ALP dark matter
and
Early Universe dynamics of PQ field

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Outline

- Introduction
- Evolution of PQ field during inflation
  - radiative corrections
  - corrections from space-time curvature
- Evolution of PQ field after inflation
  - thermal corrections
- Axions as cold and warm dark matter
- Summary

Introduction

- **Axions (QCD axions and ALPs) are interesting candidates for Dark Matter**
  - **pseudo Goldstone bosons of spontaneously broken Peccei-Quinn global $U(1)_{\rm PQ}$ symmetry**
  - **phase component of complex PQ scalar**

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- **Different mechanisms leading to relic abundance of axions**
  - misalignment — Dine . . . Sikivie . . . Wilczek 1983
  - parametric resonance — Kofman et al 1994
  - gravitational production — Ema et al 2015
  - rotating axions — Co Hall Harigaya 2019
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- Two basic scenarios for misalignment
  - $U(1)_{\rm PQ}$ broken (during inflation)

$$f_a \gg H_I$$

  - $U(1)_{\rm PQ}$ unbroken (during inflation)

$$f_a \ll H_I$$

Simple potential for PQ field

$$V(\Phi) = \lambda_\Phi \left(|\Phi|^2 - \frac{f_a}{2} \right)^2 = \frac{1}{4} \lambda_\Phi \left(S^2 - f_a^2 \right)^2$$

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- $U(1)_{\mathrm{PQ}}$ broken during and after inflation because $f_a \gg H_I$
  - $S = f_a$
  - $\langle\theta\rangle$ determined by some stochastic process
    during the phase transition from unbroken to broken $U(1)_{\mathrm{PQ}}$
  - isocurvature perturbations are generated during inflation
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- $U(1)_{\rm PQ}$ unbroken until T drops below $T_c = \mathcal{O}(f_a) \ll H_I$
  - before the phase transition: $S = 0$, θ undefined
  - just after phase transition: $S = f_a$ flat stochastic distribution of θ with $\langle \delta\theta^2 \rangle = \frac{\pi^2}{3}$
  - isocurvature white noise but only on scales smaller than the Hubble radius when the axion field starts to oscillate

Feix et al 2019

What does "unbroken $U(1)_{\mathrm{PQ}}$" mean?

- Potential with (global) minimum at $\Phi_{\mathrm{PQ}} = 0$ does not guarantee that Φ_{PQ} vanishes
- Light enough fields fluctuate during inflation
- Heavy enough fields displaced from a minimum of their potential oscillate

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In this talk I will concentrate on models in which PQ-like field has nontrivial dynamics during and after inflation

PQ field during inflation

Axion field a is massless (or at least very light) during inflation $\Rightarrow$ it fluctuates on average by $H_I/2\pi$ during each Hubble time

If saxion field S is light enough it also fluctuates during inflation

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After (long enough) inflation the initial values of both fields:

- **have average values, $\langle S_i \rangle$ and $\langle \theta_i \rangle$, determined by stochastic processes**
- **have dispersions, $\langle \delta S_i^2 \rangle$ and $\langle \delta \theta_i^2 \rangle$, generated by quantum fluctuations during last ~ 50 e-folds of inflation**

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Dynamics after inflation may lead to production of axions as

- cold dark matter (CDM)
  - e.g. misalignment
- warm dark matter (WDM)
  - e.g. parametric resonance production

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Bounds on isocurvature perturbations lead to very strong upper bounds on some couplings

e.g. kinetic misalignment: $10^{-36} \lesssim \lambda_\Phi \lesssim 10^{-22} \lll 1$

