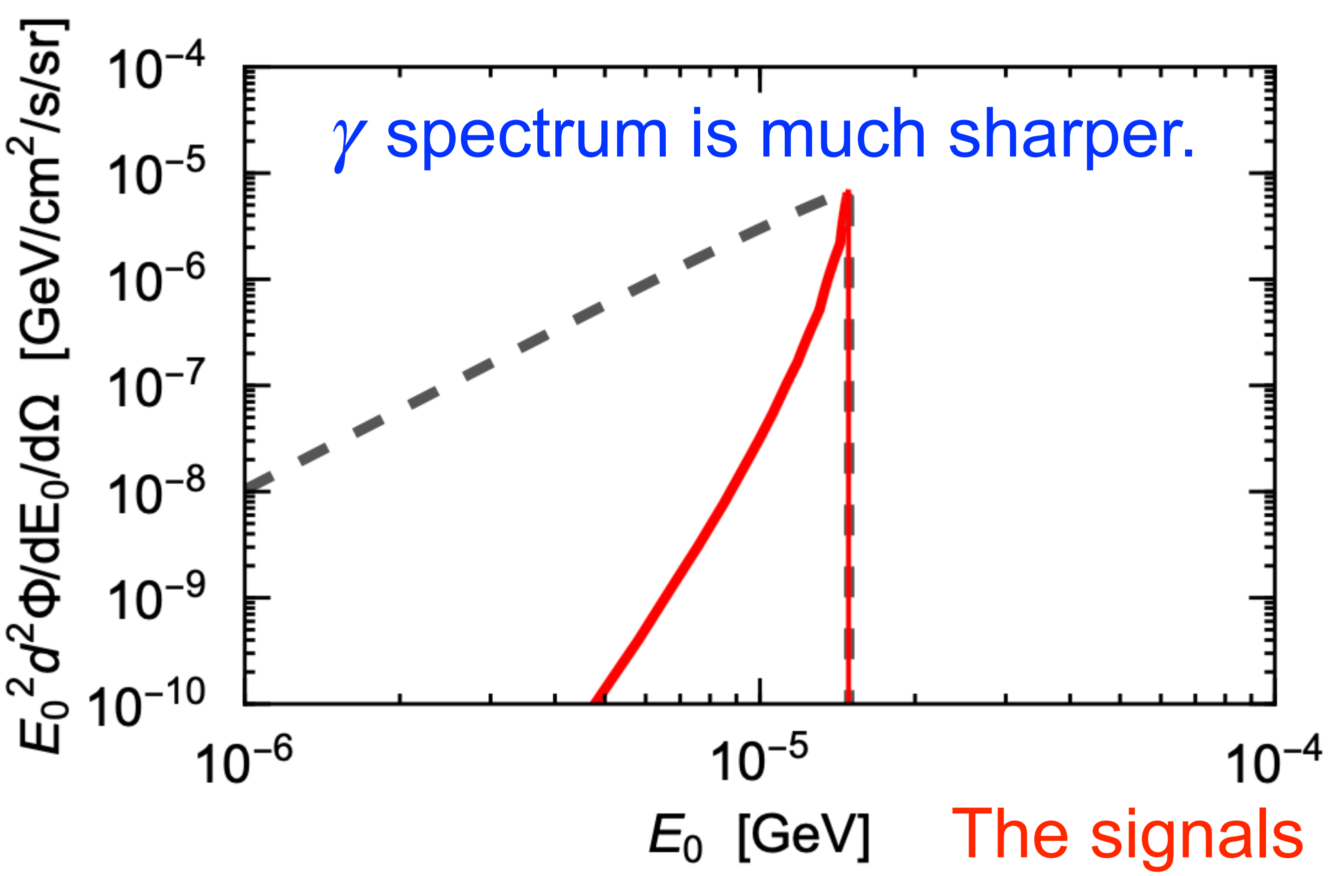
Time Flow →

Early Universe $(m_X < 2m_e)$



