Centro de Física Teórica de Partículas
Instituto Superior Técnico
Universidade de Lisboa

# Split NMSSM from dimensional reduction of a 10D, $N = 1$, $E_8$ gauge theory over a modified flag manifold

Gregory Patellis

GP, W. Porod, G. Zoupanos

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## Motivation

Unified description of Nature

- Extra Gauge Symmetry (i.e. GUTs)
- Supersymmetry
- Extra Dimensions
  - Unify gauge and Higgs sectors
  - Also unify fermion interactions with the above sectors
  - SUSY can unify all the above in one vector supermultiplet
  - Less free parameters

# Coset Space Dimensional Reduction

1. Compactification

   $B$ - compact space

   $\dim B = D - 4 = d$

   $D$ dims $\rightarrow$ 4 dims

$$\begin{array}{ccccc} M^D & \rightarrow & M^4 & \times & B \\ | & & | & & | \\ x^M & & x^\mu & & y^a \end{array}$$

2. Dimensional Reduction

   $\mathcal{L}$ independent of the extra coordinates $y^a$:

   - "Naive" way: Discard the field dependence on $y^a$ coordinates
   - Elegant way: Allow field dependence on $y^a$
     $\rightarrow$ compensated by a symmetry of the Lagrangian $\rightarrow$ Gauge Symmetry

3. Coset Space Dimensional Reduction

   *Witten (1977); Forgacs, Manton (1980); Chapline, Slansky (1982); Kapetanakis, Zoupanos - Phys.Rept. (1992) Kubyshin, Mourao, Rudolph, Volobujev - Book (1989)*

   – $B = S/R$

   – Allow a non-trivial dependence on $y^a$

   – impose the condition that a symmetry transformation by an element of the isometry group $S$ of $B$ is compensated by a gauge transformation

   $\rightarrow$ Gauge invariant $\mathcal{L} \rightarrow \mathcal{L}$ independent of $y^a$ !

## Reduction of a D-dimensional Yang-Mills Lagrangian

Consider a Yang-Mills-Dirac theory in $D$ dims based on group $G$ defined on $M^D \rightarrow M^4 \times S/R$ , $D = 4 + d$

$$S = \int d^4x d^d y \sqrt{-g} \left[ -\frac{1}{4} Tr(F_{MN}F_{K\Lambda}) g^{MK} g^{N\Lambda} + \frac{i}{2} \overline{\psi} \Gamma^M D_M \psi \right]$$

Demand: any transformation by an element of $S$ acting on $S/R$ is compensated by gauge transformations.

$\rightarrow$ Constraints on the fields of the theory $A_\alpha$ and $\psi$

Solution of constraints:

- The 4D gauge group: $H = C_G(R_G)$ (i.e. $G \supset R_G \times H$)
- 4D (surviving) fields — $D = 4n + 2$ Weyl + Majorana fermions in vector-like rep $\rightarrow$ 4D chiral theory.
- Scalar Potential

## The 4D Theory

Integrate out the extra coordinates ( + take into account constraints):

$$S = C \int d^4x \, \mathrm{tr}\left[ -\frac{1}{8} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} (D_\mu \phi_a)(D^\mu \phi^a) \right] + V(\phi) + \frac{i}{2} \bar{\psi} \Gamma^\mu D_\mu \psi - \frac{i}{2} \bar{\psi} \Gamma^a D_a \psi$$

where

$$V(\phi) = -\frac{1}{8} g^{ac} g^{bd} \mathrm{Tr} \left\{ (f_{ab}^C \phi_C - ic[\phi_a, \phi_b])(f_{cd}^D \phi_D - ic[\phi_c, \phi_d]) \right\}$$

$V(\phi)$ still only formal since $\phi_a$ must satisfy one more constraint.

