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New Challenges in Unification

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- Higher Dimensional Unified Theories and Coset Space Dimensional Reduction
- Fuzzy Extra Dimensions and Renormalizable Chiral Unified Theories
- Remarks on the predictions of Finite Unified Theories vs LHC results

Coset Space Dimensional Reduction
(CSDR)

Original motivation

Use higher dimensions

- to unify the gauge and Higgs sectors
- to unify the fermion interactions with gauge and Higgs fields

Supersymmetry provides further unification (fermions in adj reps)

Forgacs + Manton ; Manton

Chapline + Slansky

Kubyshin + Mourao + Rudolph + Volobujev - book

Kapetanakis + G. Z. - Phys. Rept

Manousselis + G. Z., Phys. Lett. B504, 122 (01); PLB518, 171 (01); JHEP03, 002 (02); JHEP11, 025 (04)

Further successes

(a) chiral fermions in 4 dims from vector-like reps in the higher dim thy.

(b) the metric can be deformed (in certain non-symmetric coset sp) and more than one scales can be introduced

(c) Wilson flux breaking can be used

ADD

- Softly broken susy chiral ths in 4 dims can result from a higher dimensional susy theory

Theory in D dims $\Rightarrow$ Thy in 4 dims

1. Compactification $\underset{x^M}{M^D} \longrightarrow \underset{x^\mu}{M^4} \times \underset{y^a}{B}$

B - a compact space

$\dim B = D - 4 = d$

2. Dimensional Reduction

Demand that $\mathcal{L}$ is independent of the extra y^a coordinates

• One way: Discard the field dependence on y^a coordinates

•• An elegant way: Allow field dependence on y^a and employ a symmetry of the Lagrangian to compensate.

Obvious choice: Gauge Symmetry

Allow a non-trivial dependence on y^a, but impose the condition that a symmetry transformation by an element of the isometry group S of B is compensated by a gauge transformation.

$\Longrightarrow \mathcal{L}$ independent of y^a just because is gauge invariant.

Integrate out extra coordinates

CSDR: $B = S/R$

$$S: \; Q_A = \{ \underset{\substack{| \\ R}}{Q_i} \, , \, \underset{\substack{| \\ S/R}}{Q_a} \}$$

$$[Q_i, Q_j] = f_{ij}^{\;k} Q_k \quad , \quad [Q_i, Q_a] = f_{ia}^{\;b} Q_b$$

$$[Q_a, Q_b] = f_{ab}^{\;i} Q_i + f_{ab}^{\;c} Q_c$$

vanishes in symmetric S/R

Consider a Yang-Mills-Dirac thy in D dims based on group G defined on $M^D \longrightarrow M^4 \times S/R$, $D=4+d$

$$g^{MN} = \begin{pmatrix} \eta^{\mu\nu} & 0 \\ 0 & -g^{ab} \end{pmatrix}, \quad \eta^{\mu\nu} = \mathrm{dig}(1,-1,-1,-1)$$

g^{ab} - coset space metric

$d = \dim S - \dim R$

$$A = \int d^4x\, d^dy \sqrt{-g} \left[-\frac{1}{4} \mathrm{Tr}(F_{MN} F_{K\Lambda}) g^{MK} g^{N\Lambda} + \frac{i}{2} \bar{\psi} \Gamma^M D_M \psi \right]$$

$$D_M = \partial_M - \theta_M - A_M, \quad \theta_M = \frac{1}{2} \theta_{MN\Lambda} \Sigma^{N\Lambda}$$

spin connection of M^D

ψ in rep F of G

We require that any transformation by an element of S acting on S/R is compensated by gauge transformations.

$$A_\mu(x,y) = g(s)\, A_\mu(x, s^{-1}y)\, g^{-1}(s)$$

$$A_\alpha(x,y) = g(s)\, J_\alpha^{b} A_b(x, s^{-1}y)\, g^{-1}(s) + g(s)\, \partial_\alpha\, g^{-1}(s)$$

$$\psi(x,y) = f(s)\, \Omega\, \psi(x, s^{-1}y)\, f^{-1}(s)$$

g, f – gauge transformations in the ad_j, F of G corresponding to the s transf. of S acting on S/R

J_α^{b} – Jacobian for s

Ω – Jacobian + local Lorentz rotation in tangent space

Above conditions imply constraints that D-dims fields should obey.

Solution of constraints $\left\{ \begin{array}{l} \cdot \text{ 4-dim fields} \\ \cdot \text{ Potential} \\ \cdot \text{ Remaining gauge invariance} \end{array} \right.$

Taking into account all the constraints and integrating out the extra coordinates, we obtain in 4 dims

$$A = C\int d^4x \Big(-\frac{1}{4} Tr F_{\mu\nu} F^{\mu\nu} + \frac{1}{2}\sum_\alpha Tr(D_\mu \phi_a D^\mu \phi_a)$$

$$+ V(\phi) + \frac{i}{2}\bar{\psi}\Gamma^\mu D_\mu \psi - \frac{i}{2}\bar{\psi}\Gamma^a D_a \psi \Big)$$

Kinetic terms mass terms

$D_\mu = \partial_\mu - A_\mu$, $D_a = \partial_a - \theta_a - \phi_a$, $\theta_a = \frac{1}{2}\theta_{abc}\Sigma^{bc}$

spin connection of coset space

C - volume of coset space

$$V(\phi) = -\frac{1}{4} g^{ac} g^{bd} Tr\Big\{ \left(f_{ab}^{\ \ C}\phi_C - [\phi_a, \phi_b]\right) \left(f_{cd}^{\ \ D}\phi_D - [\phi_c, \phi_d]\right) \Big\}$$

$A = 1, \ldots, \dim S$, f - structure const. of S

Still $V(\phi)$ only formal since ϕ_a must satisfy $f_{ai}^{\ \ D}\phi_D - [\phi_a, \phi_i] = 0$

- The 4-dim gauge group

$$H = C_G(R_G)$$

i.e. $G \supset R_G \times H$

higher dim group (G) 4-dim group (H)

- - Scalar fields

$$S \supset R$$

$$\text{adj}\, S = \text{adj}\, R + v$$

$$G \supset R_G \times H$$

$$\text{adj}\, G = (\text{adj}\, R, 1) + (1, \text{adj}\, H) + \sum (r_i, h_i)$$

If $v = \sum s_i$

when $s_i = r_i \Longrightarrow h_i$ survives in 4 dims

• • • fermions

$$G \supset R_G \times H$$

$$F = \sum (t_i, h_i)$$

spinor of $SO(d)$ under R

$$\sigma_d = \sum \sigma_j$$

for every $t_i = \sigma_i \Rightarrow h_i$ survives in 4 dims

Possible to obtain a chiral theory in 4 dims even starting with Weyl (+ Majorana) fermions in vector-like reps of G in $D = 4n+2$ dims.

If D is even

$$\Gamma^{D+1}\psi_\pm = \pm\psi_\pm \qquad \text{Weyl condition}$$

$$\psi = \psi_+ \oplus \psi_- = \sigma_D + \sigma_D'$$

non-self conjugate spinors of $SO(1, D-1)$

The $(SU(2)\times SU(2))\times SO(d)$ branching rule is

$$\sigma_D = (2, 1; \sigma_d) + (1, 2; \sigma_d')$$

$$\sigma_D' = (2, 1; \sigma_d') + (1, 2; \sigma_d)$$

Starting with Dirac fermions

$\rightarrow$ equal number of left- and right-handed reps of the 4-dim group H

Weyl condition selects either σ_D or σ_D'

Weyl condition cannot be applied in odd dims. In that case

$$\sigma_D = (2, 1; \sigma_d) + (1, 2; \sigma_d)$$

where σ_d is the unique spinor of $SO(d)$

$\Rightarrow$ equal number of left- and right-handed reps in 4 dims.

Most interesting case is when $D = 4n + 2$ and we start with a vectorlike rep. In that case σ_d is non-self-conjugate and $\sigma_d' = \bar{\sigma}_d$

Then the decomposition of σ_d, $\bar{\sigma}_d$ of $SO(d)$ under R is

$$\sigma_d = \sum \sigma_k \quad , \quad \bar{\sigma}_d = \sum \bar{\sigma}_k$$

Then

$$G \supset R_G \times H$$

$$\underset{\text{vectorlike}}{f} = \sum_i (r_i, h_i)$$

vectorlike — either self-conjugate or have a partner $(\bar{r}_i, \bar{h}_i)$

Then according to the rule from 6d we will obtain in 4 dims left-handed fermions $f_L = \sum h_k^L$

Since 6d is non-self-conjugate, f_L is non-self-conjugate.

