# NANOGrav spectral index γ=3 from melting domain walls


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# Outline

- Main idea and the model
- Appearance of domain walls and their melting
- Gravitational waves from domain walls
- NANOGRav spectral index from melting domain walls
- Late time evolution: Dark matter

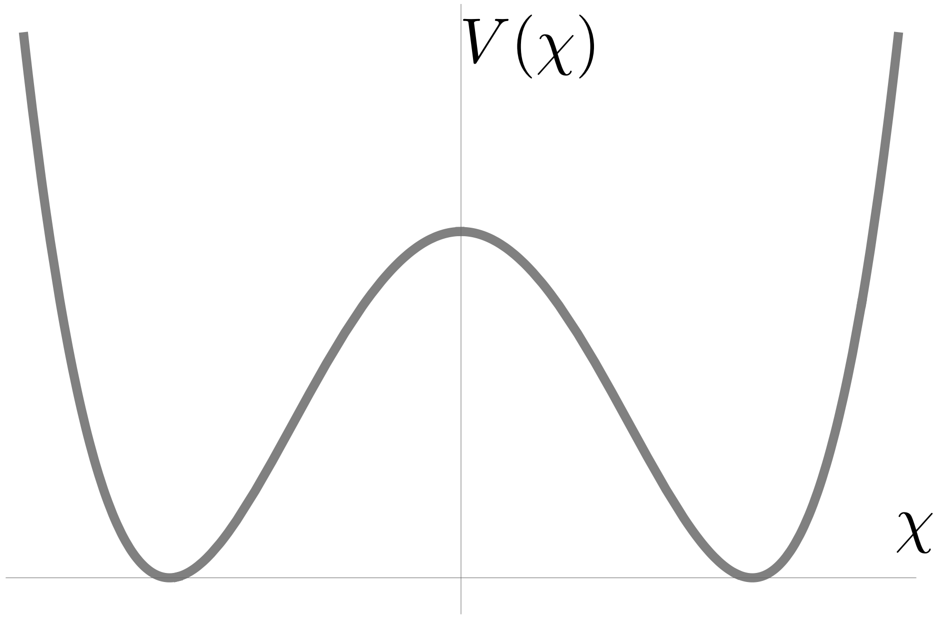
# Main idea and the model

# "Standard" domain walls

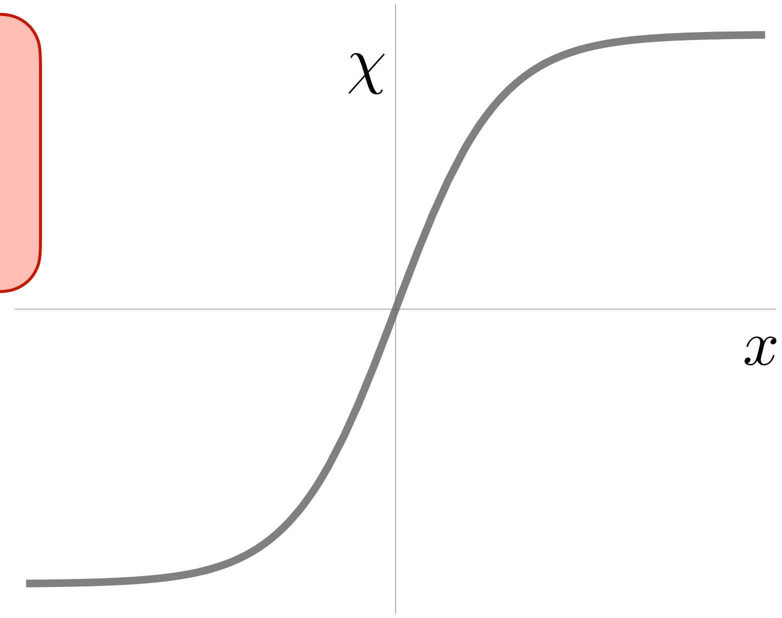
$$\mathcal{L} = \frac{1}{2}(\partial_\mu \chi)^2 - V(\chi)$$

$$V(\chi) = \frac{\lambda}{4}\left(\phi^2 - \eta^2\right)^2$$

(+---) signature




Kink solution $\chi(x) = \eta \tanh\left(\sqrt{\frac{\lambda}{2}}\eta x\right)$




Width $\delta \sim \left(\sqrt{\lambda}\eta\right)^{-1}$

Surface density $\sigma \sim \sqrt{\lambda}\eta^3$

# "Standard" domain walls in cosmology

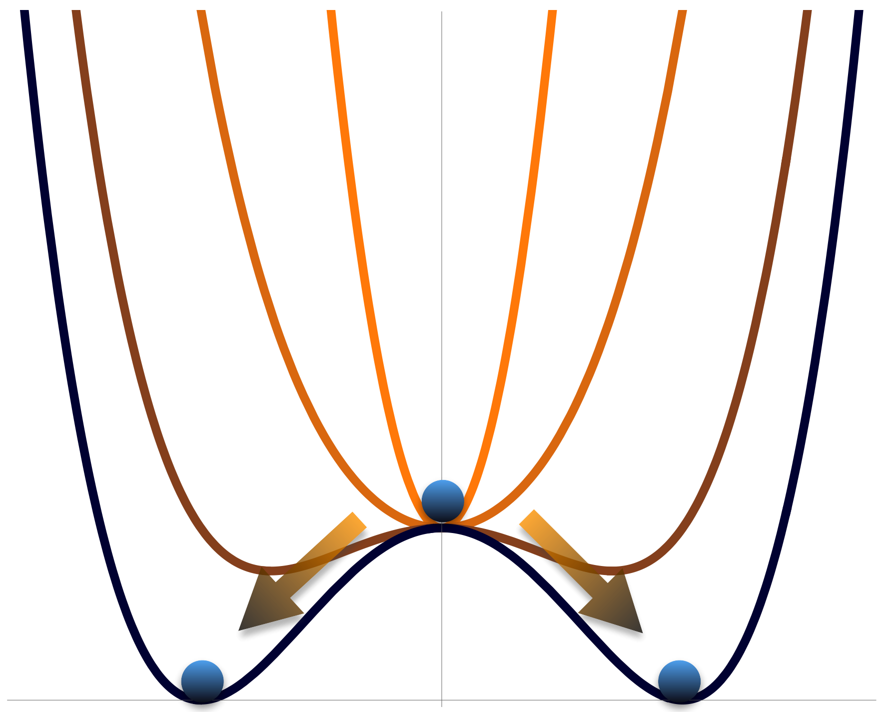
$$\mathcal{L} = \frac{1}{2}(\partial_\mu \chi)^2 - V_T(\chi)$$

