Norway grants

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Machine Learning Classification of mini Black Holes and EW sphalerons at colliders

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In collaboration with

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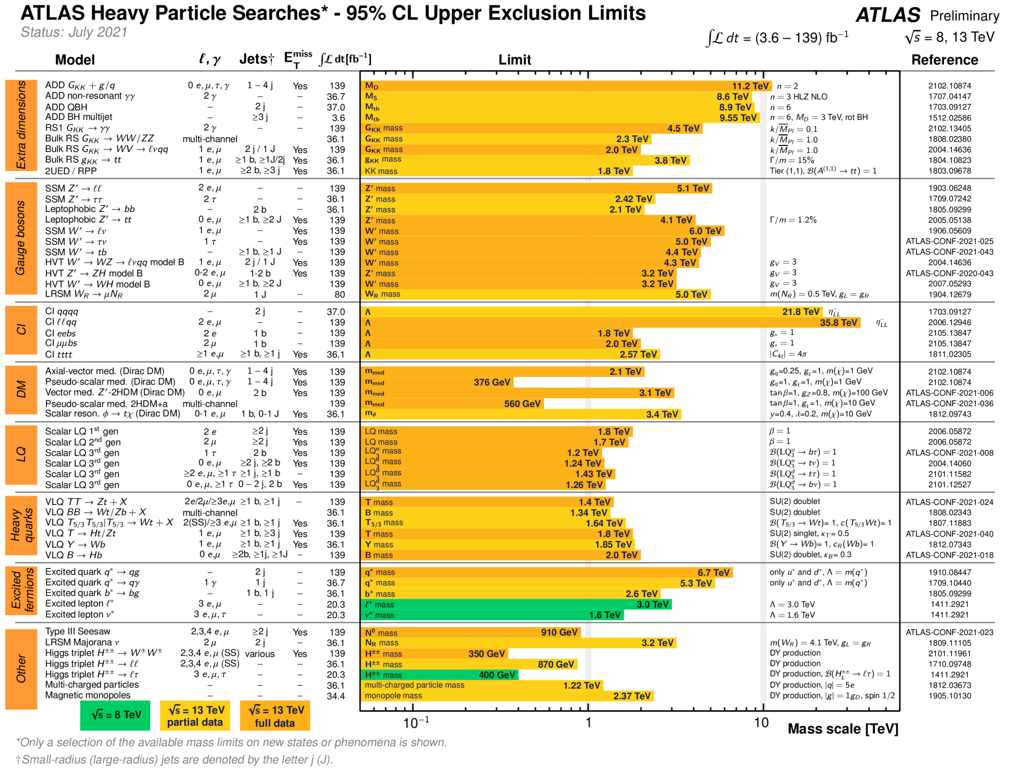
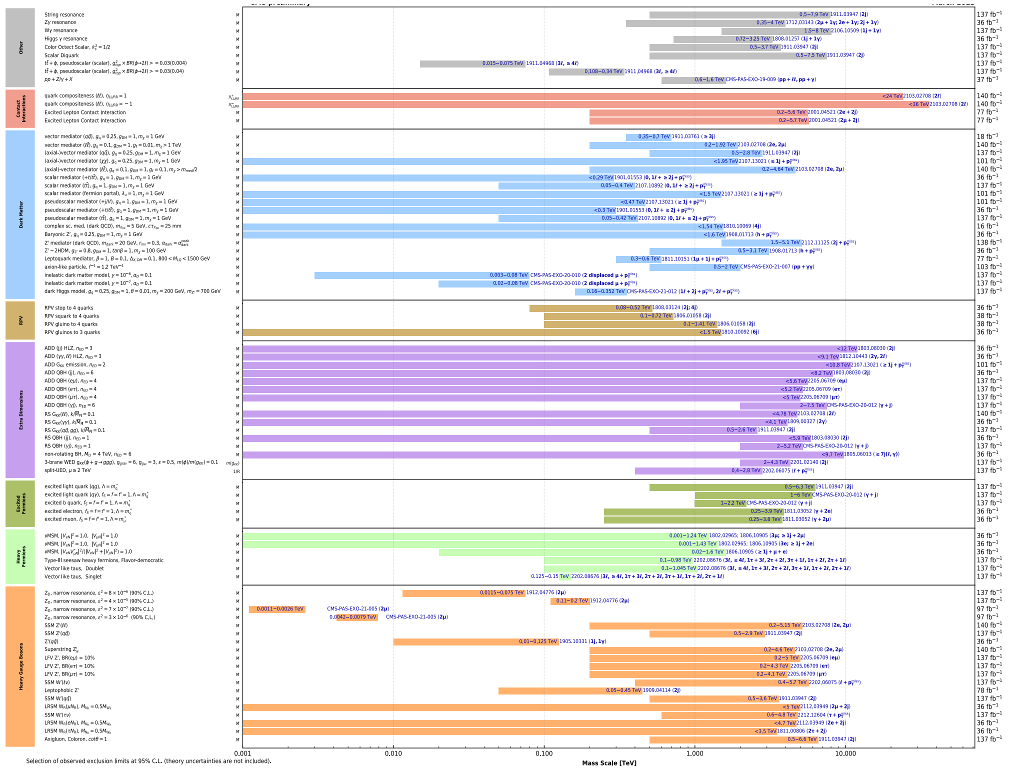
01/09/2023, Corfu Summer Institute

Introduction

- **Great effort on BSM searches @ LHC**
- **Signatures of various popular models have already been looked for** (SUSY, extra-dim, DM, LQ, W'/Z', etc..)





No New Physics Yet

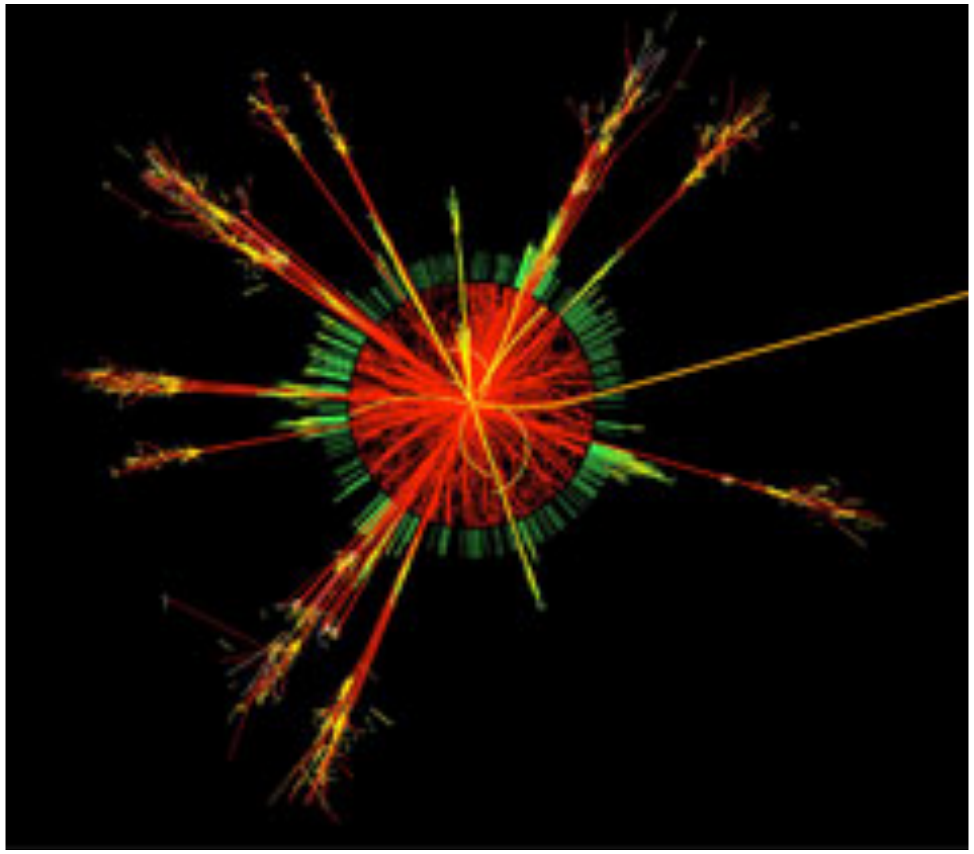
Exotic signatures

New analysis ideas

Machine Learning, …

Black holes @ LHC

- Many extra dimensional models (ADD, RS, …) suggest the possibility of producing micro black holes (BHs) at the LHC
- Once BHs ($M_{\rm BH} \gg M_*$) are produced, they decay via **Hawking radiation**, giving off the energy into all SM d.o.f. democratically
- The signature is spectacular at the LHC
  - A fireball with many, ~ O(10), high pT jets and leptons



M_* : fundamental gravitational scale

Fireball in the SM

- Fireball-like signature is NOT unique for BH.
- Expected also in the **EW sphaleron/instanton-induced process** in the SM

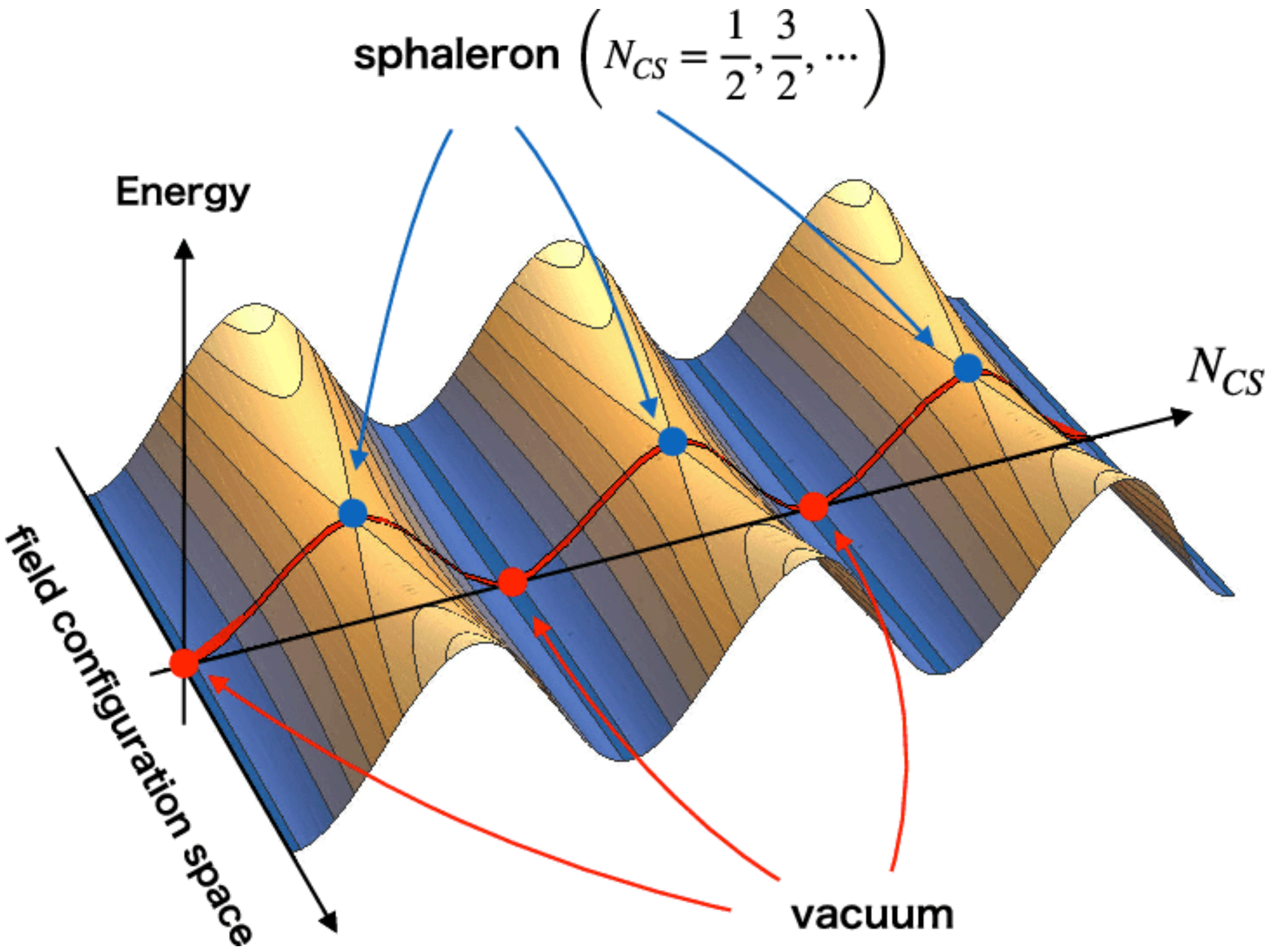
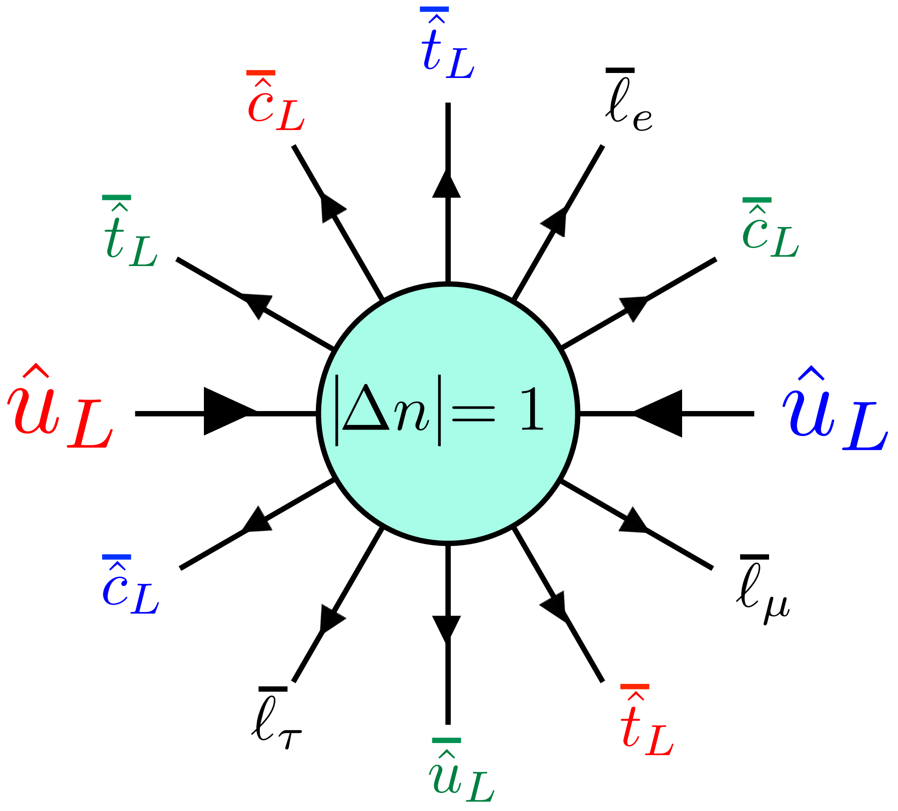


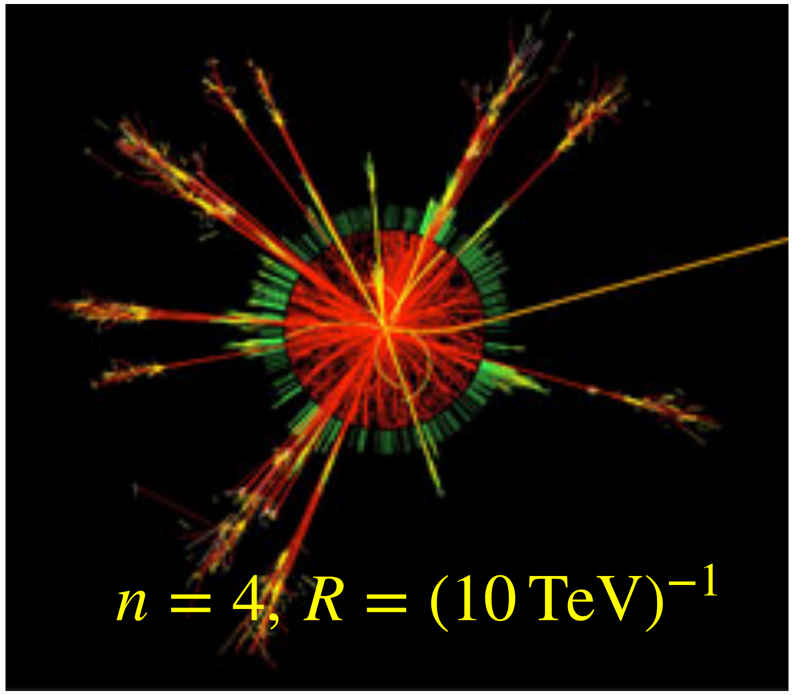
image of EW vacua [Y. Hamada]

EW sphaleron signature



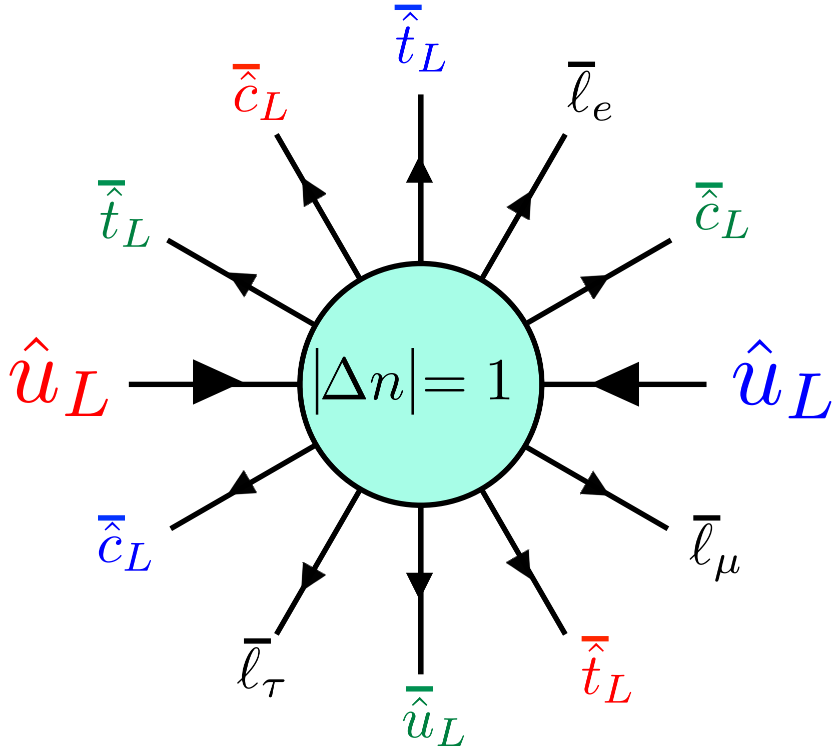
- Can we discriminate **BH** and **EW Spharelon** at colliders?

BH



?

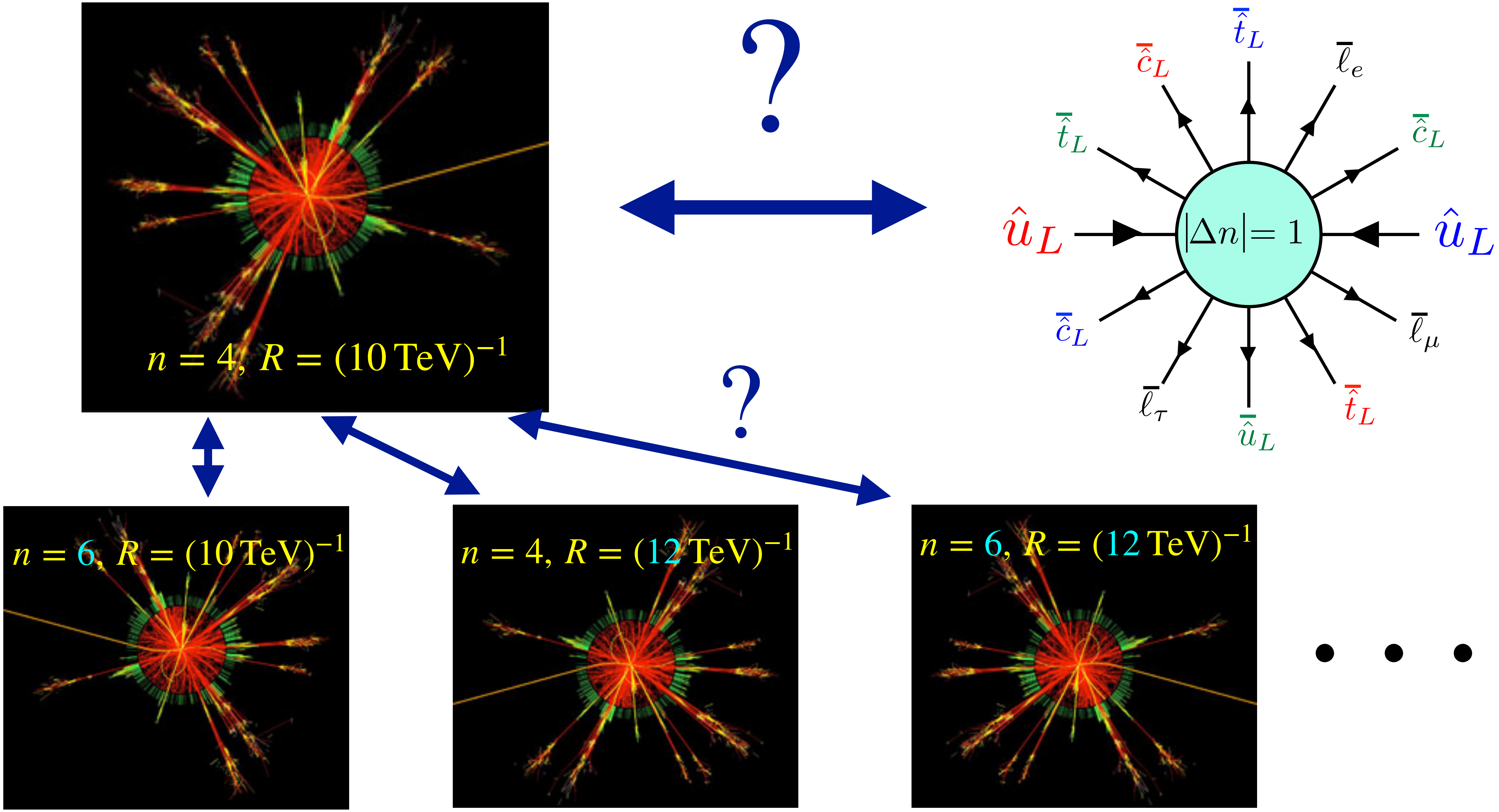
EW Sphaleron



- Can we discriminate **BH** and **EW Spharelon** at colliders?
- Can we tell **which BHs?: number** and **size** of extra-dims?

