G Zoupanos
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Developments and further Challenges in Unified Theories

Quantum Reduction of Couplings in QFT

Applications

- Finite Unified Theories
- MSSM

→ Predictions → Higgs mass
→ s-spectrum and other Higgs masses

After the discovery of the Higgs boson at the LHC, the Standard Model has been very successfully completed

⟿ low energy accessible part of a (more) fundamental Theory of Elementary Particle Physics.

However it contains

- ad hoc Higgs sector
- ad hoc Yukawa couplings

⟿ free parameters (>20)

Renormalization programme

⟹ free parameters

Traditional way of reducing the number of parameters

SYMMETRY

Celebrated example: GUTs

e.g. minimal SU(5)

↗ testable $\sin^2\theta_W$

↘ successful m_τ / m_b

However more SYMMETRY (e.g. SO(10), E(6), E(7), E(8)) does not lead necessarily to more predictions of the SM parametes.

Extreme case: Superstring Ths

On the other hand

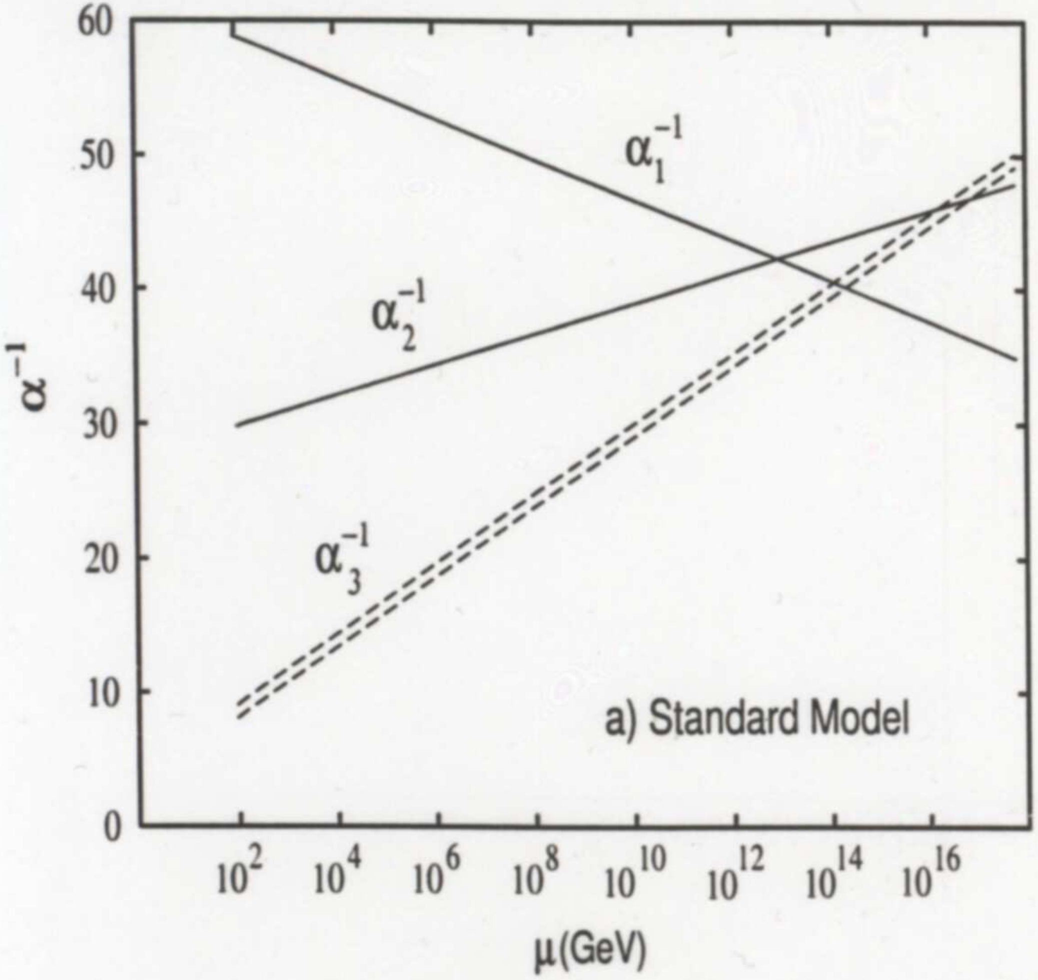
LEP data $\rightsquigarrow$ $N=1$ $SU(5)$