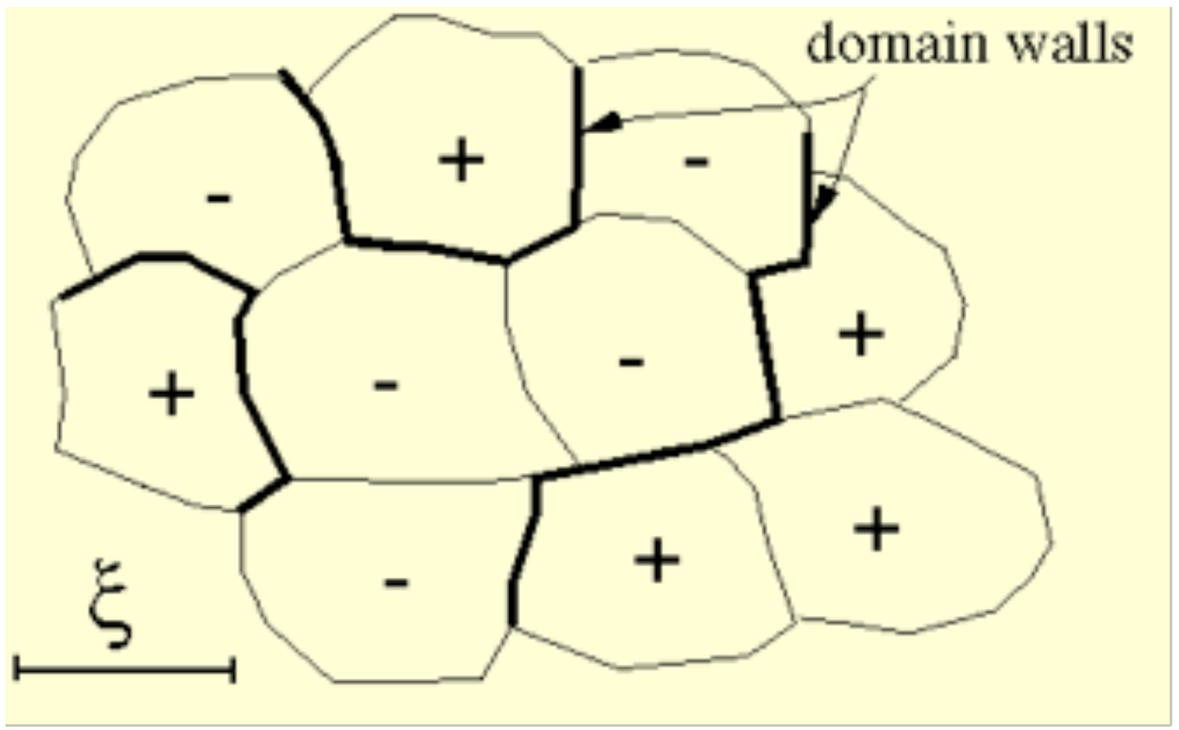
$$V(\chi) = \frac{\lambda}{4}\left(\phi^2 - \eta^2\right)^2$$

At high temperatures (early times) the potential receives temperature-dependent corrections:

$$V_T(\chi) = V(\chi) - \frac{g_* \chi^2}{90} T^4 + \frac{M^2(\chi)}{24} T^2$$

The symmetry is restored at high temperatures and it is broken at low temperatures





# Problems with domain walls in cosmology

Scaling regime (about 1 open DW within the cosmological radius):

$$\rho_{wall} \sim \frac{\sigma R_c^2}{R_c^3} \sim \frac{\sigma}{R_c} \propto T^2$$

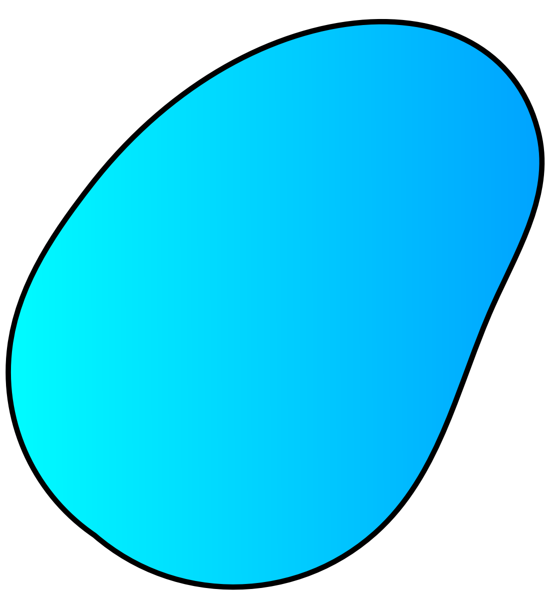
*assuming radiation domination*

$$\rho_{rad} \propto T^4$$

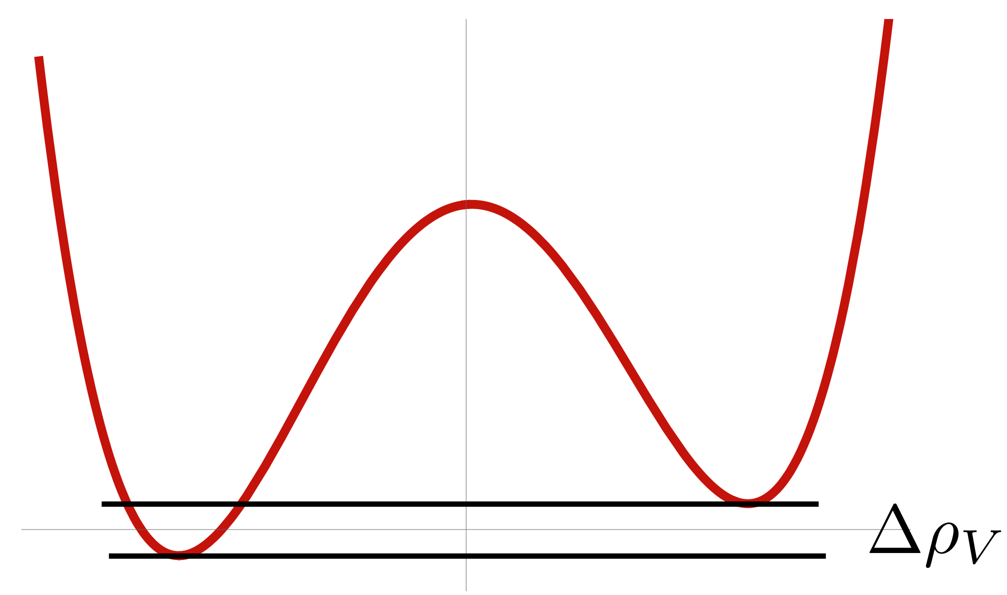
At low $T$ domain walls dominate the universe.