Similarly from $\bar{6}_d$ we will obtain the right-handed rep $f_R = \sum \bar{h}_k^R = \sum h_k^L$

But since f vectorlike, $\bar{h}_k^R \sim h_k^L$

i.e. H is chiral theory

We can still impose Majorana condition (Weyl and Majorana are compatible in $4n+2$ dims) to eliminate the doubling of fermion spectrum. Majorana cond (reverses the sign of all int. qu. nos) forces f_R to be the charge conjugate of f_L.

If f complex $\rightarrow$ chiral theory just $\bar{h}^R_K$ is different from h^L_K

An easy case in calculating the potential and its minimization

If $G \supset S \Rightarrow H$ breaks to $K = C_G(S)$

$$\begin{array}{ccccc} G & \supset & S & \times & K \\ & & \cup & & \cap \\ G & \supset & R & \times & H \end{array}$$

$K \leftarrow$ gauge group after spontaneous sym. breaking

H: gauge group in 4 dims

But

fermion masses

$$M^2 \psi = D_a D^a \psi - \frac{1}{4} R \psi - \frac{1}{2} \Sigma^{ab} F_{ab} \psi > 0$$

$F_{ab} = 0$ if $S \subset G$

comparable to the compactification scale

Supersymmetry breaking by dim reduction over Symmetric C.S.
(e.g. SO_7/SO_6)

Consider $G = E_8$ in 10 dims with Weyl-Majorana fermions in the adjoint of E_8, i.e. a susy E_8

Embedding of $R = SO(6)$ in E_8 is suggested by the decomposition

$$E_8 > SO(6) \times SO(10)$$

$$248 = (15,1) + (1,45) + (6,10) + (4,16) + (\bar{4},\overline{16})$$

$$\mathrm{adj}\, S = \mathrm{adj}\, R + v$$
$$21 = 15 + 6 \leftarrow \text{vector}$$

Spinor of $SO(6)$: 4

In 4 dims we obtain a gauge thy based on

$$H = C_{E_8}(SO(6)) = SO(10)$$

with scalars in 10

and fermions in 16

- Theorem: When S/R symmetric the potential necessarily leads to spont. breakdown of H.

- Moreover in this case we have

$$\begin{array}{ccccc} E_8 & \supset & SO(7) & \times & SO(9) \\ & & \cup & & \cap \\ E_8 & \supset & SO(6) & \times & SO(10) \end{array}$$

$\Rightarrow$ Final gauge group after breaking

$$K = C_{E_8}(SO(7)) = SO(9)$$

CSDR over symmetric coset spaces breaks completely original supersymmetry

Soft Supersymmetry Breaking by CSDR over non-symmetric CS.

We have examined the dim. red of a supersymmetric E_8 over the 3 existing 6-dim CS: G_2/SU_3, $Sp(4)/(SU(2)\times U(1))_{non-max}$, $SU(3)/U(1)\times U(1)$

$\Rightarrow$ Softly Broken Supersymmetric Theories in 4 dims without any further assumption

Non-symmetric CS admit torsion and the two latter more than one radii

Consider supersymmetric E_8 in 10 dims and $S/R = G_2/SU(3)$

We use the decomposition

$$E_8 \supset SU(3) \times E_6$$
$$248 = (8,1) + (1,78) + (3,27) + (\bar{3},\overline{27})$$

and choose $R = SU(3)$

$$\text{adj}\, S = \text{adj}\, R + v$$
$$14 = 8 + \underbrace{3 + \bar{3}}_{\text{vector}}$$

Spinor : $1 + 3$ under $R = SU(3)$

$\Rightarrow$ 4 dim thy : $H = C_{E_8}(SU(3)) = E_6$

with scalars in $27 = 6$

and fermions in $27, 78$

i.e. spectrum of a supersymmetric E_6 thy in 4 dims

The Higgs potential of the genuine Higgs σ

$$V(\sigma) = 8 - \frac{40}{3}\sigma^2 - \left[4 d_{ijk}\, \sigma^i \sigma^j \sigma^k + h.c.\right]$$

$$+ \sigma^i \sigma^j d_{ijk}\, d^{klm}\, \sigma_l \sigma_m$$

$$+ \frac{11}{4} \sum_\alpha \sigma^i (G^\alpha)_i^{\ j} \sigma_j \sigma^k (G^\alpha)_k^{\ l} \sigma_l$$

which obtains F-terms contributions from the superpotential

$$W(B) = \frac{1}{3} d_{ijk} B^i B^j B^k$$

D-term contributions

$$\frac{1}{2} D^\alpha D^\alpha \quad , \quad D^\alpha = \sqrt{\frac{11}{2}}\, \sigma^i (G^\alpha)_i^{\ j} \sigma_j$$

The rest terms belong to the SSB part of the Lagrangian

$$\mathcal{L}_{scalar\, SSB} = -\frac{1}{R^2}\frac{40}{3}\sigma^2 - \left[4 d_{ijk}\, \sigma^i \sigma^j \sigma^k + h.c.\right]\frac{9}{R}$$

$$M_{gaugino} = (1 + 3\tau)\frac{6}{\sqrt{3}}\frac{1}{R}$$

Reduction of 10-dim, $N=1$,

E_8 over $S/R = SU(3)/U(1)\times U(1)$

N. Irges, G.Z.

We use the decomposition

$$E_8 \supset E_6 \times SU(3) \supset E_6 \times U(1)_A \times U(1)_B$$

and choose $R = U(1)_A \times U(1)_B$,

$$\Rightarrow H = C_{E_8}(U(1)_A \times U(1)_B) = E_6 \times U(1)_A \times U(1)_B$$

$$E_8 \supset E_6 \times U(1)_A \times U(1)_B$$

$$\begin{aligned} 248 = {} & 1_{(0,0)} + 1_{(0,0)} + 1_{(3,1/2)} + 1_{(-3,1/2)} \\ & + 1_{(0,-1)} + 1_{(0,1)} + 1_{(-3,-1/2)} + 1_{(3,-1/2)} \\ & + 78_{(0,0)} + 27_{(3,1/2)} + 27_{(-3,1/2)} + 27_{(0,-1)} \\ & + \overline{27}_{(-3,-1/2)} + \overline{27}_{(3,-1/2)} + \overline{27}_{(0,1)} \end{aligned}$$

$\mathrm{adj}\, S = \mathrm{adj}\, R + v \Leftarrow$ vector

$\Downarrow$

$$\begin{aligned} 8 = {} & (0,0) + (0,0) + (3,1/2) + (-3,1/2) \\ & + (0,-1) + (0,1) + (-3,-1/2) + (3,-1/2) \end{aligned}$$

$SO(6) \supset SU(3) \supset U(1)_A \times U(1)_B$

$4 \quad = 1 + 3 = (0,0) + (3, 1/2) + (-3, 1/2) + (0,-1)$

spinor

$\rightsquigarrow$ 4-dim theory

$N=1, \quad E_6 \times U(1)_A \times U(1)_B$

with chiral supermultiplets

$A^i : 27_{(3,1/2)}, \; B^i : 27_{(-3,1/2)}, \; C^i : 27_{(0,-1)}$

$A : 1_{(3,1/2)}, \; B : 1_{(-3,1/2)}, \; C : 1_{(0,-1)}$

Superpotential,

$$W(A^i, B^j, C^k, A, B, C) = \sqrt{40}\, d_{ijk} A^i B^j C^k + \sqrt{40} ABC$$

E_6 symmetric tensor

D-terms, $\frac{1}{2} D^\alpha D^\alpha + \frac{1}{2} D_1 D_1 + \frac{1}{2} D_2 D_2$

where $D^\alpha = \frac{1}{\sqrt{3}} \left(\alpha^i (G^\alpha)_i^j \alpha_j + \beta^i (G^\alpha)_i^j \beta_j + \gamma^i (G^\alpha)_i^j \gamma_j \right)$

$$D_1 = \sqrt{10/3} \left(\alpha^i (3 \delta_i^j) \alpha_j + \bar{\alpha}(3)\alpha + \beta^i (-3 \delta_i^j) \beta_j + \bar{\beta}(-3)\beta \right)$$

$$D_2 = \sqrt{\frac{40}{3}} \left(\alpha^i (\tfrac{1}{2} \delta_i^j) \alpha_j + \bar{\alpha} (\tfrac{1}{2}) \alpha + \beta^i (\tfrac{1}{2} \delta_i^j) \beta_j + \bar{\beta} (\tfrac{1}{2}) \beta + \gamma^i (-1 \delta_i^j) \gamma_j + \bar{\gamma}(-1) \gamma \right)$$