BH

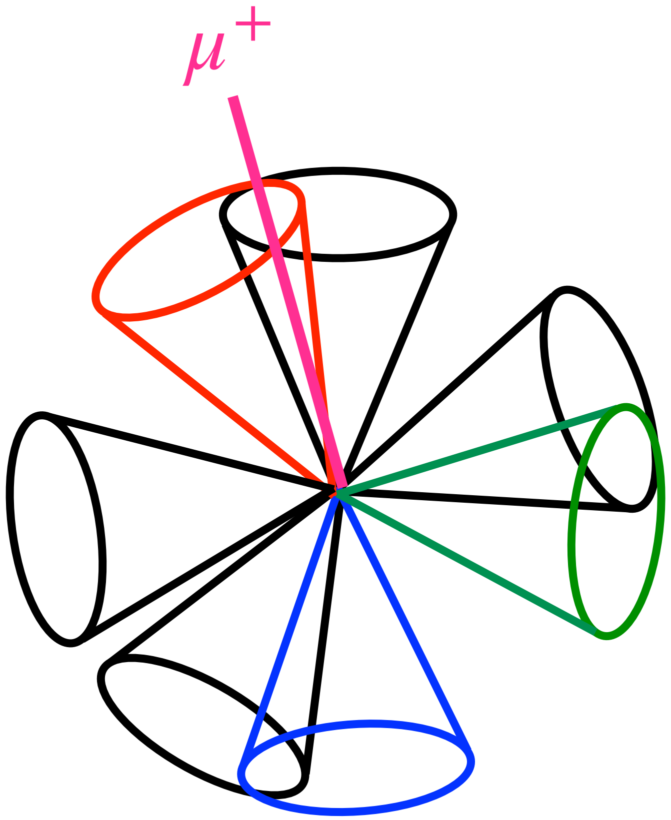
EW Sphaleron



Challenge in multi particle events

- Too many features one can look at (need some systematic guidance)
- more jets are overlapped each others
- more leptons are rejected by isolation criteria

→ jets and isolated leptons may not be a useful concept to talk about extremely busy events



➡ **Machine Learning** may be a solution

- Boosted Decision Trees (BDTs): good at finding features
- Convolutional Neutral Networks (CNNs): used for *image* recognition

↓

hits in HCAL/ECAL

no need to define jets and leptons

Plan

- Introduction
- Mini Black Holes
- EW Sphalerons
- Machine Learning classification
- Conclusion

Black Holes

- BHs will be formed if one puts enough energy in a given tiny region.
- In D=4, the Shwartzchild solution is

$$ds^2 = \left(1 - \frac{r_H}{r}\right) dt^2 - \frac{dr^2}{\left(1 - \frac{r_H}{r}\right)} - r^2 d\Omega_2^2$$

- For a given mass M, the horizon radius is

$$r_H \sim \frac{M}{M_{\rm pl}^2}$$

$$r_H^{\rm Earth} \sim 8\,{\rm mm}$$

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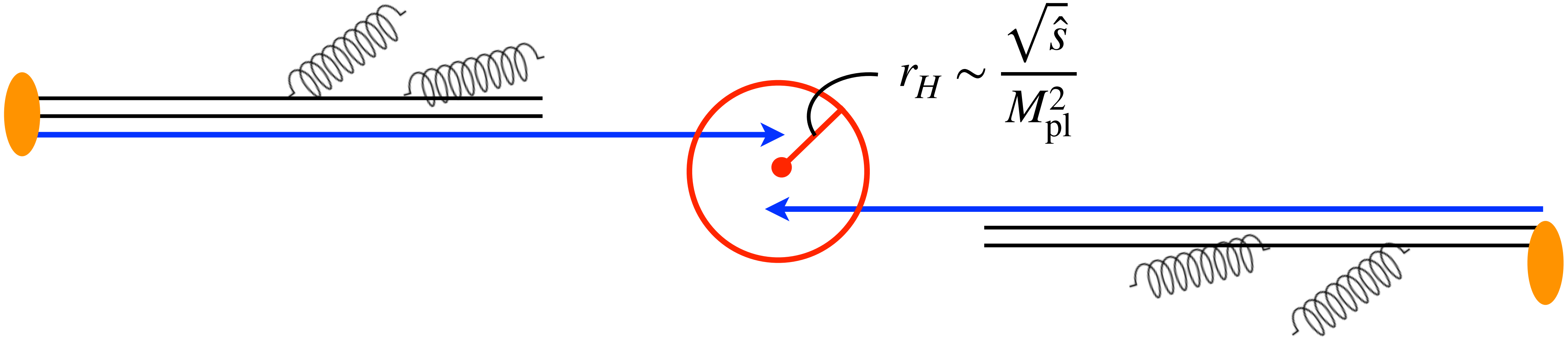
$$r_H \sim \frac{M}{M_{\text{pl}}^2} \qquad\qquad r_H^{\text{Earth}} \sim 8\,\text{mm}$$

- BHs can be formed in trans-Plancking particle collisions: ← “hoop” conjecture

[K. Thorne ’72]

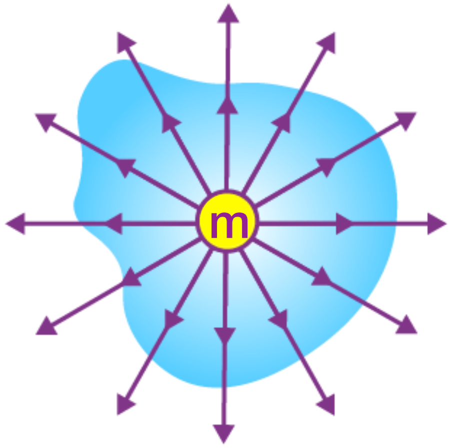


$$r_H \sim \frac{\sqrt{\hat{s}}}{M_{\text{pl}}^2}$$

Extra dimensions

Arkani-Hamed, Dimopoulos, Dvali ’98,
Randall, Sundrum ’99,
…

- In $D = n + 4$, the **Gauss law** gives the gravitational potential



$$V(r) \sim \frac{1}{M_*^{2+n}} \frac{m_1 m_2}{r^{n+1}}$$

M_*: fundamental gravity scale

- If n-extra dims are compactified with the radius R, once $r > R$, the dilution of flux into the extra dimensions stops. At large scale, the potential is effectively

$$V(r) \sim \frac{1}{M_*^{2+n} R^n} \frac{m_1 m_2}{r} = \frac{1}{M_{\rm Pl}^2} \frac{m_1 m_2}{r} \quad ➡ \quad M_* \sim \frac{1}{R} \left(R M_{\rm Pl}\right)^{\frac{2}{2+n}}$$

4D Planck scale:
$M_{\rm Pl} \sim 10^{18}\,\text{GeV}$

- For large R, the fundamental gravity scale is small!

n	1	2	3	4	…
R [mm]	10^{12}	10^{-2}	10^{-7}	10^{-10}	…

➡ $M_* \sim 5\,\text{TeV}$

Extra dimensions

Arkani-Hamed, Dimopoulos, Dvali ’98, Randall, Sundrum ’99, …

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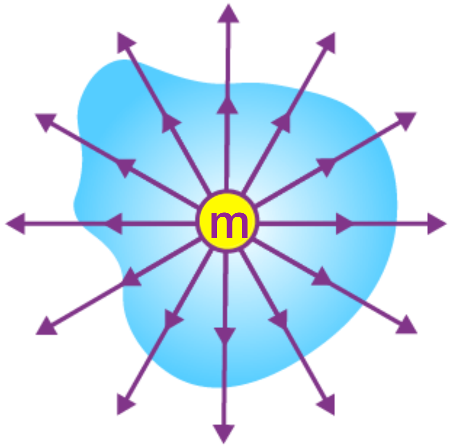

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4D Planck scale:
$M_{\rm Pl} \sim 10^{18}\,\mathrm{GeV}$

- For large R, the fundamental gravity scale is small!

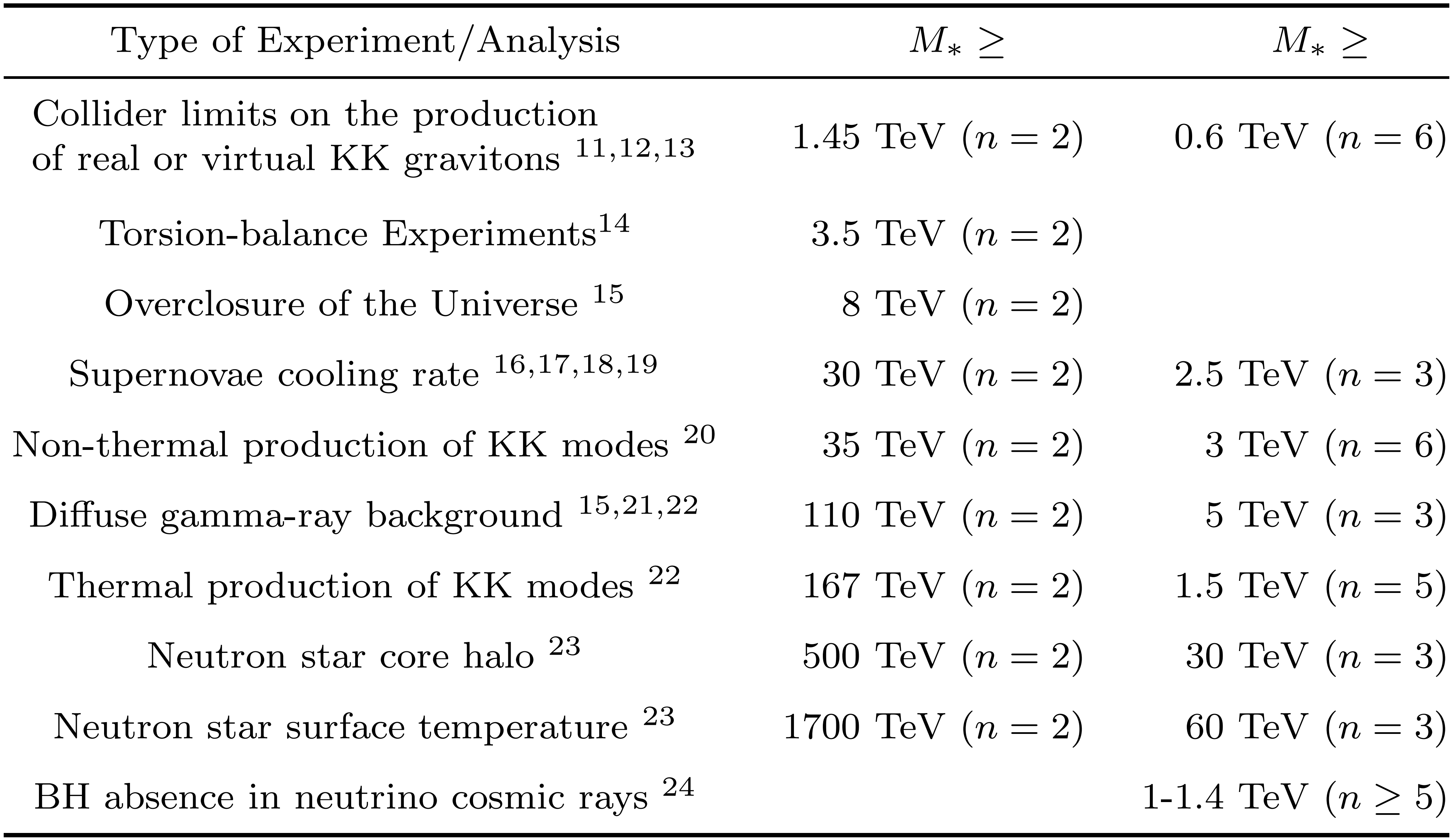
Excluded directly by gravitational experiments $R < 0.2\,[\mathrm{mm}]$ [EOT-WASH ’02]

n	1	2	3	4	…
R [mm]	10^{12}	10^{-2}	10^{-7}	10^{-10}	…

$\Rightarrow$ $M_* \sim 5\,\mathrm{TeV}$

- We assume **SM fields are (almost) confined on a 3-brane**

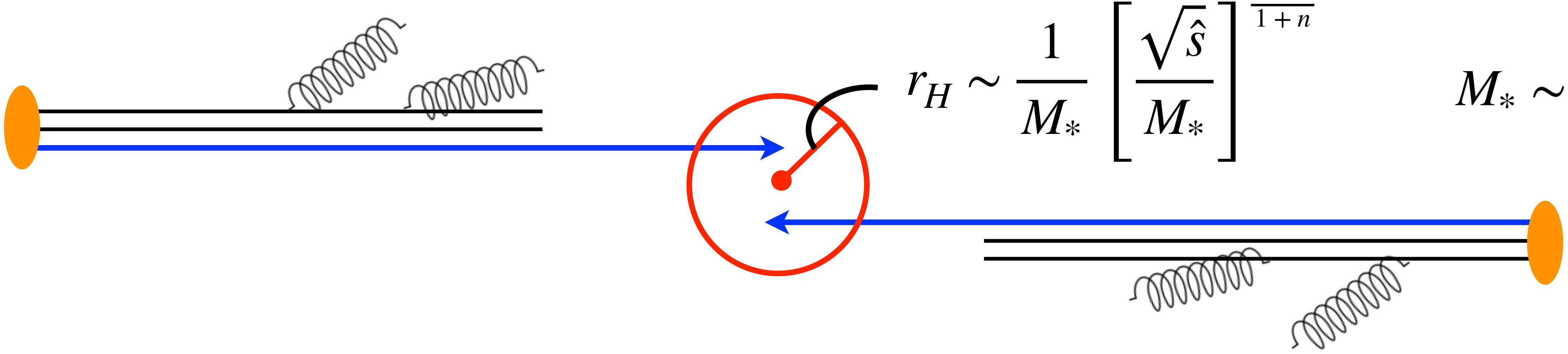
[P. Kanti '04]



Type of Experiment/Analysis	$M_* \geq$	$M_* \geq$
Collider limits on the production of real or virtual KK gravitons [11,12,13]	1.45 TeV $(n = 2)$	0.6 TeV $(n = 6)$
Torsion-balance Experiments[14]	3.5 TeV $(n = 2)$	
Overclosure of the Universe [15]	8 TeV $(n = 2)$	
Supernovae cooling rate [16,17,18,19]	30 TeV $(n = 2)$	2.5 TeV $(n = 3)$
Non-thermal production of KK modes [20]	35 TeV $(n = 2)$	3 TeV $(n = 6)$
Diffuse gamma-ray background [15,21,22]	110 TeV $(n = 2)$	5 TeV $(n = 3)$
Thermal production of KK modes [22]	167 TeV $(n = 2)$	1.5 TeV $(n = 5)$
Neutron star core halo [23]	500 TeV $(n = 2)$	30 TeV $(n = 3)$
Neutron star surface temperature [23]	1700 TeV $(n = 2)$	60 TeV $(n = 3)$
BH absence in neutrino cosmic rays [24]		1-1.4 TeV $(n \geq 5)$

We consider: $n \geq 4,\ M_* \gtrsim 8\,\mathrm{TeV}$

BH production



$$r_H \sim \frac{1}{M_*}\left[\frac{\sqrt{\hat{s}}}{M_*}\right]^{\frac{1}{1+n}}$$

$M_* \sim 8\,\mathrm{TeV}$

- **The partonic cross section is geometrical:** $M_{\mathrm{BH}}^{\mathrm{min}}$: the minimum BH mass

$$\hat{\sigma}_{ij}(\hat{s}) \;=\; c\,\pi\, r_H^2(\hat{s})\,\Theta(\sqrt{\hat{s}} - M_{\mathrm{BH}}^{\mathrm{min}})$$

$c \sim \mathcal{O}(1)$

- **The hadronic cross section:** $f_i(x)$: PDF of parton i

$$\sigma_{hh\to\mathrm{BH}}(E_{hh}) = \sum_{ij}\int dx_1 \int dx_2\, f_i(x_1) f_j(x_2)\, \hat{\sigma}_{ij}(\hat{s})\, \delta(\hat{s} - x_1 x_2 s)$$

➡ $\sigma_{pp\to\mathrm{BH}}(13\,\mathrm{TeV}) \sim 0.1\,\mathrm{fb}$ $\qquad M_{\mathrm{BH}}^{\mathrm{min}} \sim M_* \sim 10\,\mathrm{TeV}$

Thermal decay of BHs

- BHs ($M_{\rm BH} \gg M_*$) decay via Hawking radiation, with the power spectrum

$$\frac{dE^{(s)}(\omega)}{dt} = \sum_j \sigma^{(s)}_{j,n}(\omega) \frac{\omega^3}{\exp\left(\omega/T_H\right) \pm 1} \frac{d\omega}{2\pi^2}$$

s : spin

j : angular momentum

n : # of extra dim.

$$T_H = \frac{(n+1)}{4\pi\, r_H}$$: Hawking temperature

Emitted particles must travel through the strong gravitational potential of the BH. For observers far away from the BH, the spectrum is distorted from the black body. The distortion is parameterised by the **"greybody" factor**

$$\sigma^{(s)}_{j,n}(\omega) = \frac{(2\omega r_H)^{2j-2s}\,(2j+1)\,4\pi r_H^2}{|\Gamma(1-s+2\alpha)|^2\,|\,C\,(\omega r_H)^{2j+1} + D\,|^2}$$

[P. Kanti '04]

$$C = \frac{2^{2j+1}\,e^{i\pi\,(s-1/2)}\,\Gamma\left(2\beta - \frac{1-2s}{n+1}\right)\Gamma(j-s+1)}{\Gamma(\alpha+\beta)\,\Gamma\left(\alpha+\beta+\frac{s+n\,(1-s)}{n+1}\right)\Gamma(2j+1)} \qquad D = \frac{\Gamma(2j+2)\,\Gamma\left(-2\beta+\frac{1-2s}{n+1}\right)}{\Gamma(\alpha-\beta+1-s)\,\Gamma\left(\alpha-\beta+\frac{1-2s}{n+1}\right)\Gamma(j+s+1)}$$

$$\alpha = -\frac{i\omega r_H}{n+1} \qquad \beta = \frac{1}{2(n+1)}\left[1-2s-\sqrt{(1+2j)^2-4\omega^2 r_H^2-8is\omega r_H}\,\right]$$

The spectrum carries the info of # of extra dim but in a intricate way!

EW sphaleron

EW Vacua

action: $S_{\mathrm{EW}} = -\frac{1}{2g^2}\int d^4x \,\mathrm{tr}\left[F_{\mu\nu}F^{\mu\nu}\right]$ $\quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i\left[A_\mu, A_\nu\right]$

gauge trans.: $A_\mu \to U^\dagger[A_\mu + i\partial_\mu]U$

a vacuum: $A_\mu = 0 \quad \leftrightarrow \quad A_\mu = U^\dagger\partial_\mu U$

- There are as many vacua as $U_{ij}(\mathbf{x})$

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$SU(2) \ni U = a + i(\mathbf{b}\cdot\sigma)$

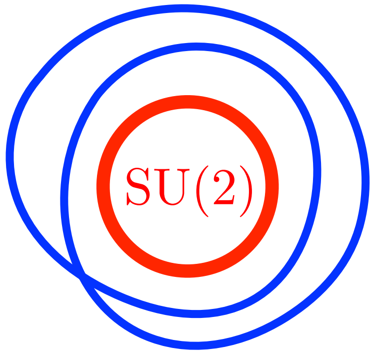
$a^2 + \mathbf{b}^2 = 1$

- There are as many vacua as $U_{ij}(\mathbf{x})$

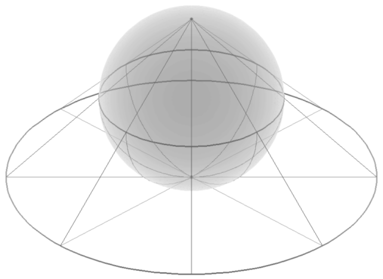
topological gauge

$U(\infty) \to \mathbf{1}$

$$\mathrm{SU}(2) \cong S^3 \xleftarrow{\text{map}} S^3 \cong \mathbf{R}^3 \cup \{\infty\}$$



$\pi_3(S^3) = \mathbb{Z}$



The map has distinctive sectors classified by the winding number!

