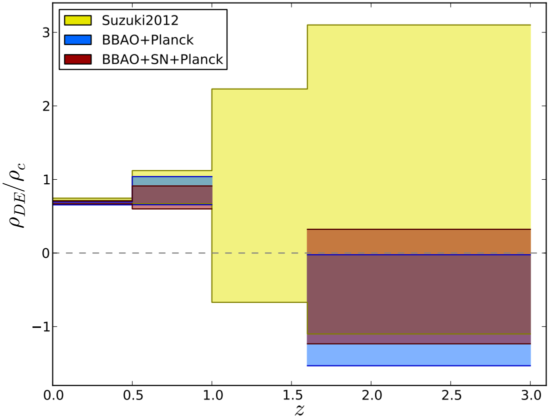
Tackling the tensions of cosmology with a negative dark energy density (and nonmonotonicity)

Emre Özülker

Istanbul Technical University

Corfu2023 Tensions in Cosmology, September 2023

Phenomenological motivation (reconstructions)



E. Auburg [BOSS] *et al.* 1411.1074

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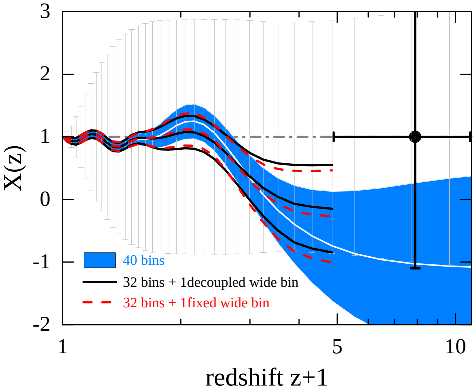

Planck+SNIa+BAO+CC+H_0

Y. Wang *et al.* 1807.03772

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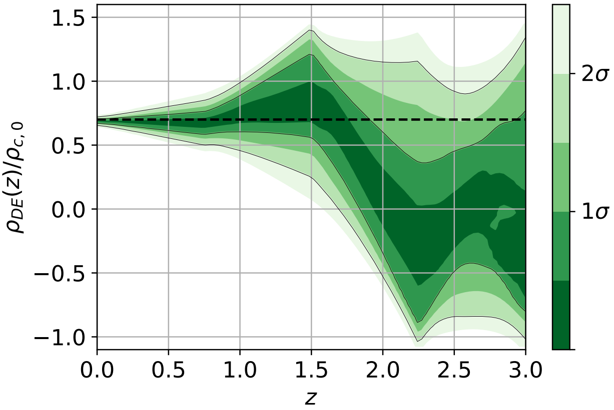

CC+SNIa+BAO+BBN

L. A. Escamilla *et al.* 2111.10457

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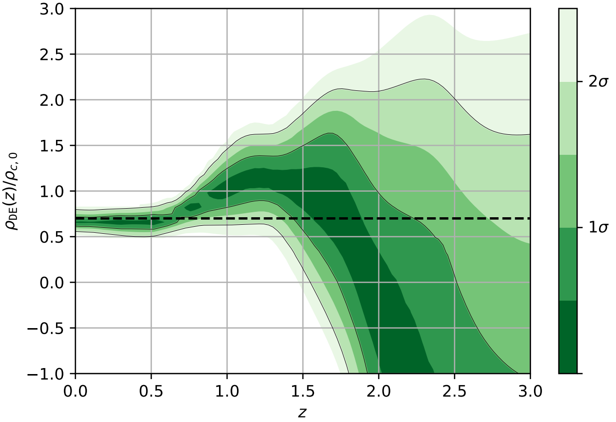


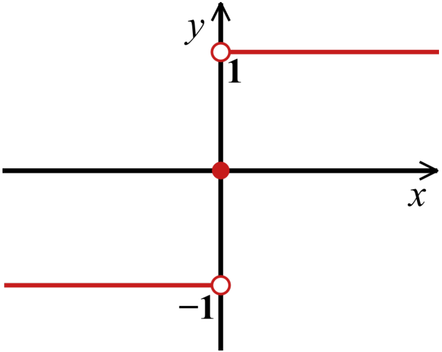
CC+SNIa+BAO+BBN

L. A. Escamilla *et al.* 2305.16290

Phenomenological motivation (Λ_sCDM)

