Particle Physics
The Standard Model

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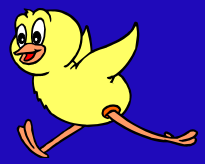
5. Electroweak Unification

- Experimental Facts
- $SU(2)_L \otimes U(1)_Y$ Gauge Theory
- Charged Current Interaction
- Neutral Current Interaction
- Gauge Self-Interactions

Quarks



up

down

charm

strange

top

beauty

Leptons

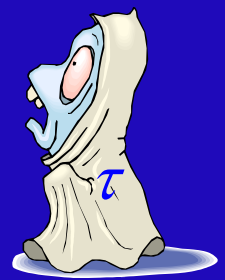
electron

neutrino e

muon

neutrino μ

tau

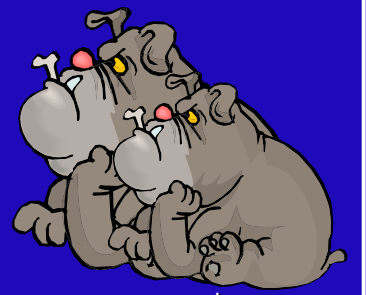


neutrino τ

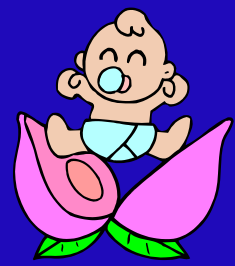
Bosons

photon

gluon



Z^0 $W^{\pm}$



Higgs

CHIRALITY

Chirality Projectors:

$$\gamma_5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 0 & I_2 \\ I_2 & 0 \end{pmatrix} \quad ; \quad (\gamma_5)^2 = I_4$$

$$P_R \equiv \frac{1+\gamma_5}{2} \quad ; \quad P_L \equiv \frac{1-\gamma_5}{2}$$

$$P_R^2 = P_R \quad ; \quad P_L^2 = P_L \quad ; \quad P_R P_L = P_L P_R = 0$$

$$\psi(x) = (P_L + P_R)\,\psi(x) \equiv \psi_L(x) + \psi_R(x)$$

$$\bar{\psi} \equiv \psi^\dagger \gamma^0$$

$$\mathcal{L} = \bar{\psi}\,(i\gamma^\mu \partial_\mu - m)\,\psi$$

$$= \bar{\psi}_L\, i\gamma^\mu \partial_\mu \psi_L + \bar{\psi}_R\, i\gamma^\mu \partial_\mu \psi_R - m\,(\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L)$$

EXPERIMENTAL FACTS

Three Families

$$\begin{bmatrix} \nu_e & u \\ e^- & d' \end{bmatrix} \quad , \quad \begin{bmatrix} \nu_\mu & c \\ \mu^- & s' \end{bmatrix} \quad , \quad \begin{bmatrix} \nu_\tau & t \\ \tau^- & b' \end{bmatrix}$$

Family Structure

$$\begin{bmatrix} \nu_l & q_u \\ l^- & q_d \end{bmatrix} \equiv \left\{ \begin{pmatrix} \nu_l \\ l^- \end{pmatrix}_L , (\nu_l)_R , l_R^- \right\} ; \left\{ \begin{pmatrix} q_u \\ q_d \end{pmatrix}_L , (q_u)_R , (q_d)_R \right\}$$

Charged Currents $W^\pm$

- Left–handed Fermions only
- Flavour Changing: $\nu_l \Leftrightarrow l$, $q_u \Leftrightarrow q_d$

Neutral currents γ , Z Flavour Conserving

Universality (Family – Independent Couplings)

$(\nu_l)_R$?

$SU(2)_L \otimes U(1)_Y$ GAUGE THEORY

Fields	$\psi_1(x)$	$\psi_2(x)$	$\psi_3(x)$
Quarks	$\begin{pmatrix} q_u \\ q_d \end{pmatrix}_L$	$(q_u)_R$	$(q_d)_R$
Leptons	$\begin{pmatrix} \nu_l \\ l^- \end{pmatrix}_L$	$(\nu_l)_R$	$(l^-)_R$

Free Lagrangian for Massless Fermions: $\mathcal{L}_0 = \sum_j i\, \bar{\psi}_j\, \gamma^\mu\, \partial_\mu\, \psi_j$