Soft scalar supersymmetry breaking terms, $\mathcal{L}_{scalar\,SSB}$

$$\mathcal{L}_{SSB} = \left(\frac{4R_1^2}{R_2^2R_3^2} - \frac{8}{R_1^2}\right)\alpha^i\alpha_i + \left(\frac{4R_1^2}{R_2^2R_3^2} - \frac{8}{R_1^2}\right)\bar{\alpha}\alpha$$

$$+ \left(\frac{4R_2^2}{R_1^2R_3^2} - \frac{8}{R_2^2}\right)b^ib_i + \left(\frac{4R_2^2}{R_1^2R_3^2} - \frac{8}{R_2^2}\right)\bar{b}b$$

$$+ \left(\frac{4R_3^2}{R_1^2R_2^2} - \frac{8}{R_3^2}\right)\gamma^i\gamma_i + \left(\frac{4R_3^2}{R_1^2R_2^2} - \frac{8}{R_3^2}\right)\bar{\gamma}\gamma$$

$$+ \left[\sqrt{280}\left(\frac{R_1}{R_2R_3} + \frac{R_2}{R_1R_3} + \frac{R_3}{R_2R_1}\right)d_{ijk}\,\alpha^i b^j \gamma^k\right.$$

$$\left. + \sqrt{280}\left(\frac{R_1}{R_2R_3} + \frac{R_2}{R_1R_3} + \frac{R_3}{R_2R_1}\right)\alpha b \gamma + h.c.\right]$$

where $\alpha^i, b^i, \gamma^i, \alpha, b, \gamma$ are the scalar components of the A^i, B^i, C^i, A, B, C

Gaugino mass, $M = (1 + 3\tau)\dfrac{R_1^2 + R_2^2 + R_3^2}{8\sqrt{R_1^2R_2^2R_3^2}}$

torsion coef.

Potential, $V = V_F + V_D + V_{soft}$

The Wilson flux breaking

$M^4 \times B_0 \longrightarrow M^4 \times B$, $B = B_0 / F^{S/R}$

$F^{S/R}$ – a freely acting discrete symmetry of B_0

1. B becomes multiply connected
2. For every element $g \in F^{S/R}$,

$\rightsquigarrow$ $$U_g = P\exp\left(-i\int_{\gamma_g} T^a A^a_M(x)\, dx^M\right) \in H$$

3. If the contour is non-contractible

$\rightsquigarrow$ $U_g \neq 1$ and then $f(g(x)) = U_g f(x)$ which leads to a breaking of H to $K' = C_H(T^H)$, where T^H is the image of the homomorphism of $F^{S/R}$ into H.

4. Matter fields invariant under $F^{S/R} \oplus T^H$

In the case of $SU(3)/U(1)\times U(1)$ a freely acting discrete group is

$$F^{S/R} = Z_3 \subset W \,, \quad W = \frac{W_S}{W_R}$$

$W_{S,R}$: Weyl group of S, R

$\rightsquigarrow \gamma_3 = \mathrm{diag}\left(\mathbb{1}, \omega \mathbb{1}, \omega^2 \mathbb{1}\right), \; \omega = e^{2i\pi/3} \in Z_3$

The fields that are invariant under $F^{S/R} \oplus T^H$ survive, i.e.

$$A_\mu = \gamma_3 A_\mu \gamma_3^{-1}$$

$$A^i = \omega \gamma_3 A^i, \; B^i = \omega^2 \gamma_3 B^i, \; C^i = \omega^3 \gamma_3 C^i$$

$$A = \omega A \,, \quad B = \omega^2 B \,, \quad C = \omega^3 C$$

$\rightsquigarrow N=1, \; SU(3)_c \times SU(3)_L \times SU(3)_R$

with matter superfields in

$$(\overline{3}, 1, 3)_{(3,1/2)}, \; (3, \overline{3}, 1)_{(0,-1)}, \; (1, 3, \overline{3})_{(-3,1/2)}$$

$$\begin{pmatrix} d^c & d^c & d^c \\ u^c & u^c & u^c \\ h^c & h^c & h^c \end{pmatrix} = q^c, \; q = \begin{pmatrix} d & u & h \\ d & u & h \\ d & u & h \end{pmatrix}, \; \lambda = \begin{pmatrix} N & E^c & \nu \\ E & N^c & e \\ \nu^c & e^c & S \end{pmatrix}$$

- Introducing non-trivial windings in R can appear 3 identical flavours in each of the bifundamental matter superfields.

Supersymmetry and gauge symmetry breaking

Consider the vevs in the scalars of $\lambda^{(1)}$, $\lambda^{(2)}$

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & V \end{pmatrix}, \quad \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ V & 0 & 0 \end{pmatrix}$$

$$\lambda^{(1)}: SU(3)_L \times SU(3)_R \times U(1)_A \times U(1)_B \to SU(2)_L \times SU(2)_R \times U(1)$$

$$\lambda^{(2)}: SU(3)_L \times SU(3)_R \times U(1)_A \times U(1)_B \to SU(2)_L \times SU(2)_R \times U(1)'$$

their combination gives

$$SU(3)_L \times SU(3)_R \times U(1)_A \times U(1)_B \longrightarrow SU(2)_L \times U(1)_Y$$

electroweak breaking proceeds by

$$\begin{pmatrix} v & 0 & 0 \\ 0 & v & 0 \\ 0 & 0 & V \end{pmatrix}$$

For a certain relation among vevs the potential vanishes at the min.

Note that before EW breaking, supersymmetry is broken by D and F-terms, in addition to its breaking by soft terms.

- there is no proton decay
- the Froggatt-Nielsen mechanism is naturally realised.

Fuzzy CSDR

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$$M^D = M^4 \times (S/R)_F$$

finite matrix manifold
e.g. fuzzy sphere S^2_F

Instead of considering the algebra of functions

$$\mathrm{Fun}(M^D) \sim \mathrm{Fun}(M^4) \times \mathrm{Fun}(S/R)$$

we consider the algebra

$$A = \mathrm{Fun}(M^4) \times M_N$$

M_N – finite dim NC (non-com) algebra of matrices that approximates the functions on $(S/R)_F$

On A we still have the action of symmetry group S $\rightarrow$ we can apply CSDR

Fuzzy Sphere

Madore

Nice example of $(S^1/R)_F$ is the fuzzy sphere S^2_F, a matrix approximation of S^2. The algebra of functions on S^2 (spanned by spherical harmonics) is truncated at a given angular momentum and becomes finite dimensional. The algebra becomes that of $N \times N$ matrices.