*Avoiding domain walls domination*

DW are attached to cosmic strings



Only *approximate* shift symmetry



# Melting DW: main idea and the model

(+−−−) signature

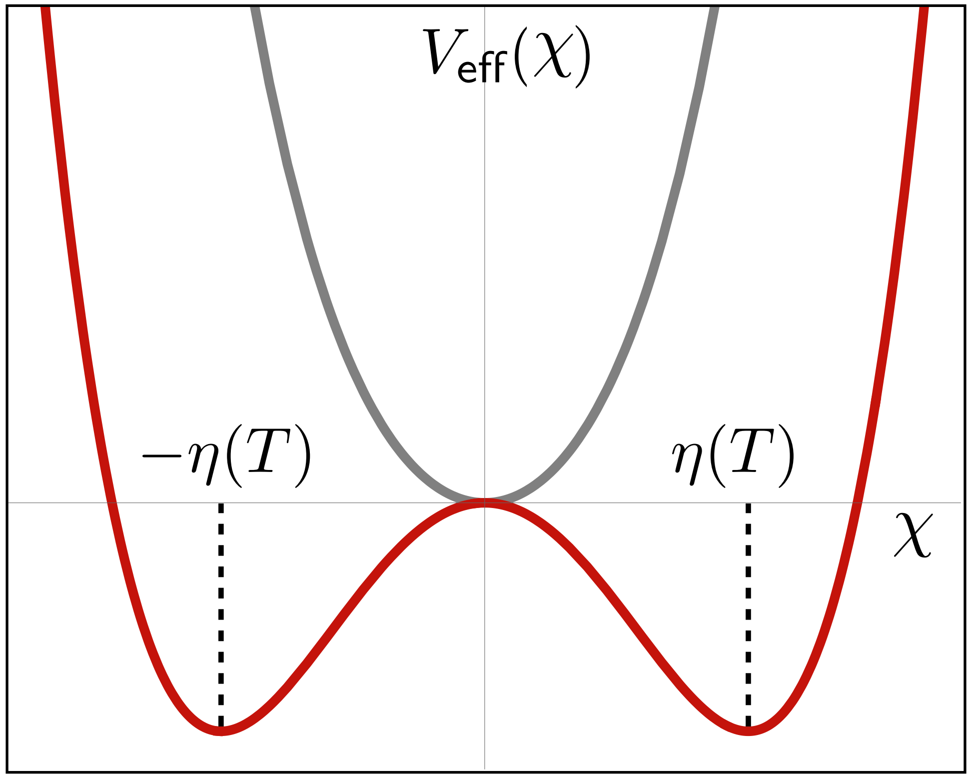
$$\mathcal{L} = \frac{(\partial_\mu \chi)^2}{2} - V_{\text{eff}}(\chi) + \frac{(\partial\phi^\dagger\partial\phi)^2}{2} - \frac{m_\phi^2\phi^2}{2} - \frac{\lambda_\phi\phi^4}{4} + \frac{\xi R\chi^2}{2}$$

$$V_{\text{eff}} = \frac{M^2\chi^2}{2} + \frac{\lambda\chi^4}{4} - \frac{g^2\chi^2\phi^\dagger\phi}{2}$$

- $\chi$ is the field of domain walls and the dark matter field. $Z_2$ symmetry protects stability.
- $\phi$ is in thermal equilibrium with hot plasma (could be Higgs field).
- "Normal" sign of the mass squared.
- "Wrong" sign of the coupling term.
- Thermal fluctuations of $\phi$ feed into the effective mass of $\chi$. The coupling between $\chi$ and $\phi$ provides symmetry breaking:

$$\langle\phi^\dagger\phi\rangle_T \approx \frac{NT^2}{12} \quad \Rightarrow \quad \eta^2(T) \approx \frac{Ng^2T^2}{12\lambda}$$

$N$ is the number of d.o.f. in $\phi$. $\eta(T)$ is a non-zero expectation value of $\chi$ at high temperatures.