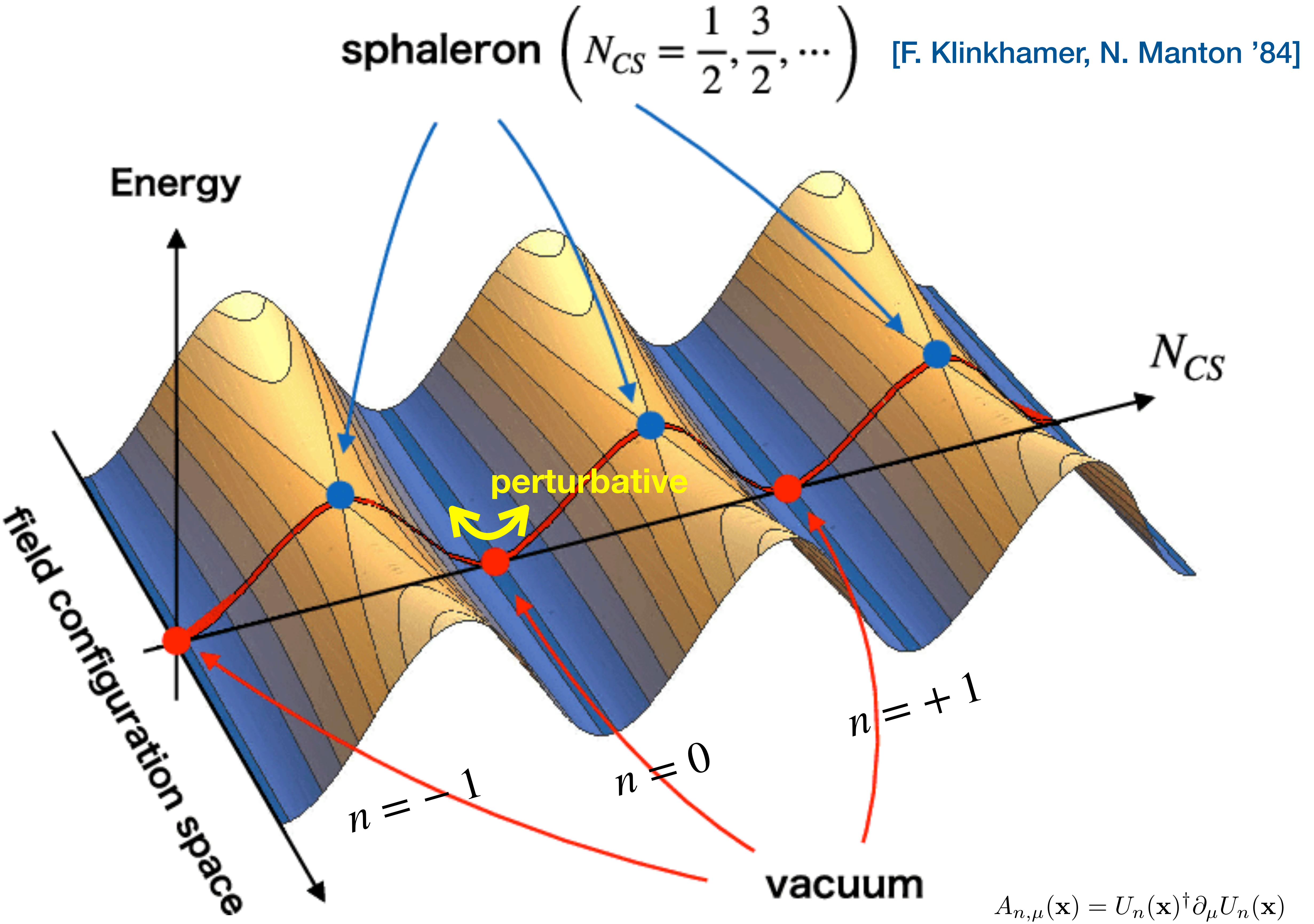
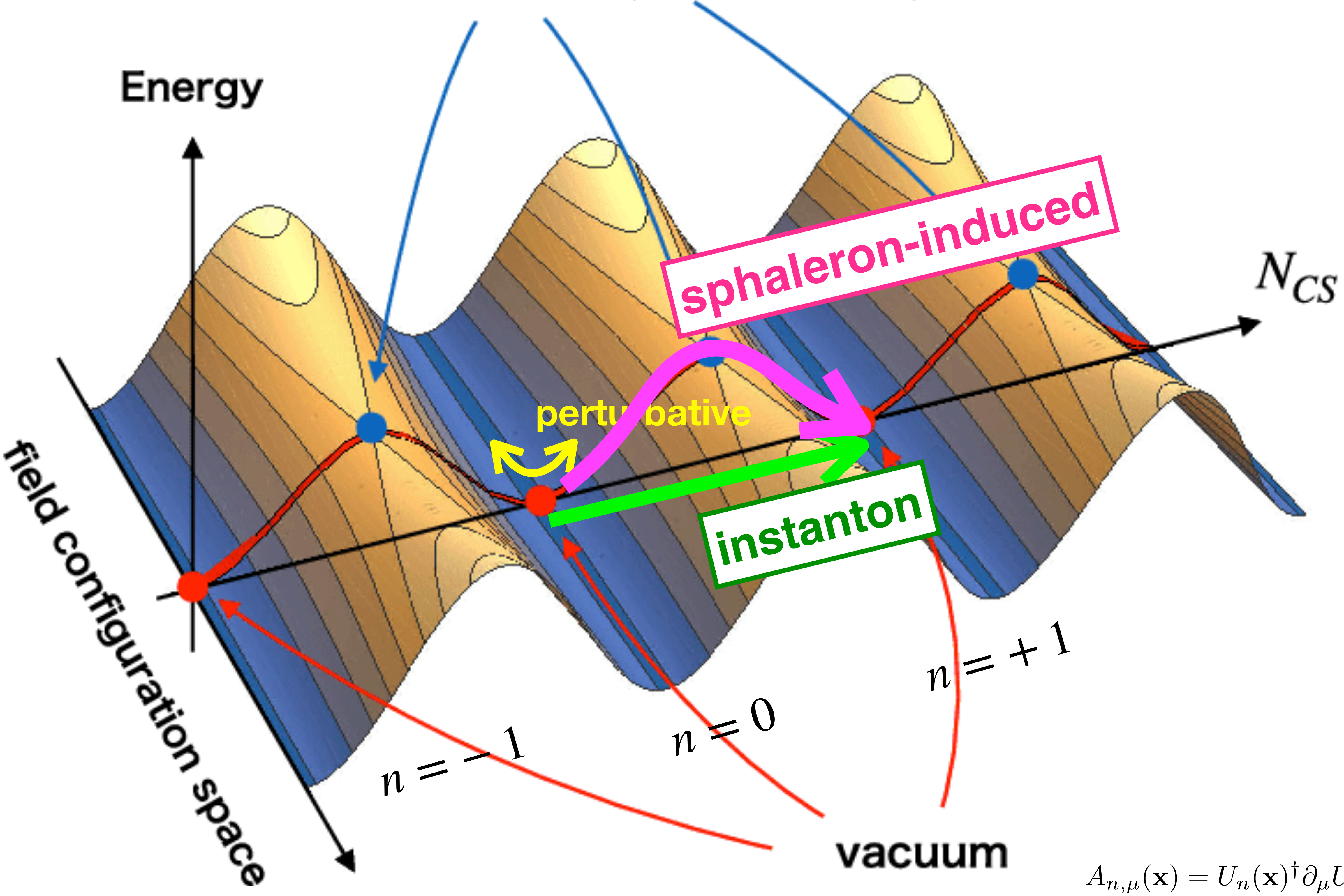


Image of EW vacua [Y. Hamada]

$$A_{n,\mu}(\mathbf{x}) = U_n(\mathbf{x})^\dagger \partial_\mu U_n(\mathbf{x})$$

$$U_n(\mathbf{x}) = \exp\left(in\pi \frac{\mathbf{x}\cdot\sigma}{\sqrt{\mathbf{x}^2 - \rho^2}}\right)$$

sphaleron $\left(N_{CS} = \frac{1}{2}, \frac{3}{2}, \cdots\right)$ [F. Klinkhamer, N. Manton '84]



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EW Sphaleron events

[G. 't Hooft '76]

$$\Delta n \; = \frac{g^2}{16\pi^2} \int \mathrm{Tr} \left[F_{\mu\nu} \tilde{F}^{\mu\nu} \right] d^4x$$

EW Sphaleron evnets

[G. 't Hooft ’76]

$$\Delta n = \frac{g^2}{16\pi^2}\int \mathrm{Tr}\left[F_{\mu\nu}\tilde{F}^{\mu\nu}\right] d^4x \underset{\text{anomaly}}{=} \Delta N_F$$

SU(2) charged fermion

EW Sphaleron evnets

[G. 't Hooft '76]

$$\Delta n = \frac{g^2}{16\pi^2}\int \mathrm{Tr}\left[F_{\mu\nu}\tilde{F}^{\mu\nu}\right] d^4x \underset{\text{anomaly}}{=} \begin{cases} \Delta N_{q_1^r} \\ \Delta N_{q_1^g} \\ \Delta N_{q_1^b} \\ \Delta N_{\ell_1} \end{cases} \times 3\,\text{flavour}$$

EW Sphaleron evnets

[G. 't Hooft '76]

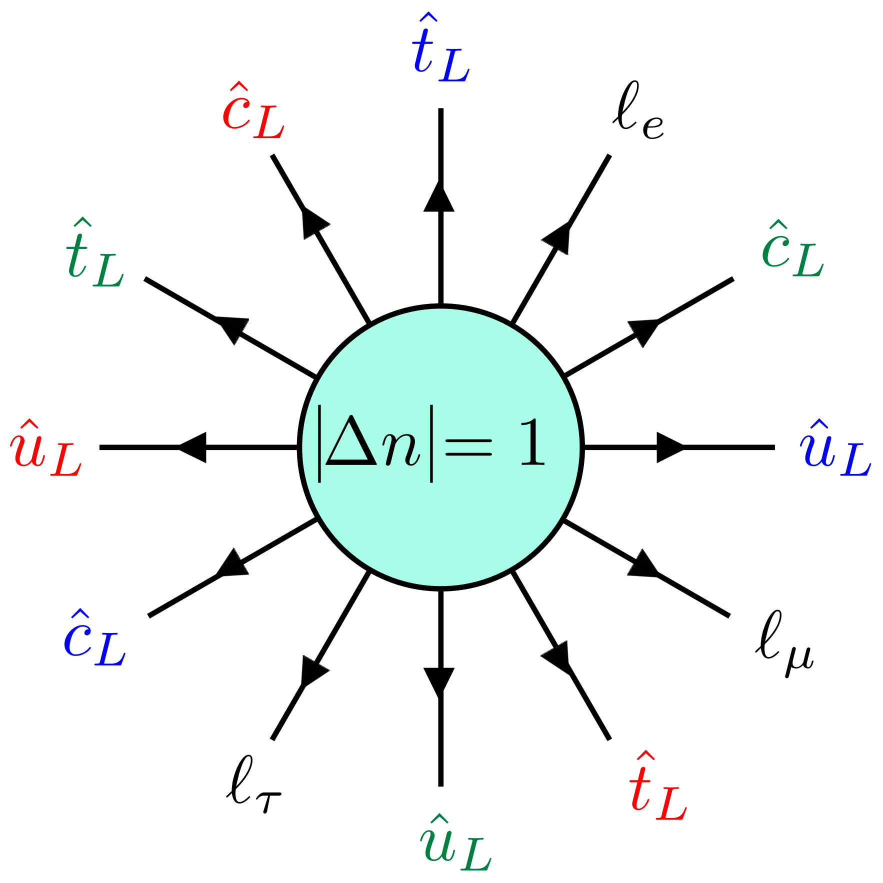
$$\Delta n = \frac{g^2}{16\pi^2}\int \mathrm{Tr}\left[F_{\mu\nu}\tilde{F}^{\mu\nu}\right] d^4x \underset{\text{anomaly}}{=} \left\{ \begin{array}{l} \Delta N_{q_1^r} \\ \Delta N_{q_1^g} \\ \Delta N_{q_1^b} \\ \Delta N_{\ell_1} \end{array} \right. \times 3\,\text{flavour}$$

- Δn is related to the change of SU(2) charged fermion numbers.

$$\Delta B = \Delta L = 3\Delta N_{CS}$$

$$\Delta(B+L) \neq 0$$

$$\Delta(B-L) = 0$$



|Δn| = 1 transition creates 12 fermions altogether!

ex)

$$uu \to e^{+}\bar{\nu}_{\mu}\bar{\nu}_{\tau}\bar{d}\,\bar{c}\,\bar{s}\,\bar{s}\,\bar{t}\,\bar{b}\,\bar{b} \quad \Rightarrow 1e^{+} + 4j + 1\bar{t} + 2b + E^{T}_{\text{miss}}$$

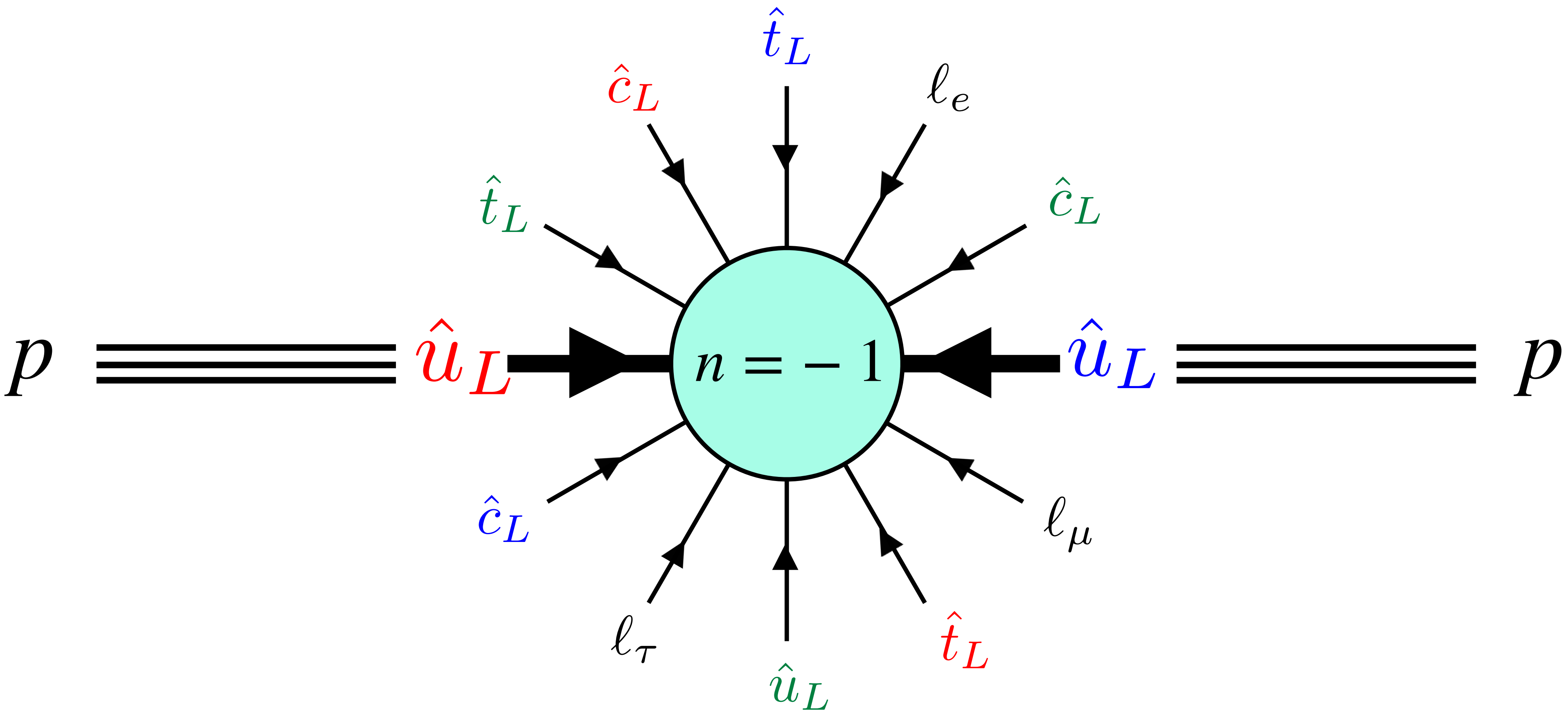
$$uu \to \bar{\nu}_{e}\,\mu^{+}\tau^{+}\bar{d}\,\bar{c}\,\bar{s}\,\bar{s}\,\bar{b}\,\bar{b}\,\bar{b} \quad \Rightarrow 1\mu^{+} + 1\tau^{+} + 4j + 3b + E^{T}_{\text{miss}}$$

$$ud \to \bar{\nu}_{e}\,\mu^{+}\bar{\nu}_{\tau}\bar{d}\,\bar{c}\,\bar{s}\,\bar{s}\,\bar{t}\,\bar{t}\,\bar{b} \quad \Rightarrow 1\mu^{+} + 4j + 2\bar{t} + 1b + E^{T}_{\text{miss}}$$

+ some EW bosons

Party at the LHC!

Confused with BH events?

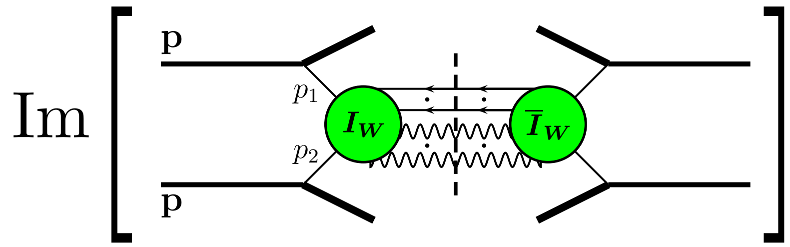


Cross Section

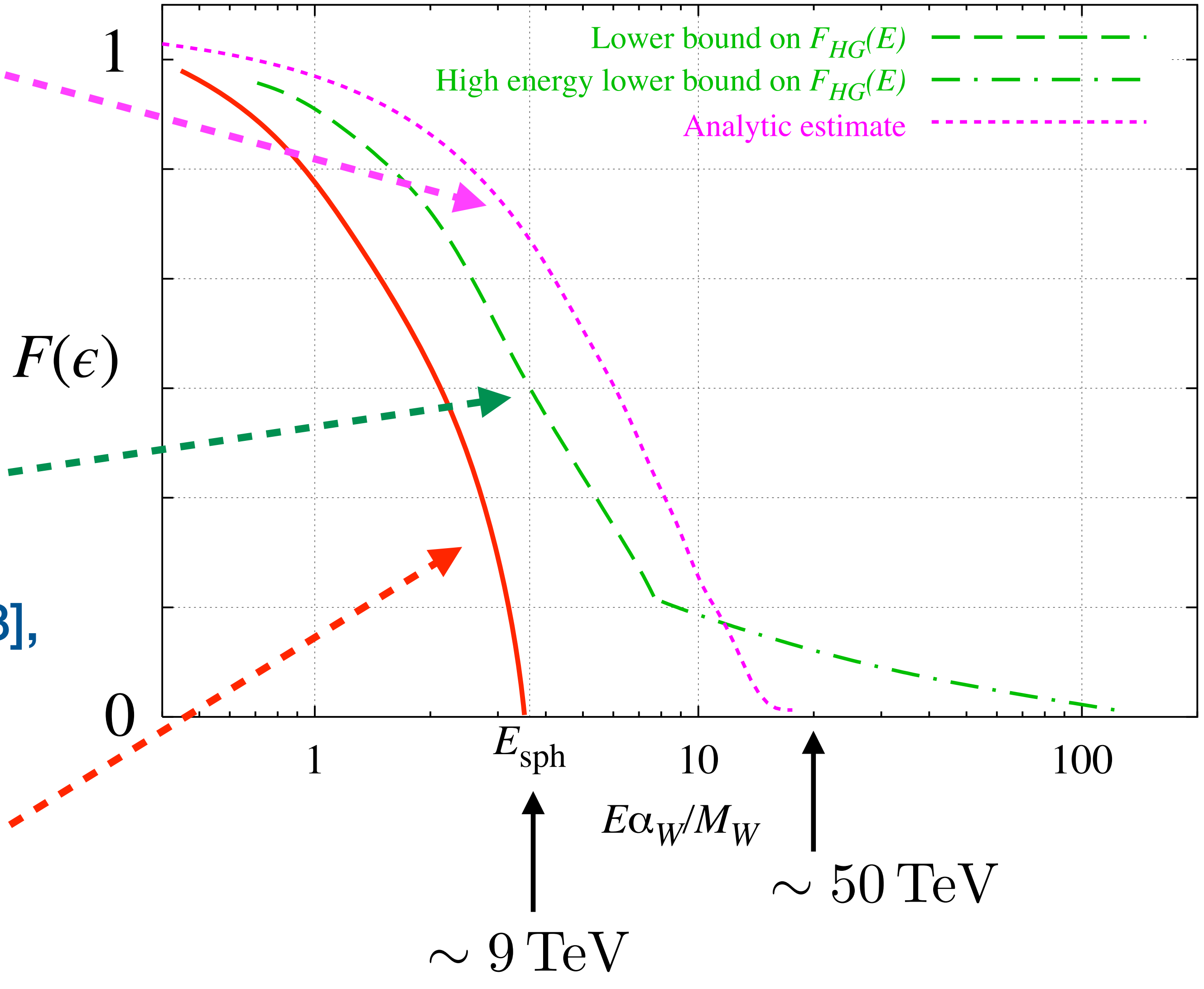
$$\boxed{\hat{\sigma}^{\mathrm{BV}}_{\mathrm{incl}} \equiv \sum_{n_W,n_h} \hat{\sigma}^{\mathrm{BV}}_{n_W,n_h} = \frac{P(\epsilon)}{m_W^2} \exp\left[-\frac{4\pi}{\alpha_W} \underline{F\left(\epsilon\right)}\right]} \qquad \epsilon \equiv E \cdot \frac{\alpha_W}{4\pi m_W}$$