- If $G \supset S \Rightarrow H$ breaks to $K = C_G(S)$:

$$\begin{array}{ccccl} G \supset & S & \times & K & \leftarrow \text{ gauge group after SSB} \\ & \cup & & \cap & \\ G \supset & R & \times & H & \leftarrow \text{ gauge group in 4 dims} \end{array}$$

*Harnad, Shnider, Tafel (1980)*

## Reduction of 10D, $N = 1$ $E_8$ over $S/R = SU(3)/U(1) \times U(1)$

Manousselis, Zoupanos (2001-2004)

The non-symmetric (nearly-Kähler) coset space $SU(3)/U(1) \times U(1)$:

- admits torsion and may have different radii
- naturally produces soft supersymmetry breaking terms
- preserves the supersymmetric multiplets

We use the decomposition

$$E_8 \supset E_6 \times SU(3) \supset E_6 \times U(1)_A \times U(1)_B$$

and choose $R = U(1)_A \times U(1)_B$

$$\rightarrow H = C_{E_8}(U(1)_A \times U(1)_B) = E_6 \times U(1)_A \times U(1)_B$$

- $N = 1,\ E_6 \times U(1)_A \times U(1)_B$ gauge group
- Three chiral supermultiplets $A^i : 27_{(3,1/2)}\ ,\ B^i : 27_{(-3,1/2)}\ ,\ C^i : 27_{(0,-1)}$
- Three chiral supermultiplets $A : 1_{(3,1/2)}\ ,\ B : 1_{(-3,1/2)}\ ,\ C : 1_{(0,-1)}$
- Gaugino mass $M = (1 + 3\tau)\frac{R_1^2+R_2^2+R_3^2}{8\sqrt{R_1^2 R_2^2 R_3^2}}$

## The Wilson Flux Breaking

Hosotani (1983); Witten (1985); Zoupanos (1988);
Kozimirov, Kuzmin, Tkachev (1989); Kapetanakis, Zoupanos (1989)

$$M^4 \times B_0 \to M^4 \times B, \qquad B = B_0 / F^{S/R}$$

– $F^{S/R}$ is a freely acting discrete symmetry of $B_0$.

$B$ becomes multiply connected $\to$ breaking of $H$ to $H' = C_H(T^H)$

$T^H$ is the image of the homomorphism of $F^{S/R}$ into $H$

In our case

- $F^{S/R} = \mathbb{Z}_3 \to B = SU(3)/U(1) \times U(1) \times \mathbb{Z}_3$
- $H = E_6 \times U(1)_A \times U(1)_B$
- $H' = SU(3)_c \times SU(3)_L \times SU(3)_R \times U(1)_A \times U(1)_B$ , still $N = 1$

Matter fields invariant under $F^{S/R} \oplus T^H$ survive

$\to \gamma_3 = \mathrm{diag}(\mathbf{1}, \omega\mathbf{1}, \omega^2\mathbf{1}), \quad \omega = e^{2i\pi/3} \in \mathbb{Z}_3$

Irges, Zoupanos (2011)

The surviving matter fields are given by:

- $A^i = \gamma_3 A^i, \quad B^i = \omega\gamma_3 B^i, \quad C^i = \omega^2\gamma_3 C^i$
- $A = A, \quad B = \omega B, \quad C = \omega^2 C$

$$E_6 \supset SU(3)_c \times SU(3)_L \times SU(3)_R \qquad 27 = (1,3,\bar{3}) \oplus (\bar{3},1,3) \oplus (3,\bar{3},1)$$

Surviving matter content of the projected theory:

$$A_1 \equiv L \sim (1,3,\bar{3})_{(3,\frac{1}{2})}, \quad B_2 \equiv q^c \sim (\bar{3},1,3)_{(-3,\frac{1}{2})},$$
$$C_3 \equiv Q \sim (3,\bar{3},1)_{(0,-1)}, \quad A \equiv \theta \sim (1,1,1)_{(3,\frac{1}{2})}$$

Non-trivial monopole charges in $R \rightarrow$ three generations: $L^{(l)}, q^{c(l)}, Q^{(l)}, \theta^{(l)}$