$\cancel{N=1}$ $\cancel{SU(5)}$ $\longrightarrow$ MSSM

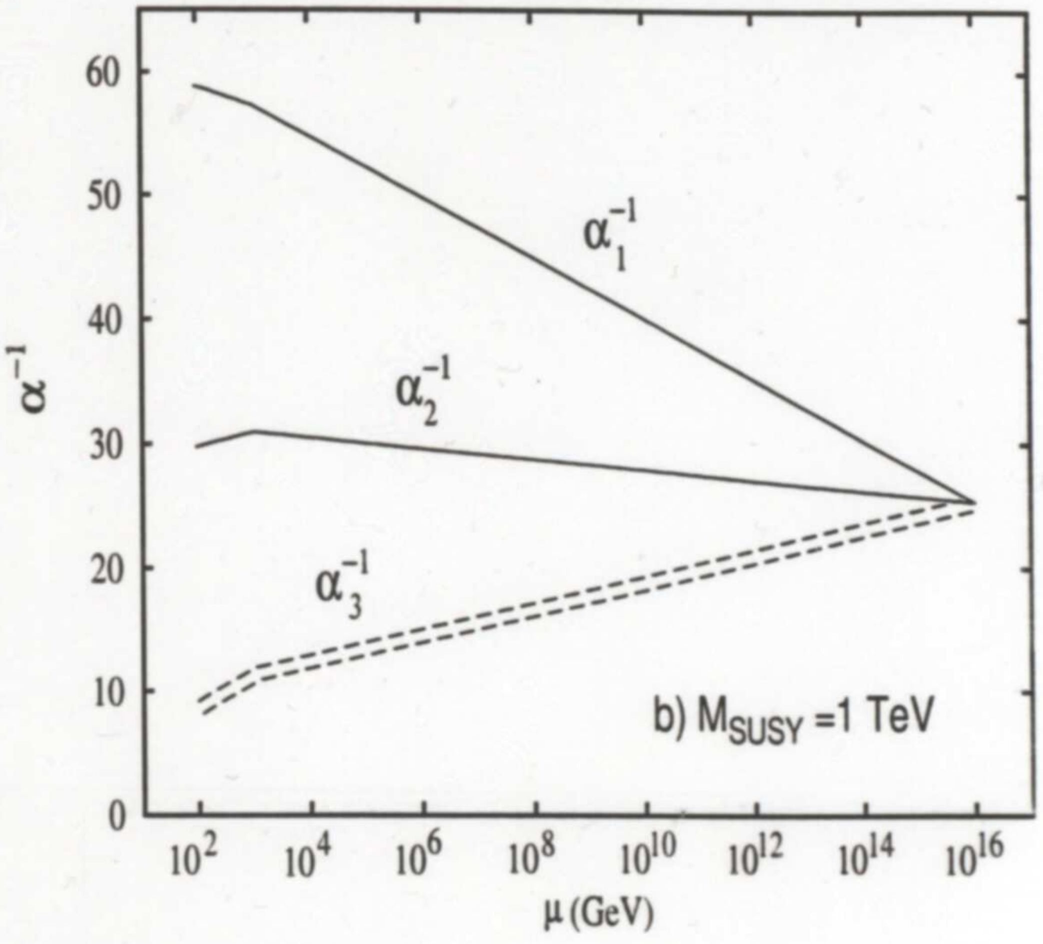
MSSM best candidate for Physics Beyond SM

But with > 100! free parameters mostly in its SSB sector.

- Cures problem of quadratic divergencies of the SM (hierarchy problem)
- Restricts the Higgs sector leading to approximate prediction of the Higgs mass



60
50
40
30
20
10
0
α^{-1}
α_1^{-1}
α_2^{-1}
α_3^{-1}
a) Standard Model
10^2
10^4
10^6
10^8
10^{10}
10^{12}
10^{14}
10^{16}
μ(GeV)



60
50
40
30
20
10
0
α^{-1}
α_1^{-1}
α_2^{-1}
α_3^{-1}
b) M_{SUSY} =1 TeV
10^2
10^4
10^6
10^8
10^{10}
10^{12}
10^{14}
10^{16}
μ(GeV)

Consider the SM with 2 Higgsdoublets

$$V = -1/2\, m_1^2 (H_1^+ H_1) - 1/2\, m_2^2 (H_2^+ H_2) - 1/2\, m_3^2 (H_1^+ H_2 + h.c.)$$

$$+ 1/2\, \lambda_1 (H_1^+ H_1) + 1/2\, \lambda_2 (H_2^+ H_2)^2$$

$$+ \lambda_3 (H_1^+ H_1)(H_2^+ H_2) + \lambda_4 (H_1^+ H_2)(H_2^+ H_1)$$

$$+ \left\{ 1/2\, \lambda_5 (H_1 H_2)^2 + \left[\lambda_6 (H_1^+ H_1) + \lambda_7 (H_2^+ H_2) \right] (H_1^+ H_2) + h.c. \right\}$$

Supersymmetry imposes tree level relations among couplings,

$$\lambda_1 = \lambda_2 = 1/4\, (g^2 + g'^2)$$

$$\lambda_3 = 1/4\, (g^2 - g'^2), \quad \lambda_4 = -\frac{1}{4} g^2$$

$$\lambda_5 = \lambda_6 = \lambda_7 = 0$$

With $v_1 = \langle \mathrm{Re}\, H_1^0 \rangle$, $v_2 = \langle \mathrm{Re}\, H_2^0 \rangle$

and $v_1^2 + v_2^2 = (246\ \mathrm{GeV})^2$, $\frac{v_2}{v_1} = \tan\beta$

$$\Longrightarrow h^0, H^0, H^\pm, A^0$$

At tree level

$$M^2_{h^0,H^0} = \frac{1}{2}\left\{M_A^2 + M_Z^2 \mp \left[(M_A^2+M_Z^2)^2 - 4M_A^2M_Z^2\cos^2 2\beta\right]^{1/2}\right\}$$

$$M^2_{H^\pm} = M_W^2 + M_A^2$$

$$\Longrightarrow \begin{cases} M_{h^0} < M_Z\,|\cos 2\beta| \\ M_{H^0} > M_Z \\ M_{H^\pm} > M_W \end{cases}$$

Radiative corrections

$$M^2_{h^0} \simeq M_Z^2\cos^2 2\beta + \frac{3g^2 m_t^4}{16\pi^2 M_W^2}\log\frac{\tilde{m}^2_{t_1}\tilde{m}^2_{t_2}}{m_t^4}$$