PQ field during inflation

If $\lambda_\Phi \lll 1$ one should consider corrections

- **radiative**
- **thermal**
- **geometric (curvature of space-time)**

Radiative corrections

We use Coleman-Weinberg (CW) potential adopting Gildener-Weinberg approach

The PQ scalar Φ couples to some scalars ϕ_i and some fermions ψ_j

$$\mathcal{L} \supset \sum_i \left(\frac{1}{2} m_i^2 \phi_i^2 + \frac{1}{2} \lambda_i |\Phi|^2 \phi_i^2 \right) + \sum_j y_j \Phi \overline{\psi}_j \psi_j$$

which gives the CW potential

$$V = \frac{1}{64\pi^2} \left\{ \sum_i M_{\phi_i}^4 \left[\ln\left(\frac{M_{\phi_i}^2}{\mu^2} \right) - \frac{3}{2} \right] - 4 \sum_j M_{\psi_j}^4 \left[\ln\left(\frac{M_{\psi_j}^2}{\mu^2} \right) - \frac{3}{2} \right] \right\}$$

$$M_{\phi_i}^2 = m_i^2 + \frac{1}{2} \lambda_i S^2 \qquad M_{\psi_j}^2 = \frac{1}{2} y_j^2 S^2$$

μ – scale at which running PQ self-coupling vanishes: $\lambda_\Phi(\mu) = 0$

Bosonic contribution must dominate for large values of S

Radiative corrections

- **Simple model:**

  N_f fermions ψ_j and $4N_f$ scalars ϕ_i with

  $y_j = y, \quad m_i = m, \quad \lambda_i = \lambda$

  (λ should not be confused with λ_Φ)

  and $y^2 = (1-\delta)\lambda$ with $0 \leq \delta \leq 1$

$$V_{\rm CW}(S) \approx \frac{N_f}{16\pi^2}\left\{\left(m^2+\frac{1}{2}\lambda S^2\right)^2\left[\ln\left(\frac{m^2+\frac{1}{2}\lambda S^2}{\mu^2}\right)-\frac{3}{2}\right] -\frac{1}{4}(1-\delta)^2\lambda^2 S^4\left[\ln\left(\frac{(1-\delta)\lambda S^2}{2\mu^2}\right)-\frac{3}{2}\right]\right\}$$

- **SUSY limit:** $m \to 0$ and $\delta \to 0 \Rightarrow V_{\rm CW} \to 0$
- quasi-SUSY when $\delta \ll 1$

Geometric corrections

CW potential in curved space-time

Markkanen et al 2018

$$V = \frac{1}{64\pi^2}\sum_i \left\{ M_{\phi_i}^4 \left[\ln\left(\frac{M_{\phi_i}^2}{\mu^2}\right) - \frac{3}{2}\right] + \frac{R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - R_{\mu\nu}R^{\mu\nu}}{90}\ln\left(\frac{M_{\phi_i}^2}{\mu^2}\right)\right\}$$

$$-\frac{4}{64\pi^2}\sum_j \left\{ M_{\psi_j}^4 \left[\ln\left(\frac{M_{\psi_j}^2}{\mu^2}\right) - \frac{3}{2}\right] + \frac{\frac{7}{8}R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} + R_{\mu\nu}R^{\mu\nu}}{90}\ln\left(\frac{M_{\psi_j}^2}{\mu^2}\right)\right\}$$

$$M_{\phi_i}^2 = m_i^2 + \lambda_i |\Phi|^2 + \left(\xi_i - \frac{1}{6}\right) R$$

$$M_{\psi_j}^2 = y_j^2 |\Phi|^2 + \frac{1}{12} R$$

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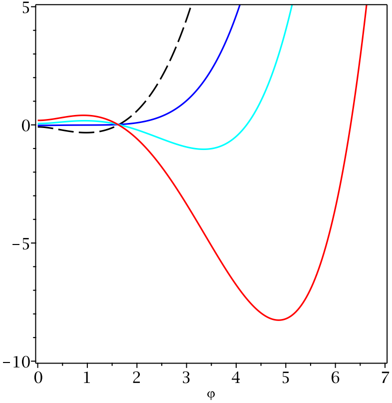
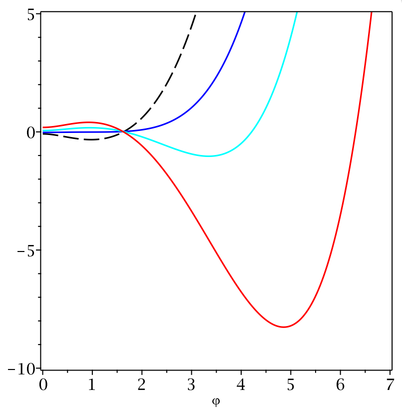
	inflation	MD	RD
R	$12H^2$	$3H^2$	0
$R_{\mu\nu}R^{\mu\nu}$	$36H^4$	$9H^4$	$12H^4$
$R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$	$24H^4$	$15H^4$	$24H^4$