Dolan (2003)

$$L = \begin{pmatrix} H_d^0 & H_u^+ & \nu_L \\ H_d^- & H_u^0 & e_L \\ \nu_R^c & e_R^c & N \end{pmatrix}_{(3,1/2)}, \quad q^c = \begin{pmatrix} d_R^{1c} & u_R^{1c} & D_R^{1c} \\ d_R^{2c} & u_R^{2c} & D_R^{2c} \\ d_R^{3c} & u_R^{3c} & D_R^{3c} \end{pmatrix}_{(-3,1/2)}, \quad Q = \begin{pmatrix} d_L^1 & d_L^2 & d_L^3 \\ u_L^1 & u_L^2 & u_L^3 \\ D_L^1 & D_L^2 & D_L^3 \end{pmatrix}_{(0,-1)}$$

## The Effective Unified Theory

If an effective *4D* theory is renormalizable by power counting, it is consistent to consider it as a renormalizable theory. *Polchinski (1984)*

$\rightarrow$ We respect the symmetries and model structure that are derived by the higher-dimensional theory and its dimensional reduction.

$\rightarrow$ We treat all the parameters of the effective theory as free parameters, to the extend allowed by symmetries.

i.e.

- one coupling for all kinetic terms + *D*-terms
- one coupling for all superpotential terms
- independent couplings for soft SUSY + R-symmetry breaking terms

$$\mathcal{W}^{(l)} = C^{(l)} d^{abc} L_a^{(l)} q_b^{c(l)} Q_c^{(l)}$$

$$\begin{aligned}
V_{soft}^{(l)} = & \left( \frac{c_{L_1}^{(l)} R_1^2}{R_2^2 R_3^2} - \frac{c_{L_2}^{(l)}}{R_1^2} \right) \langle L^{(l)} | L^{(l)} \rangle + \left( \frac{c_{\theta_1}^{(l)} R_1^2}{R_2^2 R_3^2} - \frac{c_{\theta_2}^{(l)}}{R_1^2} \right) |\theta^{(l)}|^2 \\
& + \left( \frac{c_{q_1^c}^{(l)} R_2^2}{R_1^2 R_3^2} - \frac{c_{q_2^c}^{(l)}}{R_2^2} \right) \langle q^{c(l)} | q^{c(l)} \rangle + \left( \frac{c_{Q_1}^{(l)} R_3^2}{R_1^2 R_2^2} - \frac{c_{Q_2}^{(l)}}{R_3^2} \right) \langle Q^{(l)} | Q^{(l)} \rangle \\
& + \left( \frac{R_1}{R_2 R_3} + \frac{R_2}{R_1 R_3} + \frac{R_3}{R_1 R_2} \right) \left( c_\alpha^{(l)} d^{abc} L_a^{(l)} q_b^{c(l)} Q_c^{(l)} + c_b^{(l)} d^{abc} L_a^{(l)} L_b^{(l)} L_c^{(l)} + h.c \right)
\end{aligned}$$

$$\begin{aligned}
= & \, m^2_{L^{(l)}} \langle L^{(l)} | L^{(l)} \rangle + m^2_{q^{c(l)}} \langle q^{c(l)} | q^{c(l)} \rangle + m^2_{Q^{(l)}} \langle Q^{(l)} | Q^{(l)} \rangle + m^2_{\theta^{(l)}} |\theta^{(l)}|^2 \\
& + \left( \alpha^{(l)abc} L_a^{(l)} q_b^{c(l)} Q_c^{(l)} + b^{(l)abc} L_a^{(l)} L_b^{(l)} L_c^{(l)} + h.c \right)
\end{aligned}$$

where $c_i^{(l)}$ are free parameters of $\mathcal{O}(1)$.

# Choice of Radii

Two main possible directions:

- Large $R_i$ $\rightarrow$ calculation of the Kaluza-Klein contributions of the 4D theory
  $\times$ Eigenvalues of the Dirac and Laplace operators unknown.

