Higher-Spin extended Gravity from IKKT Matrix Model

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Outline

- IKKT Matrix Model
- Quantized Embedded Branes
- Low Energy Physics, YM and pre-gravity
- One-loop effective action
- (Extended) Einstein Gravity
- Covariant Cosmological Quantum Spacetime
- One-loop dynamics and Stabilization
- Conclusions and Outlook

IKKT Matrix Model

- 1996, Matrix Regularization of type IIB superstring theory in Schild gauge (Ishibashi-Kawai-Kitazawa-Tsuchiya '97)

$$S = \frac{1}{g^2} \operatorname{tr}\left(\left[X^a, X^b\right] \left[X^c, X^d\right] \eta_{ac}\eta_{bd} + \overline{\Psi}\Gamma^a \left[X^b, \Psi\right] \eta_{ab} \right) \tag{1}$$

$a = 0, 1, \ldots, 9$, X^a $N \times N = \mathrm{End}(\mathcal{H})$ hermitian ($\mathcal{H} = \mathbb{C}^N$).

- X^a as quantized embedding coordinates of a brane.
- Classical Equations of motion

$$\Box_X X^a = -\frac{1}{4}\left[\overline{\Psi}, \Gamma^a \Psi\right], \ \Gamma_a[T^a, \Psi] = 0, \ \ \Box_X = [X^a, [X^b, \cdot]]\eta_{ab} \tag{2}$$

- Maximal SUSY

Properties

- Describes quantum dynamics of non-commutative spaces, matrix integral
$$Z = \int \mathrm{d}X\mathrm{d}\Psi e^{iS} \tag{3}$$
- Manifest $SO(1,9)$ invariance.
- Gauge $U(N)$
$$X^a \mapsto UX^aU^{-1} \tag{4}$$
- Exhibits holographic properties
- We are interested in low energy modes <u>on</u> the brane. No target space $\mathbb{R}^{1,9}$ physics, no holography.

Quantized Embedded Branes

- *Almost* commutative configurations: quasi-coherent states $|x\rangle \in \mathcal{H}$
- Define embedding coordinates in target space

$$x^a = \langle x| X^a |x\rangle \in \mathbb{R}^{1,9} \tag{5}$$

  defining symplectic brane $\mathcal{M}$.

- Associate classical functions to matrices via quantization map

$$\mathcal{Q} : \mathcal{C}(\mathcal{M}) \to \mathrm{End}(\mathcal{H}) \tag{6}$$

$$\phi(x) \mapsto \Phi = \int \Omega \phi(x) |x\rangle\langle x| \tag{7}$$

Quantized Embedded Branes

- Commutator $\sim$ Poisson Bracket

$$-i\left[X^a, X^b\right] = \Theta^{ab} \sim \{x^a, x^b\} = \theta^{ab} \tag{8}$$

symplectic form.

- Look for such quantized embedded spaces solving IKKT equations of motion.

Low Energy Physics

- Low energy excitations (subsector of $End(\mathcal{H}) \supset Loc(\mathcal{H}) \sim C_{IR}(\mathcal{M})$) are confined on the brane
- Find non-trivial backgrounds $\overline{T}^a \in End(\mathcal{H})$, and study the physics of fluctuations

$$T^a = \overline{T}^a + \mathcal{A}^a \tag{9}$$

- $U(N)$: $SU(N)$ gauge sector and noncommutative $U(1)$, *geometry*.