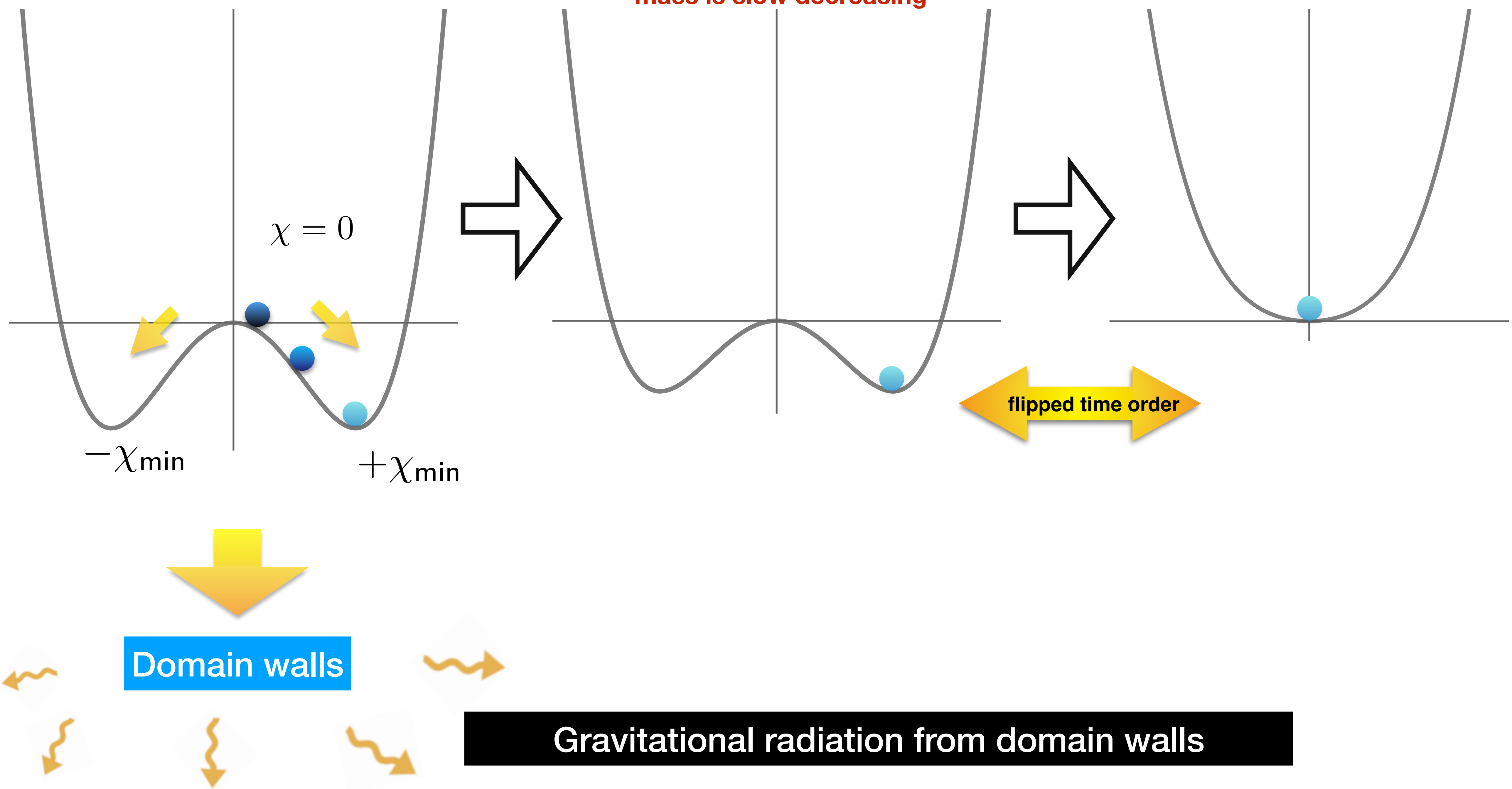
# Melting DW: main idea and the model

$$V_{\text{eff}} = \frac{M^2\chi^2}{2} + \frac{\lambda\chi^4}{4} - \frac{g^2\chi^2\phi^\dagger\phi}{2}$$

**Early Universe (after inflation)**
spontaneously Broken Phase

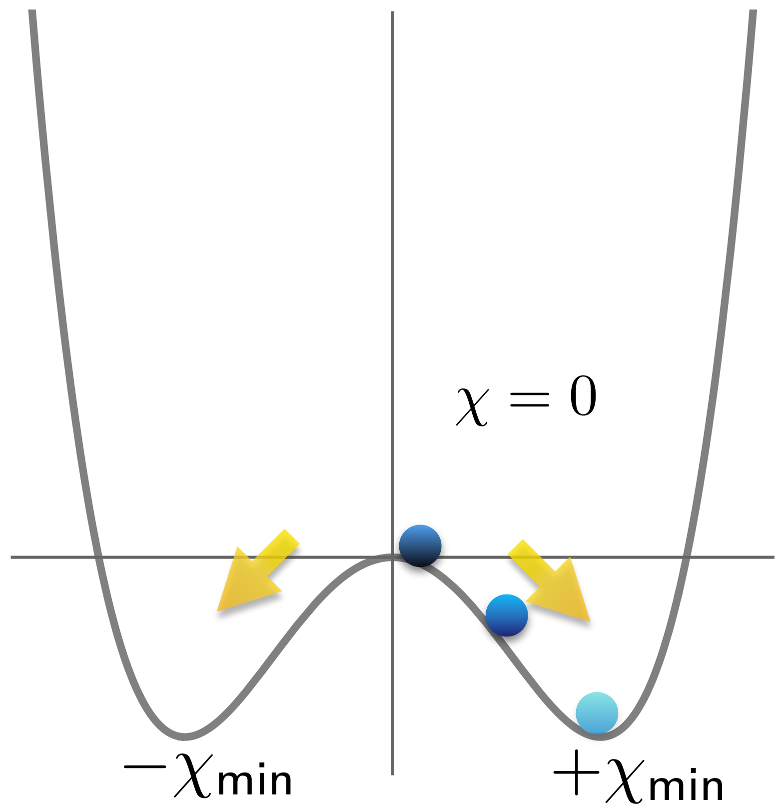
**Early Universe (radiation domination)**
spontaneously Broken Phase, the tachyonic mass is slow decreasing

**Late Universe**
Restored vacuum



## Formation of domain walls

# Formation of domain walls




- The roll starts at $t = t_h$, when

$$|M_{thermal}(T_h)| = \sqrt{\lambda}\,\eta(T) = \frac{N^{1/2} g T_h}{\sqrt{12}} \simeq H_h$$