Data set	Planck+BAOtr +PP&SH0ES+KiDS-1000
Model	Λ_sCDM ΛCDM
$z_\dagger$	$1.72^{+0.09}_{-0.12}(1.70)$ $--$
M_B[mag]	$-19.282 \pm 0.017(-19.280)$ $-19.372 \pm 0.011(-19.369)$
H_0[km/s/Mpc]	$73.16 \pm 0.64(73.36)$ $69.83 \pm 0.37(69.96)$
Ω_m	$0.2646 \pm 0.0052(0.2622)$ $0.2837 \pm 0.0045(0.2816)$
S_8	$0.774 \pm 0.009(0.773)$ $0.781 \pm 0.008(0.782)$
χ^2_{min}	4185.34 4226.50
$\ln\mathcal{B}_{ij}$	-19.77

$$\rho_{DE}(z) = \Lambda_{s0}\,\mathrm{sgn}[z_\dagger - z]$$



$y = \mathrm{sgn}[x]$

Some important functions in redshift ($8\pi G = 1$, $c = 1$)

Hubble function:

$$3H^2(z) = \rho_{\rm m0}(1+z)^3 + \rho_{\rm r0}(1+z)^4 + \rho_{\rm DE}(z)$$

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$$\nabla^\nu T_{\mu\nu} = 0 \Rightarrow \frac{\mathrm{d}\rho(z)}{\mathrm{d}z} = \frac{3}{1+z}[1+w(z)]\rho(z)$$

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Comoving angular diameter distance in flat space:

$$D_M(z) = \int_0^z \frac{\mathrm{d}z'}{H(z')}$$

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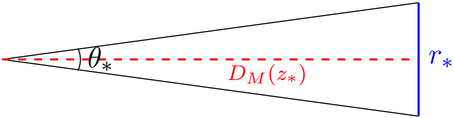
Some well-measured quantities

- $\rho_{\mathrm{m,rec}}$
- ρ_{r0}
- $H(z_i)r_{\mathrm{d}}$
- $D_M(z_i)/r_{\mathrm{d}}$
- $D_M(z_i)/f(M_B)$

Avoiding inconsistency with CMB



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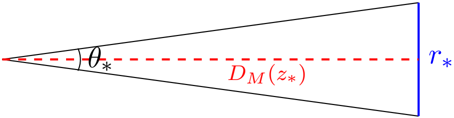

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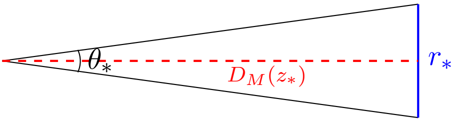

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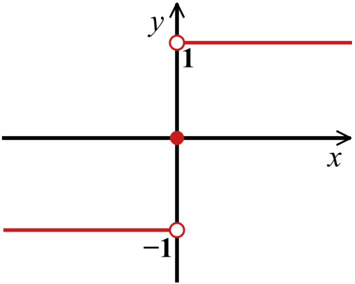

y=sgn[x]

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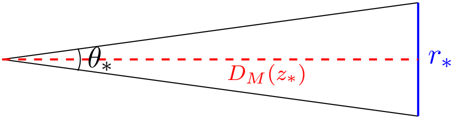

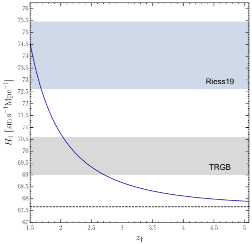

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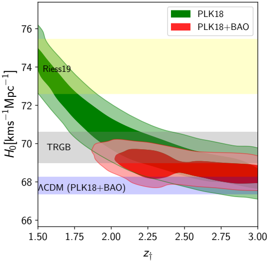
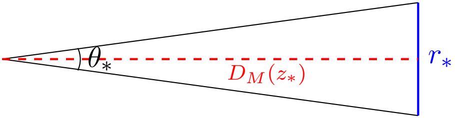