Low Energy Physics

- Tree-level action for low energy $SU(N)$ fluctuations

$$S = -\frac{1}{g^2}\,\mathrm{tr}\int \Omega\gamma^{ac}\gamma^{bd}F_{ab}F_{cd} + \cdots \tag{10}$$

using $[X^a, \cdot] = i\theta^{ab}\partial_b$, F field strength of $A_\mu = \theta^{-1}_{\mu\nu}\mathcal{A}^\nu$, γ "open string metric"

$$\gamma^{ab} = \theta^{ab}\theta^{cd}\eta_{bd} \tag{11}$$

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- Geometric interpretation of matrix d.o.f. T^a, $\mathcal{F}^{ab} = \left[T^a, T^b\right]$?
- Defining

$$E^{a\mu} = \{T^a, x^\mu\} \tag{12}$$

Typical kinetic action $[T^a, \phi]^2$,

$$S \sim -\int \Omega E_a{}^\mu E^{a\nu} \partial_\mu \phi \partial_\nu \phi = -\int \mathrm{d}^{2n} x \sqrt{G} G^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \tag{13}$$

$\Omega = \rho_M \mathrm{d}^{2n} x$, G effective metric on the brane,

$$G^{\mu\nu} = \rho^{-2} \gamma^{\mu\nu}, \quad \gamma^{\mu\nu} = E_a{}^\mu E^{a\nu}, \quad \rho_M = \rho^{-2} \sqrt{G} \tag{14}$$

- E **frame** for γ. $T_{ab}{}^\mu = \{\mathcal{F}_{ab}, x^\mu\}$ torsion of γ in **Weitzenbock** connection.

Geometry

- Gauge transformations of the background

$$\delta_\Lambda T = \{\Lambda, T\} \implies \delta_\Lambda E = \mathcal{L}_\xi E \tag{15}$$

with $\xi = \{\Lambda, \cdot\}$. They generate diffeos! Same for $G^{\mu\nu}$.

- Tree-level **not** Einstein: fluctuations $\delta E \sim \partial \mathcal{A}$

$$S \sim \mathrm{tr}\,[T, T]^2 \sim \int (\partial \mathcal{A})^2 \sim \int (\delta E)^2 \tag{16}$$

Linearized Einstein governs derivatives of the frame

$$S_{EH} \sim \int (\partial \delta E)^2 \tag{17}$$

We find (extended) Einstein at one-loop.

One-Loop Effective Action

- We compute

$$Z = \int \mathrm{d}X \mathrm{d}\Psi e^{iS} = e^{i(S_0 + \Gamma_{1-loop})} = e^{i\Gamma_{eff}} \qquad (18)$$

$$\Gamma_{1-loop} = \tfrac{i}{2}\,\mathrm{tr}\left(\log\left(\Box - i\varepsilon - \Sigma_{ab}^{(V)}[\mathcal{F}^{ab},\cdot]\right) - \tfrac{1}{2}\log\left(\Box - i\varepsilon - \Sigma_{ab}^{(\Psi)}[\mathcal{F}^{ab},\cdot]\right) - 2\log(\Box - i\varepsilon)\right)$$
$$= -\frac{i}{2}\int_0^\infty \frac{\mathrm{d}\alpha}{\alpha}\,\mathrm{tr}\, e^{-i\alpha\Box}\left[e^{i\alpha\Sigma_{ab}^{(V)}\delta\mathcal{F}^{ab}} - \frac{1}{2}e^{i\alpha\Sigma_{ab}^{(\Psi)}\delta\mathcal{F}^{ab}} - 2\right]$$
$$= -\frac{i}{2}\int_0^\infty \frac{\mathrm{d}\alpha}{\alpha}\,\mathrm{tr}\, e^{-i\alpha\Box} Q_{10}$$

- $\delta\mathcal{F} = [\mathcal{F},\cdot]$, α Schwinger parameter, Q_{10} character of $SO(1,9)$.

One-Loop Effective Action

- We want to study 4d physics, background $\mathcal{M}_4 \times \mathcal{K}$

$$T^a = (T^{\dot{\mu}}, T^I) \qquad (19)$$

With Hilbert space $\mathcal{H}_{\mathcal{M}} \times \mathcal{H}_{\mathcal{K}}$, low energy $\mathcal{C}(M) \times \mathcal{C}(\mathcal{K})$.