- **Optical theorem**

  [Khoze, Ringwald '91], …



[Bezrukov, Levkov, Rebbi, Rubakov, Tinyakov '03]



- **Semi-Classical method**

  [Bezrukov, Levkov, Rebbi, Rubakov, Tinyakov '03],

  [Rubakov, Tinyakov '92], …

- **Treating N_{CS} as a dynamical variable**

  [Tye, Wong '15 '16]

Collider event analysis

`BlackMax`: **BH** event generation (tensionless, non-rotating)

`Herwig-7`: **Sphaleron** event generation + parton shower

`Delphes`: Detector simulation

Event selection

N : # of all reconstructed objects (jets, leptons, photons) with $p_T > 70\,\mathrm{GeV},\ |\eta| < 2.5$

$$S_T = \sum_{i=1}^{N} p_T^i \ + \ E_T^{\mathrm{miss}}$$

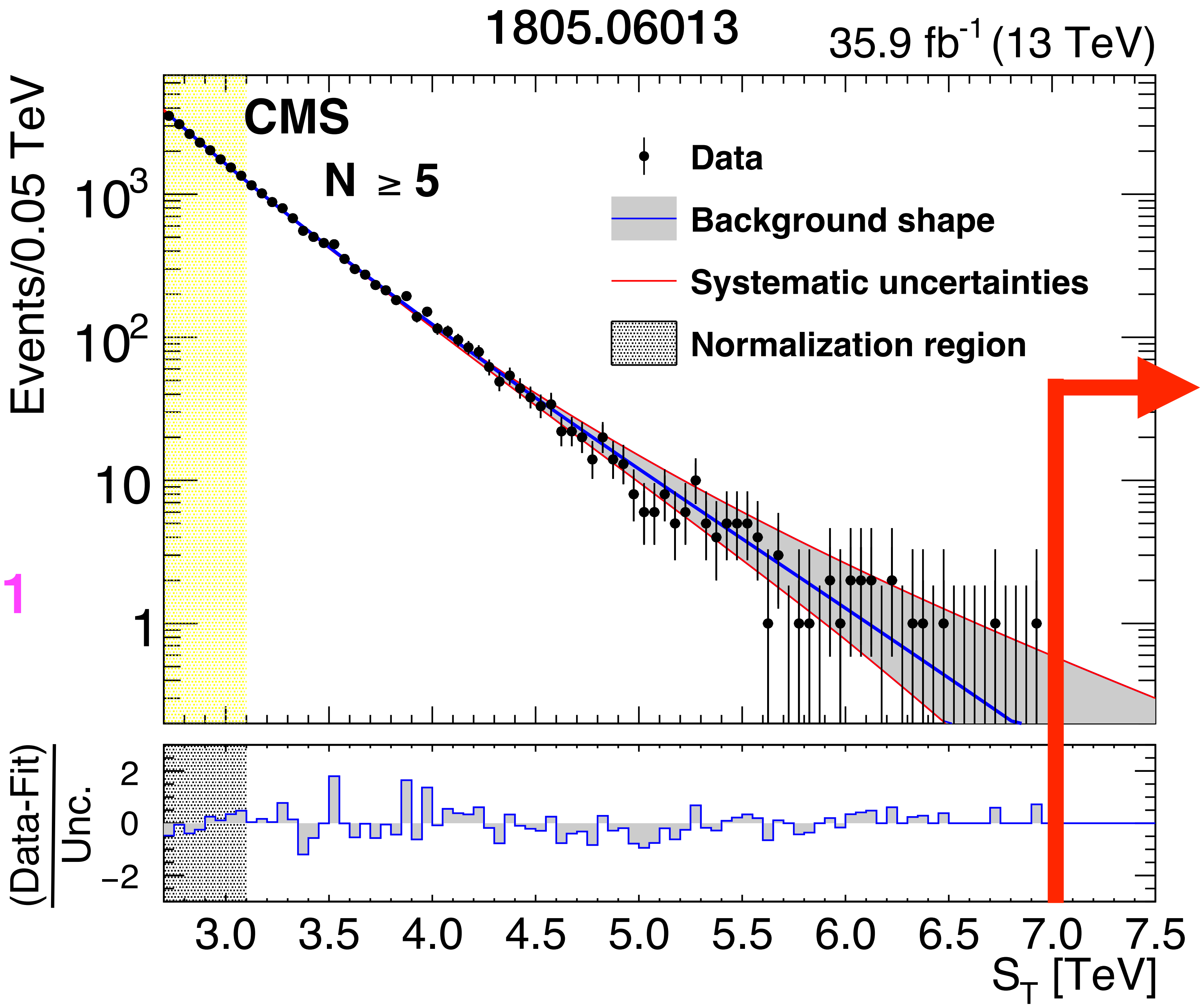
Signal Region

$$\boxed{N \geq 5,\ S_T > 7\,\mathrm{TeV}}$$

↓

SM background well below 1

In the following, we always impose this cut and do not worry about the SMBG



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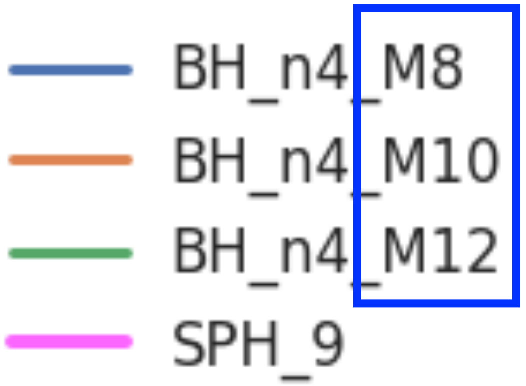
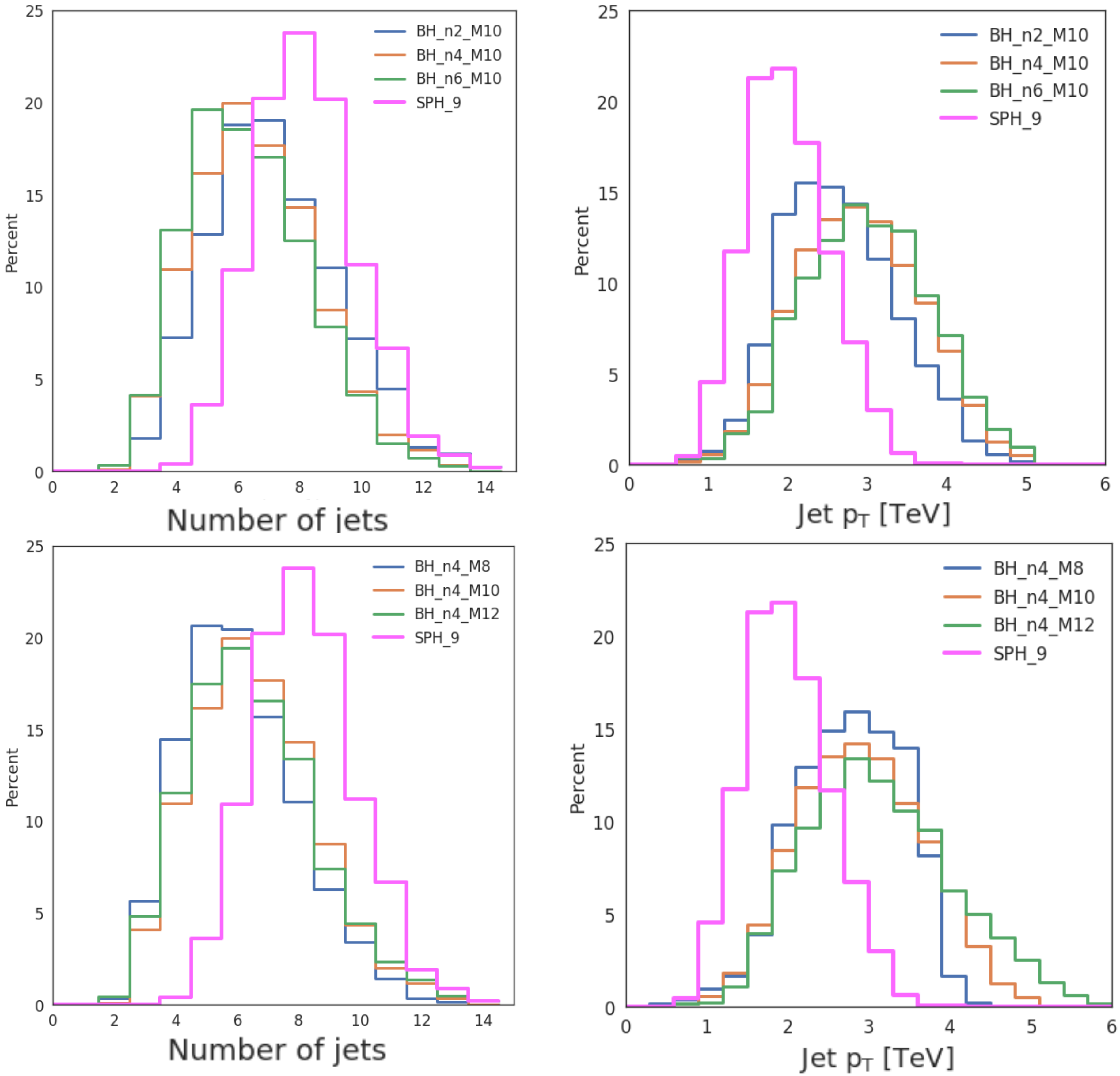
Distributions

of extra dim.

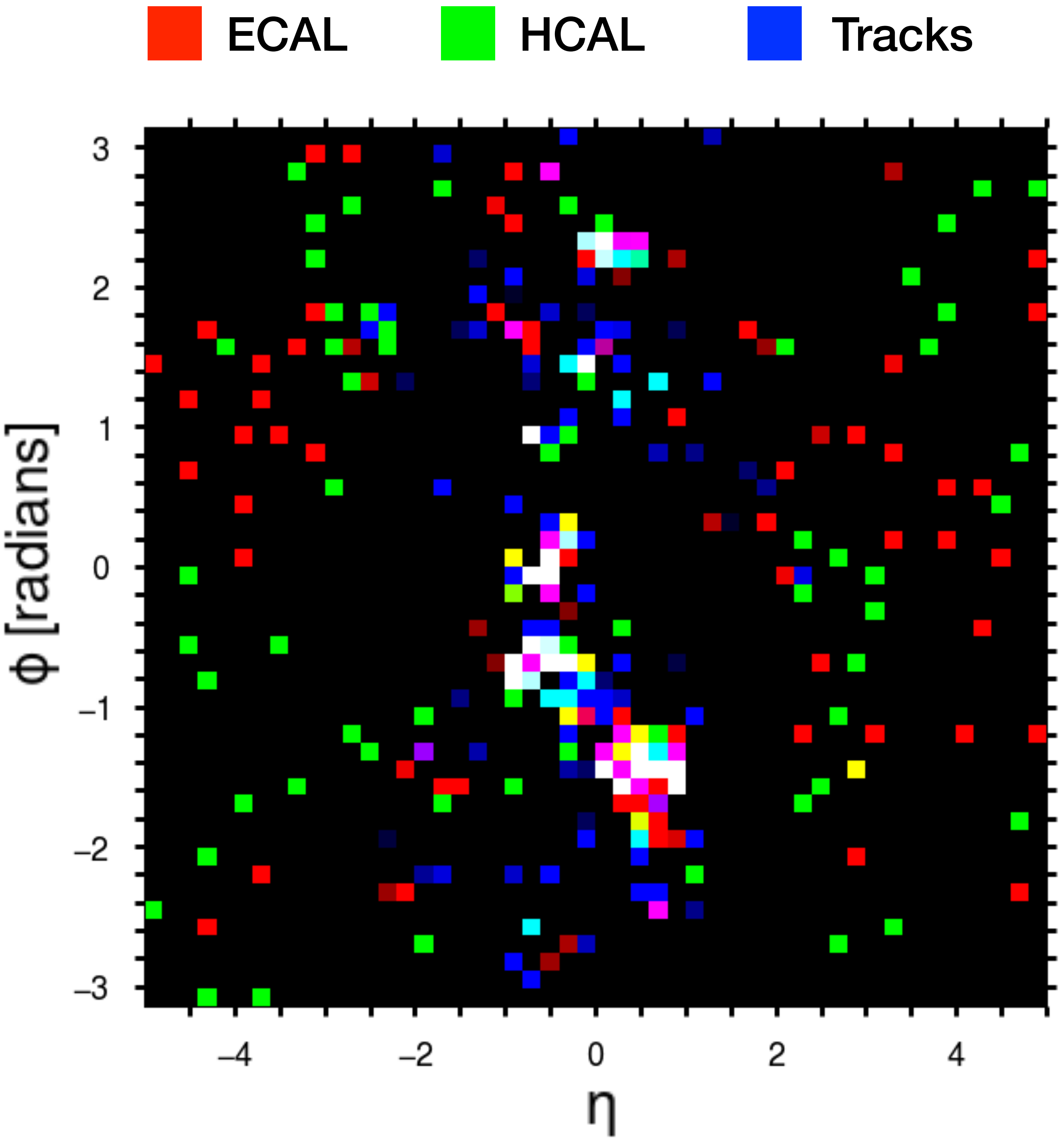
min BH mass





Low level data

- An example EW sphaleron event
- 50 x 50 pixel resolution
- Hits in ECAL, HCAL and Tracker with > 1GeV



Machine Learning Classification

- 30 highest hits (E_i, ϕ_i, η_i) in ECAL, HCAL, Tracker
- First 8 hight pT jets (p_T^i, ϕ^i, η^i)
- First 2 hight pT leptons (p_T^i, ϕ^i, η^i)
- missing pT $(p_T^{\mathrm{miss}}, \phi^{\mathrm{miss}})$

Boosted Decision Tree (BDT)
classification

[T.Chen, C.Guestrin ’16]

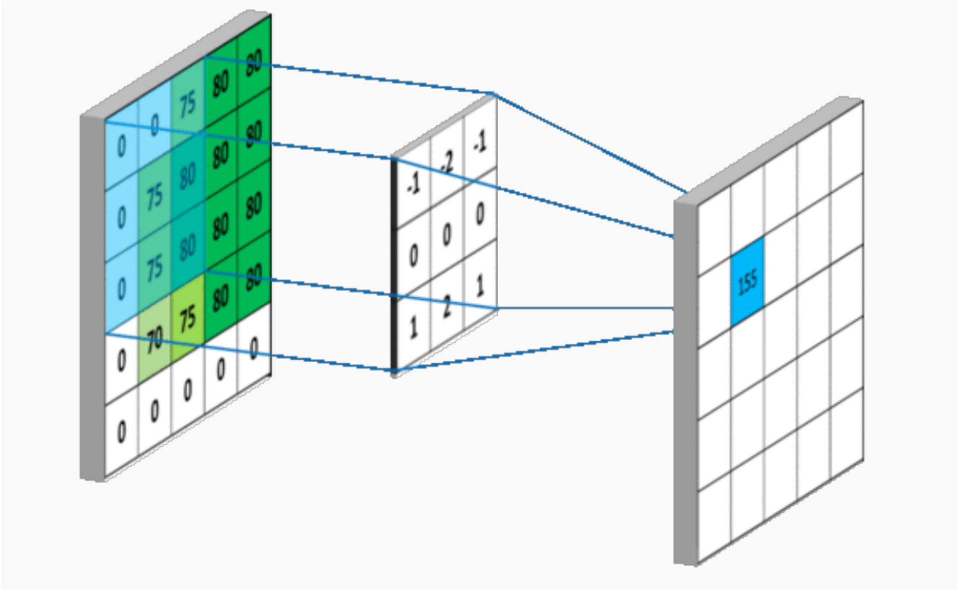
XGBoost

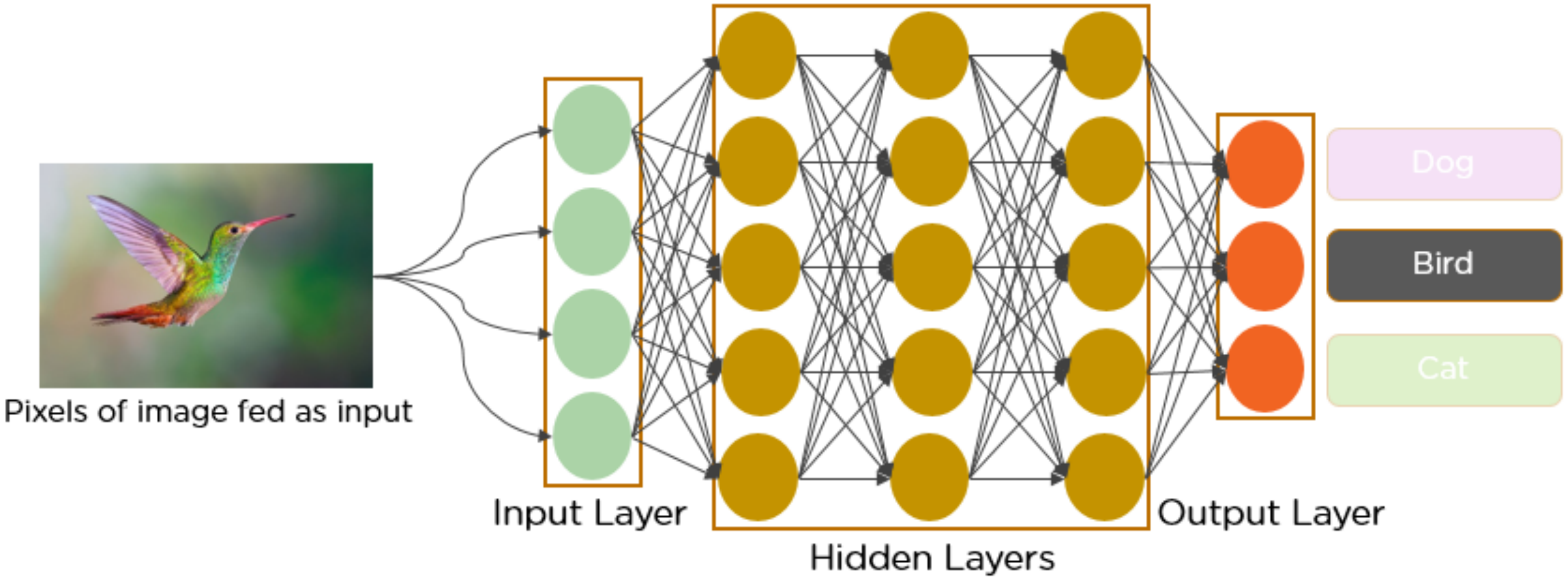
- ECAL, HCAL, Tracker hits in 50 x 50 pixel resolution
- 50 x 2 rondom shift in ϕ and $\mathrm{sng}(\eta)$ flip
  [data augmentation]

Convolutional
Neutral Network
(CNN)
classification

[K.He, X.Zhang, S.Ren, J.Sun ’15]

