Particle Physics
The Standard Model

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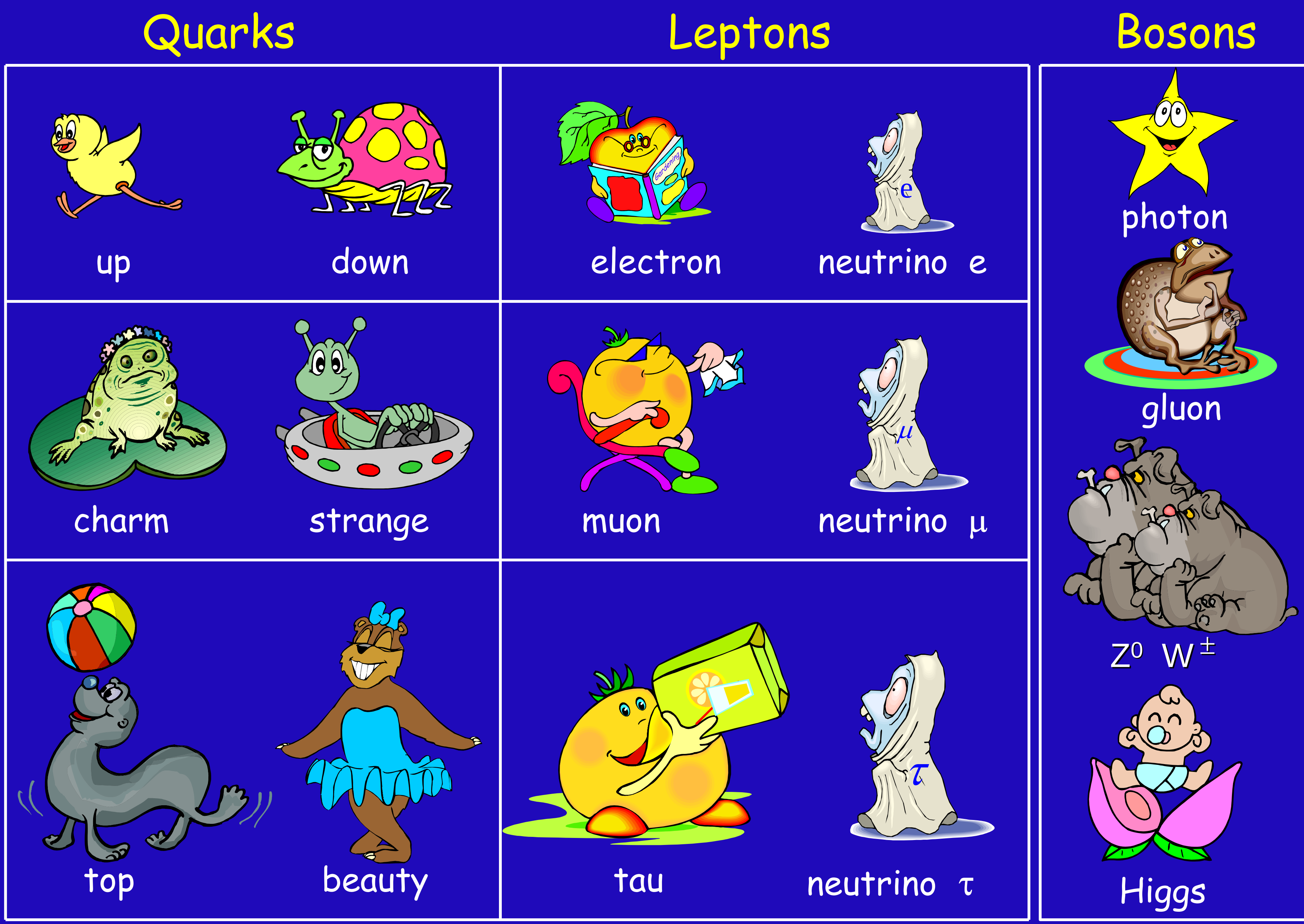
8. Flavour Dynamics

- Fermion Masses
- Fermion Generations
- Quark Mixing
- Lepton Mixing
- Standard Model Parameters
- CP Violation


Quarks
Leptons
Bosons
up
down
electron
neutrino e
photon
gluon
charm
strange
muon
neutrino μ
Z^0 $W^{\pm}$
top
beauty
tau
neutrino τ
Higgs

FERMION MASSESS

Scalar – Fermion Couplings allowed by Gauge Symmetry

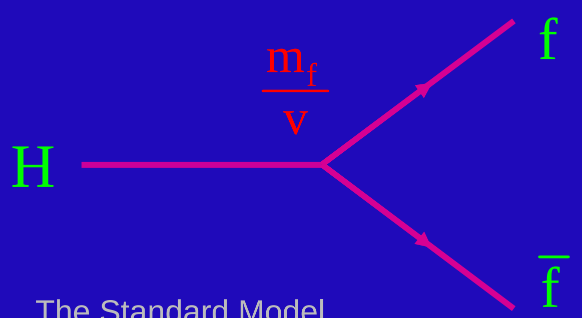
$$\mathcal{L}_Y = (\bar{q}_u , \bar{q}_d)_L \left[c^{(d)} \begin{pmatrix} \phi^{(+)} \\ \phi^{(0)} \end{pmatrix} (q_d)_R + c^{(u)} \begin{pmatrix} \phi^{(0)\dagger} \\ -\phi^{(+)\dagger} \end{pmatrix} (q_u)_R \right] + (\bar{\nu}_l , \bar{l})_L \; c^{(l)} \begin{pmatrix} \phi^{(+)} \\ \phi^{(0)} \end{pmatrix} l_R + \text{h.c.}$$

⬇ **SSB**

$$\mathcal{L}_Y = -\left(1+\frac{H}{v}\right) \left\{ m_{q_d}\; \bar{q}_d\, q_d + m_{q_u}\; \bar{q}_u\, q_u + m_l\; \bar{l}\, l \right\}$$