- Finite Unified Theories (from Quantum Reduction of Couplings)
- Higher Dimensional Unified Theories and Coset Space Dimensional Reduction (Classical Reduction of Couplings)
- Fuzzy Extra Dimensions and Renormalizable Unified Theories

Quantum
Reduction of Couplings

Consider a GUT with

g – gauge coupling

g_i – other couplings (Yukawas, self-couplings)

Any relation among the couplings

$$\Phi(g, g_1, \cdots) = \text{const}$$

which is RGI should satisfy

$$\frac{d}{dt}\Phi = 0 \quad , \quad t = \ln\mu$$

$$\frac{d}{dt}\Phi = \frac{\partial\Phi}{\partial g}\overset{\beta_g}{\overset{\shortparallel}{\frac{dg}{dt}}} + \sum_i \frac{\partial\Phi}{\partial g_i}\overset{\beta_i}{\overset{\shortparallel}{\frac{dg_i}{dt}}} = 0$$

which is equivalent to

$$\frac{dg}{\beta_g} = \frac{dg_1}{\beta_1} = \frac{dg_2}{\beta_2} = \cdots$$ characteristic system

$$\Longrightarrow \quad \beta_g \frac{dg_i}{dg} = \beta_i$$ Reduction eqs Oehme Zimmermann

Demand power series solution to the REs

$$g_i = \sum_{n=0} \rho_i^{(n+1)} g^{2n+1}$$

Remarkably, uniqueness of these solutions can be decided already at 1-loop!

Assume

$$\beta_i = \frac{1}{16\pi^2}\left[\sum_{j,k,l} \beta_i^{(1)jkl} g_j g_k g_l + \sum_{j\neq g} \beta_i^{(1)j} g_i g^2\right] + \cdots$$

$$\beta_g = \frac{1}{16\pi^2} \beta_g^{(1)} g^3 + \cdots$$

Assume $\rho_i^{(n)}$, $n \leq r$ have been uniquely determined

To obtain $\rho_i^{(r+1)}$, insert g_i in REs and collect terms of $O\left(g^{2r+1}\right)$

$$\rightsquigarrow \sum_{l \neq g} M(r)_i^{\,l} \rho_l^{(r+1)} = \text{lower order quantities}$$

Known by assumption

where

$$M(r)_i^{\,l} = 3 \sum_{j,k \neq g} b_i^{(1)jkl} \rho_j^{(1)} \rho_k^{(1)} + b_i^{(1)l} - (2r+1) b_g^{(1)} \delta_i^l$$

$$0 = \sum_{j,k,l \neq g} b_i^{(1)jkl} \rho_j^{(1)} \rho_k^{(1)} \rho_l^{(1)} + \sum_{l \neq g} b_i^{(1)l} \rho_l^{(1)} - b_g^{(1)} \rho_i^{(1)}$$

$\Longrightarrow$ for a given set of $\rho_i^{(1)}$, the $\rho_i^{(n)}$ for all $n > 1$ can be uniquely determined if

$$\det M(n)_i^{\,l} \neq 0$$

for all n

Kubo et.al.

Consider an $SU(N)$ (non-susy) theory with

$\phi^i(N)$, $\hat{\phi}_i(\bar{N})$ - complex scalars

$\psi^i(N)$, $\hat{\psi}_i(\bar{N})$ - Weyl spinors

$\lambda^a\ (a=1,\dots,N^2-1)$ - "

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} + i\sqrt{2}\, [g_Y \bar{\psi} \lambda^a T^a \phi - \hat{g}_Y \bar{\hat{\psi}} \lambda^a T^a \hat{\phi} + h.c. - V(\phi, \hat{\phi}),$$

$$V(\phi,\hat{\phi}) = \frac{1}{4}\lambda_1 (\phi^i \phi_i^*)^2 + \frac{1}{4}\lambda_2 (\hat{\phi}_i \hat{\phi}^{*i})^2 + \lambda_3 (\phi^i \phi_i^*)(\hat{\phi}_j \phi^{*j}) + \lambda_4 (\phi^i \phi_j^*)(\hat{\phi}_i \hat{\phi}^{*j})$$

Searching for power series solution of the R.E.s we find

$$g_Y = \hat{g}_Y = g;\ \lambda_1 = \lambda_2 = \frac{N-1}{N} g^2;\ \lambda_3 = \frac{1}{2N} g^2;\ \lambda_4 = -\frac{1}{2} g^2$$

i.e. SUSY

$N=1$ gauge theories

Consider a chiral, anomaly fee $N=1$ globally supersymmetric gauge th. based on a group G with gauge coupling g.

Superpotential

$$W = \frac{1}{2} m_{ij} \phi^i \phi^j + \frac{1}{6} C_{ijk} \phi^i \phi^j \phi^k$$

m_{ij}, C_{ijk} – gauge invariant tensors

ϕ^i – matter fields transforming as an ir. rep. R_i of G.

Renormalization constants associated with W

$$\phi^{0i} = (Z^i_j)^{1/2}\phi^j\,,\quad m^0_{ij} = Z^{i'j'}_{ij}\, m_{i'j'}\,,\quad C^0_{ijk} = Z^{i'j'k'}_{ijk}\, C_{i'j'k'}$$

$N=1$ non-renormalization thm ensures absence of mass and cubic-int-term infinities

$$Z^{ijk}_{i'j'k'}\, Z^{1/2\, i'}_{i''}\, Z^{1/2\, j'}_{j''}\, Z^{1/2\, k'}_{k''} = \delta^i_{(i''}\,\delta^j_{j''}\,\delta^k_{k'')}$$

$$Z^{ij}_{i'j'}\, Z^{1/2\, i'}_{i''}\, Z^{1/2\, j'}_{j''} = \delta^i_{(i''}\,\delta^j_{j'')}$$

(In the background field method)

$$Z_g Z_V^{1/2} = 1$$

$\rightsquigarrow$ Only surviving infinities are $Z^i_j, (Z_V)$

i.e. one infinity for each field.