$SU(2)_L \otimes U(1)_Y$ **Flavour Symmetry:** $\mathbf{U}_L \equiv \exp\left\{ i\, \frac{\vec{\sigma}}{2}\, \vec{\alpha} \right\}$

$$\psi_1 \to e^{i y_1 \beta}\, \mathbf{U}_L\, \psi_1 \quad ; \quad \psi_2 \to e^{i y_2 \beta}\, \psi_2 \quad ; \quad \psi_3 \to e^{i y_3 \beta}\, \psi_3$$

$$\bar{\psi}_1 \to \bar{\psi}_1\, \mathbf{U}_L^\dagger\, e^{-i y_1 \beta} \quad ; \quad \bar{\psi}_2 \to \bar{\psi}_2\, e^{-i y_2 \beta} \quad ; \quad \bar{\psi}_3 \to \bar{\psi}_3\, e^{-i y_3 \beta}$$

Gauge Principle: $\vec{\alpha} = \vec{\alpha}(x)\ ,\ \beta = \beta(x)$

$$\mathbf{D}_\mu \psi_1 \equiv \left[\partial_\mu - i\, g\, \mathbf{W}_\mu(x) - i\, g'\, y_1\, B_\mu(x)\right] \psi_1 \ \rightarrow\ e^{i\, y_1\, \beta(x)}\ \mathbf{U}_L(x)\ \mathbf{D}_\mu \psi_1$$

$$\mathbf{D}_\mu \psi_k \equiv \left[\partial_\mu - i\, g'\, y_k\, B_\mu(x)\right] \psi_k \ \rightarrow\ e^{i\, y_k\, \beta(x)}\ \mathbf{D}_\mu \psi_k \qquad (k = 2,3)$$

$$B_\mu(x) \ \rightarrow\ B_\mu(x) + \frac{1}{g'}\, \partial_\mu \beta(x)$$

$$\mathbf{W}_\mu(x) \ \rightarrow\ \mathbf{U}_L(x)\ \mathbf{W}_\mu(x)\ \mathbf{U}_L^\dagger(x) - \frac{i}{g}\, \partial_\mu \mathbf{U}_L(x)\ \mathbf{U}_L^\dagger(x)$$

$$\mathbf{U}(x) \equiv \exp\left\{ i\, \frac{\vec{\sigma}}{2}\, \vec{\alpha}(x) \right\} \quad ; \quad \mathbf{W}_\mu(x) \equiv \frac{\vec{\sigma}}{2}\, \vec{W}_\mu(x) \quad ; \quad \delta W_\mu^i = \frac{1}{g}\, \partial_\mu(\delta\alpha^i) - \varepsilon^{ijk}\, \delta\alpha^j\, W_\mu^k$$

4 Massless Gauge Bosons $W_\mu^\pm\ ,\ W_\mu^3\ ,\ B_\mu^0$

CHARGED CURRENTS

$$\sum_j i\,\bar\psi_j\,\gamma^\mu\,\mathrm{D}_\mu\,\psi_j \quad\longrightarrow\quad g\;\bar\psi_1\,\gamma^\mu\,\mathbf{W}_\mu\,\psi_1 \;+\; g'\;B_\mu\;\sum_j y_j\;\bar\psi_j\,\gamma^\mu\,\psi_j$$

$$\mathbf{W}_\mu \equiv \frac{\vec\sigma}{2}\cdot\vec{\mathrm{W}}_\mu = \frac{1}{2}\begin{pmatrix} W_\mu^3 & \sqrt{2}\,W_\mu^\dagger \\ \sqrt{2}\,W_\mu & -W_\mu^3 \end{pmatrix} \quad ; \qquad W_\mu \equiv W_\mu^1 + i\,W_\mu^2$$

$$\mathcal{L}_{\mathrm{CC}} = \frac{g}{2\sqrt{2}}\;W_\mu^\dagger\;\left[\;\bar q_u\,\gamma^\mu\,(1-\gamma_5)\,q_d \;+\; \bar\nu_l\,\gamma^\mu\,(1-\gamma_5)\,l\;\right] \;+\; \mathrm{h.c.}$$

Quark / Lepton Universality

Left – Handed Interaction

NEUTRAL CURRENTS

$$\mathcal{L}_{NC} = g\, W^3_\mu\, \bar\psi_1\, \gamma^\mu\, \frac{\sigma_3}{2}\, \psi_1 + g'\, B_\mu \sum_j y_j\, \bar\psi_j\, \gamma^\mu\, \psi_j$$

Massless Fields → Arbitrary Combination

$$\begin{pmatrix} W^3_\mu \\ B_\mu \end{pmatrix} \equiv \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix}$$