Geometric corrections



Potential during inflation is more complicated (as compared to $R = 0$ case) and usually have second (much) deeper minimum for (much) bigger value of S

Geometric corrections



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During inflation stochastic fluctuations of a light field "compete" with classical evolution caused by its potential

After long enough time the system approaches the Fokker-Planck probability distribution: $P(\Phi) \propto \exp\left[-\frac{8\pi^2}{3}\,\frac{V(\Phi)}{H_I^4}\right]$

Starobinsky, Yokoyama, 1994

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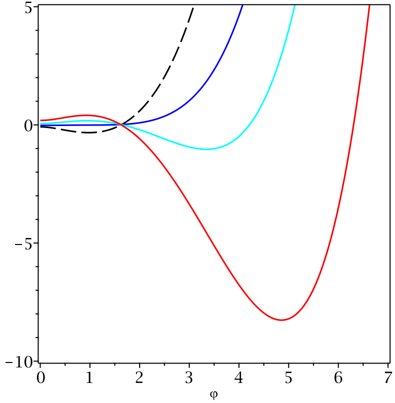

Potential during inflation is more complicated (as compared to $R = 0$ case) and usually have second (much) deeper minimum for (much) bigger value of S

Stochastic processes during inflation have somewhat different character than in the case of standard $\lambda_\Phi (S^2 - f_a^2)^2$ potential

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PQ field after inflation

Just after inflation fields S and θ are almost homogeneous

model	$\delta \approx 1$	$\delta \ll 1$ (quasi SUSY)	$\lambda_\Phi \lvert\Phi\rvert^4$
$S_i^2 \sim$	$2\frac{(2-12\xi)H_I^2 - m^2}{\lambda}$	$\frac{(3-12\xi)H_I^2 - m^2}{\lambda\delta}$	$\frac{H_I^2}{\sqrt{\lambda_\Phi}}$

θ_i - **"accidental"**

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θ_i - "accidental"

For some time PQ field is almost constant due to Hubble friction

When the Hubble parameter drops to approximately $H_i \approx m_S^{\rm eff}/3$, S starts to oscillate.

PQ field after inflation

Proposed picture in $\lambda_\Phi (S^2 - f_a^2)^2$ theory

Shtanov et al, Kofman et al 1994

- energy stored in saxion oscillations
  - redshifts due to Hubble expansion
  - transfers to particles via perturbative decays
  - transfers to particles via non-perturbative parametric resonance
- produced axions contribute to WDM

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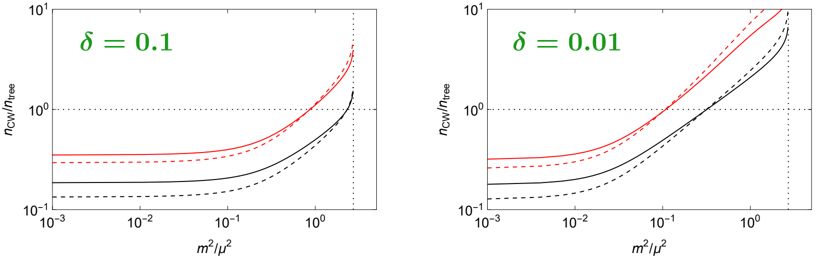
Different corrections to the simple potential may play – sometimes very important – role

Radiative corrections may change the amount for worm axions by a factor of a few



$H_I/\mu = 5$

$H_I/\mu = 10^3$

axions produced after (solid) or before (dashed) end of reheating

Thermal corrections

Our model

- **Thermal effects depend on the same fields and couplings as the CW potential does**
    - **it is not necessary to consider other sources like e.g. the Higgs portal (but it can be added)**