Inevitable wiggles and wavelets

O. Akarsu, E. O. Colgain, E. Ozulker, S. Thakur and L. Yin, Phys. Rev. D **107**, 123526 (2023). 2207.10609



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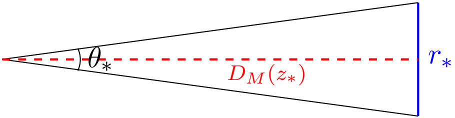

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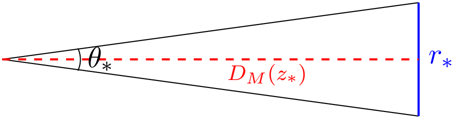

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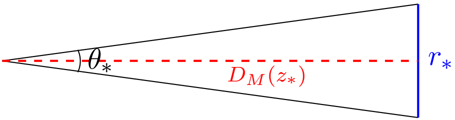

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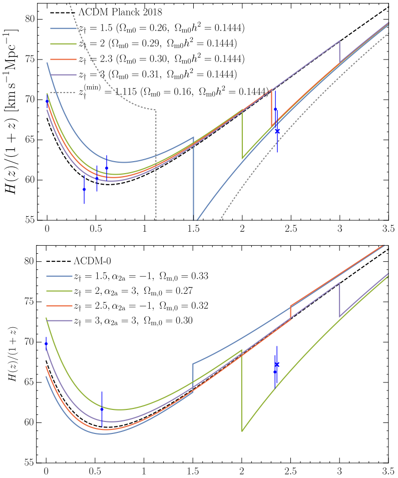

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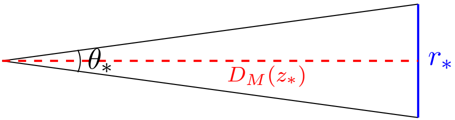

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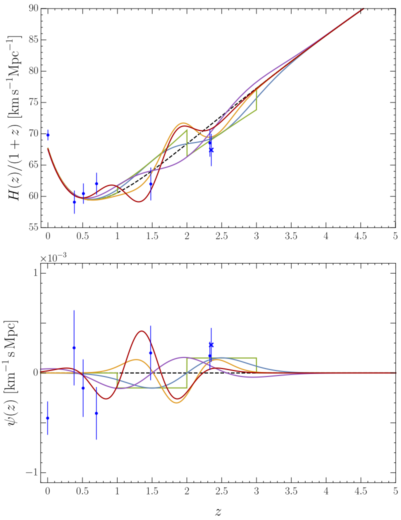


Inevitable wiggles and wavelets

Admissible wavelets can generically be obtained by derivating probability density functions, e.g.,
from Gaussian distribution: $\psi_{\mathrm{G0}}(z) = -\frac{\alpha}{2\beta} e^{-\beta(z-z_\dagger)^2}$

$$\psi_{\mathrm{G1}}(z) = -2\beta(z - z_\dagger)\psi_{\mathrm{G0}}(z),$$

$$\psi_{\mathrm{G2}}(z) = 4\beta\left[\beta(z - z_\dagger)^2 - \frac{1}{2}\right]\psi_{\mathrm{G0}}(z),$$

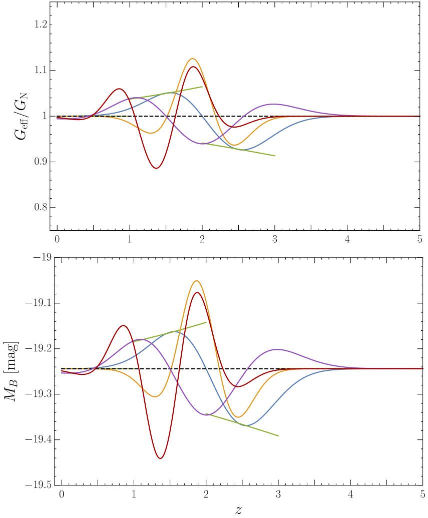
$$\psi_{\mathrm{G3}}(z) = -8\beta^2\left[\beta(z - z_\dagger)^3 - \frac{3}{2}(z - z_\dagger)\right]\psi_{\mathrm{G0}}(z),$$