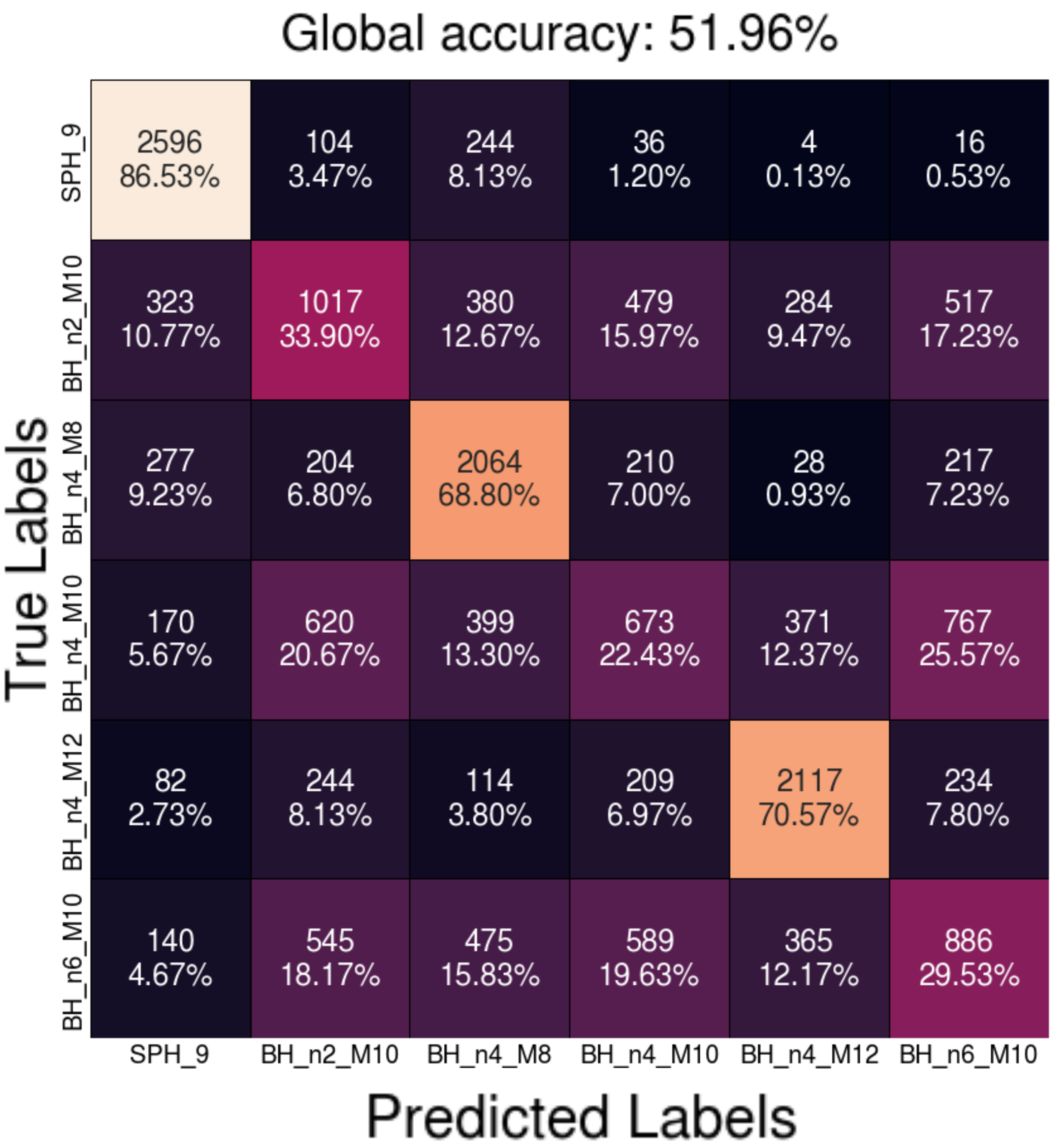
ResNet





Confusion Matrix

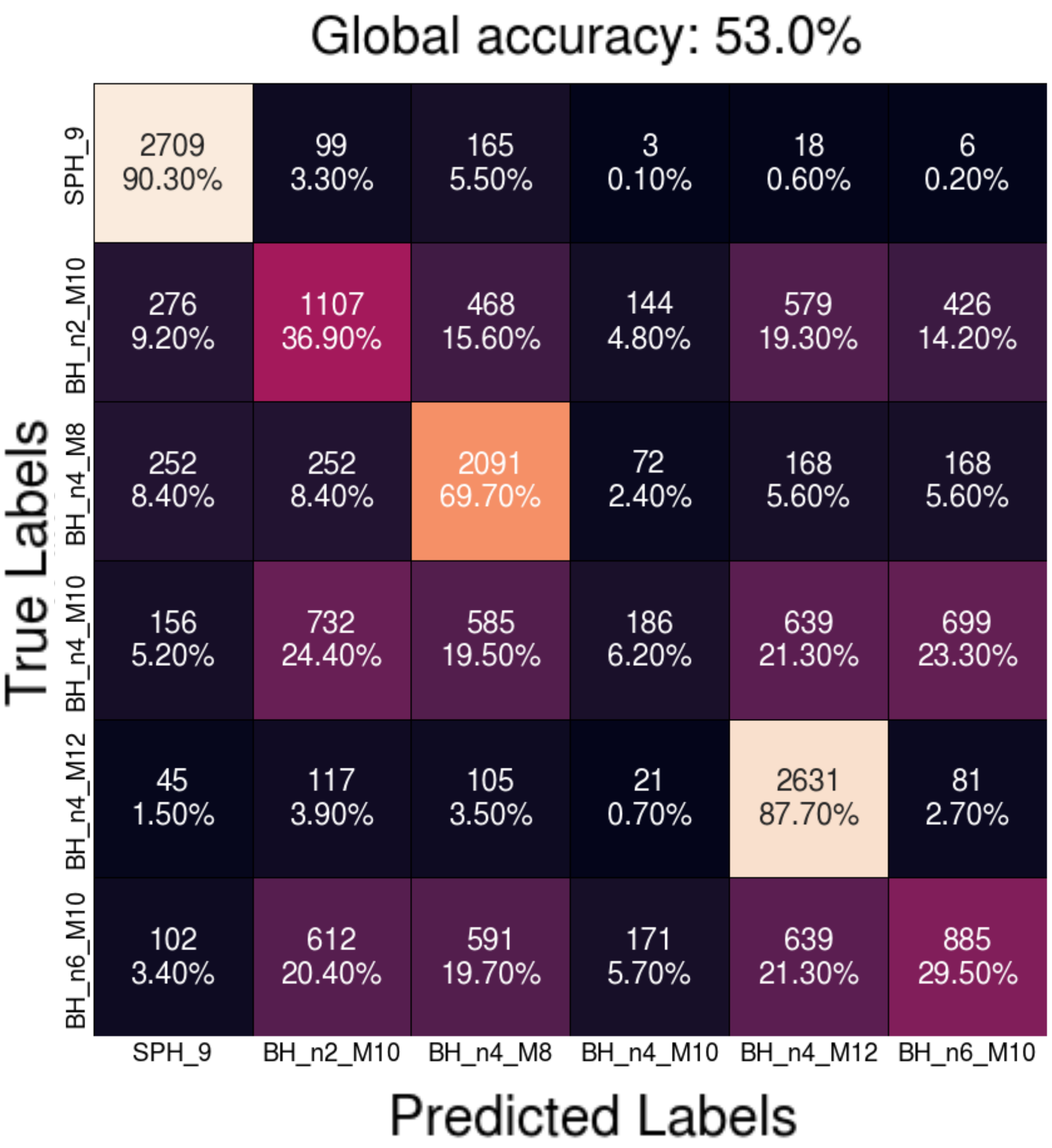
XGBoost



Global accuracy: 51.96%

True Labels \ Predicted Labels	SPH_9	BH_n2_M10	BH_n4_M8	BH_n4_M10	BH_n4_M12	BH_n6_M10
SPH_9	2596 86.53%	104 3.47%	244 8.13%	36 1.20%	4 0.13%	16 0.53%
BH_n2_M10	323 10.77%	1017 33.90%	380 12.67%	479 15.97%	284 9.47%	517 17.23%
BH_n4_M8	277 9.23%	204 6.80%	2064 68.80%	210 7.00%	28 0.93%	217 7.23%
BH_n4_M10	170 5.67%	620 20.67%	399 13.30%	673 22.43%	371 12.37%	767 25.57%
BH_n4_M12	82 2.73%	244 8.13%	114 3.80%	209 6.97%	2117 70.57%	234 7.80%
BH_n6_M10	140 4.67%	545 18.17%	475 15.83%	589 19.63%	365 12.17%	886 29.53%

ResNet

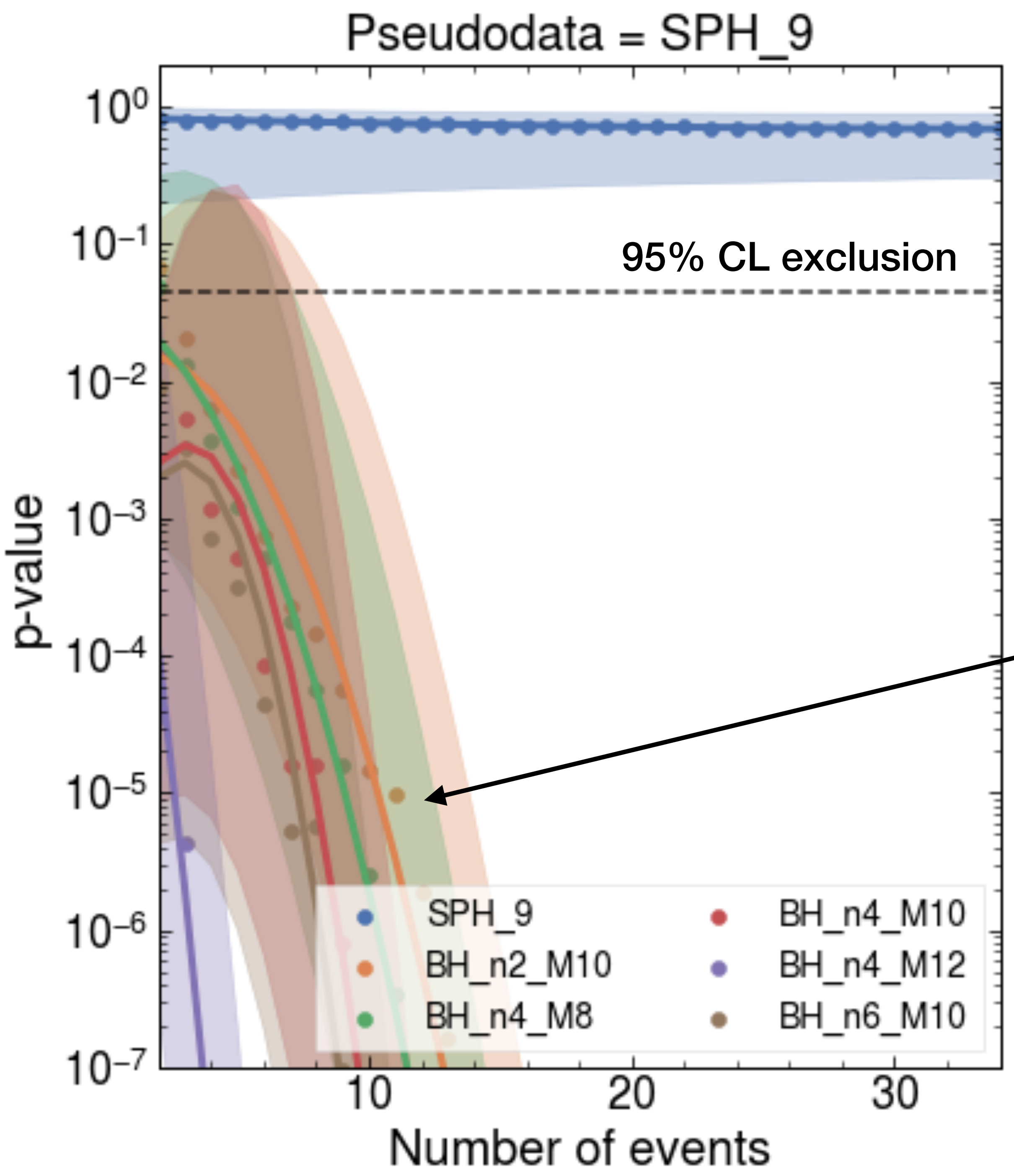


Global accuracy: 53.0%

True Labels \ Predicted Labels	SPH_9	BH_n2_M10	BH_n4_M8	BH_n4_M10	BH_n4_M12	BH_n6_M10
SPH_9	2709 90.30%	99 3.30%	165 5.50%	3 0.10%	18 0.60%	6 0.20%
BH_n2_M10	276 9.20%	1107 36.90%	468 15.60%	144 4.80%	579 19.30%	426 14.20%
BH_n4_M8	252 8.40%	252 8.40%	2091 69.70%	72 2.40%	168 5.60%	168 5.60%
BH_n4_M10	156 5.20%	732 24.40%	585 19.50%	186 6.20%	639 21.30%	699 23.30%
BH_n4_M12	45 1.50%	117 3.90%	105 3.50%	21 0.70%	2631 87.70%	81 2.70%
BH_n6_M10	102 3.40%	612 20.40%	591 19.70%	171 5.70%	639 21.30%	885 29.50%

Results with ResNet

assume LHC observes true **sphaleron** events
in the signal region

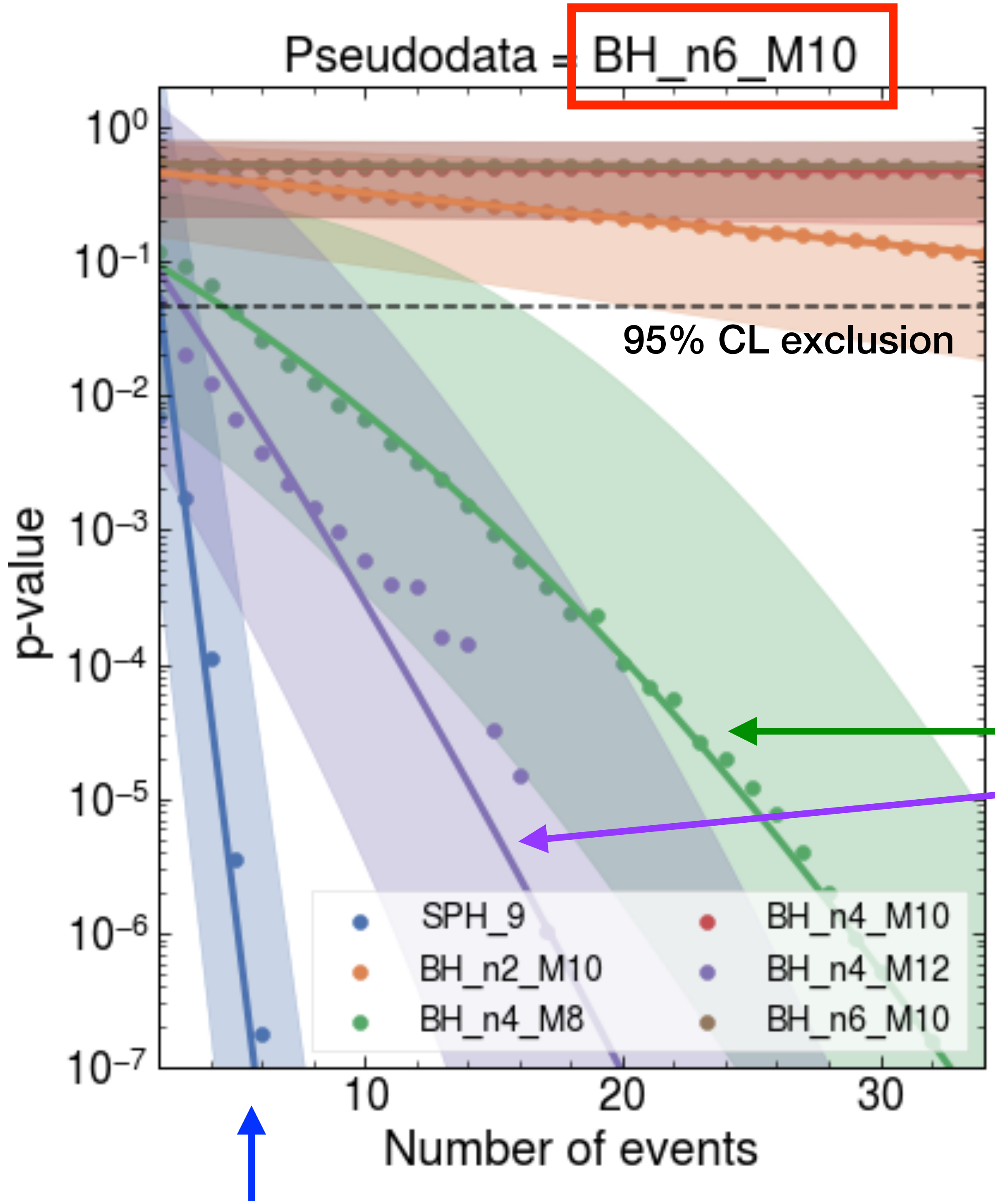


sphaleron hypothesis stays for large observed events, as it should be.

all BH hypotheses can be excluded with high confidence with a small # of events

Results with ResNet

assume LHC observes true **BH** events
with $n = 6, M_{\rm BH}^{\rm min} = 10\,{\rm TeV}$



BH hypotheses with the **same** $M_{\rm BH}^{\rm min}$ and **different** n ***cannot*** **be excluded** with ~ 30 events

BH hypotheses with the **different** $M_{\rm BH}^{\rm min}$ ***can*** **be excluded** with ~ 30 events at $\gtrsim 5\,\sigma$

sphaleron hypothesis excluded with < 10 events

Conclusion

- LHC and future high energy colliders may be able to produce **mini BHs** in extra-dim models or **EW sphalerons** in the SM.
- The signatures of these objects are spectacular but very similar.
    1. Can we discriminate **BHs** and **EW sphalerons** from observed events?
    2. Can we tell the **number** and **size** **of extra dim** by analysing BH events?
- For both objects, the final state typically contains O(10) high pT objects and the best way to analyse the data is not obvious.
- We used BDT and CNN-based ML analyses with XGBoost and ResNet.
- We found:

- Discrimination of **BHs** and **EW Sphalerons** is **possible** with a few events

- Discrimination of different $M_{\rm BH}^{\rm min}$ is **possible** with ~30 events

- Discrimination of different **# of extra dim** is **not feasible** in this method

Norway grants

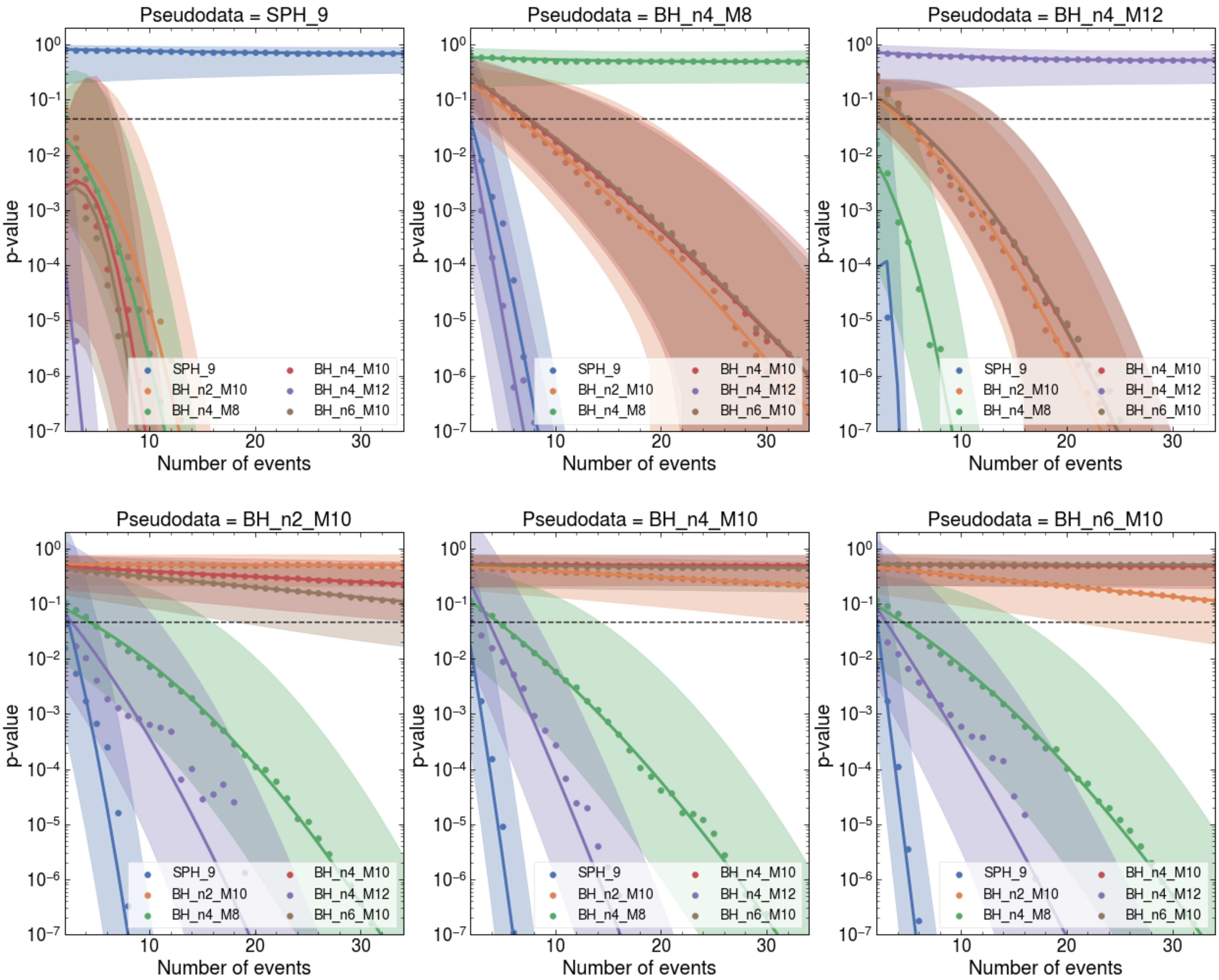
NATIONAL SCIENCE CENTRE
POLAND

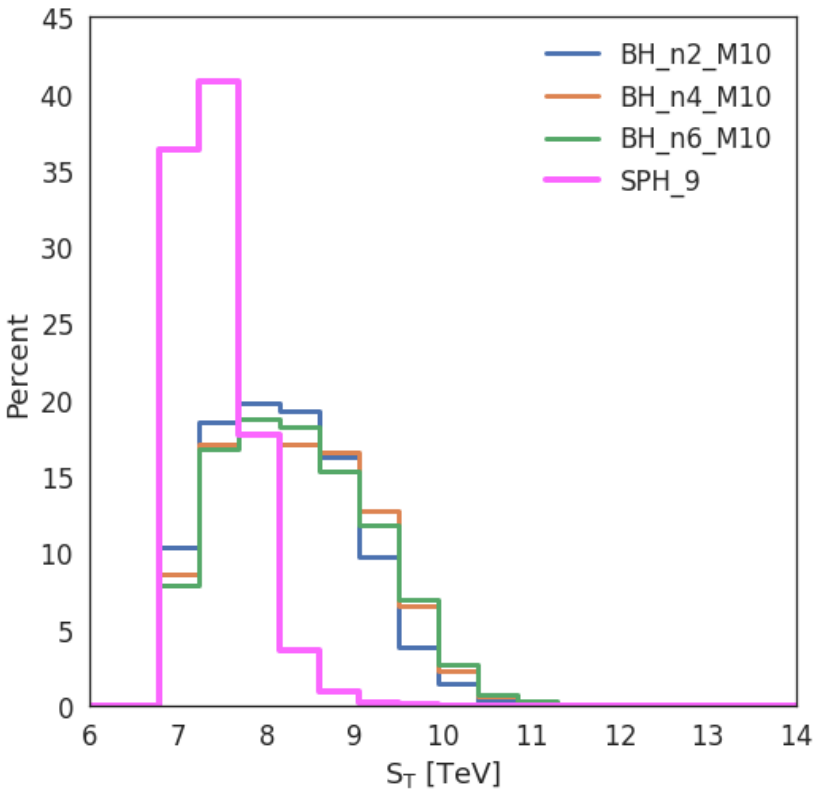
The research leading to the results presented in this talk has received funding from the Norwegian Financial Mechanism for years 2014-2021, grant nr 2019/34/H/ST2/00707