The 1-loop β-function of the gauge coupling is

$$\beta_g^{(1)} = \frac{dg}{dt} = \frac{g^3}{16\pi^2}\left[\sum_i \ell(R_i) - 3C_2(G)\right]$$

$\ell(R_i)$ – Dynkin index of R_i

$C_2(G)$ – quadratic Casimir of the adjoint rep.

β-functions of C_{ijk}, by virtue of the non-renormalization thm, are related with the anomalous dim. matrix γ_{ij} of ϕ^i

$$\beta_{ijk}^{(1)} = \frac{dC_{ijk}}{dt} = C_{ijl}\gamma^l_k + C_{ikl}\gamma^l_j + C_{jkl}\gamma^l_i$$

$$\gamma^{(1)j}_i = Z^{-\frac{1}{2}\,k}_{\ i}\frac{d}{dt}Z^{\frac{1}{2}\,j}_{\ k}$$

$$= \frac{1}{32\pi^2}\left[C^{jkl}C_{ikl} - 2g^2C_2(R_i)\delta^j_i\right]$$

$C_2(R_i)$ – quadratic Casimir of R_i

$C^{ijk} = C^*_{ijk}$

$$b_g^{(2)} = \frac{1}{(16\pi^2)^2}\, 2\, g^5 \left[\sum_i \ell(R_i) - 3C_2(G)\right]$$

$$- \frac{1}{(16\pi^2)^2}\, \frac{g^3}{r}\, C_2(R_i)\left[C^{jkl}C_{ikl} - 2g^2 C_2(R_i)\delta^j_i\right]$$

$$r : \mathrm{tr}\,\delta^{ab}$$

Parkes, West, Jones
Mezincescu, Yau
Machacek, Vaughn

$$\gamma^{(2)i}{}_j = \frac{1}{(16\pi^2)^2}\, 2\, g^4\, C_2(R_i)\left[\sum_i \ell(R_i) - 3C_2(G)\right]$$

$$- \frac{1}{(16\pi^2)^2}\, \frac{1}{2}\left[C^{ikl}C_{jkm} + 2g^2 (R^a)^i_m (R^a)^l_j\right]$$

$$\cdot \left[C^{mpq}C_{lpq} - 2\delta^m_l\, g^2 C_2(R_i)\right]$$

$$b_g^{NSVZ} = \frac{g^3}{16\pi^2}\left[\frac{\sum_i \ell(R_i)(1-2\gamma_i) - 3C_2(G)}{1 - g^2 C_2(G)/8\pi^2}\right]$$

Novikov – Shifman – Vainstein – Zakharov

Wilsonian Renorm. Group
(WRG)

Any field theory is defined with cutoff M and bare couplings λ_0^i. If we wish to change

$M \rightarrow M'$ i.e. integrate out modes between M and M'

and keep low energy physics fixed we need to change

$$\lambda_i^0 \rightarrow \lambda_i$$

Necessary changing of λ_i^0 is encoded in a WRGE

$$M \frac{d\lambda_i^0}{dM} = b_i(\lambda_i^0)$$

A $\mathcal{N}=1$ pure Yang-Mills with vector multiplet $V_h = V_h^a T^a$ can be defined at M as :

• $$\mathcal{L}_h^M(V_h) = \frac{1}{16}\int d^2\theta \, \frac{1}{g_h^2} W^\alpha(V_h) W^\alpha(V_h) + h.c.$$

where $$\frac{1}{g_h^2} = \frac{1}{g^2} + i\frac{\theta}{8\pi^2}$$

manifestly holomorphic in g_h

•• $$\mathcal{L}_c^M(V_c) = \frac{1}{16}\int d^2\theta \left(\frac{1}{g_c} + i\frac{\theta}{8\pi^2}\right) W^\alpha(g_c V_c) W^\alpha(g_c V_c) + h.c.$$

with canonical normalisation

Analyticity arguments

$$\rightsquigarrow \quad \beta\left(\frac{8\pi^2}{g_h^2}\right) = b_0$$

Holomorphic $1/g_h^2$ runs exactly at 1-loop even including non-perturbative effects.

To determine the Wilsonian b-function for the canonical g_c it is not enough to change variables of the holomorphic $\mathcal{L}_h^M(V_h)$ $V_h = g_c V_c$ to obtain the $\mathcal{L}_c^M(V_c)$ with $g_c = g_h$. There is an anomalous Jacobian in passing from V_h to V_c.

$$\Longrightarrow \frac{1}{g_c^2} = \mathrm{Re}\left(\frac{1}{g_h^2}\right) - \frac{2C_2(G)}{8\pi^2} \ln g_c$$

Arkani-Hamed
Murayama

$$\rightsquigarrow M\frac{d}{dM} g_c = b(g_c) = -\frac{3C_2(G)}{16\pi^2} \frac{g_c^3}{1 - \frac{C_2(G)\, g_c^2}{8\pi^2}}$$

In presence of matter fields generalizes to the full Novikov, Shifman, Vainshtein, Zakharov all-loop b-function.