- Small $R_i$ $\rightarrow$ high scale SUSY breaking

We choose $M_C = M_{GUT}$: *Manolakos, Patellis, Zoupanos (2020)*

— Soft trilinear terms $\sim \frac{1}{R_i} \sim \mathcal{O}(M_{GUT})$

— Soft squared scalar masses $\sim \frac{1}{R_i^2} \sim \mathcal{O}(M_{GUT}^2)$

- $R_3$ slightly different from $R_{1,2}$ in a specific configuration of $c_{\theta_i}^{(3)}$

$$\rightarrow \quad m^2_{\theta_s^{(3)}} \sim \mathcal{O}(M_{EW}^2)$$

# GUT Breaking

Babu, He, Pakvasa (1986); Ma, Mondragon, Zoupanos (2004); Leontaris, Rizos (2006); Sayre, Wiesenfeldt, Willenbrock (2006)

The GUT breaks by the following vevs:

$$\langle L_s^{(1)}\rangle = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & V_1 \end{pmatrix}, \quad \langle L_s^{(2)}\rangle = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ V_2 & 0 & 0 \end{pmatrix}, \quad \langle L_s^{(3)}\rangle = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ V_3 & 0 & V_4 \end{pmatrix}$$

$$\langle \theta_s^{(1)}\rangle = V_5 \,, \quad \langle \theta_s^{(2)}\rangle = V_6$$

— The combination of $V_1$, $V_2$ breaks trinification to the SM group

— One $V_{3,4}$ and one of $V_{5,6}$ break the abelian symmetries

EW breaking then proceeds by:

$$\langle L_s^{(3)}\rangle = \begin{pmatrix} u_d & 0 & 0 \\ 0 & u_u & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$SU(3)^3 \times U(1)^2 \xrightarrow{V_i} SU(3)_c \times SU(2)_L \times U(1)_Y \xrightarrow{u_{u,d}} SU(3)_c \times U(1)_{em}$$

# Missing Terms

- $\mu$ terms for each generation of Higgs doublets are absent
  $\rightarrow$ non-negligible higher-dimensional operators: $H_u^{(i)} H_d^{(i)} \overline{\theta}^{(i)} \frac{\overline{K}}{M}$ [3]
- The $U(1)$s forbid lepton Yukawa terms in the superpotential
  $\rightarrow$ non-negligible higher-dimensional operators: $L\bar{e}H_d \left(\frac{\overline{K}}{M}\right)^3$

Including all non-negligible higher-dimensional operators

- The $N$, $\nu_R$ and $\theta^{(1,2)}$ fermions become supermassive
- The $\theta^{(3)}$ fermion acquires an $m \sim \mathcal{O}(EW)$ due to a cancellation among vevs

[3] $\overline{K}$ is the conjugate of any field that acquires a superheavy vev

## Low Energy Effective Model

| | |
|---|---|
| $D$, $N$, $\nu_R$ masses | $\mathcal{O}(GUT)$ |
| sfermion masses | $\mathcal{O}(GUT)$ |
| soft trilinear couplings / soft B term | $\mathcal{O}(GUT)$ |
| soft Higgs mass parameters | $\mathcal{O}(GUT)$ |
| fermion & scalar $\theta^{(1,2)}$ masses | $\mathcal{O}(GUT)$ |
| fermion & scalar $\theta^{(3)}$ masses | $\mathcal{O}(EW)$ |
| unified gaugino mass | $\mathcal{O}(EW)$ |

- $\theta^{(3)} \equiv S$ singlet superfield & gauginos $\rightarrow$ light
- Trilinear term $H_u^{(3)} H_d^{(3)} \theta^{(3)} \equiv \lambda H_u H_d S \xrightarrow{<\theta^{(3)}>\sim\mathcal{O}(EW)}$ light ($\mu$-like) term
- sfermions $\rightarrow$ superheavy
- soft Higgs mass parameters, effective soft B-parameter $\rightarrow$ superheavy $\rightarrow$ light Higgs mass