Fermion Masses are New Free Parameters

$$\left[m_{q_d} , \; m_{q_u} , \; m_l \right] = -\left[c^{(d)} , \; c^{(u)} , \; c^{(l)} \right] \frac{v}{\sqrt{2}}$$



Couplings Fixed: $g_{Hf\bar{f}} = \dfrac{m_f}{v}$

FERMION GENERATIONS

$N_G = 3$ Identical Copies

Masses are the only difference

$Q=0$ / $Q=-1$ $\begin{pmatrix} \nu'_j & u'_j \\ l'_j & d'_j \end{pmatrix}$ $Q=+2/3$ / $Q=-1/3$

$(j = 1, \cdots, N_G)$

WHY ?

$$\mathcal{L}_Y = \sum_{jk} \left\{ \left(\bar{u}'_j, \bar{d}'_j\right)_L \left[c^{(d)}_{jk} \begin{pmatrix} \phi^{(+)} \\ \phi^{(0)} \end{pmatrix} d'_{kR} + c^{(u)}_{jk} \begin{pmatrix} \phi^{(0)\dagger} \\ -\phi^{(+)\dagger} \end{pmatrix} u'_{kR} \right] + \left(\bar{\nu}'_j, \bar{l}'_j\right)_L c^{(l)}_{jk} \begin{pmatrix} \phi^{(+)} \\ \phi^{(0)} \end{pmatrix} l'_{kR} \right\} + \text{h.c.}$$

SSB

$$\mathcal{L}_Y = -\left(1 + \frac{H}{v}\right) \left\{ \bar{d}'_L \cdot \mathbf{M}'_d \cdot d'_R + \bar{u}'_L \cdot \mathbf{M}'_u \cdot u'_R + \bar{l}'_L \cdot \mathbf{M}'_l \cdot l'_R + \text{h.c.} \right\}$$

Arbitrary Non-Diagonal Complex Mass Matrices

$$\left[\mathbf{M}'_d , \mathbf{M}'_u , \mathbf{M}'_l \right]_{jk} = - \left[c^{(d)}_{jk} , c^{(u)}_{jk} , c^{(l)}_{jk} \right] \frac{v}{\sqrt{2}}$$

DIAGONALIZATION OF MASS MATRICES

$$\mathbf{M}'_d = \mathbf{H}_d \cdot \mathbf{U}_d = \mathbf{S}_d^\dagger \cdot \mathcal{M}_d \cdot \mathbf{S}_d \cdot \mathbf{U}_d$$

$$\mathbf{M}'_u = \mathbf{H}_u \cdot \mathbf{U}_u = \mathbf{S}_u^\dagger \cdot \mathcal{M}_u \cdot \mathrm{S}_u \cdot \mathbf{U}_u$$

$$\mathbf{M}'_l = \mathbf{H}_l \cdot \mathbf{U}_l = \mathbf{S}_l^\dagger \cdot \mathcal{M}_l \cdot \mathbf{S}_l \cdot \mathbf{U}_l$$

$$\mathbf{H}_\mathrm{f} = \mathbf{H}_\mathrm{f}^\dagger$$

$$\mathbf{U}_\mathrm{f} \cdot \mathbf{U}_\mathrm{f}^\dagger = \mathbf{U}_\mathrm{f}^\dagger \cdot \mathbf{U}_\mathrm{f} = 1$$

$$\mathbf{S}_\mathrm{f} \cdot \mathbf{S}_\mathrm{f}^\dagger = \mathbf{S}_\mathrm{f}^\dagger \cdot \mathbf{S}_\mathrm{f} = 1$$

$$\mathcal{L}_Y = -\left(1+\frac{H}{v}\right)\left\{\bar{\mathrm{d}} \cdot \mathcal{M}_d \cdot \mathrm{d} + \bar{\mathrm{u}} \cdot \mathcal{M}_u \cdot \mathrm{u} + \bar{l} \cdot \mathcal{M}_l \cdot l\right\}$$

$$\mathcal{M}_u = \mathrm{diag}\,(m_u\,, m_c\,, m_t) \quad ; \quad \mathcal{M}_d = \mathrm{diag}\,(m_d\,, m_s\,, m_b) \quad ; \quad \mathcal{M}_l = \mathrm{diag}\,(m_e\,, m_\mu\,, m_\tau)$$

$$\mathrm{d}_L \equiv \mathbf{S}_d \cdot \mathrm{d}'_L \quad ; \quad \mathrm{u}_L \equiv \mathbf{S}_u \cdot \mathrm{u}'_L \quad ; \quad l_L \equiv \mathbf{S}_l \cdot l'_L$$

$$\mathrm{d}_R \equiv \mathbf{S}_d \cdot \mathbf{U}_d \cdot \mathrm{d}'_R \quad ; \quad \mathrm{u}_R \equiv \mathbf{S}_u \cdot \mathbf{U}_u \cdot \mathrm{u}'_R \quad ; \quad l_R \equiv \mathbf{S}_l \cdot \mathbf{U}_l \cdot l'_R$$

Mass Eigenstates

$\neq$

Weak Eigenstates

$$\bar{\mathrm{f}}'_L\, \mathrm{f}'_L = \bar{\mathrm{f}}_L\, \mathrm{f}_L \quad ; \quad \bar{\mathrm{f}}'_R\, \mathrm{f}'_R = \bar{\mathrm{f}}_R\, \mathrm{f}_R \quad \Longrightarrow \quad \mathcal{L}'_{\mathrm{NC}} = \mathcal{L}_{\mathrm{NC}}$$

$$\bar{\mathrm{u}}'_L\, \mathrm{d}'_L = \bar{\mathrm{u}}_L \cdot \mathbf{V} \cdot \mathrm{d}_L \quad ; \quad \mathbf{V} \equiv \mathbf{S}_u \cdot \mathbf{S}_d^\dagger \quad \Longrightarrow \quad \mathcal{L}'_{\mathrm{CC}} \neq \mathcal{L}_{\mathrm{CC}}$$