Thermal corrections

Our model

- Thermal effects depend on the same fields and couplings as the CW potential does
  - it is not necessary to consider other sources like e.g. the Higgs portal (but it can be added)
- We concentrate on two kinds of thermal effects
  - thermal corrections to the potential of the PQ field
  - thermalization of oscillations

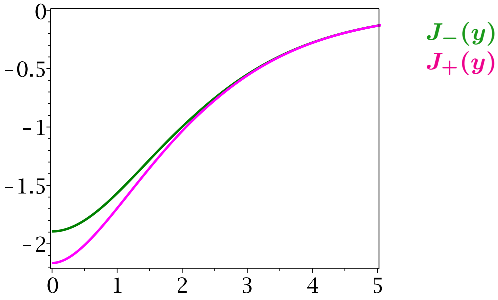
Thermal corrections

Thermal correction to the potential

$$V_T(\Phi) = \frac{T^4}{2\pi^2}\left[\sum_{\text{bosons}} J_+\left(\frac{M_{\phi_i}}{T}\right) + 4\sum_{\text{fermions}} J_-\left(\frac{M_{\psi_j}}{T}\right)\right]$$

where

$$J_\pm(y) = \pm\int_0^\infty x^2 \ln\left[1 \mp \exp\left(-\sqrt{x^2+y^2}\right)\right] dx$$



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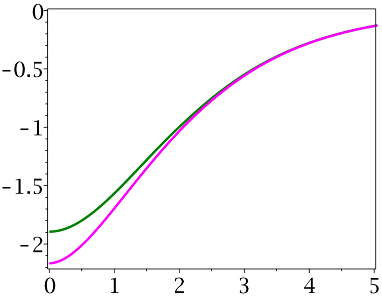

$J_-(y)$
$J_+(y)$

thermal correction to the saxion effective mass may be written as

$$\Delta\left(m_S^{\text{eff}}\right)^2 = \frac{1}{12} n_{\text{eff}} \lambda T^2$$

Saxion field starts to oscillate when H drops below $H_i \approx \frac{1}{3}\, m_S^{\rm eff}$

- If saxion effective mass is dominated by the zero-temperature $V_{\rm CW}$

$$H_i^{(0)} \approx \sqrt{\frac{\lambda N_f}{8\pi^2}\left[(1-4\xi) - \frac{2m^2}{9H_I^2}\right] \ln\left(\frac{(3-12\xi)H_I^2 - m^2}{2\delta\mu^2}\right)}\, H_I$$

- If saxion effective mass is dominated by the thermal contribution

$$H_i^{(T)} \approx \begin{cases} \dfrac{\sqrt{5}}{144\sqrt{\pi^3 g_*}}\, n_{\rm eff}\lambda M_{\rm Pl} & \text{after reheating} \\ \sqrt[6]{\dfrac{5}{g_*}}\, \dfrac{\sqrt[3]{12}}{72\sqrt{\pi}}\, (n_{\rm eff}\lambda)^{2/3}\, T_{RH}^{2/3} M_{\rm Pl}^{1/3} & \text{during reheating} \end{cases}$$

after (during) reheating if T_{RH} is bigger (smaller) than $\sqrt{\frac{5 n_{\rm eff}\lambda}{96\pi^3 g_*}}\, M_{\rm Pl}$

- Typically $\quad H_i \approx \max\left(H_i^{(0)}, H_i^{(T)}\right)$

Saxion oscillations

- Typically oscillations of the saxion field start due to the thermal mass correction
- Saxion oscillations in potential dominated by thermal mass
  - No parametric resonance production of (s)axions in quadratic potential (with constant or slowly varying mass)
  - $\Rightarrow$ particle production via parametric resonance (very) much delayed

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  - $\Rightarrow$ particle production via parametric resonance (very) much delayed
- Resonant production delayed at least until temperature drops below $\widetilde{T}$ at which thermal mass domination fades away
- How much has the amplitude of saxion oscillations A_S decreased till such time?
- Production of cold and warm axions depends strongly on $A_S(\widetilde{T})/S_{\rm min}$
  rough estimate $\frac{A_S(\widetilde{T})}{S_{\rm min}} \approx \mathcal{O}\left(\frac{1}{\sqrt{\lambda\delta}}\,\frac{m}{\mu}\,\frac{H_I}{10^{18}\,\mathrm{GeV}}\right)$
  and on details of the potential