The associativity of the algebra is nicely achieved by relaxing commutativity

The algebra of functions on S^2 can be generated by the coordinates of R^3 modulo the relation $\sum_{a=1}^{3} x_a^2 = r^2$

Scalar functions on S^2 can be expanded

$$f(\theta,\phi) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} f_{\ell m} Y_{\ell m}(\theta,\phi)$$

spherical harmonics

$Y_{\ell m}(\theta,\phi)$ can be expressed in terms of the cartesian coordinates x_a, $a=1,2,3$ in $\mathbb{R}^3$

$$Y_{\ell m}(\theta,\phi) = \sum_a f^{(\ell m)}_{a_1 \dots a_\ell} x^{a_1} \dots x^{a_\ell}$$

traceless symmetric tensor of $SO(3)$ with rank ℓ

Similarly we can expand $N \times N$ matrices of a matrix theory on a fuzzy sphere

$$\hat{f} = \sum_{\ell=0}^{N-1} \sum_{m=-\ell}^{\ell} f_{\ell m} \hat{Y}_{\ell m}$$

$$\hat{Y}_{\ell m} = r^{-\ell} \sum_a f^{(\ell m)}_{a_1 \dots a_\ell} \hat{x}^{a_1} \dots \hat{x}^{a_\ell}$$

where $f^{(\ell m)}_{a_1 \dots a_\ell}$ the same as in S^2, while

$$\hat{x}_a = r \frac{i}{\sqrt{N^2-1}} X_a \,, \qquad \hat{x}_a^+ = \hat{x}_a$$

are $N \times N$ hermitian matrices proportional to the N-dim rep of the $SU(2)$ generators

They satisfy

$$\sum_{a=1}^{3} \hat{x}_a \hat{x}_a = r^2 \quad , \quad [X_a, X_b] = \epsilon_{abc} X_c$$

$\hat{Y}_{lm}$ - fuzzy spherical harmonics

they obey $Tr_N(\hat{Y}^+_{lm} \hat{Y}_{l'm'}) = \delta_{ll'} \delta_{mm'}$

Obvious relation

$$\hat{f} = \sum_{l=0}^{N-1} \sum_{m=-l}^{l} f_{lm} \hat{Y}_{lm} \longrightarrow \sum_{l=0}^{N-1} \sum_{m=-l}^{l} f_{lm} Y_{lm}(\theta, \phi)$$

Similarly

$$\frac{1}{N} Tr_N \longrightarrow \frac{1}{4\pi} \int d\Omega \quad , \quad d\Omega = \sin\theta d\theta d\phi$$

In addition on S^2_F there is a natural $SU(2)$ covariant differential calculus. The derivations of a function f along X_a are given by

$$e_a(f) = [X_a, f] \quad , \quad a = 1, 2, 3$$

i.e. this calculus is 3-dimensional.

These are essentially the angular momentum operators

$$J_a f = i e_a f = [i X_a, f]$$

which satisfy the SU(2) Lie algebra

$$[J_a, J_b] = i \epsilon_{abc} J_c$$

In the limit $N \to \infty$ the e_a become

$$e_a = \epsilon_{abc} x_b \partial_c$$

i.e. 2-dimensional

The exterior derivative is given by

$$df = [X_a, f] \theta^a$$

θ^a - 1-forms dual to e_a, $\langle e_a, \theta^b \rangle = \delta_a^b$

1-forms are generated by θ^a

$$\omega = \sum_{a=1}^{3} \omega_a \theta^a, \quad \omega \text{ any 1-form}$$

1-form on $M^4 \times S_F^2$: $A = A_\mu dx^\mu + A_a \theta^a$

with $A_\mu = A_\mu(x^\mu, x_a)$, $A_a = A_a(x^\mu, x_a)$

Non Commutative gauge fields and transformations

Consider a field $\phi(X_a)$ on a fuzzy space described by non-com coordinates X_a. An infinitesimal gauge transformation

$$\delta\phi(X_a) = \lambda(X_a)\phi(X_a)$$

$\lambda(X_a)$ - gauge transformation parameter

$U(1)$ if $\lambda(X_a)$ antihermitian function of X_a

$U(P)$ if $\lambda(X_a)$ is valued in Lie algebra of $P \times P$ matrices

Coordinates X_a invariant under gauge transformation $\quad \delta X_a = 0$

- $\delta(X_a \phi) = X_a \lambda(X_a)\phi \neq \lambda(X_a) X_a \phi$

- $\delta(\phi_a \phi) = \lambda(X_a)\phi_a \phi$

covariant coordinates

which holds if $\delta(\phi_a) = [\lambda(X_a), \phi_a]$

- $\phi_a = X_a + A_a$

NC analogue of covariant derivative

interpreted as gauge fields

note that $\delta A_a = -[X_a, \lambda] + [\lambda, A_a]$

supporting the interpretation of A_a

Correspondingly define

- $$F_{ab} = [X_a, A_b] - [X_b, A_a] + [A_a, A_b] - C^c{}_{ab} A_c$$
$$= [\phi_a, \phi_b] - C^c{}_{ab}\phi_c$$ analogue of field strength

$\rightsquigarrow \delta F_{ab} = [\lambda, F_{ab}]$

- $\delta\psi = [\lambda, \psi]$, spinor ψ in the adjoint

Actions in higher dimensions seen as 4-dim actions (expansion in Kaluza-Klein modes)

$$G = U(P) \quad \text{on} \quad M^4 \times (S/R)_F$$

$$A_{YM} = \frac{1}{4} \int d^4x \, \mathrm{Tr} \; tr_G \, F_{MN} F^{MN}$$

integration over $(S/R)_F$

$$F_{MN} \longrightarrow (F_{\mu\nu}, F_{\mu b}, F_{ab})$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu]$$

$$F_{\mu a} = \partial_\mu A_a - [X_a, A_\mu] + [A_\mu, A_a]$$

$$= \partial_\mu \phi_a + [A_\mu, \phi_a] = D_\mu \phi_a$$

$$F_{ab} = [\phi_a, \phi_b] - C^c{}_{ab} \phi_c$$

$$\rightsquigarrow A_{YM} = \int d^4x \, \mathrm{Tr} \; tr_G \left(\frac{1}{4} F_{\mu\nu}^2 + \frac{1}{2} (D_\mu \phi_a)^2 - V(\phi) \right)$$

$$V(\phi) = -\frac{1}{4} \mathrm{Tr} \; tr_G \sum_{ab} F_{ab} F_{ab}$$

$$= -\frac{1}{4} \mathrm{Tr} \; tr_G \sum_{ab} \left([\phi_a, \phi_b] - C^c{}_{ab} \phi_c \right) \left([\phi_a, \phi_b] - C^c{}_{ab} \phi_c \right)$$

The infinitesimal G gauge transf with parameter $\lambda(x^\mu, X^a)$ can be interpreted as M^4 gauge transformation

$$\lambda(x^\mu, X^a) = \lambda^\alpha(x^\mu, X^a)\, T^\alpha$$
$$= \lambda^{h,\alpha}(x^\mu)\, T^h\, T^\alpha$$

T^α – generators of $U(P)$

$\lambda^\alpha(x^\mu, X^a)$ – $N \times N$ matrices, therefore expressible as

Kaluza-Klein modes of $\lambda(x^\mu, X^a)^\alpha$ $\Big\}$ – $\underbrace{\lambda(x^\mu)^{\alpha,h}\, T^h}$ generators of $U(N)$

Considering on equal footing the indices h and α we interpret $\lambda^{h,\alpha}(x^\mu)$ as a field valued in the tensor product $\mathrm{Lie}(U(N)) \otimes \mathrm{Lie}(U(P)) = \mathrm{Lie}(U(NP))$

Similarly we write the gauge field A_ν as

$$A_\nu(x^\mu, X^a) = A_\nu^\alpha(x^\mu, X^a)\, \mathcal{T}^\alpha$$
$$= A_\nu^{h,\alpha}(x^\mu)\, T^h\, \mathcal{T}^\alpha$$

and interpret it as $\mathrm{Lie}(U(NP))$ valued gauge field on M^4.

Similarly for ϕ_a

Then we reduce the number of gauge fields and scalars by applying the CSDR principle.

e.g. $G = U(1)$, $(S/R)_F = S^2_F$

CSDR constraints are satisfied by embedding $SU(2)$ in $U(N)$.

We find in four dimensions

- No H group (due to the fact that the differential calculus is based on $\dim S$ derivations instead of $\dim S - \dim R$ in ordinary case)
- $K = C_{U(N)}(SU(2)) = U(N-2) \times U(1)$ as the final gauge group
- a harmless (singlet) surviving Higgs

Similar results are obtained for $G = U(P)$

CSDR for more general $(S/R)_F$

(e.g. CP^M described by $N \times N$ matrices)

CSDR constraints are satisfied by embedding S in $U(N \cdot P)$

and the 4-dim gauge group is

$$K = C_{U(N\cdot P)}(S)$$

Concerning fermions, to solve the corresponding constraints we embed

$$S \subset SO(\dim S)$$

$$U(N\cdot P) \supset S_{U(N\cdot P)} \times K$$

$$\operatorname{adj} U(N\cdot P) = (\operatorname{adj} S, 1) + (1, \operatorname{adj} K) + \sum_i (s_i, k_i)$$

$$SO(\dim S) \supset S$$

$$\text{spinor } \sigma = \sum_i \sigma_i$$

for $s_i = \sigma_i \rightsquigarrow k_i$ survive in 4 dims

Major difference among ordinary and fuzzy-CSDR

- 4-dim gauge theory appears already spontaneously broken

$\rightsquigarrow$ in 4 dims appears only the physical Higgs that survives SSB

$\rightsquigarrow$ Yukawa sector

(i) massive fermions

(ii) interactions among fermions and physical Higgs fields.