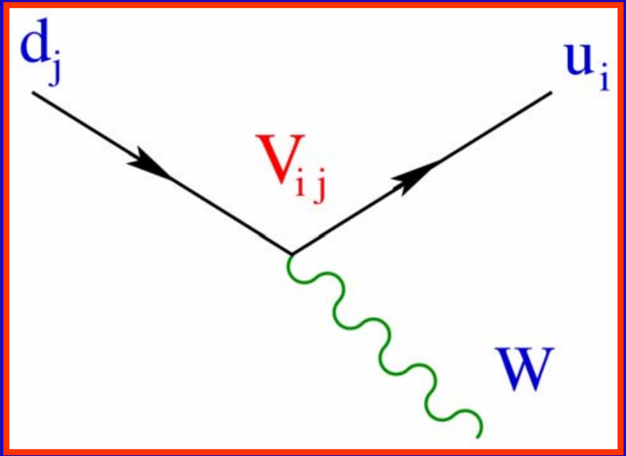
QUARK MIXING

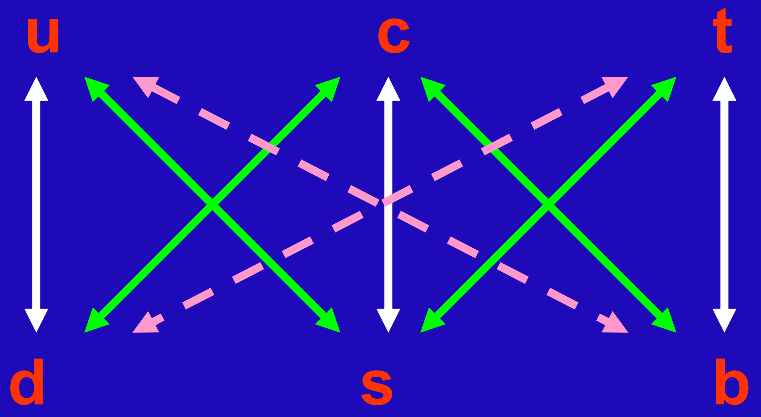
$$\mathcal{L}_{NC}^{Z} = \frac{e}{2\sin\theta_W \cos\theta_W}\, Z_\mu \sum_f \bar{f}\,\gamma^\mu\,[v_f - a_f\,\gamma_5]\, f$$

Flavour Conserving Neutral Currents

$$\mathcal{L}_{CC} = \frac{g}{2\sqrt{2}}\, W_\mu^\dagger \left[\sum_{ij} \bar{u}_i\,\gamma^\mu (1-\gamma_5)\, \mathbf{V}_{ij}\, d_j + \sum_l \bar{\nu}_l\,\gamma^\mu (1-\gamma_5)\, l \right] + \text{h.c.}$$

Flavour Changing Charged Currents





LEPTON MIXING

$$\mathcal{L}_{CC}^{(l)} = \frac{g}{2\sqrt{2}} W_\mu^\dagger \sum_{ij} \bar{\nu}_i \gamma^\mu (1-\gamma_5) \mathbf{V}_{ij}^{(l)} l_j + \text{h.c.}$$

- **IF** $m_{\nu_i} = 0$ ➡ $\mathcal{L}_{CC}^{(l)} = \frac{g}{2\sqrt{2}} W_\mu^\dagger \sum_l \bar{\nu}_l \gamma^\mu (1-\gamma_5) l + \text{h.c}$

$$\bar{\nu}_{l_j} \equiv \bar{\nu}_i \mathbf{V}_{ij}^{(l)}$$

Separate Lepton Number Conservation (Minimal SM without ν_R)

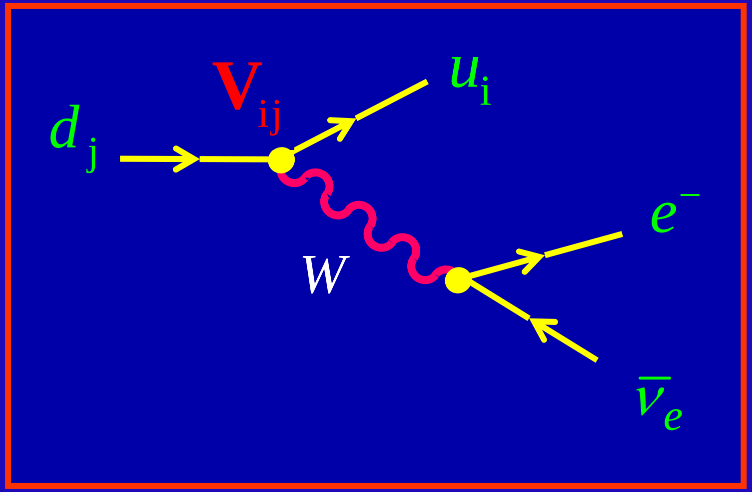
- **IF** ν_R^i exist and $m_{\nu_i} \neq 0$

$\not{L}_e , \not{L}_\mu , \not{L}_\tau$ $\qquad (L_e + L_\mu + L_\tau$ Conserved$)$

BUT $\quad \text{Br}(\mu \to e\gamma) < 1.2 \times 10^{-11} \quad ; \quad \text{Br}(\tau \to \mu\gamma) < 6.8 \times 10^{-8}$

(90 % CL)

Measurements of $\mathbf{V}_{ij}$



$$\Gamma(d_j \to u_i\, e^-\, \bar{\nu}_e) \propto \left|\mathbf{V}_{ij}\right|^2$$

We measure decays of hadrons (no free quarks)