Late Universe $(m_X > 2m_e)$

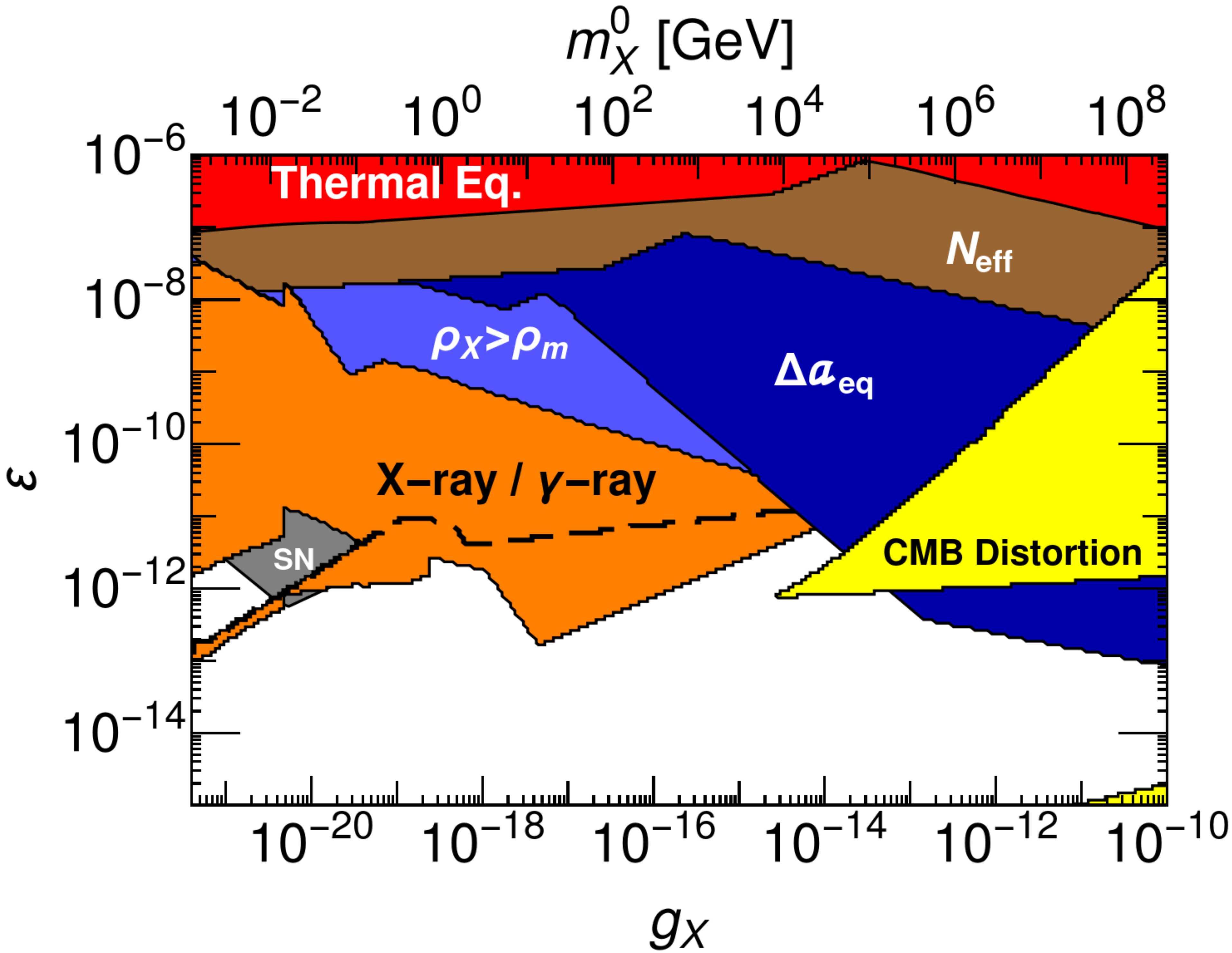




e^+e^- are highly non-relativistic.