- Fluctuations $\mathcal{A} \in End(\mathcal{H}_{\mathcal{M}}) \otimes End(\mathcal{H}_{\mathcal{K}})$
- Decompose 10d representations

$$(V) = (4) + (6), \;\; (\Psi) = ((2_-) \oplus (4_-)) \oplus ((2_+) \oplus (4_+)) \qquad (20)$$

- We expand the 4d characters to obtain

$$Q_{10} = X_6 + \alpha^2 \delta\mathcal{F}^{\dot{\mu}\dot{\nu}} \delta\mathcal{F}_{\dot{\mu}\dot{\nu}} \left(-2 + \tfrac{1}{4} \textstyle\sum_{\pm} \mathrm{tr}_{(4_\pm)}\, e^{i\alpha \Sigma^{\pm}_{IJ} \delta\mathcal{F}^{IJ}} \right) + O(\alpha^4 \mathcal{F}^4_{\dot{\mu}\dot{\nu}})$$
$$\approx X_6 + \alpha^2 \delta\mathcal{F}^{\dot{\mu}\dot{\nu}} \delta\mathcal{F}_{\dot{\mu}\dot{\nu}} G_6$$

- We also showed that generically α expansion is justified.

One-Loop Effective Action

- Two $SO(6)$ characters: X_6 effective potential for $\mathcal{K}$, G_6 determines Newton constant.
- Evaluate spacetime modes trace $End(\mathcal{H}_{\mathcal{M}})$ with basis of **string modes** $|x\rangle\langle y|$. Short string modes $|x\rangle\langle x+k| \sim$ localized wave packets $\psi_{k,x}$.

$$\begin{aligned}
&\mathrm{tr}_{End(\mathcal{H}_{\mathcal{M}})}\, \delta\mathcal{F}^{\dot\mu\dot\nu}\delta\mathcal{F}_{\dot\mu\dot\nu} e^{i\alpha\Box_4} \\
&\approx \int d^4x\sqrt{G}\int \frac{d^4k}{(2\pi)^4\sqrt{G}}\mathcal{T}^{\dot\mu\dot\nu\alpha}\mathcal{T}_{\dot\mu\dot\nu}{}^{\beta}k_\alpha k_\beta e^{-i\alpha\rho^2 G^{\mu\nu}k_\mu k_\nu} \\
&\qquad\approx \int d^4x\sqrt{G}\frac{1}{2(4\pi)^2}\frac{1}{\rho^4\alpha^3}\gamma_{\alpha\beta}\mathcal{T}^{\dot\mu\dot\nu\alpha}\mathcal{T}_{\dot\mu\dot\nu}{}^{\beta}
\end{aligned} \tag{21}$$

$$\mathcal{R} = -\tfrac{1}{2}\mathcal{T}^{\mu}{}_{\nu\alpha}\mathcal{T}_{\mu}{}^{\nu}{}_{\beta}G^{\alpha\beta} - \tfrac{1}{2}\widetilde{T}_\mu\widetilde{T}_\nu G^{\mu\nu} + 2\rho^{-2}G^{\mu\nu}\partial_\mu\rho\partial_\nu\rho - 2\nabla^\mu(\rho^{-1}\partial_\mu\rho)$$

One-Loop Effective Action

- α integral and $End(\mathcal{H}_{\mathcal{K}})$ trace: expand

$$G_6[\alpha] \approx -\frac{1}{2}\alpha^2 \delta\mathcal{F}^{IJ}\delta\mathcal{F}_{IJ} \tag{22}$$

- Approximate trace with $\mathcal{K} \sim$ flat ball of radius $m_{\mathcal{K}}$, $\delta\mathcal{F}_{IJ} \approx \Delta_{\mathcal{K}}^2 \sim m_{\mathcal{K}}^2 d^{-\frac{2}{\dim\mathcal{K}}}$. Using $\mathcal{K}$ string modes $|x\rangle\langle y|$,

$$\Box_6\big|_{|x\rangle\langle y|} \approx ((x-y)^2 + 2\Delta_{\mathcal{K}}^2) \tag{23}$$

  with $\Delta_{\mathcal{K}}$ NC scale of $\mathcal{K}$.