Resolution of the anomaly puzzle

- The anomalies under $U(1)_R$ transf. and dilations are related by holomorphy. If manifest holomorphy is kept, they have anomalies in the same supermultiplet.

- However starting with

$$\frac{1}{16}\int d^2\theta\, \frac{1}{g_c^2} W^\alpha(g_c V_c)\, W^\alpha(g_c V_c)$$

canonical normalization

dilation giving $\Big\downarrow$ anomalous Jacobian

$$\frac{1}{16}\int d^2\theta \left(\frac{1}{g_c^2} + \frac{b_0}{8\pi^2}t\right) W^\alpha(g_c V_c)\, W^\alpha(g_c V_c)$$

out of canonical normalization

change of variables $\Big\downarrow$ giving additional anomalous Jacobian

$g_c V_c = g_c' V_c'$

$$\frac{1}{16}\int d^2\theta \left(\frac{1}{g_c^2} + \frac{3C_2(G)}{8\pi^2}t - 2\frac{C_2(G)}{8\pi^2}\ln\frac{g_c'}{g_c}\right) W^\alpha(g_c' V_c')\, W^\alpha(g_c' V_c'$$

canonical normalization

$$\Rightarrow \quad \frac{1}{g_c'^2} = \frac{1}{g_c^2} + \frac{3C_2(G)}{8\pi^2}t - \frac{2C_2(G)}{8\pi^2}\ln\frac{g_c'}{g_c}$$

Performing the dilation $M \rightarrow M' = e^t M$

$\Rightarrow$ NSVZ β-function

Extension in presence of matter field to obtain the full NSVZ β-function:

$$\beta(g_c) = -\frac{1}{16\pi^2}\frac{3C_2(G) - \sum_i \ell(R_i)(1-\gamma_i)}{1 - C_2(G)g^2/8\pi^2}$$

The T^{μ}_{μ} of an all-loop finite gauge thy receives further contributions in presence of gravity. Introducing the background metric $g_{\mu\nu}(x)$

$$T^{\mu}_{\mu} = \frac{c}{16\pi^2}\left(W_{\mu\nu\rho\sigma}\right)^2 - \frac{a}{16\pi^2}\tilde{R}_{\mu\nu\rho\sigma}\tilde{R}^{\mu\nu\rho\sigma}$$

Weyl tensor

dual of curvature tensor

$\underbrace{\qquad\qquad}_{\text{Euler density}}$

$$c = \frac{1}{24}\left(3N_V + N_\chi\right)$$

$$a = \frac{1}{48}\left(9N_V + N_\chi\right)$$

Christensen + Duff

Similarly in susy ths

$$\partial_\mu\left(\sqrt{g}R^\mu\right) = \frac{c-a}{24\pi^2} R\tilde{R}$$

Anselmi et. al.

$$c = a \quad !!$$

N=1 : Only SO(10) with 2·10+54+45+16

N=2 : Non of SU(5), SO(10), E_6, E_7, E_8

N=4 All

When the condition

$$c = a \quad \|$$

is satisfied

- • R - current anomaly obviously vanishes

- •• $T^{\mu}_{\mu} \propto$ Ricci terms

Therefore vanishes in a Ricci flat background.

To see that use identity

$$W_{\mu\nu\rho\sigma} = R_{\mu\nu\rho\sigma} + \frac{R}{(n-1)(n-2)}\left(g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho}\right)$$

$$- \frac{1}{(n-2)}\left(g_{\mu\rho}R_{\nu\sigma} - g_{\mu\sigma}R_{\nu\rho} - g_{\nu\rho}R_{\mu\sigma} + g_{\nu\sigma}R_{\mu\rho}\right)$$

where $\underset{\text{Ricci tensor}}{R_{\rho\sigma}} = R_{\rho\mu\nu\sigma}\, g^{\mu\nu}$, $\underset{\text{scalar curvature}}{R} = R_{\rho\sigma}\, g^{\rho\sigma}$

MSSM

$$W = Y_t Q H_2 t^c + Y_b Q H_1 b^c$$

$$+ Y_\tau L H_1 \tau^c + \mu H_1 H_2$$

The REs for the top, bottom and tau couplings,

$$\frac{d\alpha_{t,b,\tau}}{d\alpha_3} = \frac{b_{t,b,\tau}}{b_3}$$

assuming perturbative expansion of the Yukawas in favour of α_3

$$\alpha_t = c_1 \alpha_3 + c_2 \alpha_3^2 + \cdots$$

$$\alpha_b = p_1 \alpha_3 + p_2 \alpha_3^2 + \cdots$$

$$\alpha_\tau = o_1 \alpha_3 + o_2 \alpha_3^2 + \cdots$$

(which being RGI hold at M_{GUT} where $\alpha_3 = \alpha_2 = \alpha_1$)