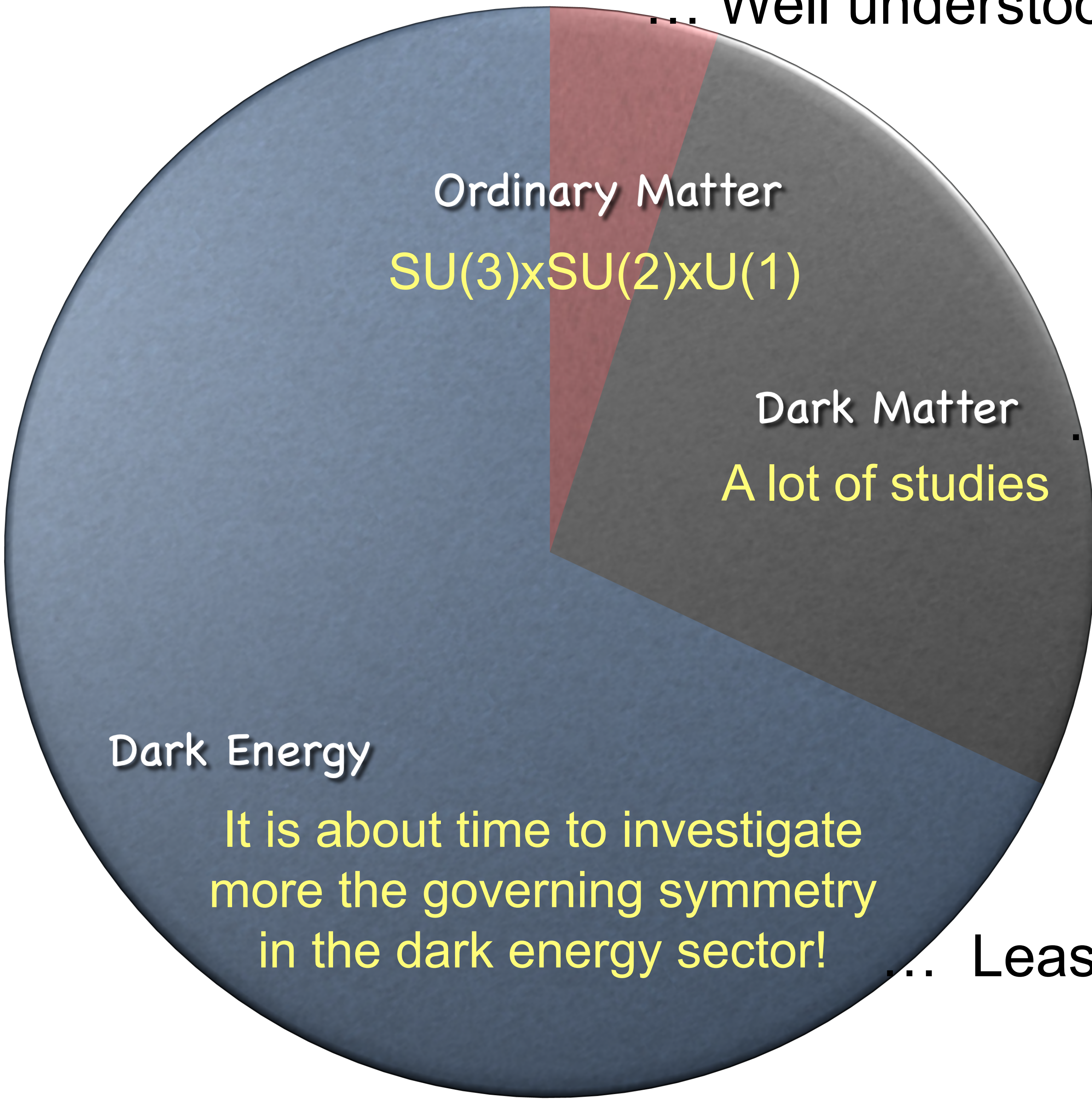
The signals have unique features in their energies, too.