$\Longrightarrow$ if we obtain in fuzzy-CSDR the SM $\rightsquigarrow$ large extra dims

Fundamental differences among ordinary and fuzzy-CSDR:

- A non-abelian gauge group is not necessary in high dims.

The presence of a $U(1)$ in the higher-dim theory is enough to obtain non-abelian gauge theories in 4 dims.

- The theory is renormalizable in the sense that divergencies can be removed by a finite number of counterterms.

Aschieri
Grammatikopoulos
Steinacker
Z
hep-th/0606021
JHEP
hep-th/07060398

We have constructed a renormalizable 4-dim $SU(N)$ gauge theory with suitable multiplet of scalar fields.

The symmetry breaking pattern and low-energy gauge group are determined dynamically in terms of a few free parameters of the potential. Depending on these parameters the final gauge group can be

$SU(n)$ or $SU(n_1) \times SU(n_2) \times U(1)$

We explicitly found the tower of massive K-K modes, consistent with an interpretation as dimensionally reduced higher-dim gauge theory over an S_F^2.

The minima of the potential where vevs of scalars, $\langle \phi_a \rangle$ form the coordinates (generators) of a NC manifold (e.g. S^2_F, CP^N_F ...)

$\rightsquigarrow$ interpreted as spontaneously generated fuzzy extra dims.

Fluctuations around the vacuum: internal components of a higher-dim gauge field

$$\underbrace{\phi_a}_{\text{covariant coordinates}} = \underbrace{\langle \phi_a \rangle}_{\text{coordinates}} + \underbrace{A_a}_{\text{gauge fields}}$$

with a finite K-K tower of massive states.

Intermediate scales

⇝ Gauge theory on $M_4 \times M_{fuzzy}$

Low energy physics governed by zero modes

At high scales the theory behaves again as a 4-dim gauge theory maintaining renormalizability.

⇒ Main features of dim red are realized within the framework of renormalizable 4-dim gauge thy.

Potential problem with chirality: In the best case only models with mirror fermions (not excluded exp)

Steinacker, Z '07
Chatzistavrakidis, Steinacker, Z '09

Chiral models demand additional requirements, e.g. orbifolding

Nice example

$SU(N)^3$ chiral models leading after further spontaneous breakings to $SU(3)^3$ and MSSM.

Chatzi/dis, Steinaker, Z
'10, '11

$N=4$ SYM

Particle content in $N=1$ language

- a $SU(3N)$ vector supermultiplet
- three adjoint chiral superplets Φ^i

and in components: $SU(3N)$ gauge bosons A_μ; 6 adjoint real scalars; ϕ_a (or 3 complex); 4 adjoint Majorana fermions

The theory has a global R-symmetry, $SU(4)_R$ under which the fieds transform:

- gauge fields as singlets
- real scalars as 6
- fermions as 4

Orbifolding by $\mathbb{Z}_3$ embeded in $SU(3)$ as

$$SU(4)_R \supset SU(3) \times U(1)_R$$

$$6 = 3_2 + \bar{3}_{-2}$$

$$4 = 1_3 + 3_{-1}$$

leads to $N=1$ theory. Kachru, Silverstein '98

$\mathbb{Z}_3$ acts non-trivially on the various fields depending on their reps under the R-symmetry and the gauge group.

Orbifold projection keeps the fields which are invariant under the combined Z_3 action (see e.g. Bailin + Love Phys. Rept '99)

The projected theory is

$N=1$, $SU(N)^3$ gauge thy

with chiral superfields in

$$3\left((N,\bar{N},1)+(\bar{N},1,N)+(1,N,\bar{N})\right)$$

i.e. chiral theory!

with 3 families!!!

However the $N=4$ superpotential,

$$W_{N=4} = \mathrm{Tr}\left(\epsilon_{ijk}\,\Phi^i\Phi^j\Phi^k\right)$$

is projected and gives the scalar pot.

$$V_{N=1}(\phi) = \frac{1}{4}\,\mathrm{Tr}\left(\left[\phi^i,\phi^j\right]^+\left[\phi_i,\phi_j\right]\right)$$

with minimum for vanishing vevs

$\rightsquigarrow$ No vacuum of NC-type!

Natural mechanism, aim for

- fuzzy vacua
- (potentially) realistic theory

require introduction of $N=1$ Soft Supersymmetry Breaking (SSB) terms, i.e. those that explicitly break $N=1$, but do not introduce quadratic divergences (Girandello-Grisaru '81): scalar mass terms, trilinear scalar interaction, gaugino masses.

$\rightsquigarrow$ Full potential is

$$V = V_{N=1} + V_{SSB} + V_D$$

D-terms ≥ 0

and can brought in the form

$$V = \frac{1}{4}\left(F^{ij}\right)^{+} F^{ij} + V_D,$$

with $F^{ij} = [\phi^i, \phi^j] - i\epsilon^{ijk}\phi^{k+}$

Vacuum

The minimum is obtained when

$$[\phi^i, \phi^j] = i\epsilon^{ijk}\phi^{k+}$$
$$\phi^i \phi^{i+} = R^2$$

compatible with Z_3 projection

Defining $\phi^i = \underline{\Omega}\tilde{\phi}^i$

with $\underline{\Omega} \neq 1$, $\underline{\Omega}^3 = 1$, $\underline{\Omega}^+ = \underline{\Omega}^{-1}$;

$\tilde{\phi}^{i+} = \tilde{\phi}^i$, i.e. $\phi^{i+} = \underline{\Omega}\phi^i$

$\rightsquigarrow [\tilde{\phi}^i, \tilde{\phi}^j] = i\epsilon^{ijk}\tilde{\phi}^k$; $\tilde{\phi}^i\tilde{\phi}^i = R^2$

i.e. ordinary fuzzy sphere.

The ϕ^is with fluctuations around the vacuum

$$\phi^i = \begin{pmatrix} \lambda^i_{(N-n)} + A^i & 0 & 0 \\ 0 & \omega(\lambda^i_{(N-n)} + A'^i) & 0 \\ 0 & & \omega^2(\lambda^i_{(N-n)} + A''^i) \end{pmatrix}$$

with $\omega = 2\pi/3$

The gauge symmetry $SU(N)^3$ is broken down to $SU(n)^3$

Moreover, there exist a finite K-K tower of massive states.