have solutions with

$$c_1 \simeq 0.892\,,\quad c_2 \simeq \frac{1}{4\pi}\,2.42$$

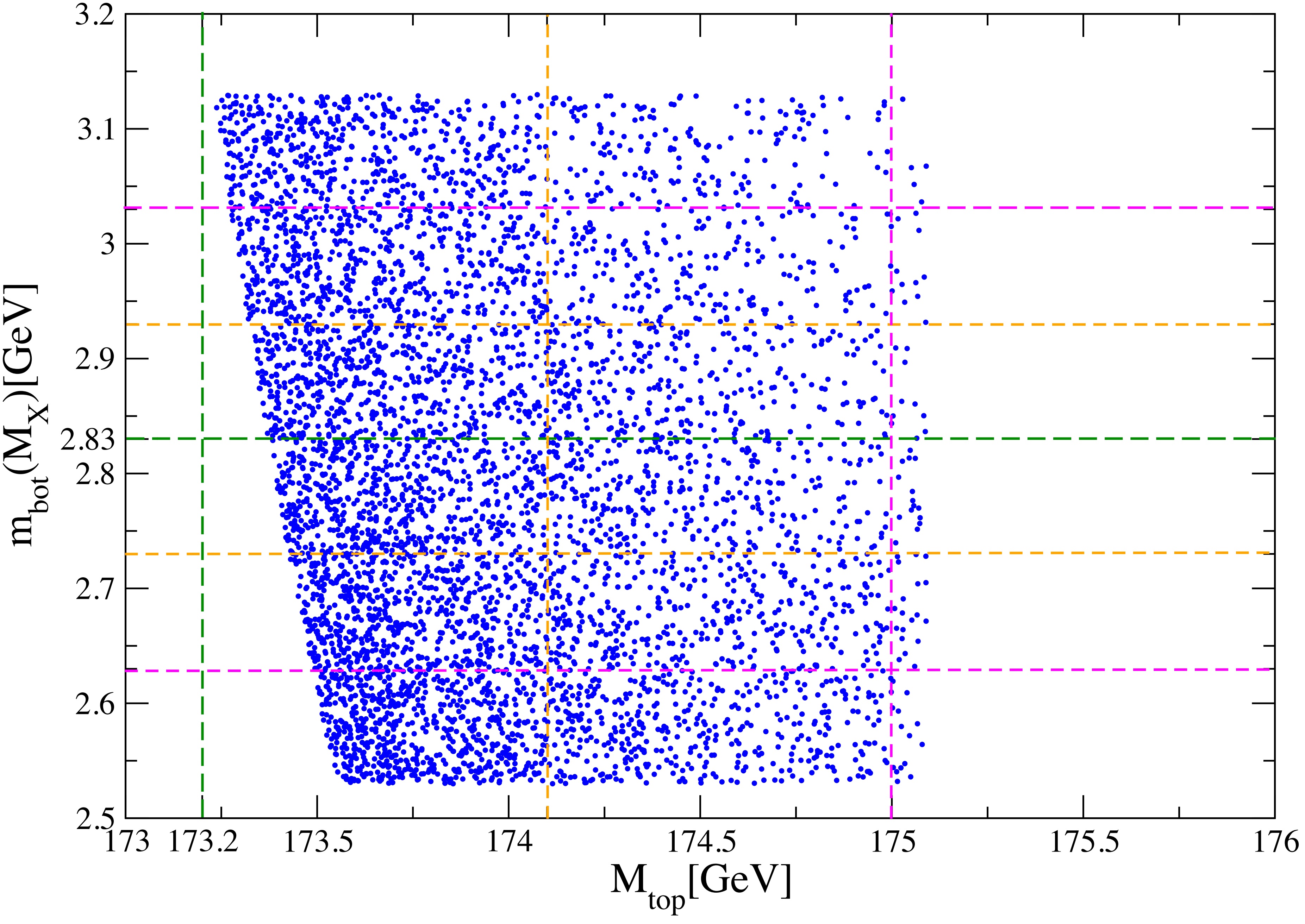
$$p_1 \simeq 0.849\,,\quad p_2 \simeq \frac{1}{4\pi}\,2.54$$

$$o_1 \simeq -0.187\,,\quad o_2 \simeq -\frac{1}{4\pi}\,1.46$$

Therefore $\alpha_t, \alpha_b, \alpha_3$ can be reduced, while α_T cannot and is left as a free parameter.

New observation

The $\alpha_t, \alpha_b, \alpha_3$ are not only reduced but they predict correctly the experimental values!


3.2
3.1
3
2.9
2.83
2.8
2.7
2.6
2.5
$m_{bot}(M_X)$[GeV]
173
173.2
173.5
174
174.5
175
175.5
176
M_{top}[GeV]

Finite Unification

Old days...

... divergences are "hidden under the carpet" (Dirac, Lects on Q.F.T., '64)

Recent years ...

... divergences reflect existence of a higher scale where new degrees of freedom are excited.

Not just artifacts of pert. th.

However the presence of quadratic divergences means that physics at one scale are very sensitive to unkown physics at higher scales.

⇝ SUSY ths which are free of quadratic divergences in spite of any experimental evidence...

⇝ Natural to expect that beyond unification scale the theory should be completely finite.

- $N=4$ ⇝ finite to all orders in pert.
- $N=2$ ⇝ only 1-loop contributions to β-function. Possible to arrange the spectrum so that theory is finite.

Multiplicities for massless irreducible reps with maximal helicity 1

N / Spin	1	1	2	2	4
1	–	1	–	1	1
1/2	1	1	2	2	4
0	2	–	4	2	6

$$N=2 \; : \; b(g) = \frac{2g^3}{(4\pi)^2}\left(\sum_i T(R_i) - C_2(G)\right)$$

e.g. $SU(N)$ with $2N$ fundamental reps $\longrightarrow b(g) = 0$

$SU(5)$: $p(5+\bar{5})$; $q(10+\overline{10})$; $r(15+\overline{15})$

with $p+3q+7r=10$

$SO(10)$: $p(10+\overline{10})$; $q(16+\overline{16})$

with $p+2q=8$

E_6 : $4(27+\overline{27})$

Finite Unified Theories

$N=1$

- 1-loop finiteness conditions

  $b_g^{(1)} = 0$

  $\gamma^{(1)i}_{\ j} = 0$ - anomalous dimensions of all chiral superfields

·· Exists complete classification of all chiral $N=1$ models with $b_g^{(1)} = 0$ Hamidi - Patera - Schwarz
Jiang - Zhou