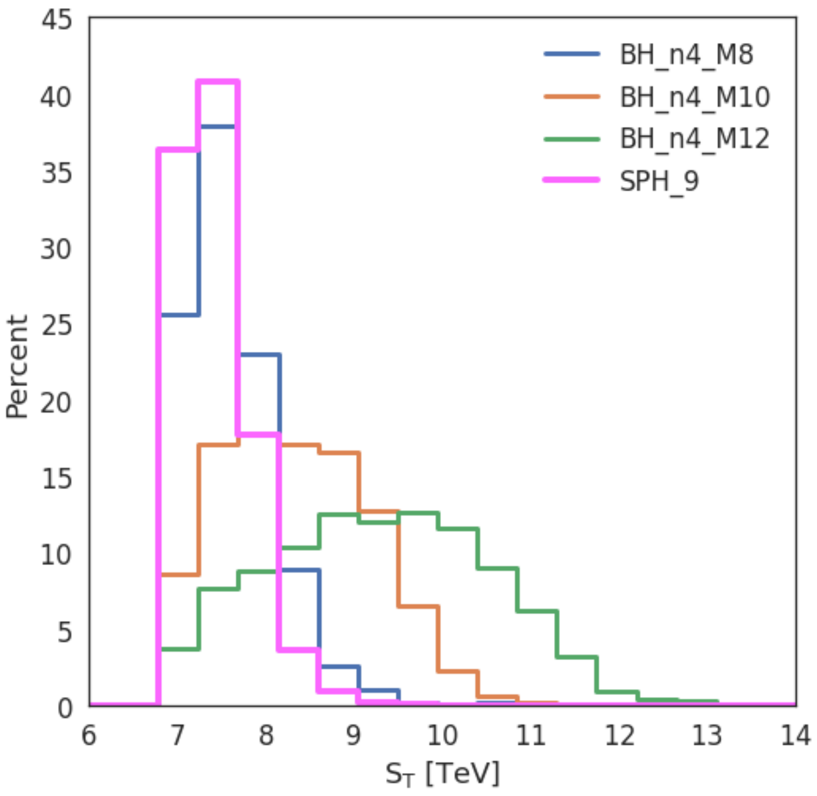
Understanding the Early Universe:
interplay of theory and collider experiments

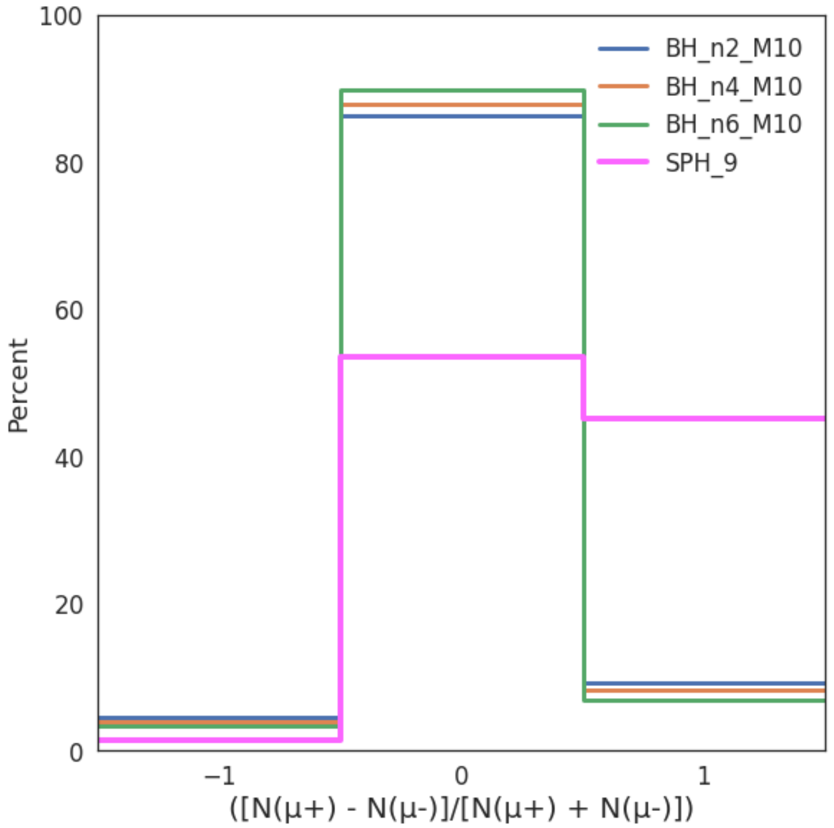
Joint research project between the University of Warsaw & University of Bergen

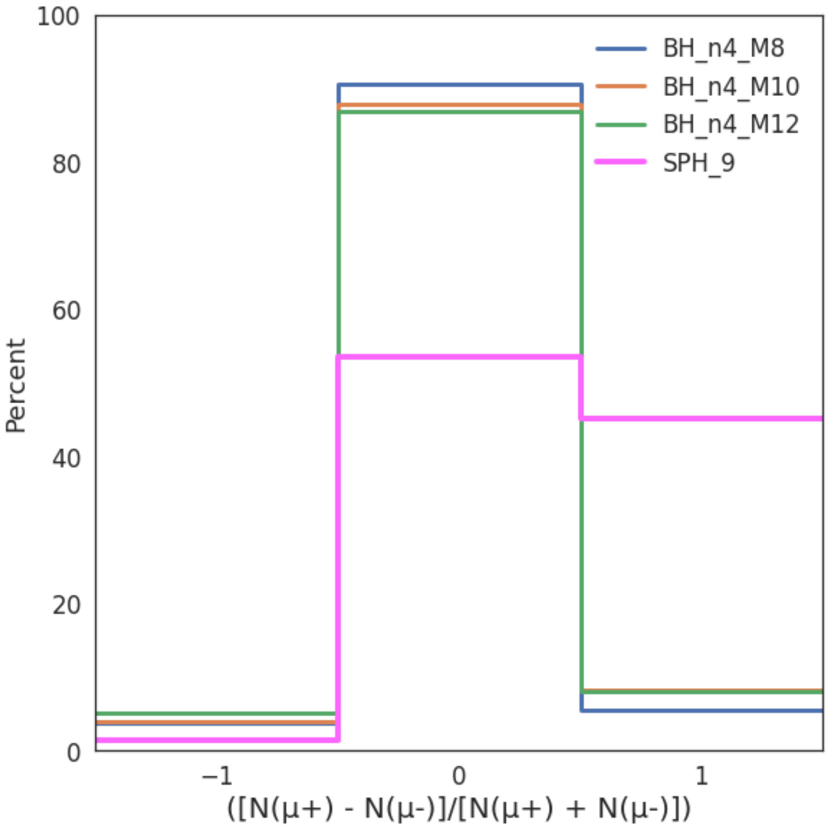
Back up

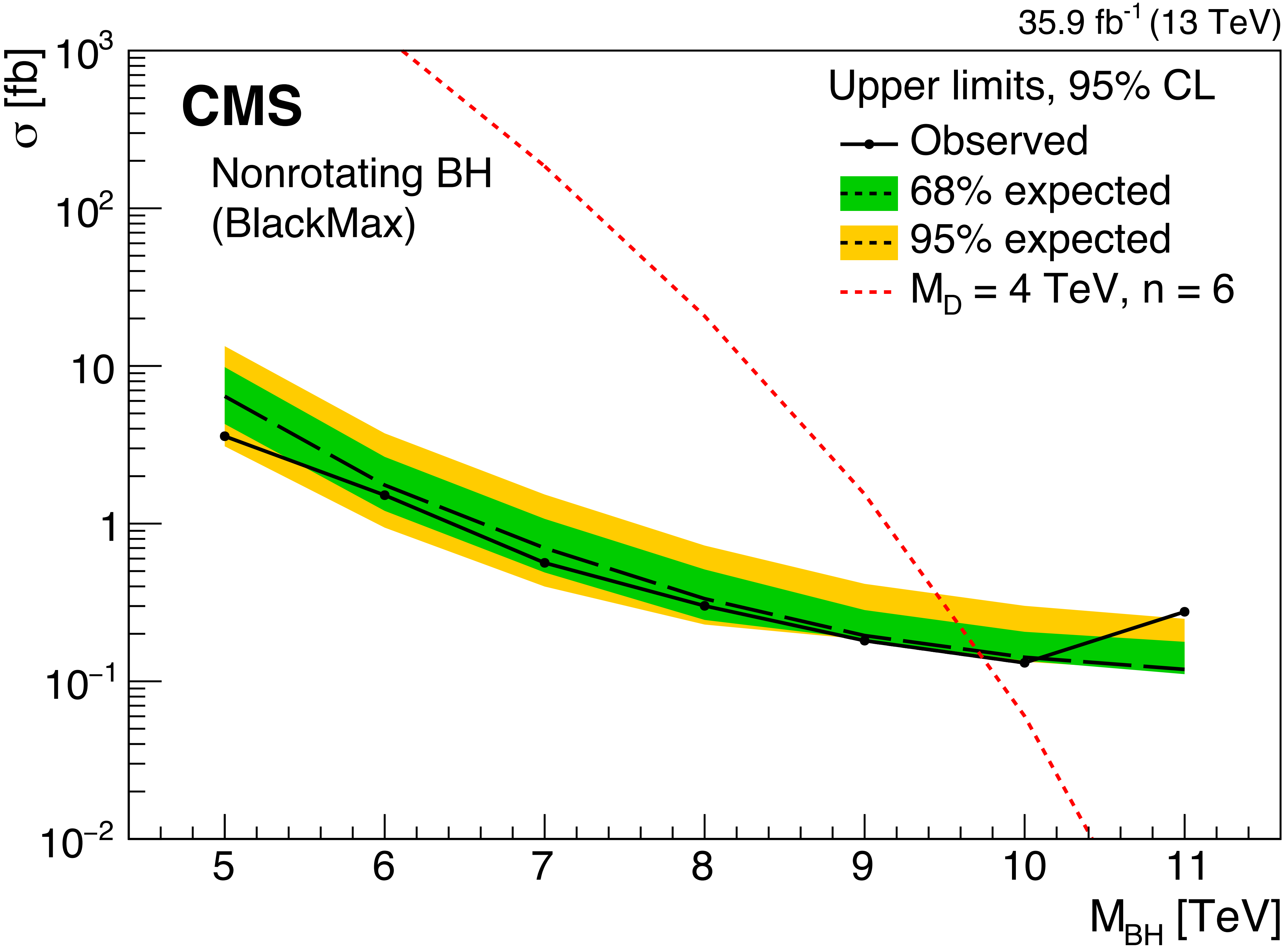

Pseudodata = SPH_9
Pseudodata = BH_n4_M8
Pseudodata = BH_n4_M12
Pseudodata = BH_n2_M10
Pseudodata = BH_n4_M10
Pseudodata = BH_n6_M10
p-value
Number of events
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10
20
30
SPH_9
BH_n2_M10
BH_n4_M8
BH_n4_M10
BH_n4_M12
BH_n6_M10


Percent
45
40
35
30
25
20
15
10
5
0
6
7
8
9
10
11
12
13
14
S_T [TeV]
BH_n2_M10
BH_n4_M10
BH_n6_M10
SPH_9


Percent
45
40
35
30
25
20
15
10
5
0
6
7
8
9
10
11
12
13
14
S_T [TeV]
BH_n4_M8
BH_n4_M10
BH_n4_M12
SPH_9


Percent
100
80
60
40
20
0
−1
0
1
([N(μ+) - N(μ-)]/[N(μ+) + N(μ-)])
BH_n2_M10
BH_n4_M10
BH_n6_M10
SPH_9


Percent
100
80
60
40
20
0
−1
0
1
([N(μ+) - N(μ-)]/[N(μ+) + N(μ-)])
BH_n4_M8
BH_n4_M10
BH_n4_M12
SPH_9


35.9 fb^{-1} (13 TeV)
CMS
Nonrotating BH
(BlackMax)
Upper limits, 95% CL
Observed
68% expected
95% expected
M_D = 4 TeV, n = 6
σ [fb]
10^3
10^2
10
1
10^{-1}
10^{-2}
5
6
7
8
9
10
11
M_{BH} [TeV]

- A “current” carrying the winding number:

$$K_\mu \;=\; \frac{1}{8\pi^2}\epsilon_{\mu\nu\rho\sigma}\mathrm{tr}A^\nu(\partial^\rho A^\sigma + \frac{2}{3}A^\rho A^\sigma)$$

- One can show

$$\int K_0(A_n(\mathbf{x}))d^3x = n\,, \qquad \frac{1}{16\pi^2}F_{\mu\nu}\tilde{F}^{\mu\nu} \;=\; \partial^\mu K_\mu$$

- This implies

$$\int \frac{1}{16\pi^2}F_{\mu\nu}\tilde{F}^{\mu\nu}d^4x \;=\; \int \partial^\mu K_\mu d^3x dt \;\;=\;\; \Big[\int K_0(t,\mathbf{x})d^3x\Big]_{t=-\infty}^{t=\infty}$$

$$=\; n(t=\infty) - n(t=-\infty) \;\;=\; \Delta n$$

The tunnelling rate can be estimated using the WKB approximation as

$$\langle n | n + \Delta n \rangle \sim e^{-\hat{S}_E}$$

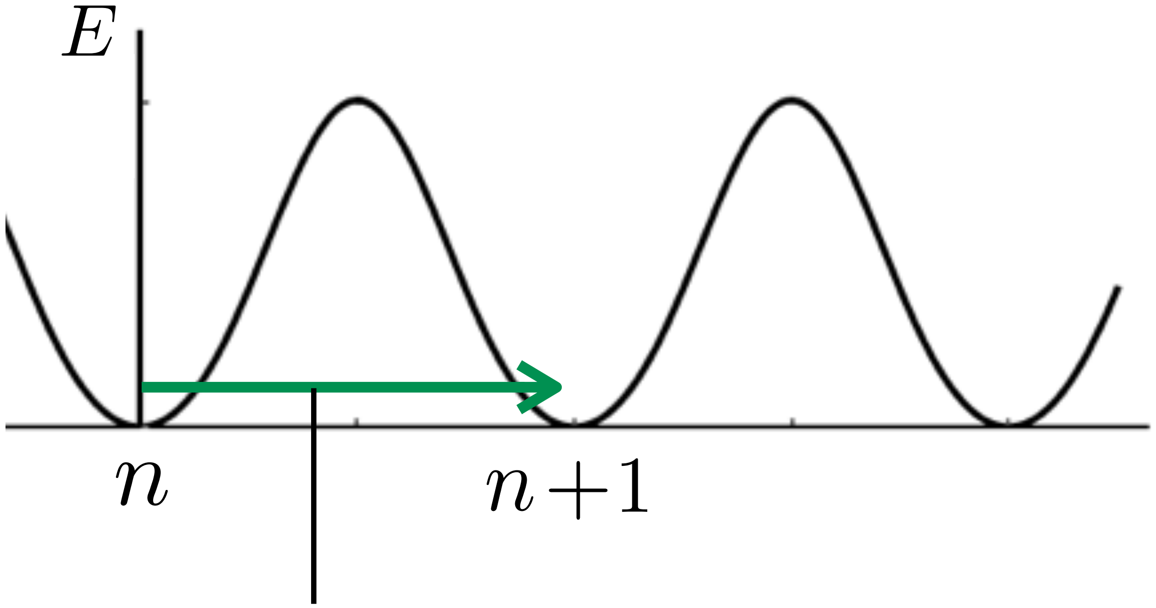
S_E is the Euclidean action at the stationary point, which is given by

$$\begin{aligned} \hat{S}_E &= \frac{1}{2g^2} \int F F d^4x \\ &= \frac{1}{2g^2} \left| \int F \widetilde{F} d^4x \right| \\ &= \frac{8\pi^2}{g^2} |\Delta n| \end{aligned}$$

Note that:

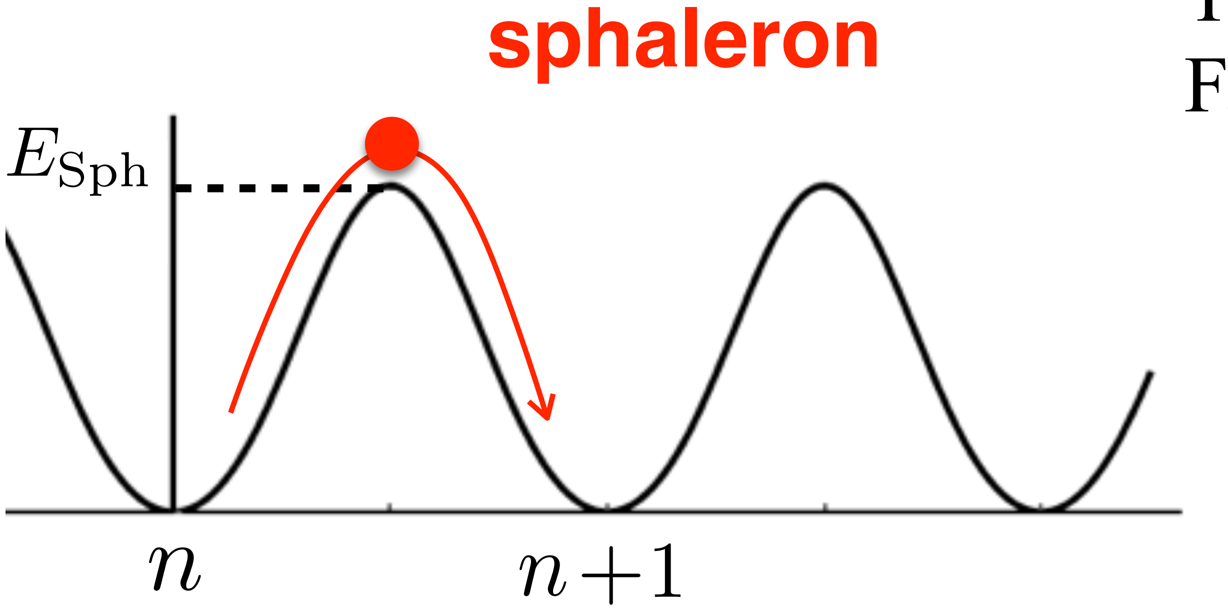
$$\int (F \pm \tilde{F})^2 d^4x \geq 0$$

$$\Longrightarrow \int F F d^4x \geq \left| \int F \tilde{F} d^4x \right|$$



$$e^{-\frac{4\pi}{\alpha_W}} \sim 10^{-170}$$

The tunnelling rate is unobservably small



The barrier hight was calculated by F.R.Klinkhamer and N.S.Manton (1984)

$$E_{\rm Sph} = \frac{2m_W}{\alpha_W} B\left(\frac{m_H}{m_W}\right)$$

$$\simeq 9\,\mathrm{TeV} \qquad (\text{for } m_H = 125\mathrm{GeV})$$

- At high temperature, the sphaleron rate may be unsuppressed.

$$\Gamma \propto \exp\left(-\frac{E_{\rm Sph}(T)}{T}\right)$$

It plays an important role in baryo(lepto)genesis.

What happens for the high energy (zero temperature) case?