Relic axions

- **scenario A:** $A_S(\widetilde{T}) \gg S_{\min}$:
  - Less warm axions produced
- **scenario B:** $A_S(\widetilde{T}) \sim S_{\min}$:
  - More warm axions produced
- in both scenarios A and B:
  - Cold (warm) axions from misalignment (parametric resonance)
  - Saxion oscillations "remember" the initial value $\theta_i \Rightarrow$ relic density of cold axions depends on stochastic processes before or during inflation
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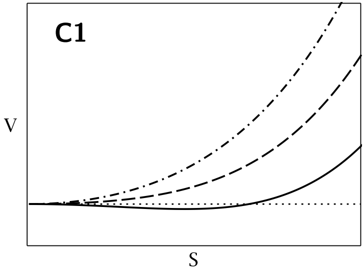
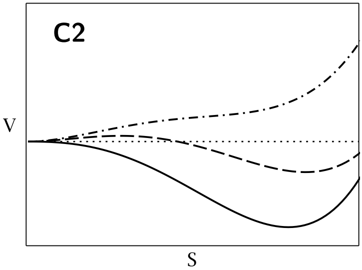
- scenario C: $A_S(\tilde{T}) \ll S_{\min}$:
  - Tachyonic instability important — Felder et al 2000
  - Dynamics may "forget" the initial value θ_i
    - if so the situation is similar to the "classical window": relic density of cold axions depends on stochastic processes after inflation $\langle \delta\theta^2 \rangle = \frac{\pi^2}{3}$, white noise isocurvature perturbations at low scales

Relic axions

- **scenario C1:**
  no barrier between global minimum and $S = 0$:
  very few warm axions produced via tachyonic instability
- **scenario C2:**
  barrier exists for some range of T:
  a lot of warm axions produced via tachyonic instability





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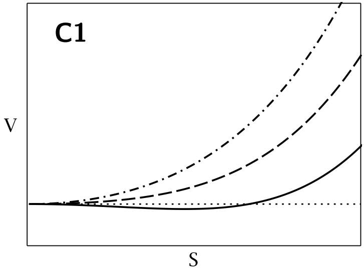

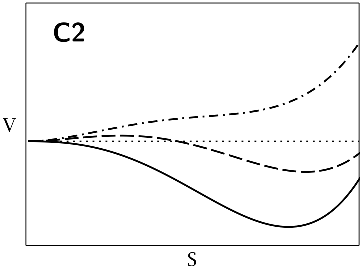

- scenario D: early thermalization

Relic axions

**Relic density of cold and warm axions
depends quite strongly on many details of a model**

Axion relic density

	λ	δ	$\frac{m}{\mu}$	μ [GeV]	H_I [GeV]	n/T^3 CW	n/T^3 $CW{+}G$	n/T^3 $CW{+}T$	n/T^3 $CW{+}T{+}G$
P_1	10^{-7}	0.1	0.1	10^9	10^{11}	0.042	0.2	$4.57 \cdot 10^5$	$4.57 \cdot 10^5$
P_2	10^{-7}	0.1	0.1	10^{10}	10^{13}	34	195	$4.33 \cdot 10^5$	$3.18 \cdot 10^5$
P_3	10^{-7}	0.1	0.1	10^{12}	10^{13}	56	223	$4.28 \cdot 10^5$	$3.04 \cdot 10^5$
P_4	10^{-7}	0.1	0.1	10^{11}	10^{13}	41.6	206	$4.3 \cdot 10^5$	$3.1 \cdot 10^5$
P_5	10^{-7}	0.1	0.5	10^{11}	10^{13}	41.9	207	$5.0 \cdot 10^3$	$2.2 \cdot 10^3$
P_6	10^{-7}	0.1	0.7	10^{11}	10^{13}	42.0	207	95	5.9
P_7	10^{-7}	0.1	0.8	10^{11}	10^{13}	42.0	207	0	0
P_8	10^{-7}	0.03	0.5	10^{11}	10^{13}	84	690	$1.9 \cdot 10^3$	$3.0 \cdot 10^3$
P_9	10^{-7}	0.01	0.5	10^{11}	10^{13}	160	2000	$1.2 \cdot 10^3$	$8.9 \cdot 10^3$
P_{10}	10^{-6}	0.01	0.5	10^{11}	10^{13}	9.0	120	490	47
P_{11}	10^{-6}	0.001	0.5	10^{11}	10^{13}	36	1100	370	940
P_{12}	10^{-7}	0.1	0.2	10^{10}	10^{12}	1.3	6.5	$8.4 \cdot 10^4$	$8.3 \cdot 10^4$
P_{13}	10^{-8}	0.1	0.2	10^{10}	10^{12}	23	120	$2.6 \cdot 10^5$	$2.5 \cdot 10^5$
P_{14}	10^{-9}	0.1	0.2	10^{10}	10^{12}	420	2100	$7.9 \cdot 10^5$	$5.8 \cdot 10^5$