··· 1-loop finiteness
$\longrightarrow$ 2-loop finiteness

Parkes-West
Jones
Mezincescu

.... Exist simple criteria that guarantee all loop finiteness (vanishing of all-loop beta functions)

Lucchesi-Piguet
Sibold

Ermushev
Kazakov
Tarasov

Leigh-Strassler

- All-loop finite SU(5) ⟹ top quark mass ✓

Kapetanakis
Mondragon
Z
'92

~~Susy~~ sector

Jones
Mezincescu
Yao

- 1-loop finiteness conds (require in particular universal soft ~~susy~~ scalar masses
$$(m^2)^i_j = \frac{1}{3} M M^* \delta^i_j$$
)

·· 1-loop finiteness

→ 2-loop finiteness — Jack, Jones

Reduction of couplings

· Extension of method in SSB sector + application in min susy SU(5) — Kubo, Mondragon^2

·· 1-loop sum rule for soft scalar masses in non-finite susy ths. — Kawamura, Kobayashi, Kubo

··· 2-loop sum rule for soft scalar masses in finite ths. — Kobayashi, Kubo, Mondragon^2

* All-loop RGI relations in finite and non-finite ths — Yamada, Hisano, Shifman, Kazakov, Jack, Jones, Pickering

* * All-loop sum rule for soft scalar masses in finite and non-finite ths

Kobayashi
Kubo
Z

● ● SU(5) FUTs

Kobayashi
Kubo
Mondragon
Z

- Prediction of s-spectrum in terms of few parameters starting from several hundreds GeV.

•• The LSP is neutralino ✓ (see e.g. Kazakov et. al. Yoshioka)

••• Radiative E-W breaking ✓ (see e.g. Brignole Ibanez, Munoz)

•••• No funny colour, charge ✓ (see e.g. Casas et. al)

* Prediction of Higgs masses

Lightest ~ 118 - 129 GeV

Similar results also for min susy SU(5)

Consider a chiral, anomaly free, $N=1$ gauge theory with group G.

The superpotential is

$$W = \frac{1}{6} Y^{ijk} \Phi_i \Phi_j \Phi_k + \frac{1}{2} \mu^{ij} \Phi_i \Phi_j$$

$$\left.\begin{matrix} Y^{ijk} \\ \mu^{ij} \end{matrix}\right\} \text{gauge invariant}$$

Yukawa couplings

Φ_i – matter superfields in irreducible reps of G

Necessary and sufficient conditions for $N=1$ 1-loop finiteness

- Vanishing of $b_g^{(1)}$ implies

$$\sum_i l(R_i) = 3 C_2(G) \qquad \|$$

$l(R_i)$ - Dynkin index of R_i

$C_2(G)$ - Quadratic Casimir of G (adjoint)

⇒ Selection of the field content (representations) of the theory

•• Vanishing of $\gamma^{(1)i}_{\;\;j}$ implies

$$\underset{\substack{\uparrow \\ \text{Yukawa}}}{Y^{ikl}}\, Y_{jkl} = 2\,\delta^i_j\, \underset{\substack{\uparrow \\ \text{gauge}}}{g^2}\, C_2(R_i) \quad \|$$

$C_2(R_i)$ – quadratic Casimir of R_i

$Y_{ijk} = (Y_{ijk})^*$

⇒ Yukawa and gauge couplings are related.

Note • μ^{ij} are not restricted

·· Appearence of U(1) is incopatible with 1st condt.

··· 2nd condt forbids the presence of singlets with nonvanishing couplings.

∷ ⇒ $\cancel{\text{Susy}}$ by G-invariant soft terms

- 1-loop finiteness condts necessary and sufficient to guarantee 2-loop finiteness

- 1-loop finiteness condts ensure that $b_g^{(3)}$ in 3-loops vanishes but in general $\gamma^{(3)}$ does not.

Grisaru - Milewski - Zanon
Parkes - West

What happens in higher loops?

So far 1-loop finiteness condts (on γs) are telling us

$$Y^{ijk} = Y^{ijk}(g)$$

$$b_Y^{(1)ijk} = 0$$

* * Necessary and sufficient condts for vanishing b_g and b_{ijk} to all orders

Lucchesi
Piquet
Sibold

1. $b_g^{(1)} = 0$
2. $\gamma^{(1)i}_{\;\;j} = 0$
3. $b_Y^{ijk} = b_g \dfrac{dY^{ijk}}{dg}$

admit power series solution which in lowest order is a solution of condt 2.

3.
- 3'. There exist solutions to $\gamma^{(1)i}_{\;\;j}=0$ of the form
  $Y^{ijk} = \rho^{ijk} g$, ρ^{ijk} – complex
- 4. These solutions are isolated and non-degenerate considered as solutions of $b_Y^{(1)ijk} = 0$

Recall

R-invariance, axial anomaly

In massless $N=1$ ths

$U(1)$ chiral transformation R:

$A_\mu \to A_\mu$, $\lambda \to e^{-i\alpha}\lambda$,

$\phi \to e^{-i\frac{2}{3}\alpha}\phi$, $\psi \to e^{i\frac{1}{3}\alpha}\psi$, ...