Cross-section estimate

LSZ formula:

$$\langle f\,|\,S\,|\,i\rangle = \left[i\int d^4x_1 e^{-ip_1x_1}(\Box_1+m_1^2)\right]\cdots\left[i\int d^4x_n e^{-ip_nx_n}(\Box_n+m_n^2)\right]\cdot\langle\Omega\,|\,T\{\phi_1(x_1)\cdots\phi_n(x_n)\}\,|\,\Omega\rangle$$

Path-integral:

$$\langle\Omega\,|\,T\{\phi_1(x_1)\cdots\phi_n(x_n)\}\,|\,\Omega\rangle = \frac{\int\mathscr{D}\phi\;\phi_1(x_1)\cdots\phi_n(x_n)\,e^{iS[\phi]}}{\int\mathscr{D}\phi\;e^{iS[\phi]}}$$

- Matrix elements for Δn = 1 processes may be obtained by

$$i\mathscr{M}(\Delta n=1) = \left.\frac{\int\mathscr{D}\phi\,|_{\Delta n=1}\;\phi_1(x_1)\cdots\phi_1(x_n)\,e^{iS}}{\int\mathscr{D}\phi\;e^{iS}}\right|_{\text{LSZ}}$$

$$\sim\left.\frac{\int\mathscr{D}\delta\phi\;\tilde{\phi}_1(x_1)\cdots\tilde{\phi}_1(x_n)\,e^{iS(\tilde{\phi}+\delta\phi)}}{\int\mathscr{D}\phi\;e^{iS}}\right|_{\text{LSZ}}$$

stationary (instanton) configuration with Δn = 1

$$\big\downarrow$$

$$\phi_i = \tilde{\phi}_i + \delta\phi_i$$

- The LO Matrix Element in the ***instanton* background**

$$i\mathcal{M} \sim \int \mathcal{D}q\mathcal{D}W\mathcal{D}\phi\, q(x_1)\cdots q(x_{12}) W(y_1)\cdots W(y_{n_W}) \phi(z_1)\cdots\phi(z_{n_h}) \exp(-S_E)\Big|_{\mathrm{LSZ}}$$

[Ringwald ’90, Espinosa ’90]

- The LO Matrix Element in the ***instanton* background**

$$i\mathcal{M} \sim \int \mathcal{D}q\mathcal{D}W\mathcal{D}\phi\, q(x_1)\cdots q(x_{12})\,W(y_1)\cdots W(y_{n_W})\,\phi(z_1)\cdots\phi(z_{n_h})\,\exp(-S_E)\Big|_{\text{LSZ}}$$

- Evaluate it at the instanton configuration:

[Ringwald '90, Espinosa '90]

$$W^{\mu a}_{\text{inst}} \simeq \frac{2\rho^2}{g}\underset{\text{orientation}}{U_{ab}}\frac{\bar{\eta}_{b\mu\nu}(x-x_0)_\nu}{(x-x_0)^2[(x-\underset{\text{position}}{x_0})^2+\underset{\text{size}}{\rho^2}]} \qquad \phi_{\text{inst}}(x) \simeq v\left[\frac{(x-x_0)^2}{(x-x_0)^2+\rho^2}\right]^{1/2}$$

- The LO Matrix Element in the ***instanton* background**

$$i\mathcal{M} \sim \int \mathcal{D}q\mathcal{D}W\mathcal{D}\phi\, q(x_1)\cdots q(x_{12})W(y_1)\cdots W(y_{n_W})\phi(z_1)\cdots\phi(z_{n_h})\,\exp(-S_E)\Big|_{\text{LSZ}}$$

- Evaluate it at the instanton configuration:

[Ringwald '90, Espinosa '90]

$$W^{\mu a}_{\text{inst}} \simeq \frac{2\rho^2}{g}U_{ab}\frac{\bar{\eta}_{b\mu\nu}(x-x_0)_\nu}{(x-x_0)^2[(x-x_0)^2+\rho^2]} \qquad \phi_{\text{inst}}(x) \simeq v\left[\frac{(x-x_0)^2}{(x-x_0)^2+\rho^2}\right]^{1/2}$$

orientation position size

- Integration over orientation, position, size and phase-space:

$$\sigma(n_W, n_h) \sim \int |\mathcal{M}|^2 \cdot d\Phi_{\text{PS}}$$

- The LO Matrix Element in the ***instanton* background**

$$i\mathcal{M} \sim \int \mathcal{D}q\mathcal{D}W\mathcal{D}\phi\, q(x_1)\cdots q(x_{12}) W(y_1)\cdots W(y_{n_W})\phi(z_1)\cdots\phi(z_{n_h}) \exp(-S_E)\Big|_{\text{LSZ}}$$

- Evaluate it at the instanton configuration:

[Ringwald '90, Espinosa '90]

$$W^{\mu a}_{\text{inst}} \simeq \frac{2\rho^2}{g}\underbrace{U_{ab}}_{\text{orientation}} \frac{\bar\eta_{b\mu\nu}(x-x_0)_\nu}{(x-x_0)^2[(x-\underbrace{x_0}_{\text{position}})^2+\underbrace{\rho^2}_{\text{size}}]} \qquad \phi_{\text{inst}}(x) \simeq v\left[\frac{(x-x_0)^2}{(x-x_0)^2+\rho^2}\right]^{1/2}$$

- Integration over orientation, position, size and phase-space:

$$\sigma(n_W, n_h) \sim \int |\mathcal{M}|^2 \cdot d\Phi_{\text{PS}}$$

- Result

[Ringwald '90, Espinosa '90]

$$\sigma_{\text{LO}}(n_W, n_h) \sim \mathscr{G}^2 2^n v^{-2n}\left[\frac{\Gamma(n+103/12)}{\Gamma(103/12)}\right]^2 \frac{1}{n_B!\,n_H!} \qquad \mathscr{G} = 1.6\times 10^{-101}\,\text{GeV}^{-14}$$

$$\times \int \prod_{i=1}^{10} \frac{\mathrm{d}^3 p_i}{(2\pi)^3 2E_i} E_i \prod_{j=1}^{n_B} \frac{\mathrm{d}^3 p_j}{(2\pi)^3 2E_j} \frac{2(4E_j^2 - m_W^2)}{m_W^2} \prod_{k=1}^{n_H} \frac{\mathrm{d}^3 p_k}{(2\pi)^3 2E_k} (2\pi)^4 \delta^{(4)}\left(P_{\text{in}} - \sum_{i=1}^{10} p_i - \sum_{j=1}^{n_B} p_j - \sum_{k=1}^{n_H} p_k\right)$$

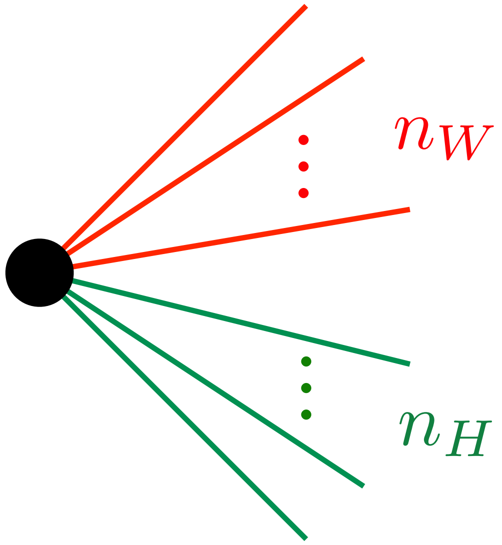
The cross-section grows with energy and the number of final state bosons.

$$i\mathcal{M} \sim \int \mathcal{D}q\mathcal{D}W\mathcal{D}\phi\, q(x_1)\cdots q(x_{12})\, W(y_1)\cdots W(y_{n_W})\,\phi(z_1)\cdots\phi(z_{n_h})\exp(-S_E)\Big|_{\text{LSZ}}$$

$$A^{\text{inst}\,a}_{\mu}(x_i) \xrightarrow{\text{FT}} \frac{4i\pi^2\rho^2}{g}\,\frac{\bar{\eta}^a_{\mu\nu}p_i^\nu}{p_i^2(p_i^2+m_W^2)}\,e^{ip_ix_0} \xrightarrow{\text{LSZ}} \boxed{\frac{4i\pi^2\rho^2}{g}\,\frac{\bar{\eta}^a_{\mu\nu}p_i^\nu}{p_i^2}\,e^{ip_ix_0}}$$

$$H^{\text{inst}}(x_j) \to -\frac{2\pi^2\rho^2 v}{(p_j^2+m_H^2)}\,e^{ip_jx_0} \to \boxed{-2\pi^2\rho^2 v\, e^{ip_jx_0}}$$

- Multi-particle interaction under the instanton BG is (almost) a point-like vertex



$$i\mathcal{M} \sim \left[\frac{4i\pi^2\rho^2}{g}\,\frac{\bar{\eta}^a_{\mu\nu}p_i^\nu}{p_i^2}\,e^{ip_ix_0}\right]^{n_W}\left[-2\pi^2\rho^2 v\, e^{ip_jx_0}\right]^{n_H}$$

$$\Phi_n(Q) \sim (Q^2)^{n-2}$$

↑

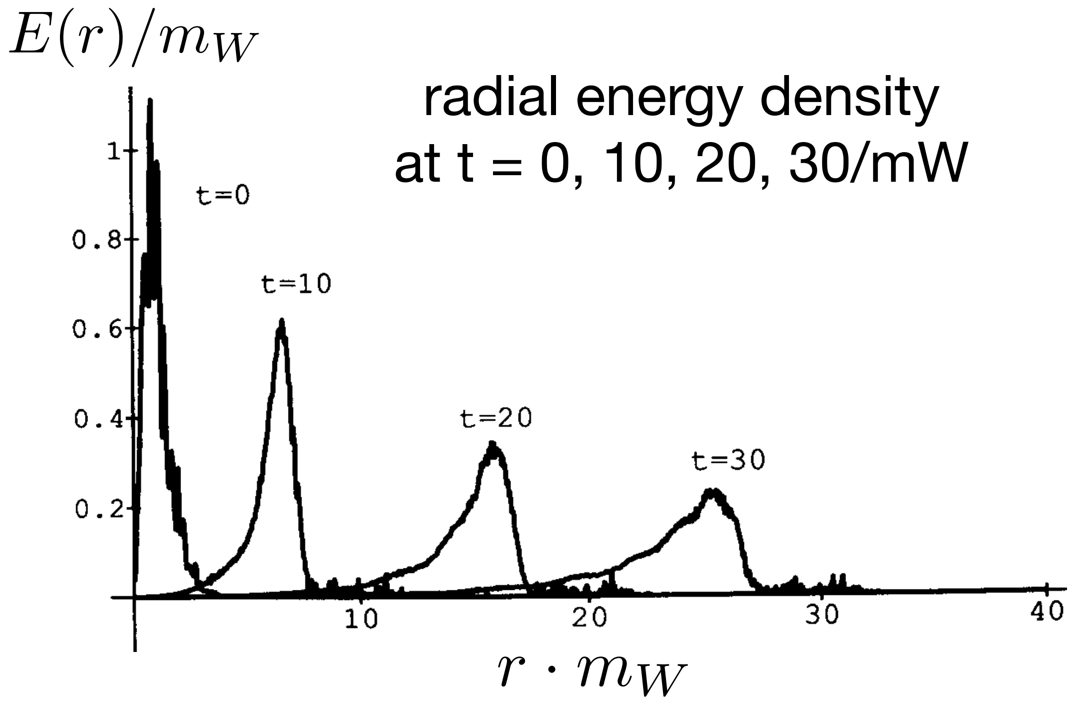
n-body phase-space

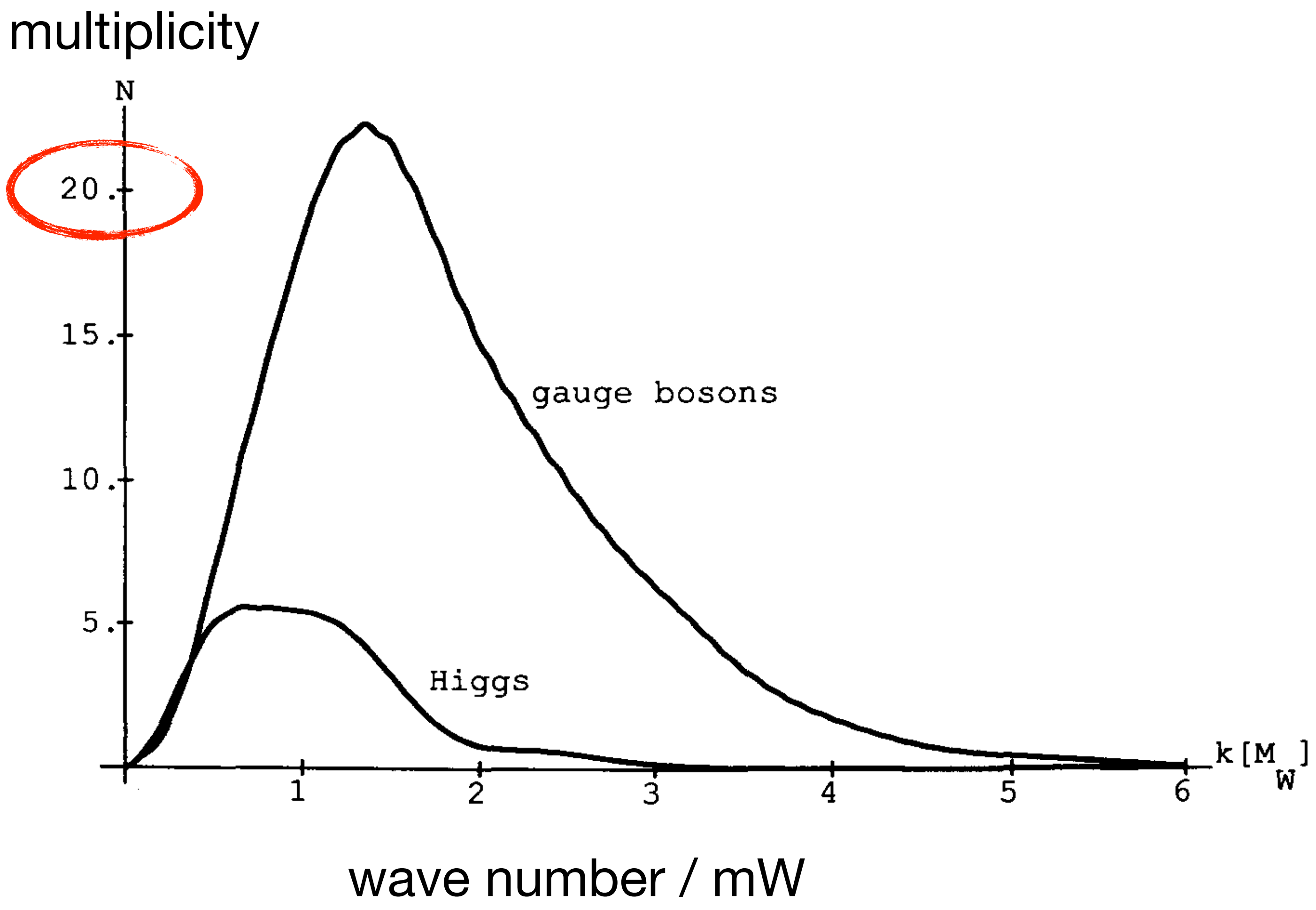
Such a vertex is highly unrenormalisable and high energy behaviour is not regulated.

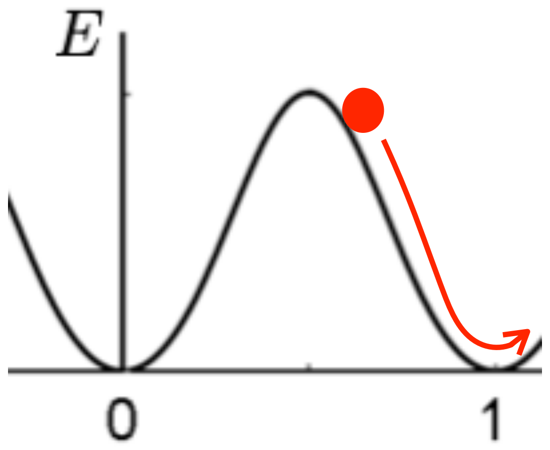
Enhancement at large *nW* and *nH*.

Real time evolution [Heilmund, Kripfganz '91]

- Prepare an *almost* sphaleron configuration, deviated slightly to the unstable direction.
- Evolve it with EoM and observe the field rump dissipates.
- Fourier expand (expansion in terms of fee particle modes) the final state and count the number of W and H bosons.

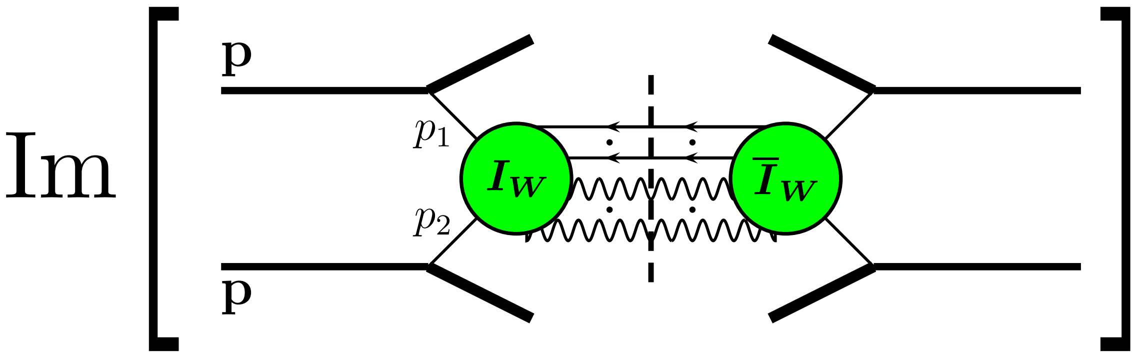

radial energy density at t = 0, 10, 20, 30/mW





- The inclusive cross-section can be estimated using the dispersion relations (optical theorem).

[Khoze, Ringwald '91]



$$\hat{\sigma}^{\rm BV}_{\rm incl} \equiv \sum_{n_W,n_h} \hat{\sigma}^{\rm BV}_{n_W,n_h} = \frac{P(\epsilon)}{m_W^2} \exp\left[-\frac{4\pi}{\alpha_W} F(\epsilon)\right]$$

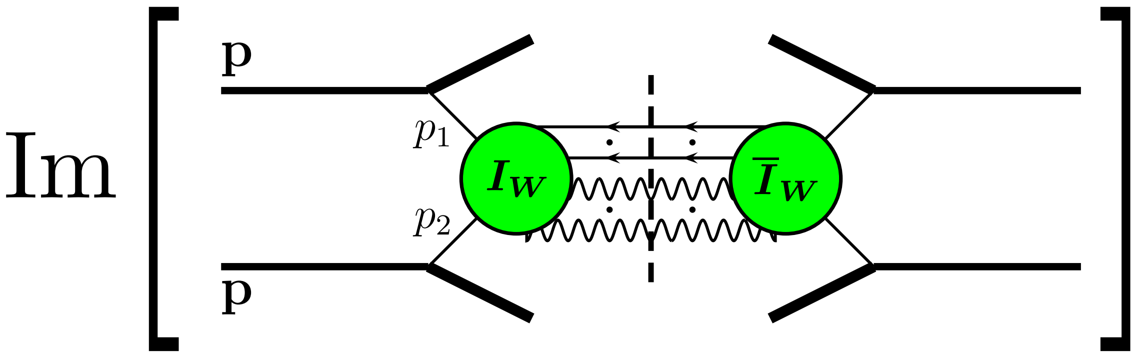
$$F(\epsilon) = 1 - \frac{9}{8}\epsilon^{4/3} + \frac{9}{16}\epsilon^2 - \frac{3}{32}\left(4 - 3\frac{m_h^2}{m_W^2}\right)\epsilon^{8/3}\ln\frac{1}{\epsilon} + \mathcal{O}(\epsilon^{8/3}\cdot\mathrm{const})$$

$$P(\epsilon) = \frac{\pi^{15/2}}{1024}\left(\frac{3}{2}\right)^{\frac{2}{3}} d^2 \left(\frac{4\pi}{\alpha_W}\right)^{7/2} \epsilon^{\frac{74}{9}}\left[1 + \mathcal{O}\left(\epsilon^{2/3}\right)\right] \qquad d \simeq 0.15$$

$$\epsilon \equiv \frac{\sqrt{\hat{s}}}{M_0} \qquad M_0 \equiv \sqrt{6}\pi\frac{m_W}{\alpha_W} \simeq 18.3\ \mathrm{TeV}$$

- The inclusive cross-section can be estimated using the dispersion relations (optical theorem).

[Khoze, Ringwald ’91]



$$\hat{\sigma}^{\rm BV}_{\rm incl} \equiv \sum_{n_W,n_h} \hat{\sigma}^{\rm BV}_{n_W,n_h} = \frac{P(\epsilon)}{m_W^2} \exp\left[-\frac{4\pi}{\alpha_W}\boxed{F\left(\epsilon\right)}\right]$$

‘Holy grail’ function →

$$\boxed{F(\epsilon)} = 1 - \frac{9}{8}\epsilon^{4/3} + \frac{9}{16}\epsilon^2 - \frac{3}{32}\left(4 - 3\frac{m_h^2}{m_W^2}\right)\epsilon^{8/3}\ln\frac{1}{\epsilon} + \mathcal{O}(\epsilon^{8/3}\cdot{\rm const})$$

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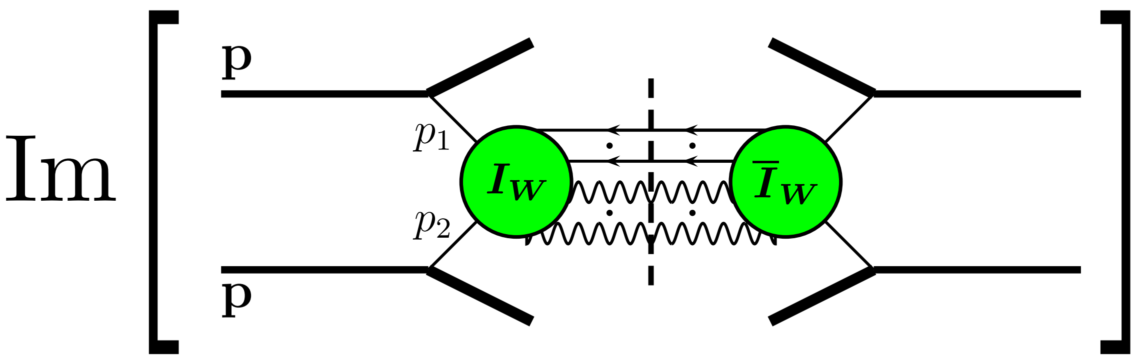
- The inclusive cross-section can be estimated using the dispersion relations (optical theorem).