A_μ has the QED Interaction IF $\quad g \sin\theta_W = g' \cos\theta_W = e$

$$y_1 = Q_u - \frac{1}{2} = Q_d + \frac{1}{2} \quad ; \quad y_2 = Q_u \quad ; \quad y_3 = Q_d$$

Electroweak Unification

$$\mathcal{L}_{NC} = e\, A_\mu \sum_j \bar\psi_j\, \gamma^\mu\, Q_j\, \psi_j + \mathcal{L}^Z_{NC}$$

$$Q_1 = \begin{pmatrix} Q_u & 0 \\ 0 & Q_d \end{pmatrix} \quad ; \quad Q_2 = Q_u \quad ; \quad Q_3 = Q_d$$

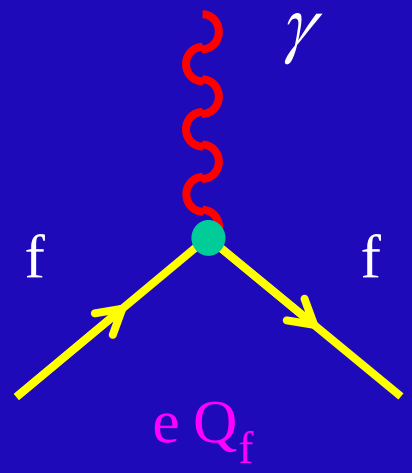
$$\mathcal{L}^{Z}_{NC} = \frac{e}{\sin\theta_W \cos\theta_W} Z_\mu \left\{ \bar{\psi}_1 \gamma^\mu \frac{\sigma_3}{2} \psi_1 - \sin^2\theta_W \sum_j \bar{\psi}_j \gamma^\mu Q_j \psi_j \right\}$$

$$= \frac{e}{2 \sin\theta_W \cos\theta_W} Z_\mu \sum_f \bar{f} \gamma^\mu \left[v_f - a_f \gamma_5 \right] f$$

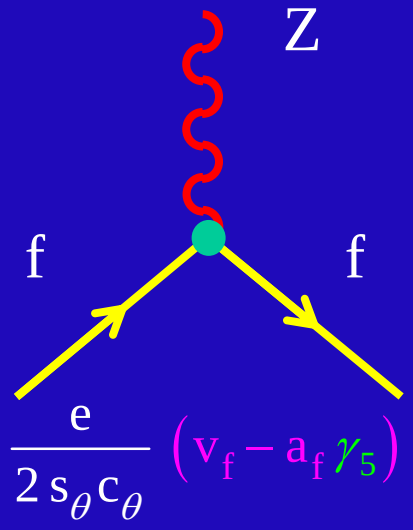
	q_u	q_d	ν_l	l^-
$2\,v_f$	$1 - \frac{8}{3}\sin^2\theta_W$	$-1 + \frac{4}{3}\sin^2\theta_W$	1	$-1 + 4\sin^2\theta_W$
$2\,a_f$	1	-1	1	-1

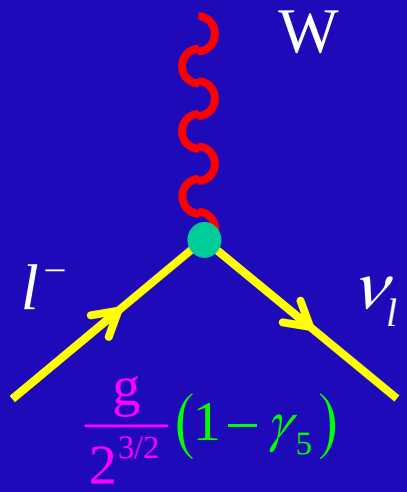
IF ν_R do exist: $y(\nu_R) = Q_\nu = 0$ ➡ No ν_R Interactions