- Find G_N as a function of $\rho, d_{\mathcal{K}}, m_{\mathcal{K}}$.

Cosmological Quantum Spacetime

- Apply above to a specific interesting background: $\mathcal{M}^{3,1}_J$, fuzzy hyperboloid. (Sperling, Steinacker 2018)
  - Quantized Coadjoint orbit of $SO(4,1)$
  - In terms of $SO(4,2)$ generators M^{AB}, $A, B = 0, 1, \ldots, 5$. Built from irreps labelled by spin J. Here $J \gg 1$.
  - Background $T^{\dot{\mu}} = \frac{1}{R} M^{\dot{\mu}4}$, *coordinate* matrices $X^\mu = \ell_p M^{\mu 5}$.
  - Semi-classical geometry (coherent states) for $J > 0 \sim$ 6d symplectic space,
    $$H^4 \tilde{\times} S^2_J \tag{24}$$
  - S^2_J harmonics define a *truncated* tower of higher spin modes over spacetime.
  - Semi-classical geometry looks FLRW-like. $\rho^2 \approx \sinh^3 \tau$.

Relevant Scales

- ℓ_p fundamental length scale of coordinates.
- $R \approx \frac{\ell_p J}{2}$, radius of hyperboloid.
- IR curvature scale $L_H = R\cosh\tau$ (τ cosmological conformal time)
- NC scale $L_{NC} = \sqrt{J}\ell_p\sqrt{\cosh\tau}$.
- $\Delta_J \sim$ average of higher spin masses: see caveats.
- KK scale $m_{\mathcal{K}}$
- $\mathcal{K}$ NC scale $\Delta_{\mathcal{K}} \approx d_{\mathcal{K}}^{-\frac{1}{\dim\mathcal{K}}} m_{\mathcal{K}}$.

- One-loop effective action (YM sector in (Steinacker, Tran 2024)), now also trace over higher spin harmonics: adds dependence on J and NC scale of S^2_J, Δ_J to G_N: for $\dim \mathcal{K} = 4$ (we will see why), with suitable approximations

$$G_N \sim \frac{\rho^2}{J^2 d_{\mathcal{K}}^{\frac{3}{2}} \Delta_{\mathcal{K}}^2} \tag{25}$$

- Significance and dynamics of HS yet to be fully understood.
- We find a mechanism for stabilizing $m_{\mathcal{K}}$ as follows: the effective action contains the following contributions to potential for single brane $\mathcal{K}$

$$\begin{aligned} V &= V_0 + V_{\mathcal{K}}^{1-loop} + V_{S^2_J} + V_{S^2-\mathcal{K}} + V_{grav} && (26) \\ &\approx V_0 + V_{S^2-\mathcal{K}} + V_{S^2_J} && (27) \end{aligned}$$

One-Loop

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- With the approximations made above, we can estimate

$$S_0 = -\frac{1}{g^2}\,\mathrm{tr}\,\mathcal{F}_{IJ}\mathcal{F}^{IJ} \approx -\frac{J}{\ell_p^4 g^2}\int d^4x \frac{\sqrt{G}}{\rho^4}\,\mathrm{tr}_{\mathcal{K}}\,\mathcal{F}_{IJ}\mathcal{F}^{IJ} \tag{28}$$

$$\approx -\frac{J}{\ell_p^4 g^2}\int d^4x \frac{\sqrt{G}}{\rho^4}\Delta_{\mathcal{K}}^4 d_{\mathcal{K}} =: -\int d^4x \sqrt{G}\, V_0 \tag{29}$$