Particle Physics Models

Considering the embedding

$$SU(N) \supset SU(N-3) \times SU(3) \times U(1)$$

$$\rightsquigarrow SU(N) \longrightarrow SU(3)^3$$

$$SU(3)_c \times SU(3)_L \times SU(3)_R$$

$$3 \cdot \big((3, \bar{3}, 1) + (\bar{3}, 1, 3) + (1, 3, \bar{3}) \big)$$

Embedding in Matrix Models

$\rightsquigarrow$ Z_3-Orbifold Matrix M. Aoki, Iso, Suyama '02

$$Z_3 \subset SU(3) \times U(1) \subset SO(6) \subset SO(9,1)$$

** Necessary and sufficient condts for vanishing β_g and β_{ijk} to all orders

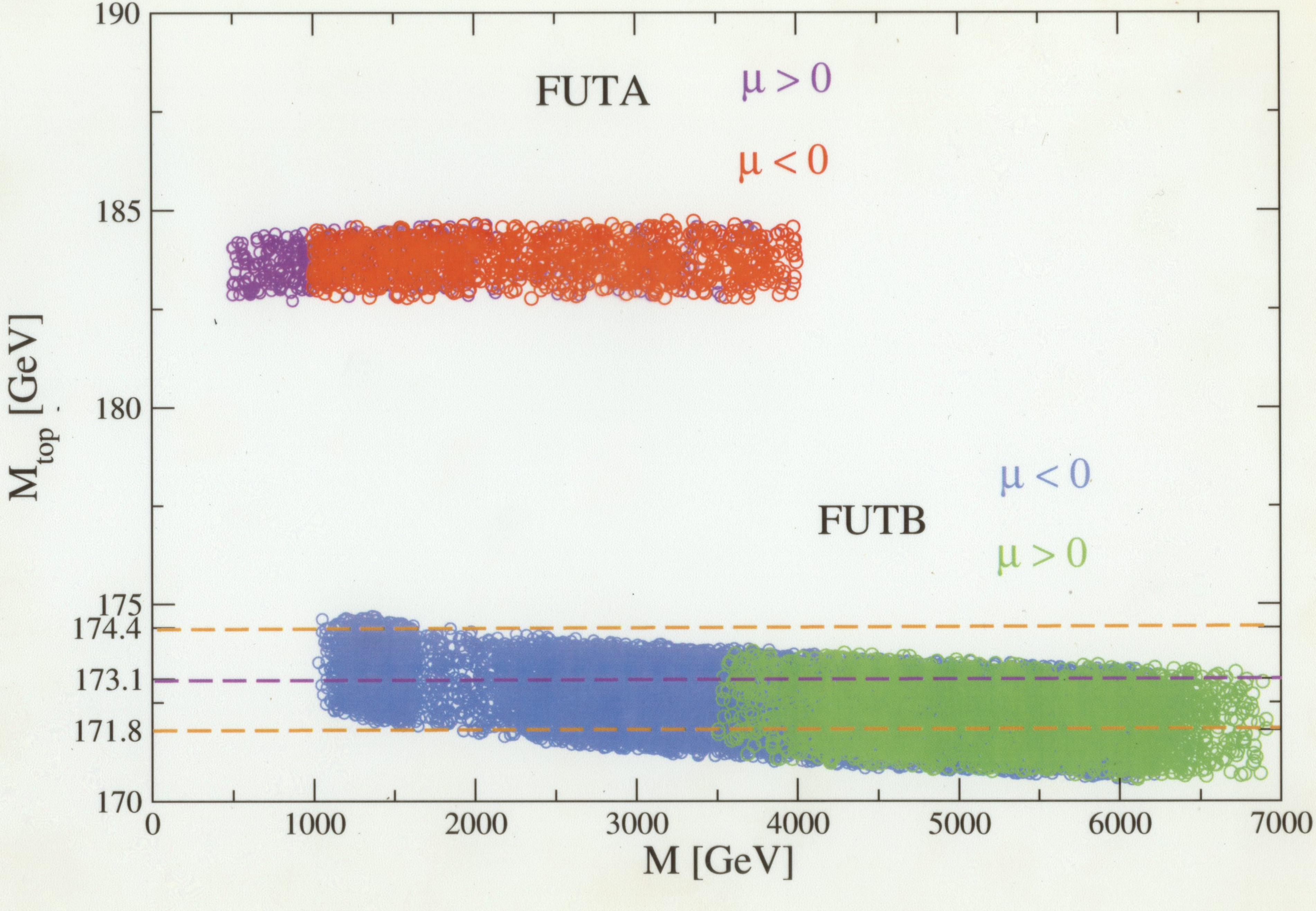
Lucchesi
Piguet
Sibold

1. $\beta_g^{(1)} = 0$
2. $\gamma^{(1)i}{}_j = 0$
3. $\beta_Y^{ijk} = \beta_g \dfrac{d Y^{ijk}}{dg}$

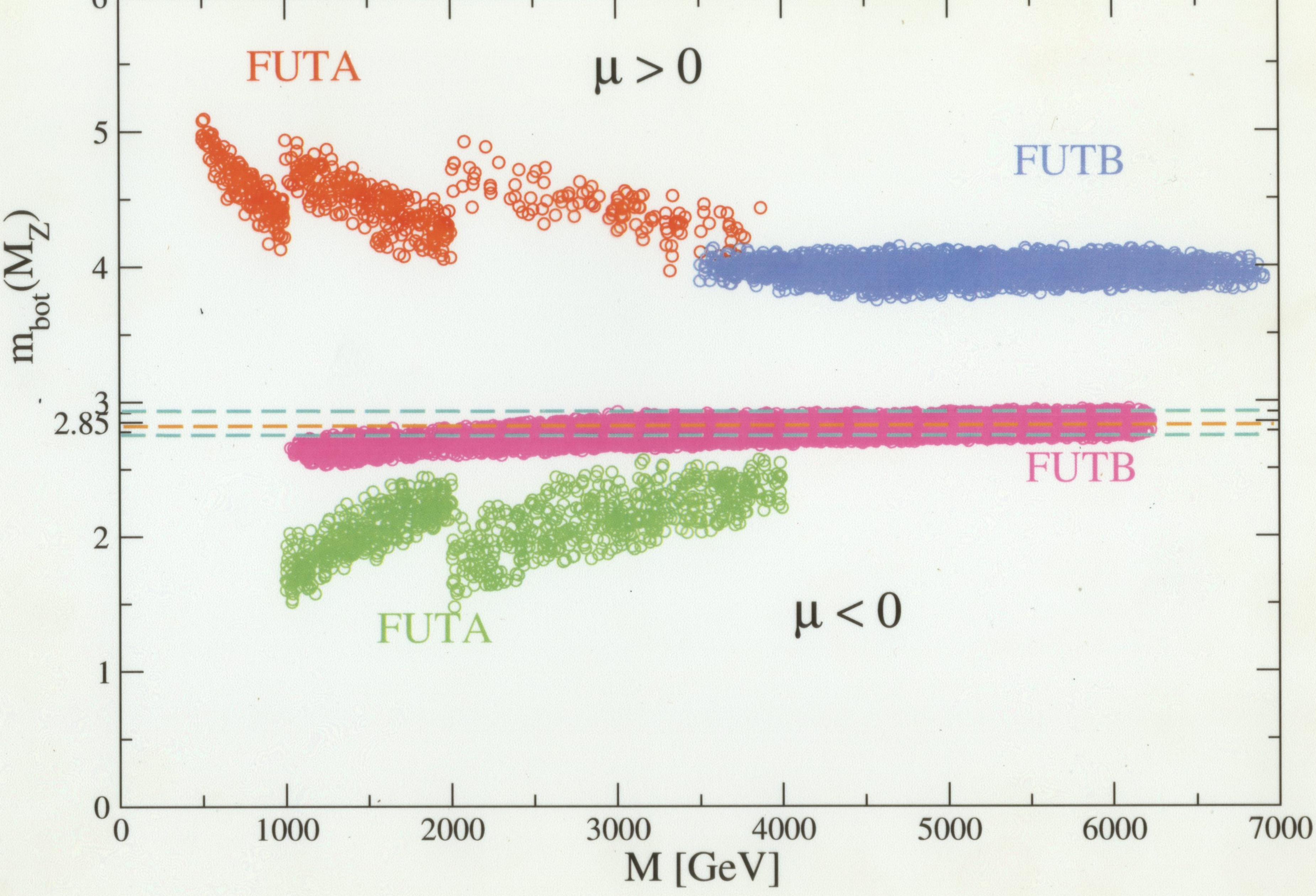
admit power series solution which in lowest order is a solution of condt 2.

3. → 3'. There exist solutions to $\gamma^{(1)i}{}_j = 0$ of the form
$$Y^{ijk} = \rho^{ijk} g, \quad \rho^{ijk}\text{-complex}$$

→ 4. These solutions are isolated and non-degenerate considered as solutions of $\beta_Y^{(1)ijk} = 0$