[Khoze, Ringwald '91]



$$\hat{\sigma}^{\mathrm{BV}}_{\mathrm{incl}} \equiv \sum_{n_W,n_h} \hat{\sigma}^{\mathrm{BV}}_{n_W,n_h} = \frac{P(\epsilon)}{m_W^2} \exp\left[-\frac{4\pi}{\alpha_W}\boxed{F\left(\epsilon\right)}\right]$$

'Holy grail' function →

$$\boxed{F(\epsilon)} = 1 - \frac{9}{8}\epsilon^{4/3} + \frac{9}{16}\epsilon^2 - \frac{3}{32}\left(4 - 3\frac{m_h^2}{m_W^2}\right)\epsilon^{8/3}\ln\frac{1}{\epsilon} + \mathcal{O}(\epsilon^{8/3}\cdot\mathrm{const})$$

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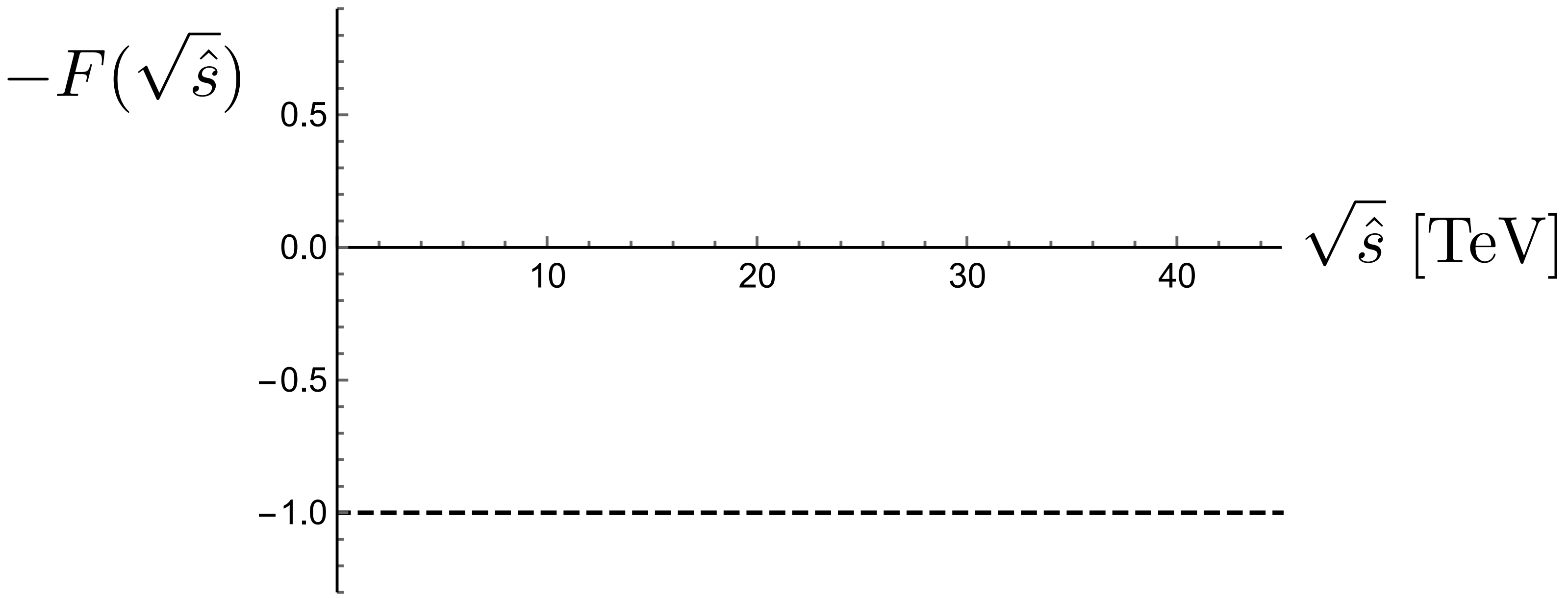
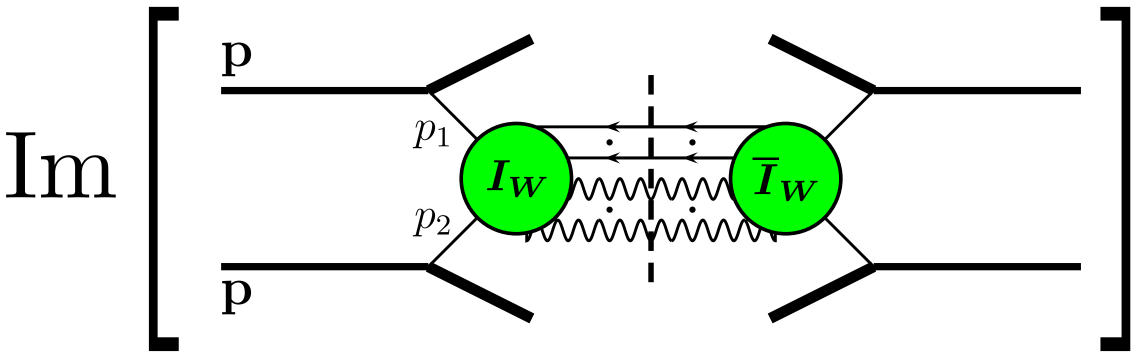
$$\epsilon \equiv \frac{\sqrt{\hat{s}}}{M_0} \qquad M_0 \equiv \sqrt{6}\pi\frac{m_W}{\alpha_W} \simeq 18.3\ \mathrm{TeV}$$



- The inclusive cross-section can be estimated using the dispersion relations (optical theorem).

[Khoze, Ringwald ’91]



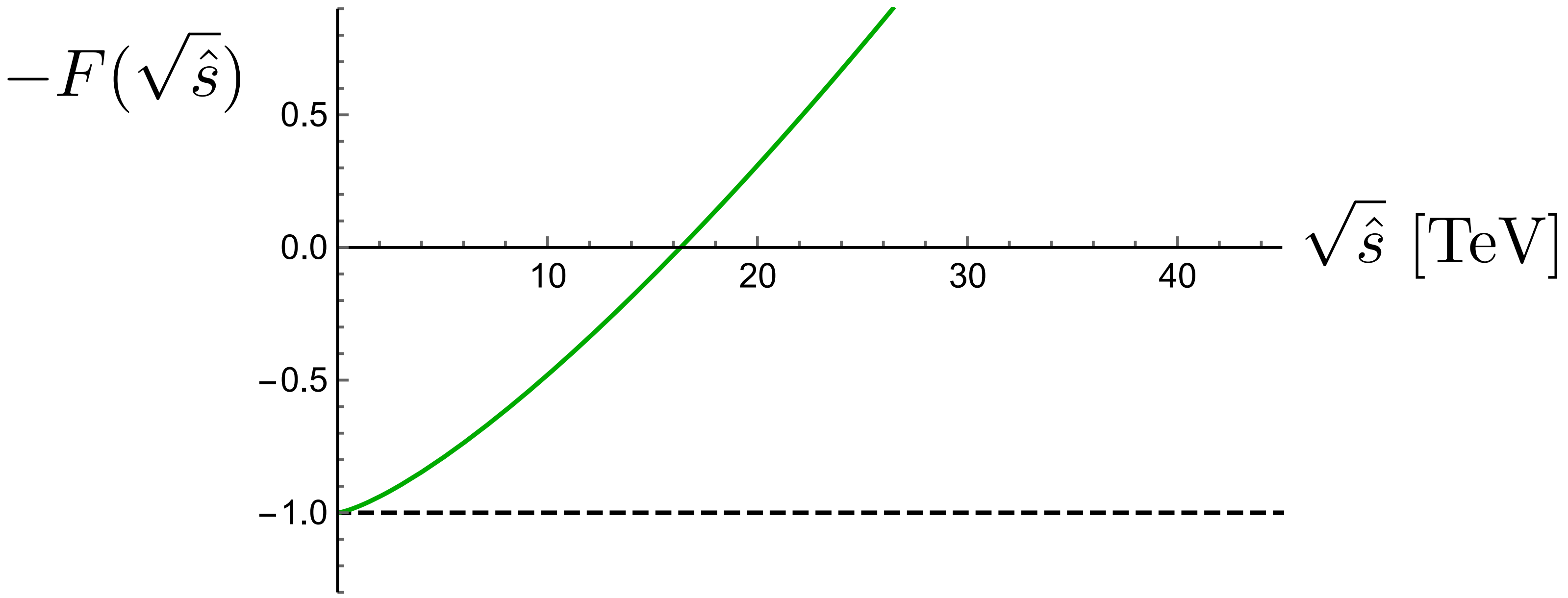
$$\hat{\sigma}^{\mathrm{BV}}_{\mathrm{incl}} \equiv \sum_{n_W,n_h} \hat{\sigma}^{\mathrm{BV}}_{n_W,n_h} = \frac{P(\epsilon)}{m_W^2} \exp\left[-\frac{4\pi}{\alpha_W}\boxed{F(\epsilon)}\right]$$

‘Holy grail’ function

$$\boxed{F(\epsilon)} = 1 - \frac{9}{8}\epsilon^{4/3} + \frac{9}{16}\epsilon^2 - \frac{3}{32}\left(4 - 3\frac{m_h^2}{m_W^2}\right)\epsilon^{8/3}\ln\frac{1}{\epsilon} + \mathcal{O}(\epsilon^{8/3}\cdot\mathrm{const})$$

$$P(\epsilon) = \frac{\pi^{15/2}}{1024}\left(\frac{3}{2}\right)^{\frac{2}{3}} d^2 \left(\frac{4\pi}{\alpha_W}\right)^{7/2} \epsilon^{\frac{74}{9}} \left[1 + \mathcal{O}\left(\epsilon^{2/3}\right)\right] \qquad d \simeq 0.15$$

$$\epsilon \equiv \frac{\sqrt{\hat{s}}}{M_0} \qquad M_0 \equiv \sqrt{6}\pi\frac{m_W}{\alpha_W} \simeq 18.3\ \mathrm{TeV}$$



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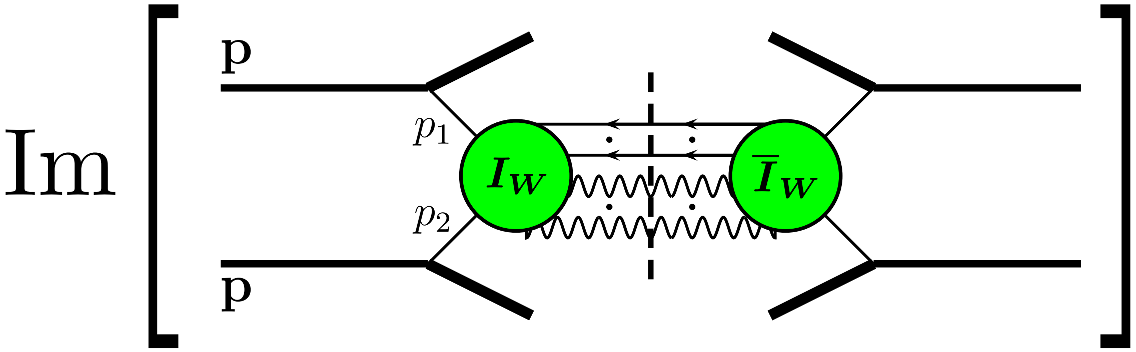

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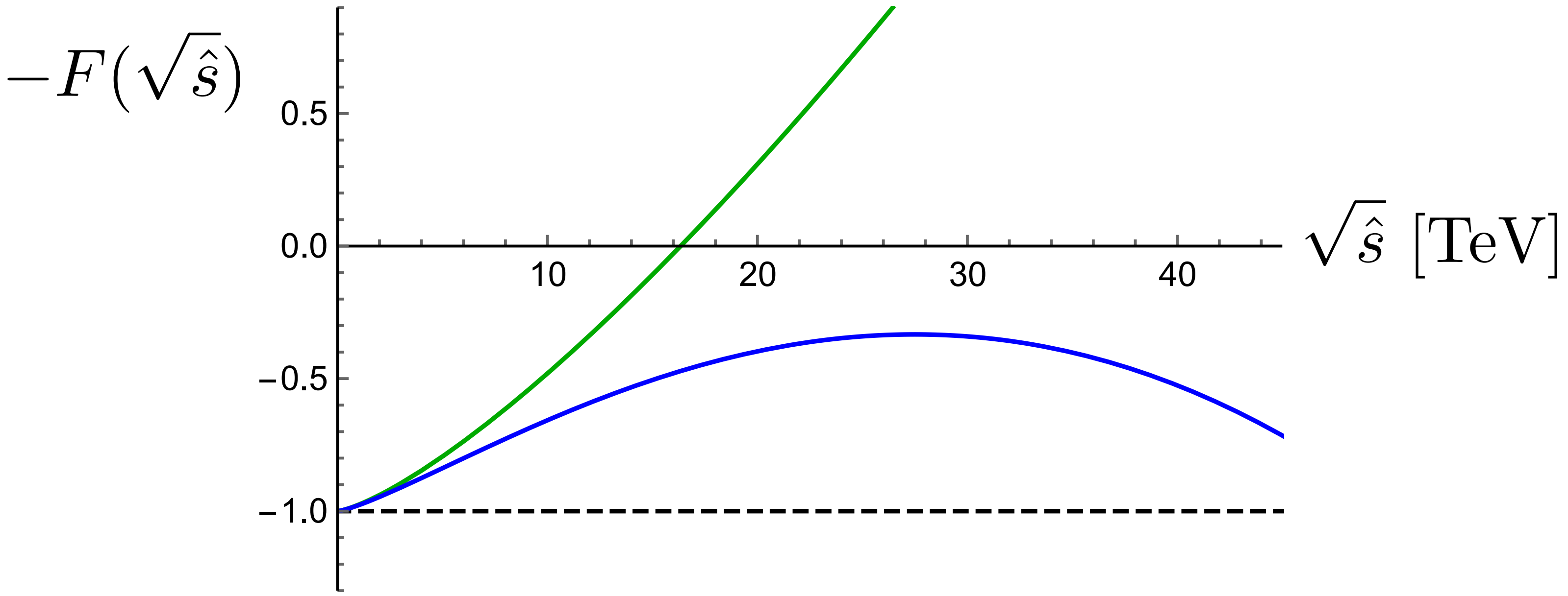

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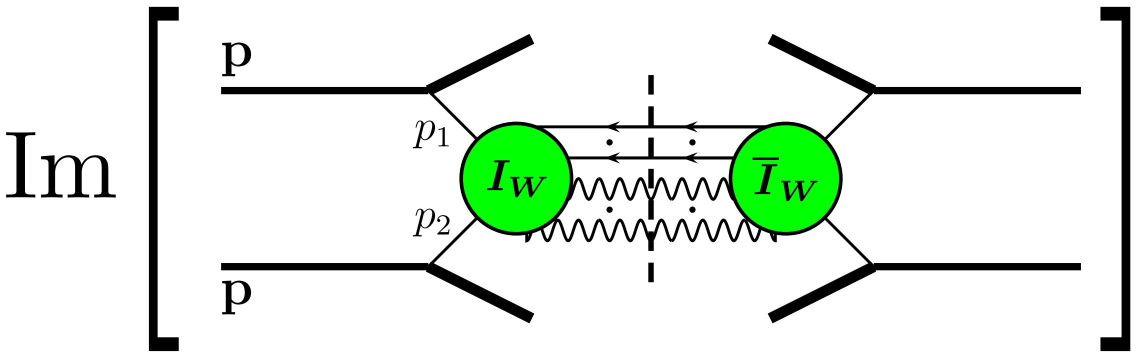

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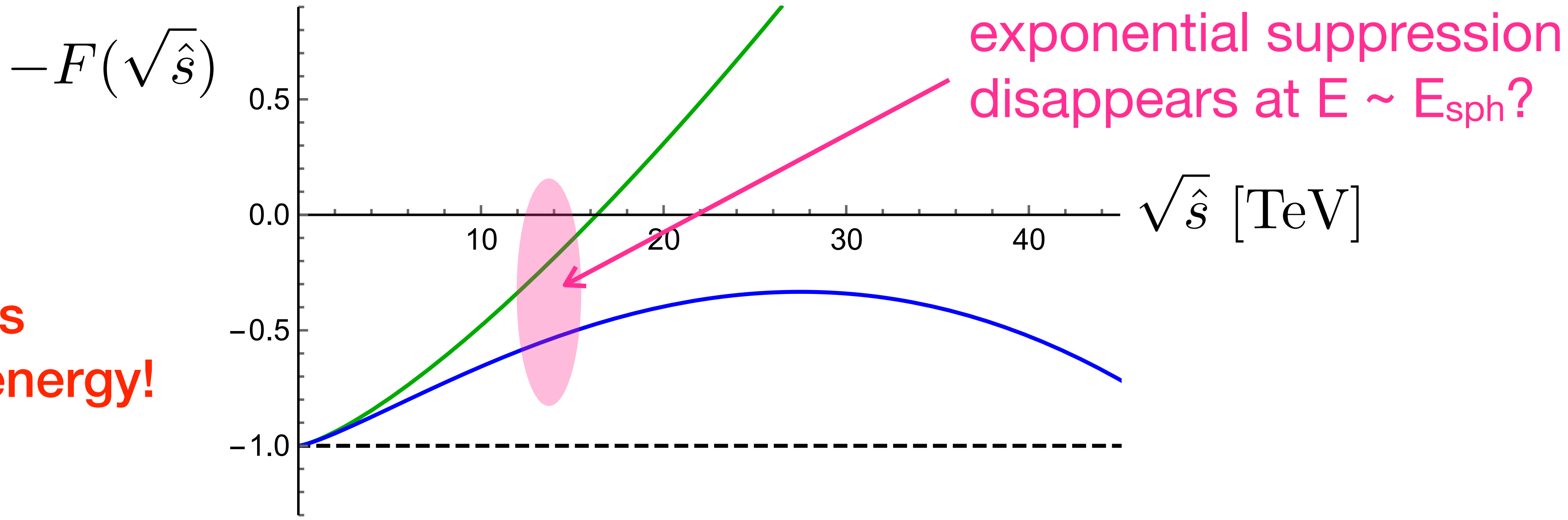

exponential suppression disappears at E ~ $E_{\rm sph}$?

Cross-section grows exponentially with energy!

4.3 p-values

To estimate the practical usefulness of our CNN classifier, we want to calculate how many events we would need to detect to reject the hypothesis that the detected events are of a certain type. We can do this by using the Pearson χ^2 hypothesis test to calculate the probability that the difference between an observed distribution and a theoretical distribution is due to random fluctuations. Our theoretical prediction distribution is found by predicting across 15000 test events of each type. The result is 6 histograms, each with 6 bins which are then all normalized. These histograms can be compared against predictions over a pseudo experiment of m particles of a certain type by calculating the χ^2 test statistic of the sum of the squared differences between the two distributions.

$$\chi^2 = N \sum_{i=0}^{k} (O_i/N - \rho_i)^2/\rho_i \tag{4.1}$$

N is the number of observations, O_i is the number of observations classified as class i, k is the number of classes and ρ_i is the predicted probability of making an observation classified as i. From the χ^2-value the p-value can be obtained directly, setting the number of degrees of freedom in the distribution to k-1. For this study $k = 6$ as we have 6 samples.

To estimate the number of events we would have to observe at the LHC to exclude each class using the CNN classifier we perform $m = 3000$ pseudo experiments for each N number of particles detected in the range 2 to 35 and calculate the average p-value and the upper and lower error ($\bar{\sigma}_{p>\bar{p}}$, $\bar{\sigma}_{p<\bar{p}}$) of the average p-value:

$$\bar{p} = \frac{1}{m} \sum_{i=0}^{m} (p_i)^2 \tag{4.2}$$

$$\bar{\sigma}_{p>\bar{p}} = \frac{1}{m_{p>\bar{p}}} \sum_{i=0}^{m_{p>\bar{p}}} (\bar{p} - p_{i,>\bar{p}})^2 \tag{4.3}$$

$$\bar{\sigma}_{p<\bar{p}} = \frac{1}{m_{p<\bar{p}}} \sum_{i=0}^{m_{p<\bar{p}}} (\bar{p} - p_{i,<\bar{p}})^2 \tag{4.4}$$

$m_{p>\bar{p}}$ and $m_{p<\bar{p}}$ are the number of pseudo experiments where the p-value was above/below the average. $p_{i,>\bar{p}}$ and $p_{i,<\bar{p}}$ are the i-th p-value of the p-values that are above/below the average.