Constraints on the kinetic mixing and dark gauge coupling



Concluding remarks

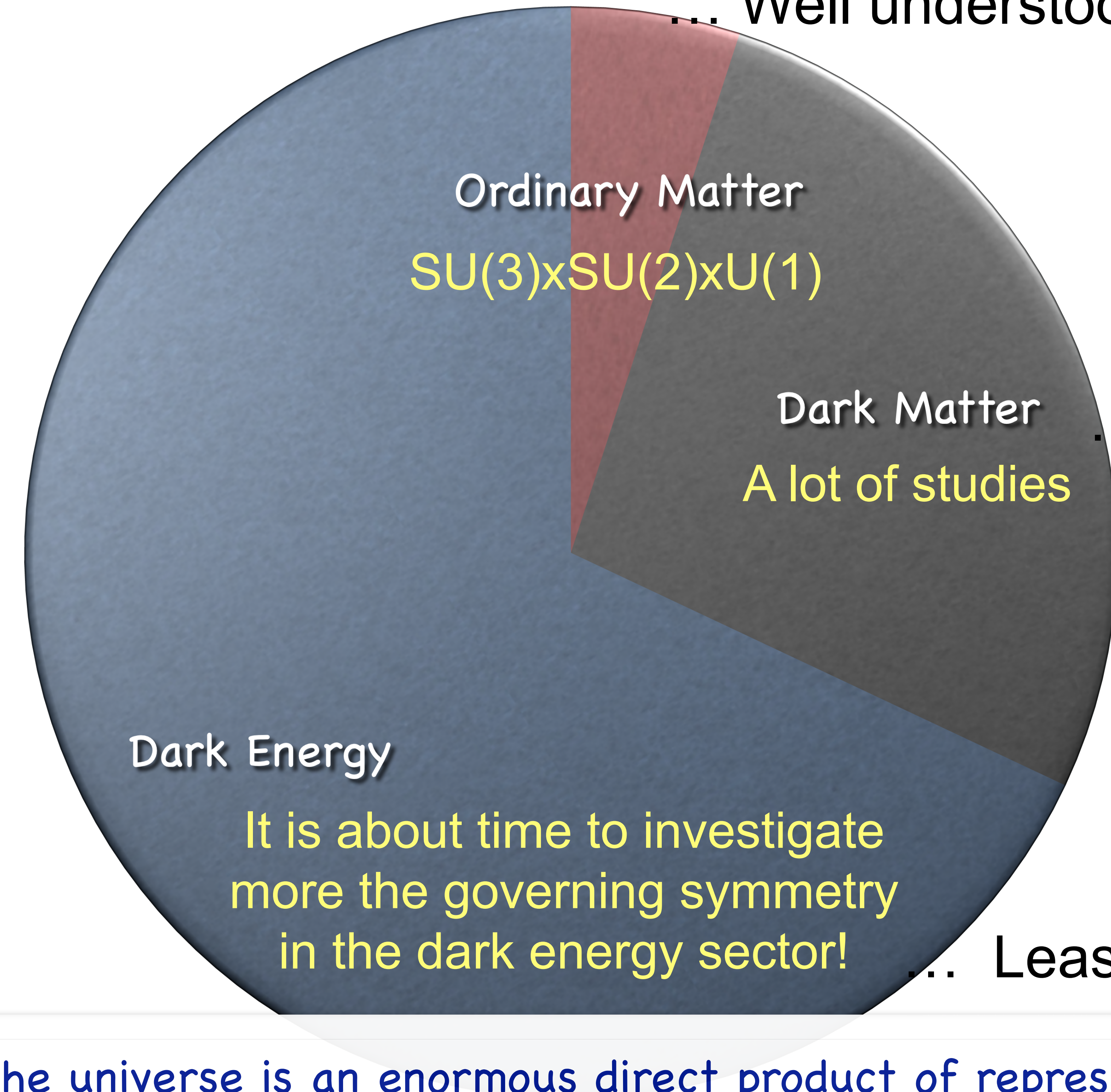
... Well understood



... Global competitions

... Least understood

We can use many skills/methods we developed for the DM sector in the DE sector.



... Well understood

... Global competitions

... Least understood

The universe is an enormous direct product of representations of symmetry groups.