- We assume radiation stage

$$3M_{Pl}^2 H^2 = \rho = g_*(T)\frac{\pi^2 T^4}{30}$$

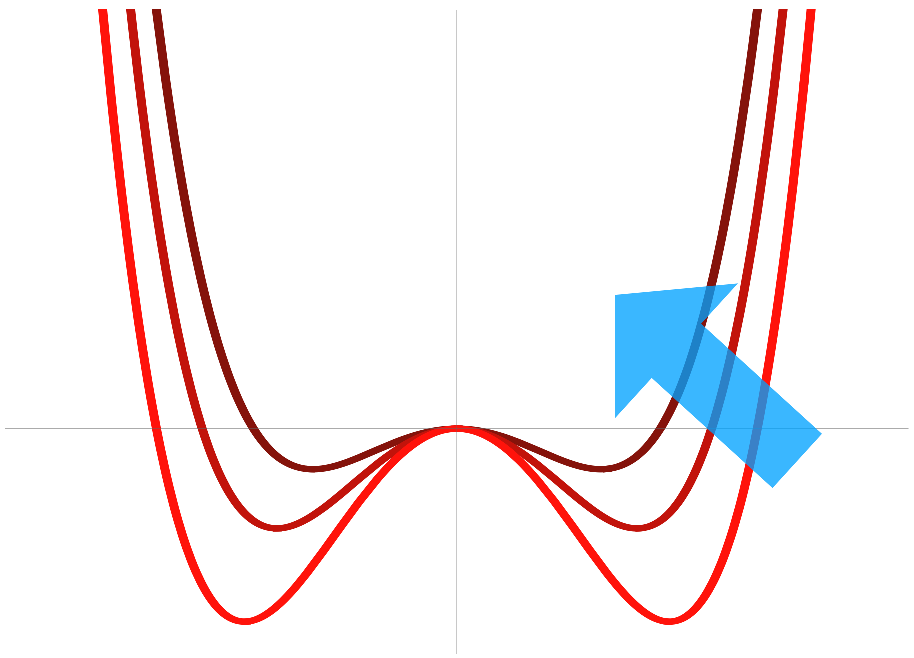
- We can take into account a finite duration of the roll:

$$B \equiv \frac{T_h^2}{T_i^2} \gtrsim 1, \quad T_i = \frac{T_h}{\sqrt{B}} \simeq \frac{N^{1/2} g M_{Pl}}{\sqrt{B g_*(T_i)}}$$

'$i$' is the end of the roll. $B$ can be calculated.

Domain walls start growing before $t_i$. However we neglect the effects of presence of 'pre-domain walls' and we consider $t_i$ (when the temperature of the Universe is $T_i$) to be the moment of formation of domain walls.

# Evolution of domain walls



Domain walls with varying tension!

$$\sigma_{wall} = \frac{2\sqrt{2\lambda}\eta^3(T)}{3}$$

so that $\sigma_{wall}(T) \propto T^3$, thus the tension decreases with time.

- Scaling regime, so that one domain wall in a Hubble patch. The total mass of domain wall inside the Hubble volume: $M_{wall} \sim \sigma_{wall}/H^2$
- the domain wall energy density is $\rho_{wall} \sim M_{wall}H^3 \sim \sigma_{wall}H$.
- the fraction of energy in domain walls:

$$\frac{\rho_{wall}}{\rho_{rad}} \sim \frac{N^2}{30\sqrt{B}g_*(T)\beta} \cdot \frac{T}{T_i}$$

$$\beta \equiv \frac{\lambda}{g^4} \geq 1$$

**NB** Constant tension domain walls: $\frac{\rho_{wall}}{\rho_{rad}} \propto \frac{T^2}{T^4} \propto t$

Known problem in such a scenario

# Gravitational waves from domain walls

# Gravitational waves from domain walls

- The power of gravitational radiation:

$$P \sim \frac{\dddot{Q}_{ij}\dddot{Q}_{ij}}{40\pi M_{Pl}^2}$$

- The quadrupole moment $Q_{ij}$ is related to the wall mass $M_{wall}$ inside the patch of the size $H^{-1}$:

$$|Q_{ij}| \sim \frac{M_{wall}}{H^2}$$

- An estimate of the energy density of GWs produced at the time $t$:

$$\rho_{gw} \sim (P \cdot t) \cdot H^3 \sim \frac{\sigma_{wall}^2}{40\pi M_{Pl}^2} \sim \frac{\lambda \eta^6}{40\pi M_{Pl}^2} \; .$$

$$\rho_{gw} \sim \frac{5N^3 g^6 T^6}{10^6 \lambda^2 M_{Pl}^2}$$

- Most energetic GWs are emitted at high temperatures $T \simeq T_i$ (close to the moment of domain wall formation) !

- This is a direct consequence of the temperature-dependent tension $\sigma_{wall}(T)$.

- Were the tension $\sigma_{wall}$ constant, we would end up with the constant energy density of GWs.

# Gravitational waves from domain walls

For more precise analysis we rely on lattice simulations [Hiramatsu +'14] which investigates the properties of GWs in the scenario with constant tension domain walls.

- The numerical simulation of [Hiramatsu +'14] supports the rough estimate.
- In our scenario domain walls have *varying tension*, unlike the scenario in [Hiramatsu+'14].
- We expect that the qualitative results on total power do not change.
- The spectrum is however modified.

- Numerical simulations of [Hiramatsu+'14] suggest the fit for the quantity $d\rho_{gw}/d\ln f$ at its peak:

$$\Omega_{gw,peak}(t_i) \approx \frac{\lambda \tilde{\epsilon}_{gw} \mathcal{A}^2 \eta_i^6}{27\pi H_i^2 M_{Pl}^4}$$

$\tilde{\epsilon}_{gw} \sim 0.7$ and $\mathcal{A} \sim 0.8$ measure efficiency of GW emission and the scaling property

- The peak frequency $f_{peak}$ is approximately equal to the Hubble rate at the instance of emission $t_i$, i.e., $f_{peak}(t_i) \simeq H_i$ [Hiramatsu+'14]

# Radiated energy at peak frequency (now)

$$\Omega_{gw,peak}h^2 \approx \frac{4 \cdot 10^{-14} \cdot N^4}{B \cdot \beta^2} \cdot \left(\frac{100}{g_*(T_i)}\right)^{7/3}, \qquad \beta \equiv \frac{\lambda}{g^4}$$

$$f_{peak}(t_0) \simeq 60\ \text{Hz} \cdot \left(\frac{N}{B}\right)^{1/2} \cdot \left(\frac{g}{10^{-8}}\right) \cdot \left(\frac{100}{g_*(T_i)}\right)^{1/3}$$