Important QCD Uncertainties

V_{ij} DETERMINATIONS

CKM entry	Value	Source
$\lvert V_{ud}\rvert$	0.9740 ± 0.0005 0.9745 ± 0.0019 0.9740 ± 0.0005	Nuclear β decay $n \to p e^- \bar{\nu}_e$
$\lvert V_{us}\rvert$	0.2233 ± 0.0028 0.2208 ± 0.0034 0.2219 ± 0.0025 0.2221 ± 0.0016	$K \to \pi e^- \bar{\nu}_e$ τ decays $K/\pi \to \mu\nu$, Lattice
$\lvert V_{cd}\rvert$	0.224 ± 0.012	$\nu d \to c X$
$\lvert V_{cs}\rvert$	0.97 ± 0.11 0.974 ± 0.013	$W^+ \to c\bar{s}$ $W^+ \to$ had, V_{uj}, $V_{cd,cb}$
$\lvert V_{cb}\rvert$	0.0413 ± 0.0021 0.0416 ± 0.0010 0.0415 ± 0.0010	$B \to D^* l \bar{\nu}_l$ $b \to c l \bar{\nu}_l$
$\lvert V_{ub}\rvert$	0.0038 ± 0.0009 0.0044 ± 0.0005 0.0043 ± 0.0005	$B \to \rho l \bar{\nu}_l$ $b \to u l \bar{\nu}_l$
$\lvert V_{tb}\rvert / \sqrt{\sum_q \lvert V_{tq}\rvert^2}$	$0.97 \,{}^{+\,0.16}_{-\,0.12}$	$t \to bW / qW$

$$|\mathbf{V_{ud}}|^2 + |\mathbf{V_{us}}|^2 + |\mathbf{V_{ub}}|^2 = \mathbf{0.9980 \pm 0.0017}$$

$$\sum_{\mathbf{j}} \left(|\mathbf{V_{uj}}|^2 + |\mathbf{V_{cj}}|^2 \right) = \mathbf{1.999 \pm 0.025} \quad \text{(LEP)}$$

QUARK MIXING MATRIX

- **Unitary** $N_G \times N_G$ **Matrix:** N_G^2 **parameters**

$$\mathbf{V} \cdot \mathbf{V}^\dagger = \mathbf{V}^\dagger \cdot \mathbf{V} = \mathbf{1}$$

- $2 N_G - 1$ **arbitrary phases:**

$$u_i \to e^{i\phi_i} u_i \quad ; \quad d_j \to e^{i\theta_j} d_j \quad \Longrightarrow \quad \mathbf{V}_{ij} \to e^{i(\theta_j - \phi_i)} \mathbf{V}_{ij}$$

$\Downarrow$

$\mathbf{V}_{ij}$ **Physical Parameters:**

$\frac{1}{2} N_G (N_G - 1)$ **Moduli** ; $\frac{1}{2} (N_G - 1) (N_G - 2)$ **phases**

- **$N_f = 2$:** 1 angle, 0 phases (Cabibbo)

$$\mathbf{V} = \begin{bmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{bmatrix} \Longrightarrow \text{No } \not{CP}$$

- **$N_f = 3$:** 3 angles, 1 phase (CKM) $\qquad c_{ij} \equiv \cos\theta_{ij} \;;\; s_{ij} \equiv \sin\theta_{ij}$

$$\mathbf{V} = \begin{bmatrix} c_{12}\, c_{13} & s_{12}\, c_{13} & s_{13}\, e^{-i\delta_{13}} \\ -s_{12}\, c_{23} - c_{12}\, s_{23}\, s_{13}\, e^{i\delta_{13}} & c_{12}\, c_{23} - s_{12}\, s_{23}\, s_{13}\, e^{i\delta_{13}} & s_{23}\, c_{13} \\ s_{12}\, s_{23} - c_{12}\, c_{23}\, s_{13}\, e^{i\delta_{13}} & -c_{12}\, s_{23} - s_{12}\, c_{23}\, s_{13}\, e^{i\delta_{13}} & c_{23}\, c_{13} \end{bmatrix}$$

$$\approx \begin{bmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{bmatrix} + \mathcal{O}\left(\lambda^4\right)$$

$\lambda \approx \sin\theta_C \approx 0.222 \;;\; A \approx 0.84 \;;\; \sqrt{\rho^2 + \eta^2} \approx 0.41$ $\qquad \delta_{13} \neq 0 \quad (\eta \neq 0) \Longrightarrow \not{CP}$

Standard Model Parameters

QCD: $\alpha_S(M_Z)$ **1**

EW Gauge / Scalar Sector: **4**

$$g\,,\,g'\,,\,\mu^2\,,\,h \;\Leftrightarrow\; \alpha\,,\,\theta_W\,,\,M_W\,,\,M_H \;\Leftrightarrow\; \alpha\,,\,G_F\,,\,M_Z\,,\,M_H$$

Yukawa Sector: **13**

$$m_e\,,\; m_\mu\,,\; m_\tau$$

$$m_d\,,\; m_s\,,\; m_b$$

$$m_u\,,\; m_c\,,\; m_t$$

$$\theta_1\,,\,\theta_2\,,\,\theta_3\,,\,\delta$$

➡ **18 Free Parameters** (+ Neutrino Masses / Mixings ?)

TOO MANY !