$\rightarrow$ **Split NMSSM**

Demidov, Gorbunov 2007;
Gabelmann, Mühlleitner, Staub 2019

# 2-loop Analysis

Analysis focused on 3rd gen: SARAH → SPHENO

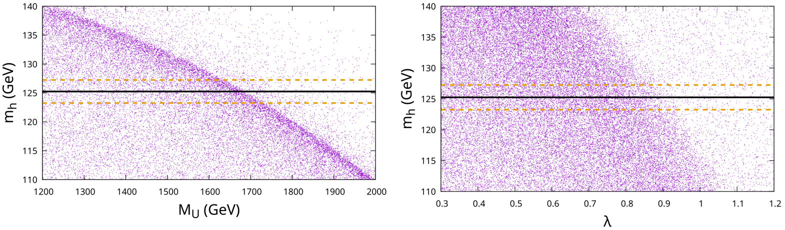
Porod 2003; Staub 2010

- At $M_{GUT} \to Y_t = Y_b$ from boundary conditions for the 2-loop RGEs that run down to $M_{EW}$. In order to satisfy experimental limits:

$$m_t = (172.69 \pm 0.30)\text{ GeV} \quad , \qquad m_b(m_b) = (4.18 \pm 0.03)\text{ GeV}$$

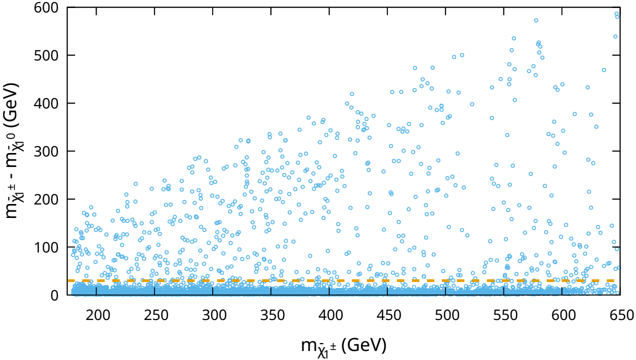
$$\to 70 < tan\beta < 80$$

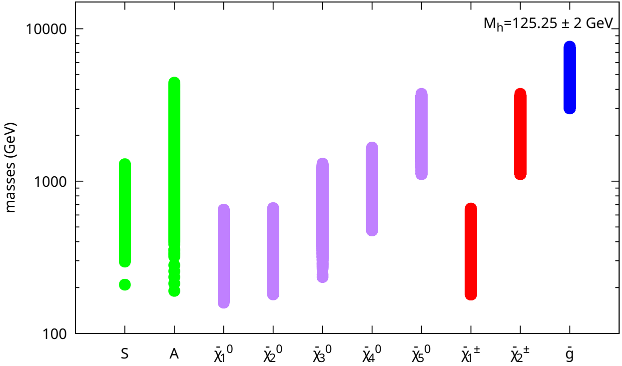
- The light Higgs mass is calculated with a 2 GeV theoretical uncertainty:



- Lightest neutralino → LSP
- Lightest chargino > 180 GeV
- The majority of points satisfy $m_{\tilde{\chi}_1^\pm} - m_{\tilde{\chi}_1^0} < 30$ GeV
- LSP mostly Higgsino-like (with singlino contribution)

→ LSP survives lower bounds from ATLAS and CMS






- *S* and *A* gauge singlets $\rightarrow$ no production at the LHC
- Gluino $>$ 2 TeV, consistent with non-detection
  $\rightarrow$ model difficult to probe in coming LHC runs
- HL-LHC could detect $m_{\tilde{\chi}_1^\pm} < 200$ GeV

## Conclusions

The Model:

- $10D$, $N = 1$, $E_8$ reduced over $SU(3)/U(1)^2 \times \mathbb{Z}_3$
  $\rightarrow SU(3)_c \times SU(3)_L \times SU(3)_R \times U(1)_A \times U(1)_B$
- Soft terms of geometrical origin due to CSDR
- Special choice of small coset radii for Split NMSSM below GUT

Results

- Top, bottom and light Higgs mass in agreement with latest LHC data
- (Light) SUSY spectrum consistent with non-detection
- HL-LHC could see the lower part of the spectrum

Next:

- Neutralino LSP could be CDM
- Detailed study for collider searches