Axion relic density

	λ	δ	$\frac{m}{\mu}$	μ [GeV]	H_I [GeV]	n/T^3 CW	n/T^3 $CW{+}G$	n/T^3 $CW{+}T$	n/T^3 $CW{+}T{+}G$
P_1	10^{-7}	0.1	0.1	10^{9}	10^{11}	0.042	0.2	$4.57 \cdot 10^5$	$4.57 \cdot 10^5$
P_2	10^{-7}	0.1	0.1	10^{10}	10^{13}	34	195	$4.33 \cdot 10^5$	$3.18 \cdot 10^5$
P_3	10^{-7}	0.1	0.1	10^{12}	10^{13}	56	223	$4.28 \cdot 10^5$	$3.04 \cdot 10^5$
P_4	10^{-7}	0.1	0.1	10^{11}	10^{13}	41.6	206	$4.3 \cdot 10^5$	$3.1 \cdot 10^5$
P_5	10^{-7}	0.1	0.5	10^{11}	10^{13}	41.9	207	$5.0 \cdot 10^3$	$2.2 \cdot 10^3$
P_6	10^{-7}	0.1	0.7	10^{11}	10^{13}	42.0	207	95	5.9
P_7	10^{-7}	0.1	0.8	10^{11}	10^{13}	42.0	207	0	0
P_8	10^{-7}	0.03	0.5	10^{11}	10^{13}	84	690	$1.9 \cdot 10^3$	$3.0 \cdot 10^3$
P_9	10^{-7}	0.01	0.5	10^{11}	10^{13}	160	2000	$1.2 \cdot 10^3$	$8.9 \cdot 10^3$
P_{10}	10^{-6}	0.01	0.5	10^{11}	10^{13}	9.0	120	490	47
P_{11}	10^{-6}	0.001	0.5	10^{11}	10^{13}	36	1100	370	940
P_{12}	10^{-7}	0.1	0.2	10^{10}	10^{12}	1.3	6.5	$8.4 \cdot 10^4$	$8.3 \cdot 10^4$
P_{13}	10^{-8}	0.1	0.2	10^{10}	10^{12}	23	120	$2.6 \cdot 10^5$	$2.5 \cdot 10^5$
P_{14}	10^{-9}	0.1	0.2	10^{10}	10^{12}	420	2100	$7.9 \cdot 10^5$	$5.8 \cdot 10^5$