$$\psi_{\mathrm{G4}}(z) = 16\beta^2\left[\frac{3}{4} + (z - z_\dagger)^4\beta^2 - 3\beta(z - z_\dagger)^2\right]\psi_{\mathrm{G0}}(z),$$

$$\vdots$$

Inevitable wiggles and wavelets

Varying gravitational coupling



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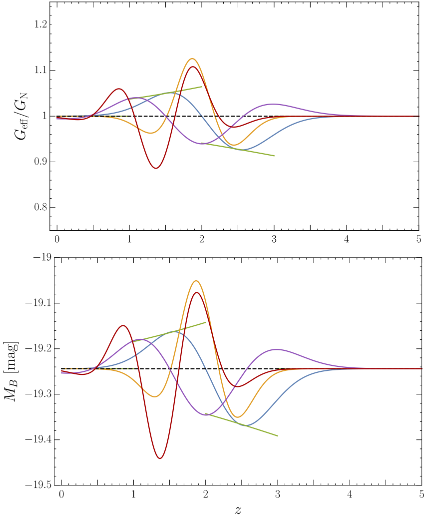
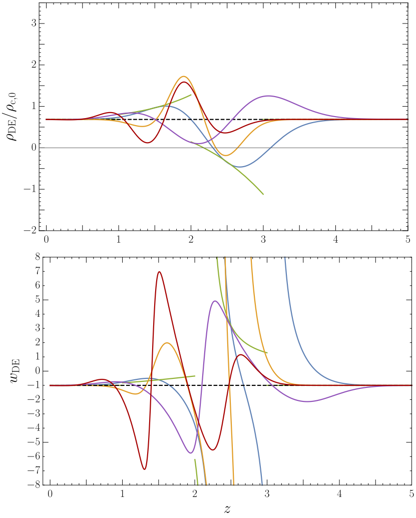
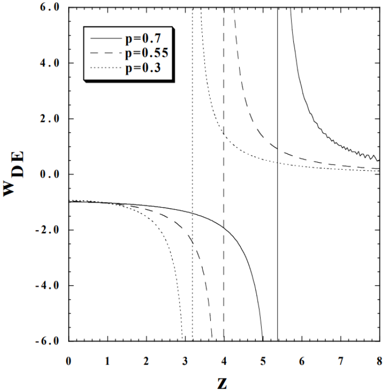


Dark energy



Negative DE and singular EoS



Scalar-Tensor Theory
S. Tsujikawa *et al.* 0803.1106

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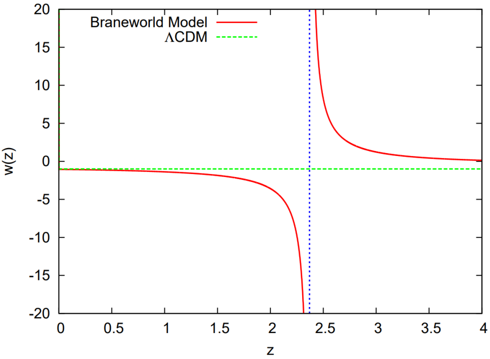
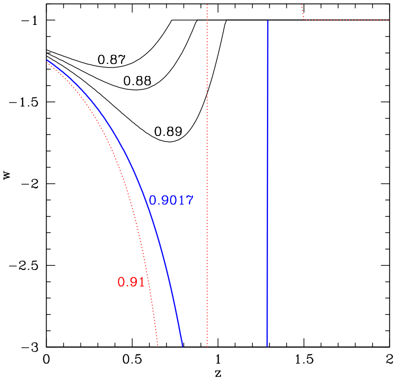