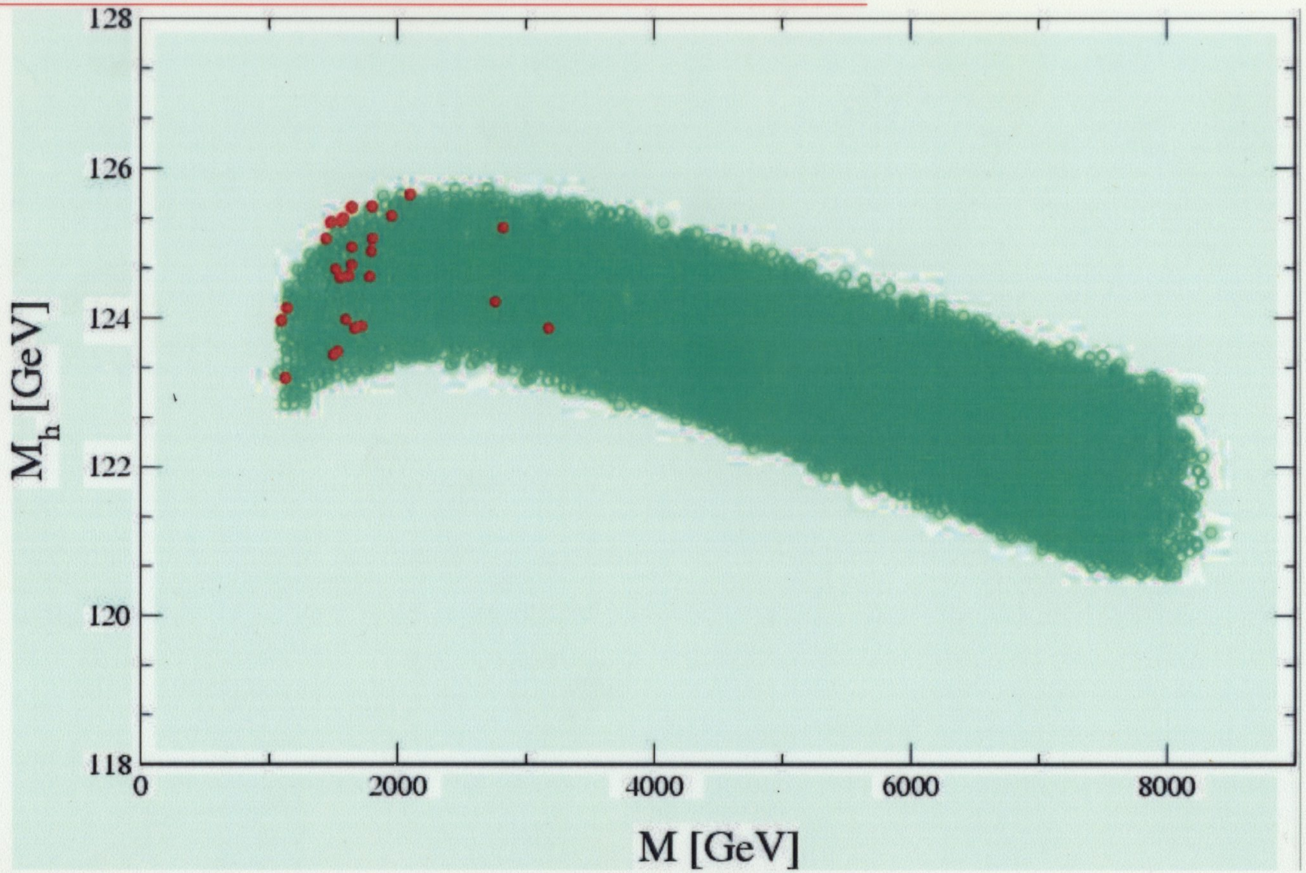


FUTA
$\mu > 0$
$\mu < 0$
FUTB
$\mu < 0$
$\mu > 0$
M_{top} [GeV]
M [GeV]
190
185
180
175
174.4
173.1
171.8
170
0
1000
2000
3000
4000
5000
6000
7000



FUTA
μ > 0
FUTB
FUTB
FUTA
μ < 0
2.85
$m_{bot}(M_Z)$
M [GeV]
0
1
2
3
4
5
6
0
1000
2000
3000
4000
5000
6000
7000

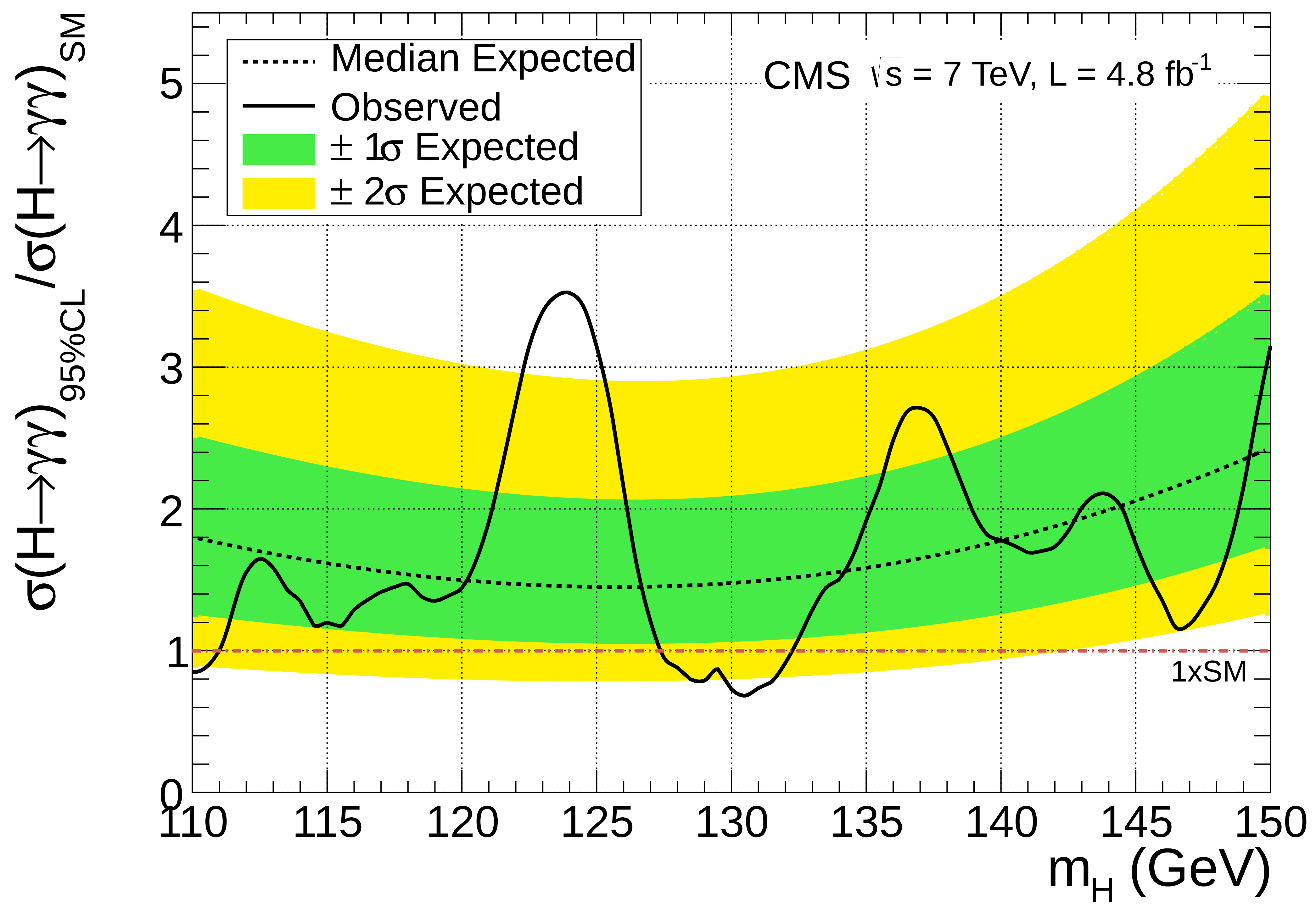
3D) Predictions for the light Higgs boson



green: consistent with B physics constraints
red: agreement with (loose) CDM bound

118 GeV $\leq M_h \leq$ 129 GeV (incl. theor. unc.)

$\Rightarrow$ "easy" to find for LHC (but "only" SM-like . . .)