$n_{CW+G} > n_{CW}$ **(bigger change for smaller δ)**

Axion relic density

	λ	δ	$\frac{m}{\mu}$	μ [GeV]	H_I [GeV]	n/T^3 CW	n/T^3 $CW+G$	n/T^3 $CW+T$	n/T^3 $CW+T+G$
P_1	10^{-7}	0.1	0.1	10^9	10^{11}	0.042	0.2	$4.57\cdot10^5$	$4.57\cdot10^5$
P_2	10^{-7}	0.1	0.1	10^{10}	10^{13}	34	195	$4.33\cdot10^5$	$3.18\cdot10^5$
P_3	10^{-7}	0.1	0.1	10^{12}	10^{13}	56	223	$4.28\cdot10^5$	$3.04\cdot10^5$
P_4	10^{-7}	0.1	0.1	10^{11}	10^{13}	41.6	206	$4.3\cdot10^5$	$3.1\cdot10^5$
P_5	10^{-7}	0.1	0.5	10^{11}	10^{13}	41.9	207	$5.0\cdot10^3$	$2.2\cdot10^3$
P_6	10^{-7}	0.1	0.7	10^{11}	10^{13}	42.0	207	95	5.9
P_7	10^{-7}	0.1	0.8	10^{11}	10^{13}	42.0	207	0	0
P_8	10^{-7}	0.03	0.5	10^{11}	10^{13}	84	690	$1.9\cdot10^3$	$3.0\cdot10^3$
P_9	10^{-7}	0.01	0.5	10^{11}	10^{13}	160	2000	$1.2\cdot10^3$	$8.9\cdot10^3$
P_{10}	10^{-6}	0.01	0.5	10^{11}	10^{13}	9.0	120	490	47
P_{11}	10^{-6}	0.001	0.5	10^{11}	10^{13}	36	1100	370	940
P_{12}	10^{-7}	0.1	0.2	10^{10}	10^{12}	1.3	6.5	$8.4\cdot10^4$	$8.3\cdot10^4$
P_{13}	10^{-8}	0.1	0.2	10^{10}	10^{12}	23	120	$2.6\cdot10^5$	$2.5\cdot10^5$
P_{14}	10^{-9}	0.1	0.2	10^{10}	10^{12}	420	2100	$7.9\cdot10^5$	$5.8\cdot10^5$

scenario B: $n_{CW+T} > n_{CW}$, $n_{CW+T+G} > n_{CW+T}$

Axion relic density

	λ	δ	$\frac{m}{\mu}$	μ [GeV]	H_I [GeV]	n/T^3 CW	n/T^3 $CW+G$	n/T^3 $CW+T$	n/T^3 $CW+T+G$
P_1	10^{-7}	0.1	0.1	10^9	10^{11}	0.042	0.2	$4.57 \cdot 10^5$	$4.57 \cdot 10^5$
P_2	10^{-7}	0.1	0.1	10^{10}	10^{13}	34	195	$4.33 \cdot 10^5$	$3.18 \cdot 10^5$
P_3	10^{-7}	0.1	0.1	10^{12}	10^{13}	56	223	$4.28 \cdot 10^5$	$3.04 \cdot 10^5$
P_4	10^{-7}	0.1	0.1	10^{11}	10^{13}	41.6	206	$4.3 \cdot 10^5$	$3.1 \cdot 10^5$
P_5	10^{-7}	0.1	0.5	10^{11}	10^{13}	41.9	207	$5.0 \cdot 10^3$	$2.2 \cdot 10^3$
P_6	10^{-7}	0.1	0.7	10^{11}	10^{13}	42.0	207	95	5.9
P_7	10^{-7}	0.1	0.8	10^{11}	10^{13}	42.0	207	0	0
P_8	10^{-7}	0.03	0.5	10^{11}	10^{13}	84	690	$1.9 \cdot 10^3$	$3.0 \cdot 10^3$
P_9	10^{-7}	0.01	0.5	10^{11}	10^{13}	160	2000	$1.2 \cdot 10^3$	$8.9 \cdot 10^3$
P_{10}	10^{-6}	0.01	0.5	10^{11}	10^{13}	9.0	120	490	47
P_{11}	10^{-6}	0.001	0.5	10^{11}	10^{13}	36	1100	370	940
P_{12}	10^{-7}	0.1	0.2	10^{10}	10^{12}	1.3	6.5	$8.4 \cdot 10^4$	$8.3 \cdot 10^4$
P_{13}	10^{-8}	0.1	0.2	10^{10}	10^{12}	23	120	$2.6 \cdot 10^5$	$2.5 \cdot 10^5$
P_{14}	10^{-9}	0.1	0.2	10^{10}	10^{12}	420	2100	$7.9 \cdot 10^5$	$5.8 \cdot 10^5$