- Steven Weinberg -

Summary

1. We introduced the first gauge symmetry model for a popular quintessence dark energy scalar field.

2. The interaction between the quintessence and the gauge boson ($V_{\text{gauge}} = \frac{1}{2} g_X^2 \phi^2 X_\mu X^\mu$) brings many interesting features to the universe evolution.

3. The mass-varying effect of the X gauge boson may overcome the problem of the vector boson misalignment mechanism (scaling factor suppression).

4. Hubble tension might be alleviated. (Need quantitative study)

5. Our study may serve as a proof of concept that the dark energy sector can be studied using the gauge principle. (Gauge interaction for the dark energy)

Back-up

Gauged Quintessence

Equations of motion for ϕ and X (coupled via V_{gauge})

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V_0}{\partial \phi} + g_X^2 X_\mu X^\mu \phi = 0$$

$$\partial_\mu X^{\mu\nu} + 3H X^{0\nu} - g_X^2 \phi^2 X^\nu = 0$$

$$V_{\text{gauge}} = \frac{1}{2} g_X^2 \phi^2 X_\mu X^\mu$$

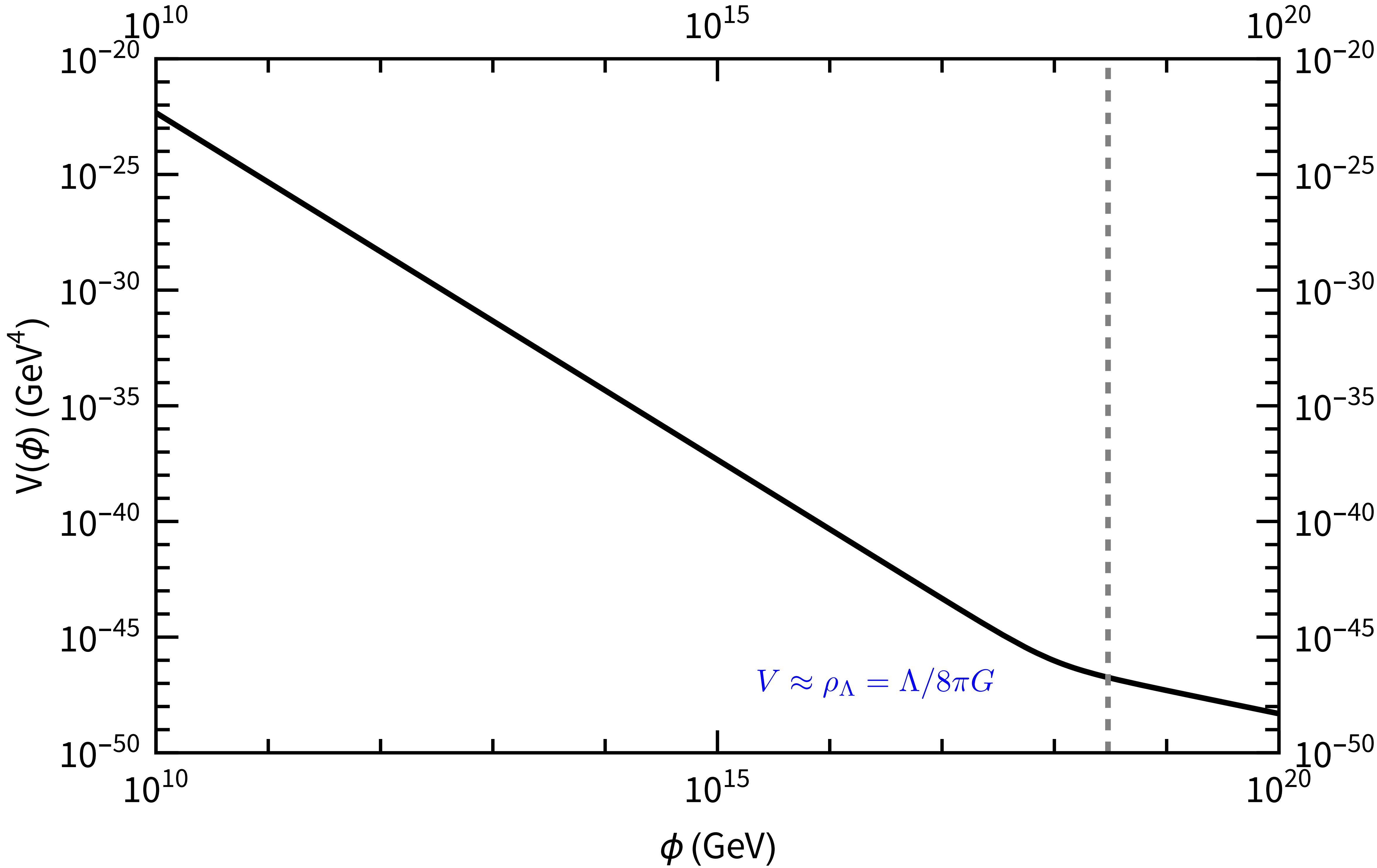
Energy-momentum tensor

$$T_{\mu\nu} = (\partial_\mu \phi)(\partial_\nu \phi) - \frac{1}{2} g_{\mu\nu} (\partial_\alpha \phi)(\partial^\alpha \phi) - g_{\mu\nu} V_0(\phi)$$
$$- \frac{1}{2} g_{\mu\nu} g_X^2 \phi^2 X_\alpha X^\alpha + g_X^2 \phi^2 X_\mu X_\nu + X_{\mu\alpha} X_\nu^\alpha - \frac{g_{\mu\nu}}{4} X_{\alpha\beta} X^{\alpha\beta}$$

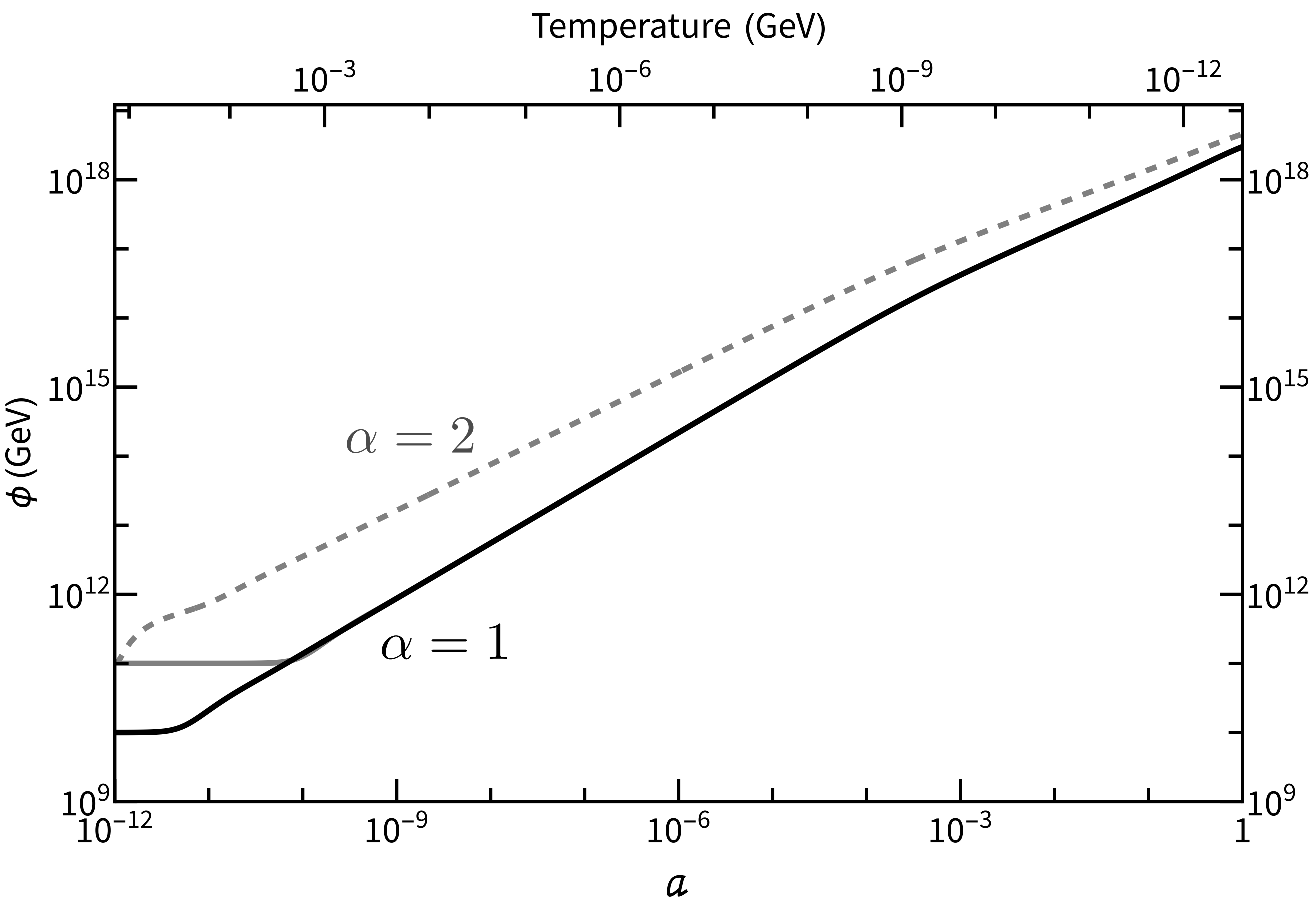
$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V_0(\phi)\ , \quad p_X = p_{\phi+X} - p_\phi$$