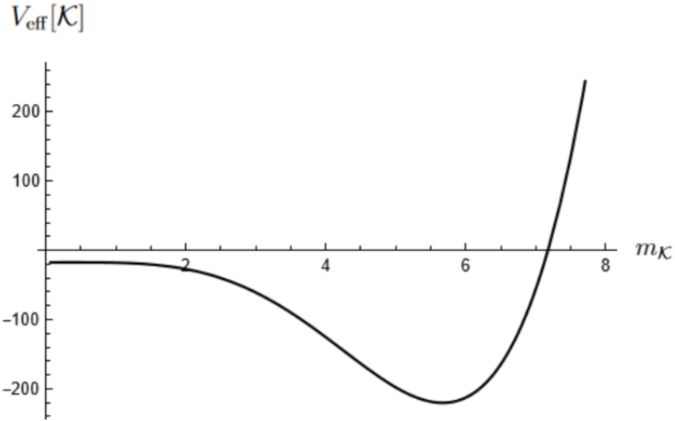
$V_0 > 0$, $\propto \Delta_{\mathcal{K}}^4$.

- $V_{S_J^2}$ and $V_{S^2-\mathcal{K}}$ give a decreasing dependence on $\Delta_{\mathcal{K}}$ for $\dim \mathcal{K} = 2, 4$, thus a **minimum** for the effective potential altogether. $\dim \mathcal{K} = 2$ seems pathological, therefore focus on $\dim \mathcal{K} = 4$. Find minimum, $m_{\mathcal{K}}^\star$ and $G_N(m^\star)$

$$m_{\mathcal{K}}^\star(\ell_p, J, g, \rho, \Delta_J) \tag{30}$$

Potential for $m_{\mathcal{K}}$

- For suitable parameter ranges*

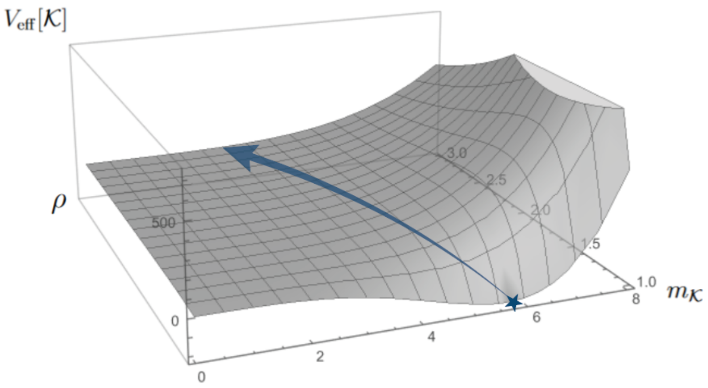


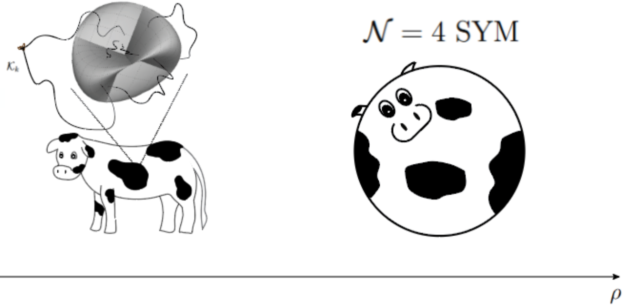
Caveats

I made it too simple, exhibited nice behaviors, but some things to fix.

- Background $T^{\dot{\mu}}$ solves **massive** IKKT: we are finding deformations of this background that solve IKKT.
- That's good, because solution above is problematic (we expect general results to hold):
  - $\Delta_J^2 < 0$, problems with stability.
  - $m^{\star}_{\mathcal{K}}$ depends strongly on time (and does not exist at very late times!)
  - Consistent ranges of validity for parameters?
- Most of these problems seem to be fixable with deformed backgrounds, with added complications to be studied.

Minimum




$\mathcal{N} = 4$ SYM
ρ

Conclusions

- IKKT as quantum theory of spacetime.
- Emergent (Extended) Einstein-Hilbert action from 1-loop with fuzzy extra dimensions.
- Cosmological Background: Mechanism for stabilization of the background at 1-loop.
- This background is problematic: work to do to solve problems.

Thank you for the attention