Braneworld Model

V. Sahni *et al.* 1406.2209

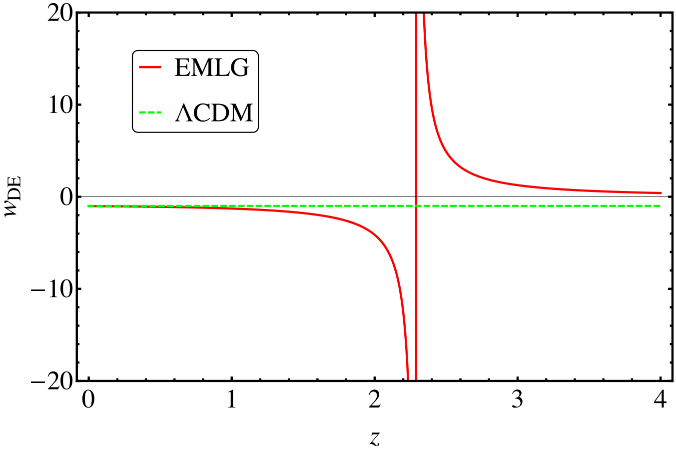
Negative DE and singular EoS



Vacuum Phase Transition

E. Di Valentino *et al.* 1710.02153

Negative DE and singular EoS



$f(T^{\mu\nu}T_{\mu\nu})$ **Gravity**
O. Akarsu *et al.* 1903.11519

Negative DE and singular EoS

E. Ozulker, Phys. Rev. D **106**, 063509 (2022). 2203.04167

Recall:

$$\frac{\mathrm{d}\rho(z)}{\mathrm{d}z} = \frac{3}{1+z}[1+w(z)]\rho(z)$$

Negative DE and singular EoS

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Recall:

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$$\Downarrow$$

$$\rho_{\mathrm{DE}}(z_2) = \rho_{\mathrm{DE}}(z_1) e^{\int_{z_1}^{z_2} 3\frac{1+w_{\mathrm{DE}}(z)}{1+z}\mathrm{d}z}$$

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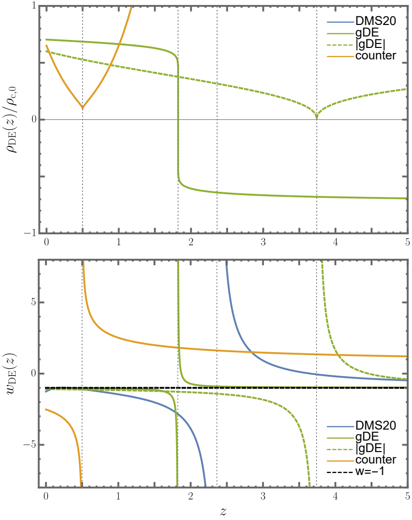
$$\lim_{z\to z_\dagger^\pm} w_{\mathrm{DE}}(z) = \pm\infty$$



Omnipotent DE

S. A. Adil, O. Akarsu, E. Di Valentino, R. C. Nunes, E. Ozulker, A. A. Sen and E. Specogna, 2306.08046

Combining negative DE and nonmonotonicity

Density	EoS	Scaling in z	Scaling in a	Naming
$\rho > 0$	$w > -1$	$\mathrm{d}\rho\,/\,\mathrm{d}z > 0$	$\mathrm{d}\rho\,/\,\mathrm{d}a < 0$	p-quintessence
	$w = -1$	$\mathrm{d}\rho\,/\,\mathrm{d}z = 0$	$\mathrm{d}\rho\,/\,\mathrm{d}a = 0$	positive-CC
	$w < -1$	$\mathrm{d}\rho\,/\,\mathrm{d}z < 0$	$\mathrm{d}\rho\,/\,\mathrm{d}a > 0$	p-phantom
$\rho < 0$	$w > -1$	$\mathrm{d}\rho\,/\,\mathrm{d}z < 0$	$\mathrm{d}\rho\,/\,\mathrm{d}a > 0$	n-quintessence
	$w = -1$	$\mathrm{d}\rho\,/\,\mathrm{d}z = 0$	$\mathrm{d}\rho\,/\,\mathrm{d}a = 0$	negative-CC
	$w < -1$	$\mathrm{d}\rho\,/\,\mathrm{d}z > 0$	$\mathrm{d}\rho\,/\,\mathrm{d}a < 0$	n-phantom