Sterile Neutrinos

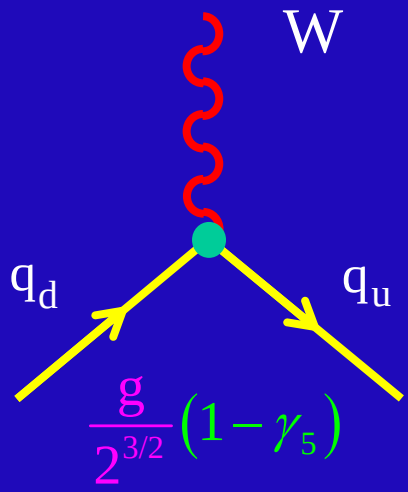


NEUTRAL CURRENTS





CHARGED CURRENTS



$$\mathbf{W}_{\mu\nu} \equiv \frac{i}{g}\left[\mathbf{D}_\mu , \mathbf{D}_\nu\right] \equiv \frac{\vec{\sigma}}{2}\cdot\vec{W}_{\mu\nu} \;\rightarrow\; \mathbf{U}_L\, \mathbf{W}_{\mu\nu}\, \mathbf{U}_L^\dagger \qquad ; \qquad B_{\mu\nu} \equiv \partial_\mu B_\nu - \partial_\nu B_\mu \;\rightarrow\; B_{\mu\nu}$$

$$W^i_{\mu\nu} = \partial_\mu W^i_\nu - \partial_\nu W^i_\mu + g\, \varepsilon^{ijk}\, W^j_\mu\, W^k_\nu$$

$$\mathcal{L}_K = -\frac{1}{4} B_{\mu\nu}\, B^{\mu\nu} - \frac{1}{2}\mathrm{Tr}\left(\mathbf{W}_{\mu\nu}\, \mathbf{W}^{\mu\nu}\right) = -\frac{1}{4} B_{\mu\nu}\, B^{\mu\nu} - \frac{1}{4}\vec{W}_{\mu\nu}\vec{W}_{\mu\nu} = \mathcal{L}_{\text{kin}} + \mathcal{L}_3 + \mathcal{L}_4$$

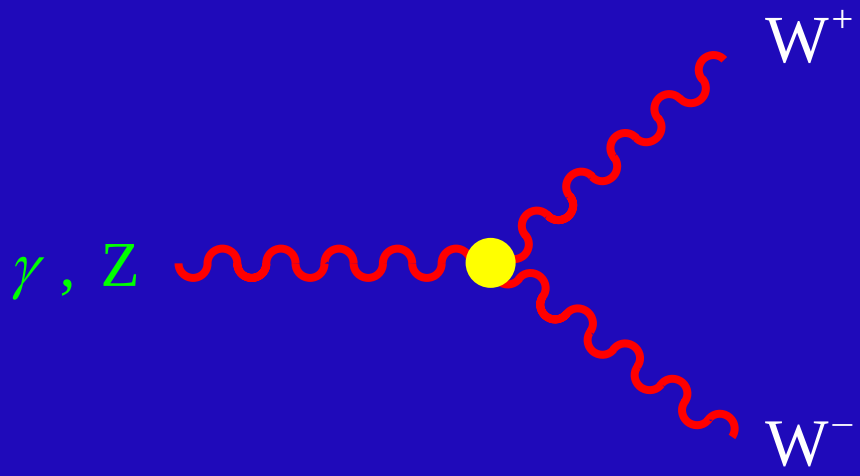
$$\mathcal{L}_3 = -i\,e\,\cot\theta_W \left\{\left(\partial^\mu W^\nu - \partial^\nu W^\mu\right) W^\dagger_\mu Z_\nu - \left(\partial^\mu W^{\nu\dagger} - \partial^\nu W^{\mu\dagger}\right) W_\mu Z_\nu + W_\mu W^\dagger_\nu \left(\partial^\mu Z^\nu - \partial^\nu Z^\mu\right)\right\}$$

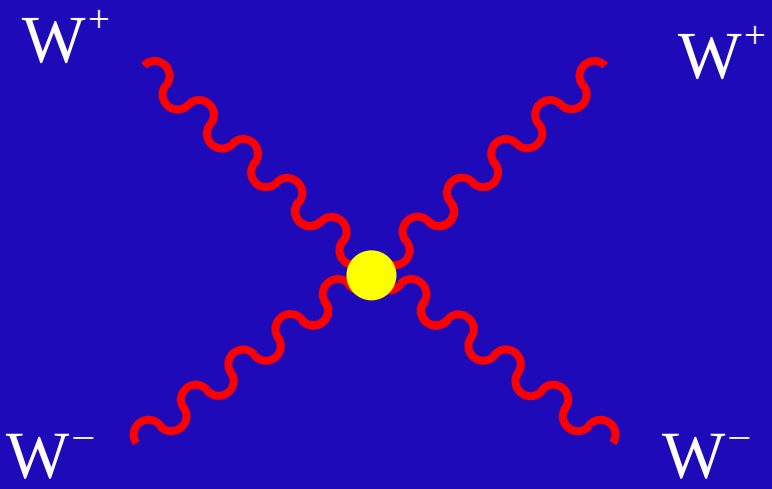
$$-i\,e\,\left\{\left(\partial^\mu W^\nu - \partial^\nu W^\mu\right) W^\dagger_\mu A_\nu - \left(\partial^\mu W^{\nu\dagger} - \partial^\nu W^{\mu\dagger}\right) W_\mu A_\nu + W_\mu W^\dagger_\nu \left(\partial^\mu A^\nu - \partial^\nu A^\mu\right)\right\}$$