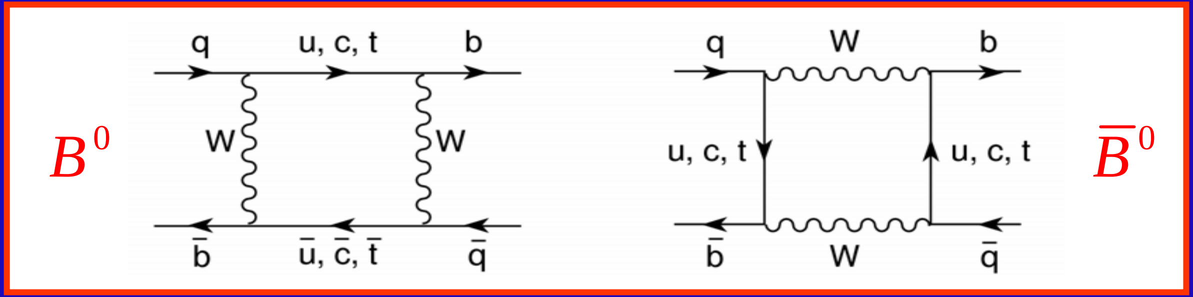
- $\mathcal{C}$, $\mathcal{P}$: Violated maximally in weak interactions
- $\mathcal{CP}$: Symmetry of nearly all observed phenomena
- Slight (~ 0.2 %) $\not{\mathcal{CP}}$ in K^0 decays (1964)
- Sizeable $\not{\mathcal{CP}}$ in B^0 decays (2001)
- Huge Matter— Antimatter Asymmetry in our Universe → Baryogenesis

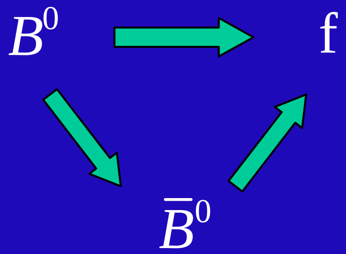
$\mathcal{CPT}$ Theorem: $\not{\mathcal{CP}} \Longleftrightarrow \not{\mathcal{T}}$

Thus, $\not{\mathcal{CP}}$ requires:

- Complex Phases
- Interferences

Meson – Antimeson Mixing

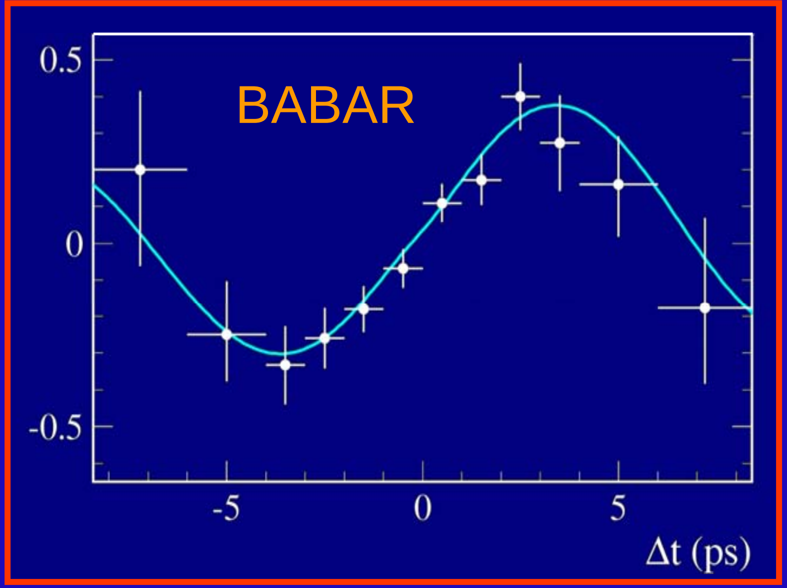




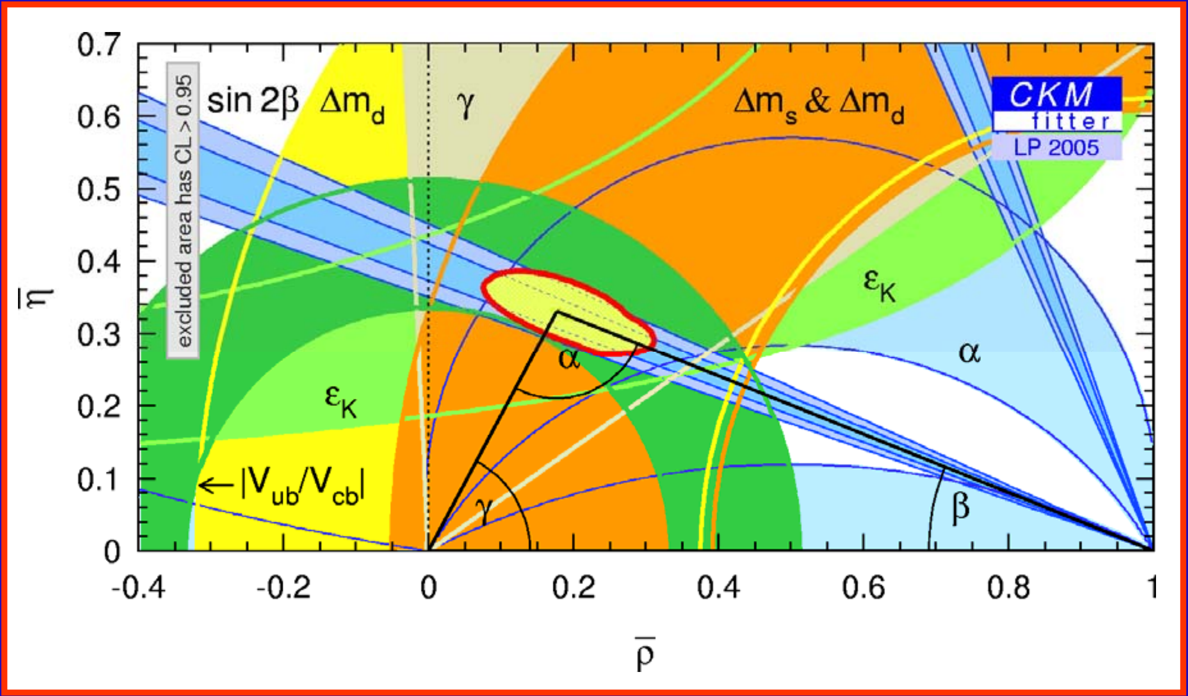
2 Interfering Amplitudes

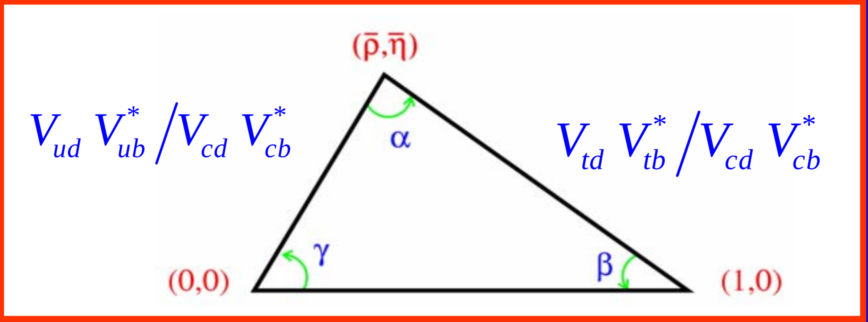
$\not{C}\not{P}$ Signal

$$\frac{\Gamma(B^0 \to J/\psi\, K_S) - \Gamma(\overline{B}^0 \to J/\psi\, K_S)}{\Gamma(B^0 \to J/\psi\, K_S) + \Gamma(\overline{B}^0 \to J/\psi\, K_S)} \neq 0$$



$$V_{ud}\, V_{ub}^{*} + V_{cd}\, V_{cb}^{*} + V_{td}\, V_{tb}^{*} = 0$$





UT*fit*

$$\bar{\eta} \equiv \eta \left(1 - \frac{1}{2}\lambda^2\right) = 0.342 \pm 0.022$$

$$\bar{\rho} \equiv \rho \left(1 - \frac{1}{2}\lambda^2\right) = 0.216 \pm 0.036$$

$$\alpha = 98.2 \pm 7.7^\circ \;;\; \beta = 23.7 \pm 1.4^\circ \;;\; \gamma = 57.9 \pm 7.3^\circ$$

Standard Model Mechanism of $\not{CP}$

Complex phases in Yukawa couplings only:

$$\mathcal{L}_Y = \sum_{jk} (\bar{u}'_j, \bar{d}'_j)_L \left[c^{(d)}_{jk} \begin{pmatrix} \phi^{(+)} \\ \phi^{(0)} \end{pmatrix} d'_{kR} + c^{(u)}_{jk} \begin{pmatrix} \phi^{(0)\dagger} \\ -\phi^{(+)\dagger} \end{pmatrix} u'_{kR} \right] + \text{h.c.}$$

SSB $\left[\langle \phi^{(0)} \rangle = v/\sqrt{2} \right]$

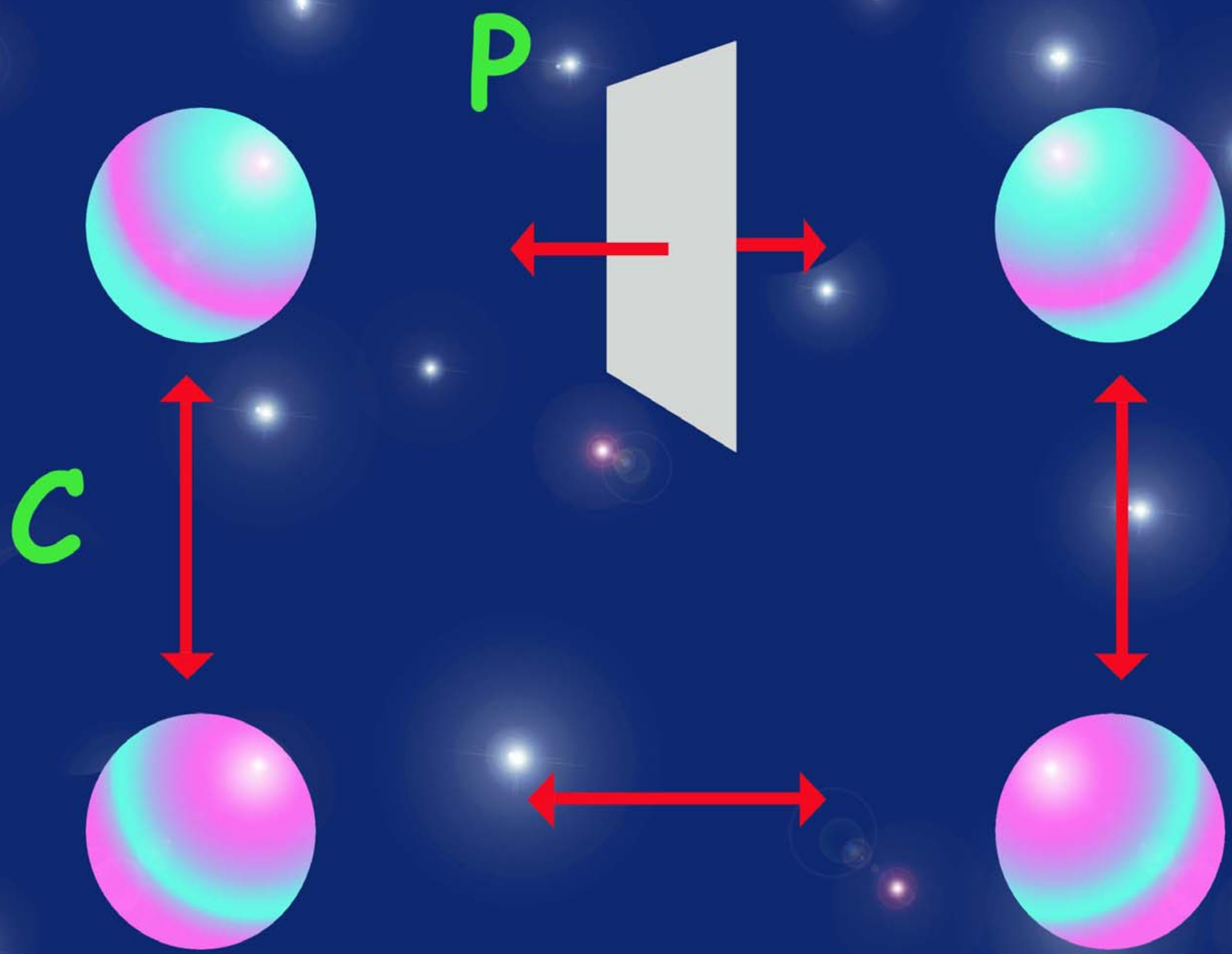
$$\mathcal{L}_Y = -\left(1 + \frac{H}{v}\right) \frac{v}{\sqrt{2}} \left\{ \bar{d}'_{jL}\, c^{(d)}_{jk}\, d'_{kR} + \bar{u}'_{jL}\, c^{(u)}_{jk}\, u'_{kR} + \text{h.c.} \right\}$$