scenarios C1 $\rightarrow$ C2: $n_{CW+T+G} < n_{CW+T}$

Axion relic density

	λ	δ	$\frac{m}{\mu}$	μ [GeV]	H_I [GeV]	n/T^3 CW	n/T^3 $CW+G$	n/T^3 $CW+T$	n/T^3 $CW+T+G$
P_1	10^{-7}	0.1	0.1	10^9	10^{11}	0.042	0.2	$4.57 \cdot 10^5$	$4.57 \cdot 10^5$
P_2	10^{-7}	0.1	0.1	10^{10}	10^{13}	34	195	$4.33 \cdot 10^5$	$3.18 \cdot 10^5$
P_3	10^{-7}	0.1	0.1	10^{12}	10^{13}	56	223	$4.28 \cdot 10^5$	$3.04 \cdot 10^5$
P_4	10^{-7}	0.1	0.1	10^{11}	10^{13}	41.6	206	$4.3 \cdot 10^5$	$3.1 \cdot 10^5$
P_5	10^{-7}	0.1	0.5	10^{11}	10^{13}	41.9	207	$5.0 \cdot 10^3$	$2.2 \cdot 10^3$
P_6	10^{-7}	0.1	0.7	10^{11}	10^{13}	42.0	207	95	5.9
P_7	10^{-7}	0.1	0.8	10^{11}	10^{13}	42.0	207	0	0
P_8	10^{-7}	0.03	0.5	10^{11}	10^{13}	84	690	$1.9 \cdot 10^3$	$3.0 \cdot 10^3$
P_9	10^{-7}	0.01	0.5	10^{11}	10^{13}	160	2000	$1.2 \cdot 10^3$	$8.9 \cdot 10^3$
P_{10}	10^{-6}	0.01	0.5	10^{11}	10^{13}	9.0	120	490	47
P_{11}	10^{-6}	0.001	0.5	10^{11}	10^{13}	36	1100	370	940
P_{12}	10^{-7}	0.1	0.2	10^{10}	10^{12}	1.3	6.5	$8.4 \cdot 10^4$	$8.3 \cdot 10^4$
P_{13}	10^{-8}	0.1	0.2	10^{10}	10^{12}	23	120	$2.6 \cdot 10^5$	$2.5 \cdot 10^5$
P_{14}	10^{-9}	0.1	0.2	10^{10}	10^{12}	420	2100	$7.9 \cdot 10^5$	$5.8 \cdot 10^5$

Approximate analytical calculations give the following leading dependence on the parameters

$$\frac{n_{CW}}{T^3} \propto \delta^{-5/8}\lambda^{-5/4}H_I^{3/2}$$

$$\frac{n_{CW+G}}{T^3} \propto \delta^{-1}\lambda^{-5/4}H_I^{3/2}$$

$$\frac{n_{CW+T}}{T^3}, \frac{n_{CW+T+G}}{T^3} \propto \lambda^{-1/2}\left(\frac{m}{\mu}\right)^{-2}$$

Summary

- Peccei-Quinn field with non-trivial dynamics during and after inflation must have extremely small self-coupling
- Corrections to its potential may be crucial
  - radiative, thermal, geometric
- During inflation
  - saxion potential has (second) minimum at $S \gg f_a$
  - $\langle S_i \rangle$, $\langle \theta_i \rangle$, $\langle \delta S_i^2 \rangle$, $\langle \delta \theta_i^2 \rangle$ determined by stochastic processes
  - but $\langle S_i \rangle$ close to the position of the minimum at $S \gg f_a$
- After inflation
  - thermal corrections very important for the evolution of saxion field
  - evolution of S and production of particles via parametric resonance or tachyonic instability depend quite strongly on details of a model
- Relic abundance of axions (also model-dependent)
  - contribution to WDM may vary by many orders of magnitude
  - contribution to CDM may be stochastic or not
  - isocurvature perturbations may be standard (generated during inflation) or may have the form of white noise at small scales
- Numerical simulations necessary to get precise predictions