Model parameters

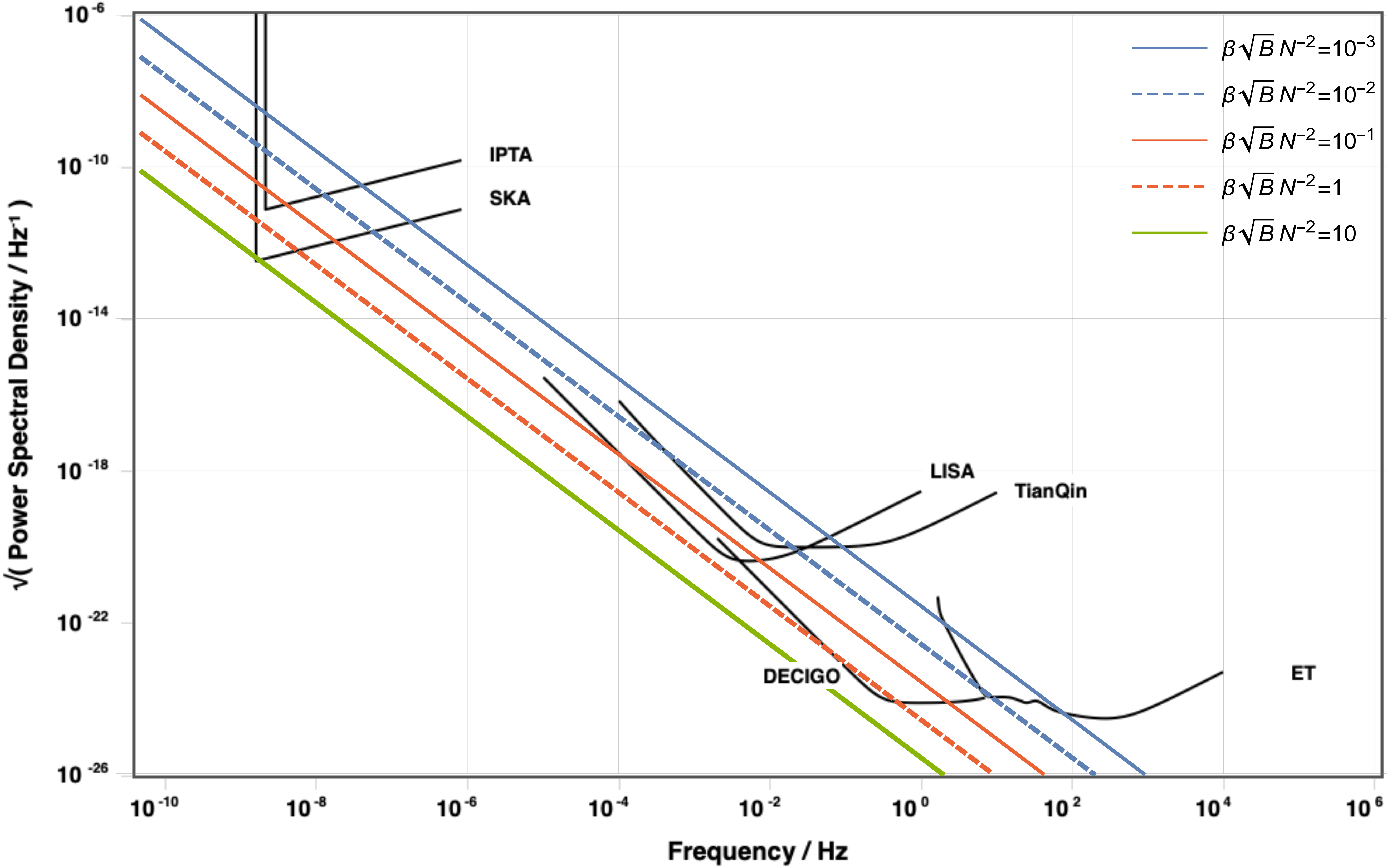
We should are able to determine $g$ and $\lambda$ by studying the peak characteristics of the GW background.

Strain of GWs versus the peak frequency, using `gwplotter.com`:

$$\Omega_{gw} H_0^2 = \frac{2\pi^2 f^3}{3} \cdot S_h$$

Spectral density

Strain: $\sqrt{S_h}$




Strain as a function of peak frequency. $\beta \simeq 1-10$, $B \simeq 1-100$, $N = 4-24$. One point on each colored line corresponds to a particular set of the parameters of the model and it represents the strain at the peak frequency. Not the spectrum!

# Spectrum of GWs (low frequencies)

- Simulations of [Hiramatsu +'14] reveal the behaviour $\Omega_{gw}(f) \propto f^3$ at $f < f_{peak}(t_0)$ (supported by considerations of causality).
- In [Hiramatsu +'14] the largest contribution to GWs comes just before the domain walls collapse.
- In our scenario the situation is reverse: most energetic GWs are emitted close to the time of wall formation $t_i$.
- GWs continue being emitted at $t > t_i$. This later time emission gives the lower frequency part of the GW spectrum. *(The Hubble rate is smaller at later times; hence the characteristic frequency is also lower.)*

# Spectrum of GWs (low frequencies)

- $f \sim H \propto T^2$ while $f_{peak} \sim H_i \propto T_i^2$.
- One has to take into account a relative redshift of waves with frequency $f_i$ when compared at the time of radiation of waves with frequency $f$:

$$f_{peak}(t_f) = f_{peak}\frac{T_f}{T_i} \quad \Rightarrow \frac{f}{f_{peak}} = \frac{T_f}{T_i}.$$

- The present day frequency $f$ of GWs emitted at the time, when the temperature of the Universe $T_f$ (where $T_f < T_i$):

$$\frac{f(t_0)}{f_{peak}(t_0)} \simeq \frac{T_f}{T_i}$$

- The low frequency tail of GWs:

$$\Omega_{gw}(f) \simeq \Omega_{gw,peak} \cdot \left(\frac{f}{f_{peak}(t_0)}\right)^2 \qquad f < f_{peak}(t_0)$$

# Spectrum of GWs (high frequencies)

- In the high frequency range $f > f_{peak}(t_0)$ the results of [Hiramatsu +'14] are applicable:

$$\Omega_{gw}(f) \simeq \Omega_{gw,peak} \cdot \left(\frac{f_{peak}(t_0)}{f}\right) \qquad f_{peak}(t_0) < f < f_{cut}(t_0)$$

$f_{cut}(t_0)$ is the cutoff frequency defined by the domain wall width, i.e.,

$$f_{cut}(t_0) \simeq (\delta_{w,i})^{-1} \cdot \frac{a_i}{a_0} \simeq f_{peak}(t_0)\sqrt{B}$$

- The difference of $f_{cut}(t_0)$ and $f_{peak}(t_0)$ is a factor $\sqrt{B}$ (not very large). The spectrum of GWs has a very short tale at high frequencies.
In the standard scenario with constant tension domain walls, DW thickness is normally much smaller than the horizon size. $\Rightarrow$ An extended high-frequency tale.