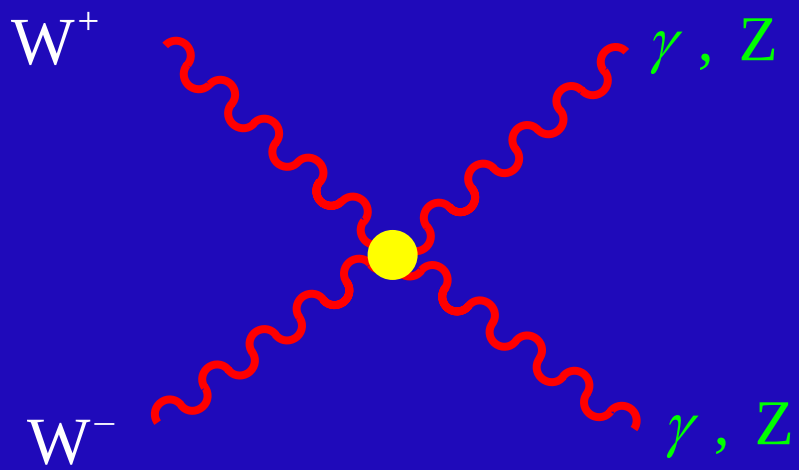
$$\mathcal{L}_4 = -\frac{e^2}{2\sin^2\theta_W}\left\{\left(W^\dagger_\mu W^\mu\right)^2 - W^\dagger_\mu W^{\mu\dagger} W_\nu W^\nu\right\} - e^2\cot^2\theta_W \left\{W^\dagger_\mu W^\mu Z_\nu Z^\nu - W^\dagger_\mu Z^\mu W_\nu Z^\nu\right\}$$

$$-e^2\cot\theta_W \left\{2W^\dagger_\mu W^\mu Z_\nu A^\nu - W^\dagger_\mu Z^\mu W_\nu A^\nu - W^\dagger_\mu A^\mu W_\nu Z^\nu\right\} - e^2\left\{W^\dagger_\mu W^\mu A_\nu A^\nu - W^\dagger_\mu A^\mu W_\nu A^\nu\right\}$$

GAUGE SELF-INTERACTIONS







PROBLEM WITH MASS SCALES

Gauge Symmetry $\longrightarrow$ $m_\gamma = 0$ **Good**

$M_W = M_Z = 0$ **Bad!**

$M_W = 80.43\ \text{GeV}$

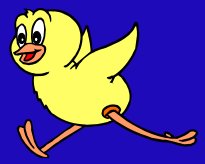
$M_Z = 91.19\ \text{GeV}$

Moreover $\mathcal{L}_{m_f} \equiv -m_f\ \bar{f}\ f = -m_f\ (\bar{f}_L\ f_R + \bar{f}_R\ f_L)$

Also Forbidden by Gauge Symmetry $\longrightarrow$ $m_f = 0 \quad \forall\ f$

All Particles Massless

Quarks



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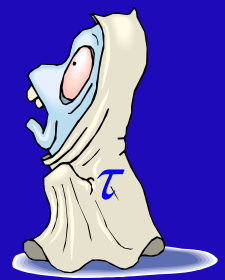

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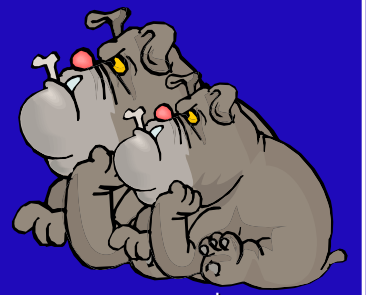
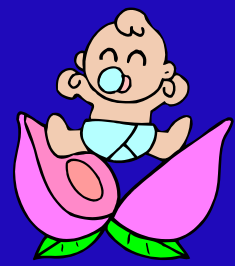


Z^0 $W^{\pm}$



Higgs