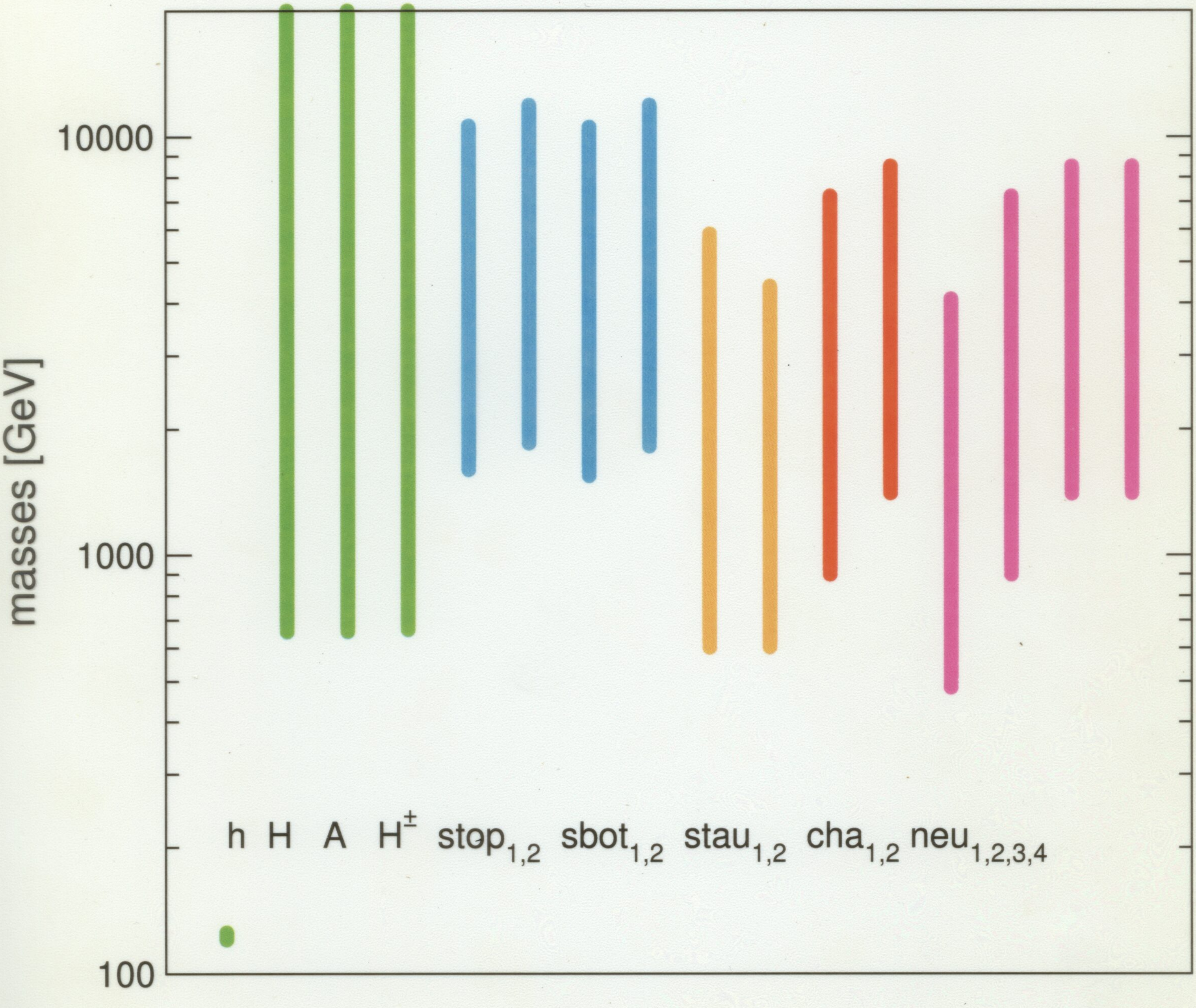
Median Expected
Observed
± 1σ Expected
± 2σ Expected
CMS √s = 7 TeV, L = 4.8 fb-1
1xSM
σ(H→γγ)95%CL/σ(H→γγ)SM
mH (GeV)
0
1
2
3
4
5
110
115
120
125
130
135
140
145
150

Typical mass spectrum for FUTB- :

m_t	172	$\overline{m_b}(M_Z)$	2.7
$\tan\beta =$	46	α_s	0.116
$m_{\tilde{\chi}_1^0}$	796	$m_{\tilde{\tau}_2}$	1268
$m_{\tilde{\chi}_2^0}$	1462	$m_{\tilde{\nu}_3}$	1575
$m_{\tilde{\chi}_3^0}$	2048	μ	-2046
$m_{\tilde{\chi}_4^0}$	2052	B	4722
$m_{\tilde{\chi}_1^\pm}$	1462	M_A	870
$m_{\tilde{\chi}_2^\pm}$	2052	$M_{H^\pm}$	875
$m_{\tilde{t}_1}$	2478	M_H	869
$m_{\tilde{t}_2}$	2804	M_h	124
$m_{\tilde{b}_1}$	2513	M_1	796
$m_{\tilde{b}_2}$	2783	M_2	1467
$m_{\tilde{\tau}_1}$	798	M_3	3655

M1	580 GeV
M2	1077 GeV
Mgluino	2754 GeV
Stop1	1876 GeV
Stop2	2146 GeV
Sbot1	1849 GeV
Sbot2	2117 GeV
Mstau1	635 GeV
Mstau2	867 GeV
Char1	1072 GeV
Char2	1597 GeV
Neu1	579 GeV
Neu2	1072 GeV
Neu3	1591 GeV
Neu4	1596 GeV
Mh	123.1 GeV
MH	679 GeV
MA	680 GeV
$MH^{\pm}$	685 GeV
Mtop	172.2 GeV
Mbot(M_Z)	2.71 GeV

FUTB, $\mu < 0$


masses [GeV]
10000
1000
100
h H A H±
stop1,2 sbot1,2 stau1,2 cha1,2 neu1,2,3,4

E I S A

European Institute for Sciences and Their Applications

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12th Hellenic School and Workshops on Elementary Particle Physics and Gravity

Corfu, Greece 2012

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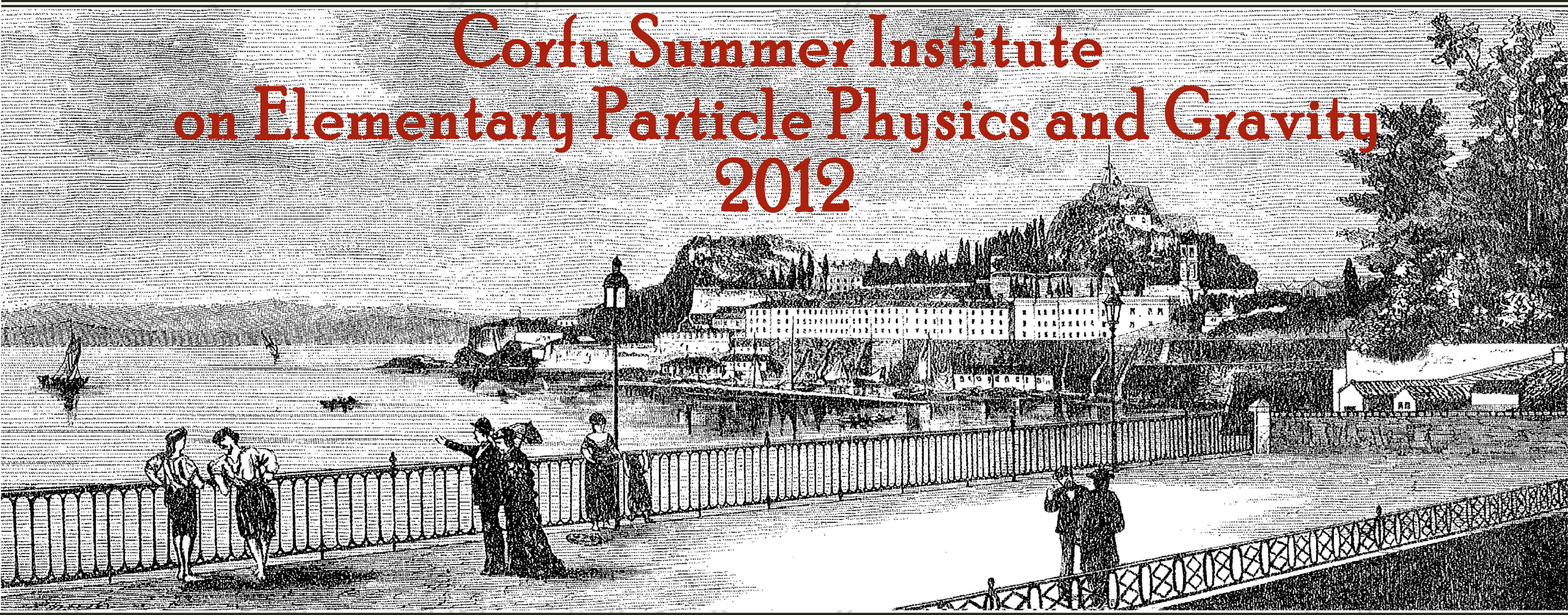

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