$c^{(q)}_{jk}$ diagonalization

$$\mathcal{L}_Y = -\left(1 + \frac{H}{v}\right) \left\{ \bar{d}_{jL}\, m_{d_j}\, d_{jR} + \bar{u}_{jL}\, m_{u_j}\, u_{jR} + \text{h.c.} \right\}$$

$$\mathcal{L}_{CC} = \frac{g}{2\sqrt{2}} W^\dagger_\mu \sum_{ij} \bar{u}_i \gamma^\mu (1-\gamma_5)\, \mathbf{V}_{ij}\, d_j + \text{h.c.}$$

The CKM matrix $\mathbf{V}_{ij}$ is the only source of $\not{CP}$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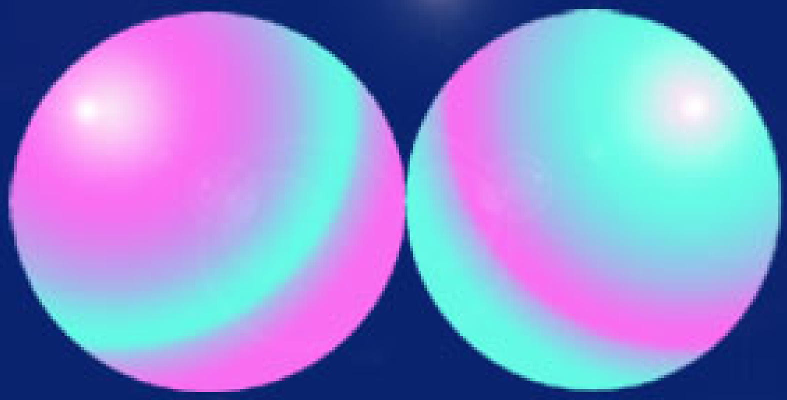
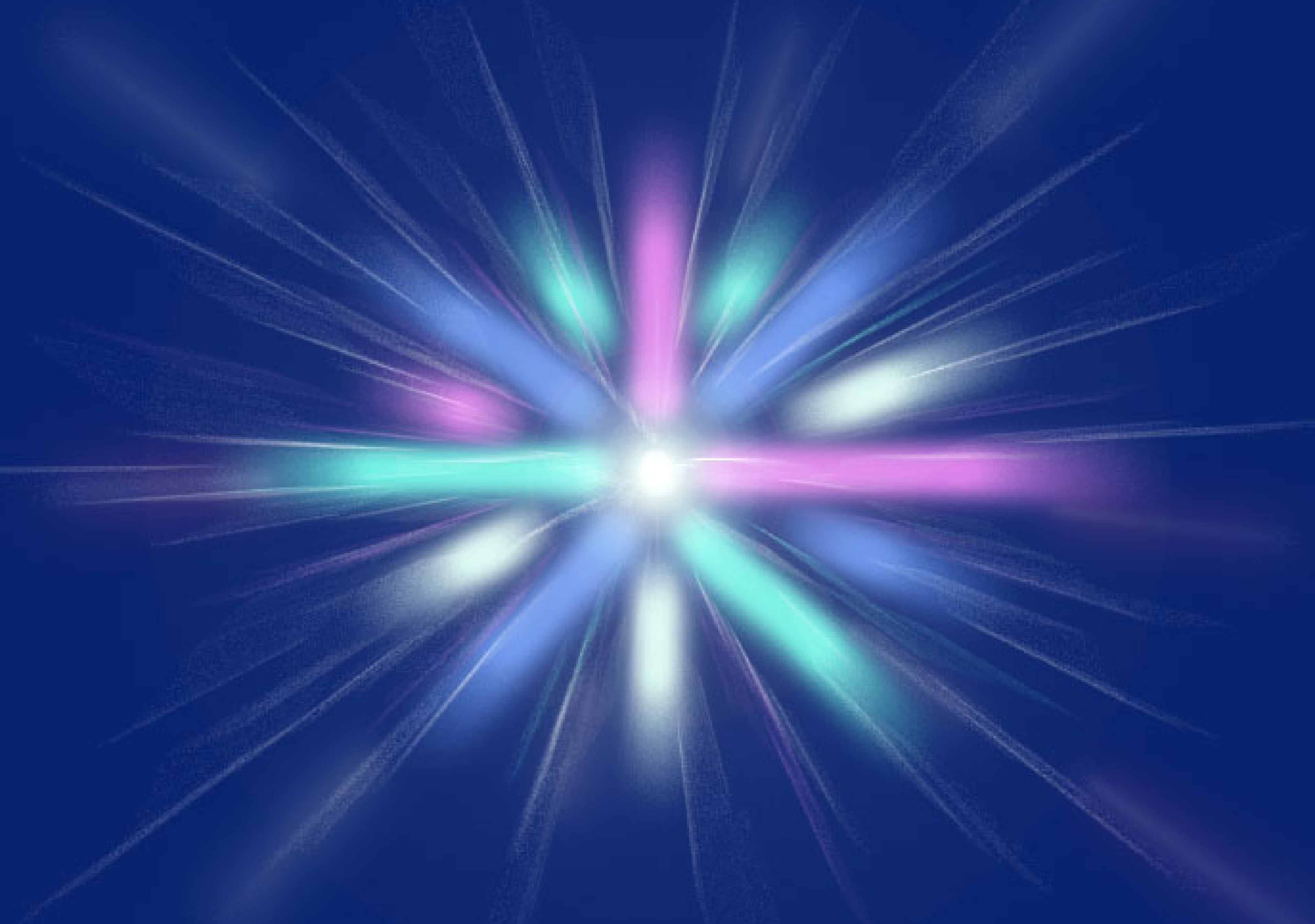
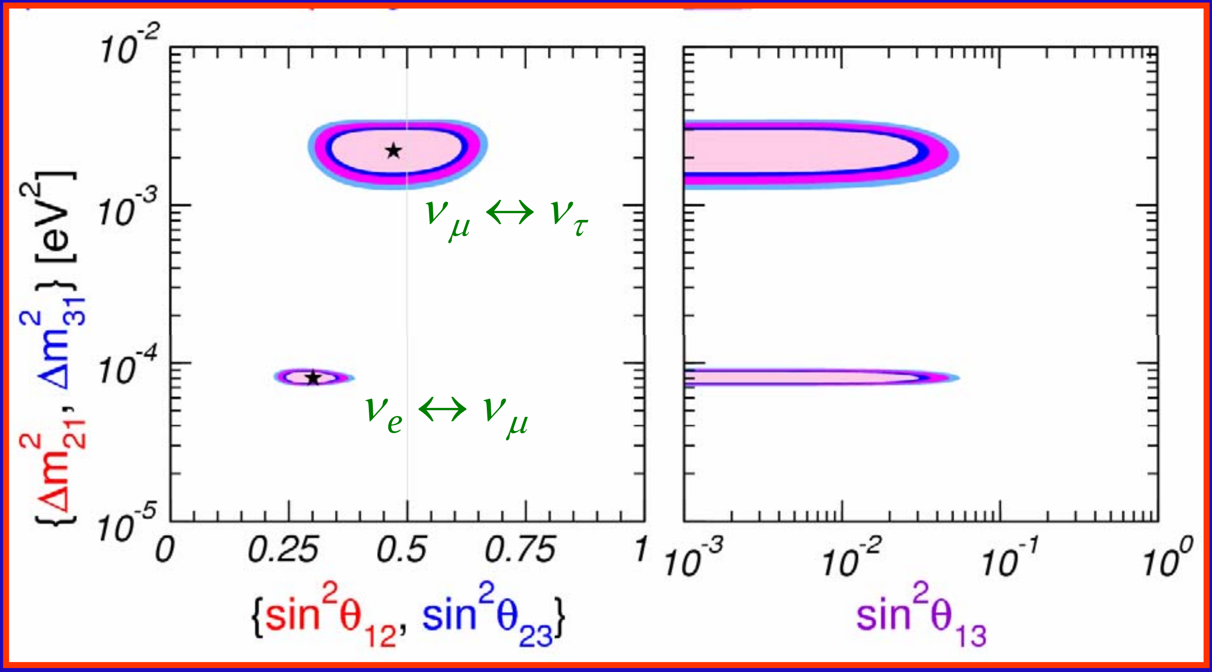