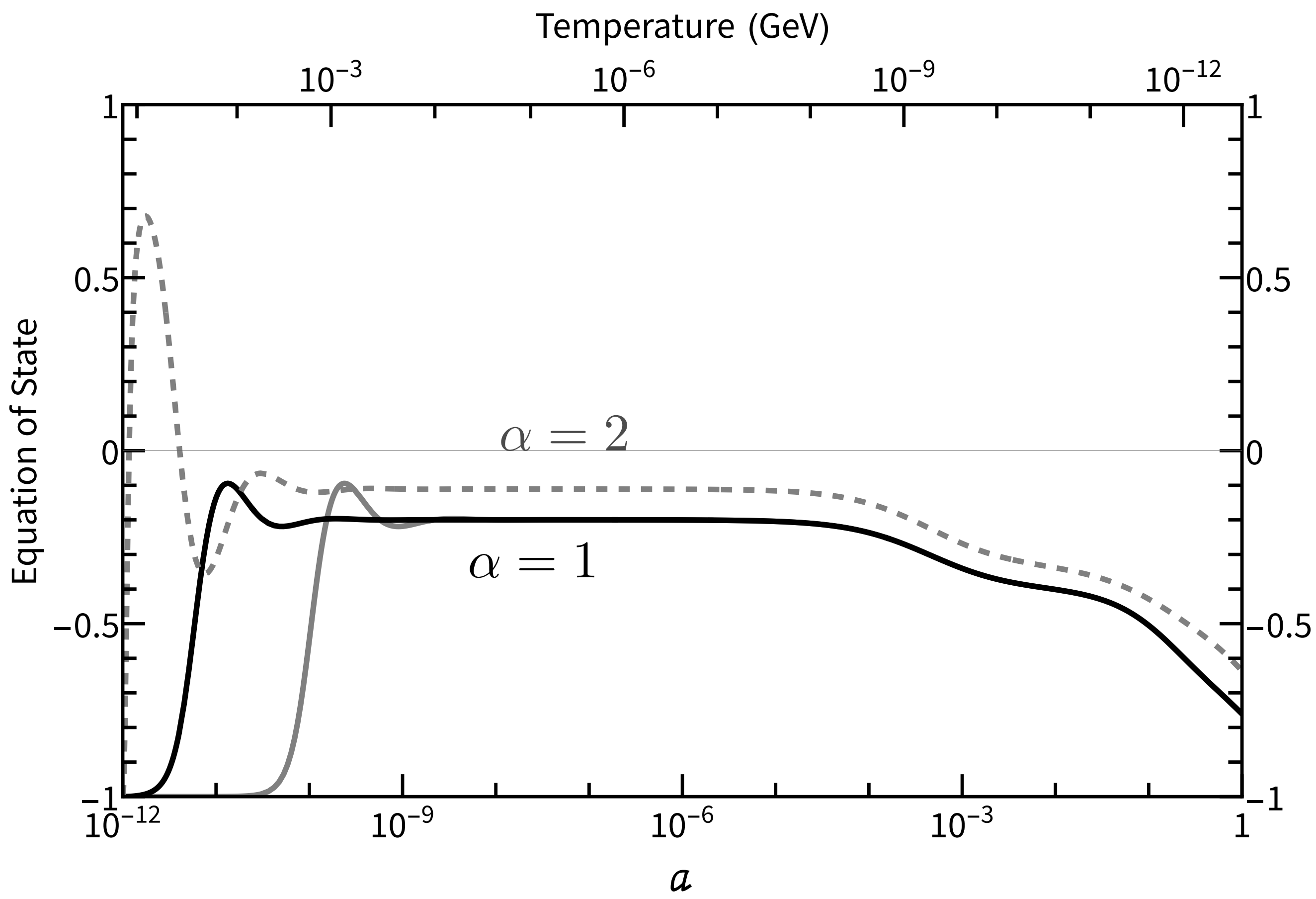
$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V_0(\phi)\ , \quad \rho_X = \rho_{\phi+X} - \rho_X$$