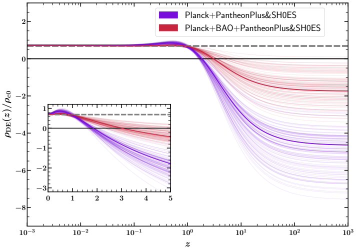
DMS20 Parametrization

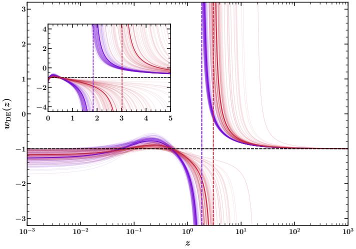
S. A. Adil, O. Akarsu, E. Di Valentino, R. C. Nunes, E. Ozulker, A. A. Sen and E. Specogna, 2306.08046

Parameters	Planck +PantheonPlus&SH0ES	Planck+BAO +PantheonPlus&SH0ES
$\Omega_c h^2$	0.1176 ± 0.0011	0.1198 ± 0.0011
$10^2\Omega_b h^2$	2.257 ± 0.014	2.243 ± 0.014
$100\theta_{MC}$	1.04207 ± 0.00029	1.04191 ± 0.00029
τ	$0.0490^{+0.0081}_{-0.0071}$	0.0502 ± 0.0076
n_s	0.9714 ± 0.0041	0.9659 ± 0.0038
$\ln(10^{10}A_s)$	$3.027^{+0.016}_{-0.014}$	3.035 ± 0.015
a_m	$0.957^{+0.016}_{-0.023}$	$0.922^{+0.041}_{-0.035}$
α	$7.0^{+1.6}_{-2.0}$	< 2.77
β	$16.5^{+3.4}_{-4.3}$	$6.5^{+1.9}_{-3.4}$
H_0 [km/s/Mpc]	73.49 ± 0.98	70.05 ± 0.64
Ω_m	0.2610 ± 0.0077	0.2912 ± 0.0057
σ_8	0.861 ± 0.012	0.835 ± 0.010
S_8	0.803 ± 0.011	0.823 ± 0.011
r_{drag} [Mpc]	147.50 ± 0.26	147.07 ± 0.25
t_0 [Gyr]	13.454 ± 0.056	13.679 ± 0.031

$$\rho_{DE}(a) = \rho_{DE0}\frac{1+\alpha(a-a_m)^2+\beta(a-a_m)^3}{1+\alpha(1-a_m)^2+\beta(1-a_m)^3}$$

E. Di Valentino, *et al.* 2005.12587





Should we worry?

E. Ozulker, Phys. Rev. D **106**, 063509 (2022). 2203.04167

Energy conditions for a perfect fluid

NEC: $\rho + p \geq 0 \Rightarrow w \geq -1$

WEC: $\rho \geq 0$ & $[\rho + p \geq 0 \Rightarrow w \geq -1]$

SEC: $\rho \geq |p| \Rightarrow w\frac{|p|}{p} \leq 1$

DEC: $[\rho + p \geq 0 \Rightarrow w \geq -1]$ & $[\rho + 3p \geq 0 \Rightarrow w \geq -1/3]$

Should we worry?

Theoretical mechanisms realizing negative DE Background Dynamics

Simplest:

- Closed universe $\Omega_k < 0$
- AdS+p-phantom field

Modified Gravity:

- Kaluza-Klein
- Unimodular Gravity $\nabla^\nu T_{\mu\nu} = \frac{1}{4}\nabla^\nu(R+T)g_{\mu\nu}$
- Brans-Dicke Theory (Scalar-Tensor Theories)
- Braneworld models
- $f(T^{\mu\nu}T_{\mu\nu})$ gravity
- $f(T)$ gravity
- Bimetric gravity

Quantum Gravity:

- String Theory Landscape
- Loop Quantum Gravity $3H^2 = \rho(1-\rho/M)$
- Everpresent Λ (causal set theory)

From Entropy:

- Tsallis entropy
- Running Barrow entropy

Others:

- Nonminimally IDE $\nabla_\nu T^{\mu\nu}_{\rm DE} = -\nabla_\nu T^{\mu\nu}_{\rm DM}$
- Lifshitz Cosmology