Neutrino Oscillations

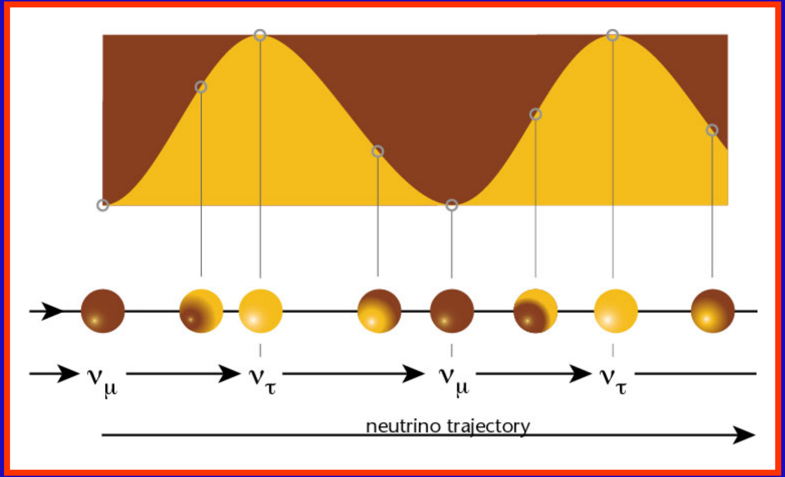


M. Maltoni

Lepton Mixing

ν_R , ~~CP~~ ?

NEW PHYSICS



THE STANDARD THEORY OF FUNDAMENTAL INTERACTIONS

$$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$$

Electroweak + Strong Forces

- Gauge Symmetry ➡ Dynamics
- 3 Gauge Parameters: $\alpha_s(M_Z^2)$, α , θ_W
- All Known Experimental Facts Explained
- Problem with Mass Scales / Mixings:

- 15 Additional Parameters
- Why 3 Families ?
- Why Left $\neq$ Right ?
- Why $m_t > M_Z$?
- Does the Higgs Exist ?
- Flavour Mixing
- $\mathcal{CP}$ Violation
- Neutrino Masses / Oscillations


WANTED
Higgs
GREAT REWARD
STOCKHOLM
e
μ
τ