Ratra-Peebles potential (with quantum corrections)



Quintessence dynamics (without a gauge symmetry)





$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V}{\partial \phi} = 0$$

$$V(\phi) = \frac{M^{\alpha+4}}{|\phi|^{\alpha}}$$

Balancing between the potential slope and Hubble friction results in the common tracking solution for the quintessence. The present values are not sensitive to its initial values (quintessence tracking behavior).

Effective DE density

$$w_{\rm eff}(\widetilde{DE}) = -1 + \frac{1}{\rho_{\widetilde{DE}}}\left((1+w_0)\rho_\phi + \left(\frac{m_X}{m_X^0} - 1\right)\frac{\rho_X^0}{a^3}\right)$$

$$\dot\rho + 3H(1+w)\rho = 0$$

: $w_{\rm eff}$ for the effective DE density in the gauged quintessence

Developed in the DE-DM interaction model. [Das, Corasaniti, Khoury (2006)]

Take the effective DM density ($\rho_{\widetilde{CDM}}$) for the constant mass with a^{-3} scaling part.
The remaining mass-varying part is absorbed in the effective DE density ($\rho_{\widetilde{DE}}$).

$$\frac{a^3\rho_X}{m_X} = \frac{\rho_X^0}{m_X^0}$$

$$\begin{aligned}\rho_{\rm CDM} + \rho_X + \rho_\phi &= \rho^0_{\rm CDM}a^{-3} + \frac{m_X}{m_X^0}\rho_X^0 a^{-3} + \rho_\phi \\ &= \left[\left(\rho^0_{\rm CDM} + \rho^0_X\right)a^{-3}\right] + \boxed{\left[\left(\frac{m_X}{m_X^0} - 1\right)\rho_X^0 a^{-3} + \rho_\phi\right]} \\ &= \rho_{\widetilde{CDM}} + \rho_{\widetilde{DE}}\end{aligned}$$

$\rho_{\widetilde{DE}}$

Adiabatic condition

It should be noted that in both cases we will only consider the case the adiabatic condition [77], which can be written as

$$\frac{dm_X}{dt} \ll m_X^2 \,, \tag{4.1}$$

is always satisfied. If this condition is violated, the WKB-like solution for the wave function of X_μ cannot be used. In other words, non-perturbative X_μ production could be non-negligible in such a case. The adiabatic condition may break in some cases[6], which can bring intriguing phenomenology. Furthermore, the violation of the adiabaticity indicates that the fragmentation of a condensate (ϕ and/or X_μ) may take place through gauge or self interactions[7] in a similar manner of other coherent states, such as inflaton [78] and axion-like particle [79]. However, in this paper, we will investigate the simple cases in which this condition is valid.