# Another variant of the scenario: GWs from domain walls at preheating

- We assumed before that domain walls emerge at the radiation dominated epoch. The scenario is self-consistent for $T_{reh} > T_h$.

- For sufficiently low reheating temperatures and/or small constants $g$, this inequality cannot be fulfilled.
- One should consider the domain wall emergence, evolution and GWs emission separately.
- The conclusion that generically the signal is lower than in the case of radiation domination.
- The peak frequency is different. Hence, for the same values of the model parameters, cosmological models with low reheating temperatures may have better chances to be probed with future GW detectors.

# NANOGRav spectral index from melting domain walls

# Recent results from PTAs

- Several pulsar timing arrays (PTAs: NANOGrav, EPTA, InPTA, PPTA, CPTA) reported evidence pointing to a nHz stochastic gravitational wave (GW) background.
- Signals from all the PTAs are in a good agreement.
- Focus here on the NANOGrav 15yrs data, as being more stringent and with the largest statistical significance.
- No clear hints on the origin of the observed signal.
- Disfavor simple GW-driven models of supermassive black hole binaries (SMBHBs) with $\Omega_{gw} \propto f^{2/3}$.
- A power law signal $\Omega_{gw} \propto f^{1.2-2.4}$ is preferred.

**Melting domain walls is a perfect candidate**

$$\Omega_{gw} \propto f^2$$

# Recent results from PTAs

- Conventional parameterization of the PTA GW signal:

$$\Omega_{gw} = \Omega_{yr} \left( \frac{f}{f_{yr}} \right)^{5-\gamma}$$

where $\gamma$ is the spectral index and $f_{yr} = 1$ yr$^{-1} \simeq 32$ nHz.

- The NANOGrav best-fit value of the spectral index $\gamma = 3.2 \pm 0.6$.

- $\Omega_{yr} = 5.8 \times 10^{-8}$. We assume $f_{peak} \simeq f_{yr}$, so $\Omega_{gw,peak} \simeq \Omega_{yr}$.

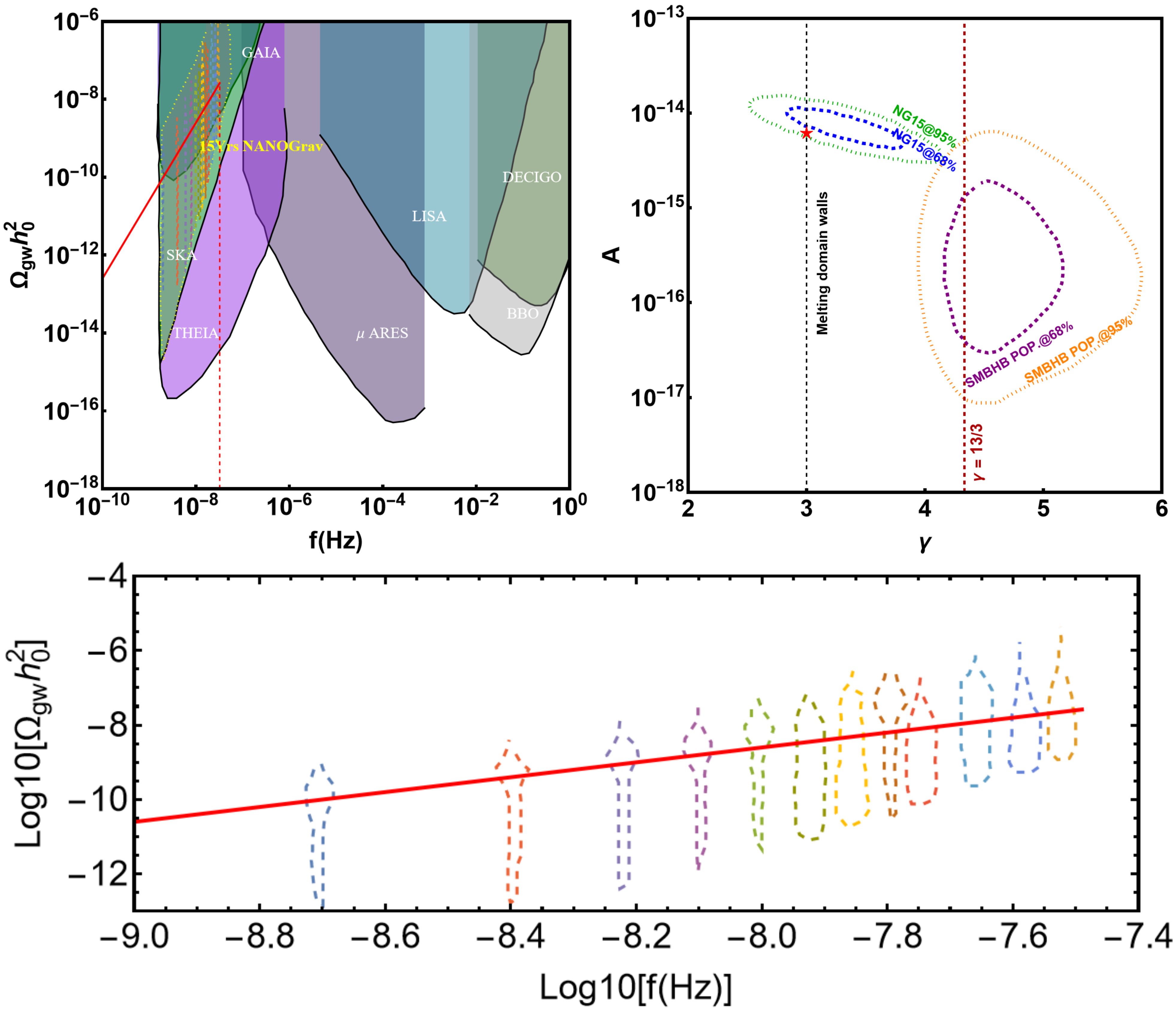
- The temperature of DW formation:

$$T_i \simeq 1.2 \text{ GeV}$$

- Let us fix the parameters of the model:

$$\beta = 1, \;\; B = 1, \;\; N = 24 \quad \Rightarrow \quad g = 10^{-18}$$

# Recent results from PTAs

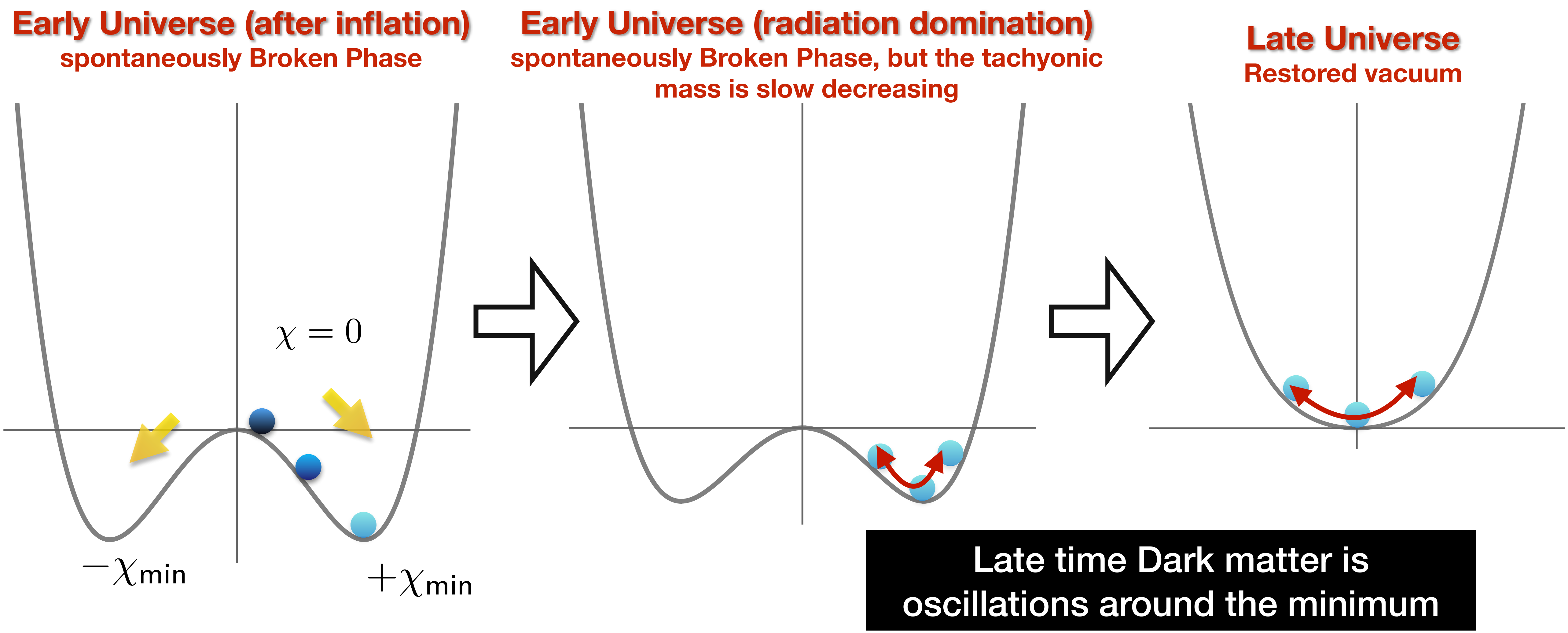


# Dark Matter

# Dark matter: no freeze-in or freeze-out

- $|g^2| \simeq 0.1 - 10^{-8} \Rightarrow$ freeze-out
- $|g^2| \simeq 10^{-11} \Rightarrow$ freeze-in
  [Chu, Hambye, Tytgat'11, Yaguna'11, Lebedev & Toma'19]
- $0 < g^2 \lesssim 10^{-11} \Rightarrow$ second order phase transition

# Dark matter generation




**Two scenarios of DM generation:**

1. Oscillations are generated at the phase transition. They persist during the whole evolution, and can have the right DM abundance.

2. For some reason oscillations gets dumped after the phase transition, but DM with observable abundance is generated at the *inverse* phase transition.

# DM for parameters favored by NANOGrav

**Two scenarios of DM generation:**

1. $$M_\chi \simeq 6.5 \cdot 10^{-17}\ \mathrm{eV} \cdot \left(\frac{f_{peak}}{30\ \mathrm{nHz}}\right) \cdot \left(\frac{g_*(T_i)}{100}\right)^{1/6} \cdot \sqrt{\frac{10^{-8}}{\Omega_{gw,peak} \cdot h_0^2}}$$

2. $$M_\chi \simeq 10^{-12}\ \mathrm{eV} \cdot B^{9/20} \cdot \left(\frac{g_*(T_{sym})}{100}\right)^{1/5} \cdot \left(\frac{g_*(T_i)}{100}\right)^{1/20} \cdot \left(\frac{m_\phi}{10\ \mathrm{MeV}}\right)^{1/2} \times \left(\frac{f_{peak}}{30\ \mathrm{nHz}}\right)^{6/5} \cdot \left(\frac{10^{-8}}{\Omega_{gw,peak} h_0^2}\right)^{3/20},$$

- In both cases GW parameters favoured by NANOGrav data imply ultra-light dark matter masses $M_\chi$.
- With these values of $M_\chi$, our scenario predicts super-radiance instability of rotating black holes with astrophysical masses.
- Complementary way of testing the model:
  - future LISA observations will probe the masses of dark matter particles corresponding to the direct phase transition;
  - LIGO data may be used to test the masses involved in the inverse phase transition.

# Conclusions

- **Properties of GWs emitted by the network of melting domain walls are consistent with the signal detected at PTAs. (Melting domain walls do not overclose the Universe.)**

- **Constituent field $\chi$ serves as a suitable dark matter candidate (Complimentary way of testing the model).**

- **Numerical study of melting domain wall evolution: peak frequency, energy density, the spectral shape, formation and scaling of DWs?**