$\psi_D = \begin{pmatrix} \psi \\ \bar{\chi} \end{pmatrix} \longrightarrow e^{i\alpha\gamma_5}\psi_D$

Noether current $J^\mu_R = \bar{\lambda}_D \gamma^\mu \gamma^5 \lambda_D + \cdots$

$\leadsto \partial_\mu J^\mu_R = r\left(\epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} + \cdots\right)$

$r = b_g^{(1)}$!

Only 1-loop contributions due to non-renormalization thm.

Adler, Bardeen, Jackiw, Pi, Shei, Zee

Supercurrent

$$J \equiv \{ J'^{\mu}_{R} \, , \, Q^{\mu}_{\alpha} \, , \, T^{\mu}_{\nu} \} , \dots$$ vector super multiplet

J'^{μ}_{R} associated to R-invariance

Q^{μ}_{α} associated to susy

T^{μ}_{ν} associated to translation inv.

Ferrara + Zumino

(supercurrent is represented as vector superfield

$$V_\mu(x,\theta,\bar\theta) = R_\mu(x) - i\theta^\alpha Q_{\mu\alpha}(x) + i\bar\theta_{\dot\alpha}\bar Q^{\dot\alpha}_{\mu}(x) - 2(\theta\sigma^\nu\bar\theta)T_{\mu\nu}(x) + \cdots$$)

- $J'^{\mu}_{R} \neq J^{\mu}_{R}$
- $J'^{\mu}_{R} = J^{\mu}_{R} + O(\hbar)$

In addition

Clark
Piguet
Sibold

$$S = \{ b_g F^{\mu\nu} F_{\mu\nu} + \cdots , \; b_g \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} + \cdots ,$$

$$b_g \lambda^\beta \sigma^{\mu\nu}_{\alpha\beta} F_{\mu\nu} + \cdots , \; \cdots \}$$ chiral super multiplet

S: Super trace anomaly

$b_g F^{\mu\nu} F_{\mu\nu} + \cdots$: trace anomaly of T^{μ}_{ν}

$b_g \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} + \cdots$: anomaly of R-current

$b_g \lambda^\beta \sigma^{\mu\nu}_{\alpha\beta} F_{\mu\nu} + \cdots$: trace anomaly of susy current

There is a relation, whose structure is independent from the renormalization scheme, although individual coefficients (except the 1-loop values of b-functions) may be scheme dependent

$$r = b_g \left(1 + x_g\right) + b_{ijk}\, x^{ijk} - \gamma_A\, r^A$$

unrenormalized coefficients of anomalies associated to chiral inv. of superpotential

radiative corrections

linear combinations of anomalous dims

Thm: If (i) no gauge anomaly
(ii) $b^{(1)}(g) = 0$ i.e. no R-current anomaly
(iii) $\gamma^{(1)i}{}_{j} = 0$ implies also $r^A = 0$
(iv) exist solutions to $\gamma^{(1)} = 0$ of the form $c_{ijk} = \rho_{ijk}\, g$, ρ_{ijk} - complex
(v) these solutions are isolated + non-degenerate

when considered as solutions of $\beta^{(1)}_{ijk} = 0$.

- Then each of the solutions can be uniquely extended to a formal power series in g, and the $N = 1$ Y-M models depend on the single coupling constant g with a β-function vanishing to all orders.

Proof: Inserting $\beta_{ijk} = \beta_g \dfrac{d f_{ijk}}{dg}$ in the identity and taking into account the vanishing of γ, γ^A

$$\rightsquigarrow \quad 0 = \beta_g \left(1 + O(\hbar) \right)$$

Its solution (as formal power series in $\hbar$) is: $\beta_g = 0$

and $\beta_{ijk} = 0$ too. //

2-loop RGEs for SSB parameters

Martin - Vaughn - Yamada - Jack - Jones
'94

Consider $N=1$ gauge thy with

$$W = \frac{1}{6} Y^{ijk} \Phi_i \Phi_j \Phi_k + \frac{1}{2} \mu^{ij} \Phi_i \Phi_j$$

and SSB terms

$$-\mathcal{L}_{SB} = \frac{1}{6} h^{ijk} \phi_i \phi_j \phi_k + \frac{1}{2} b^{ij} \phi_i \phi_j$$

$$+ \frac{1}{2} (m^2)^i_j \phi^{*i} \phi_j + \frac{1}{2} M \lambda\lambda + h.c.$$

- 1-loop finiteness conditions

$$h^{ijk} = - M Y^{ijk}$$

$$(m^2)^i_j = \frac{1}{3} M M^* \delta^i_j$$ universality

in addition to $\beta_g^{(1)} = \gamma^{(1)i}_j = 0$

- - 1-loop finiteness
$\rightarrow$ 2-loop finiteness

Assuming

- $b_g^{(1)} = \gamma^{(1)j}_{\ i} = 0$
- the reduction eq

$$b_Y^{ijk} = b_g \, dY^{ijk}/dg$$

admits power series solution

$$Y^{ijk} = g \sum_{n=0} \rho^{ijk}_{(n)} g^{2n}$$

- $(m^2)^i_{\ j} = m_j^2 \delta^i_{\ j}$

$$\rightsquigarrow (m_i^2 + m_j^2 + m_k^2)/MM^* = 1 + \frac{g^2}{16\pi^2} \Delta^{(2)}$$

for i, j, k with $\rho^{ijk}_{(0)} \neq 0$

where $\Delta^{(2)} = -2 \sum_l \left[(m_l^2/MM^*) - \frac{1}{3} \right] \ell(R_l)$

- $\Delta^{(2)} = 0$ for $N=4$ with STr cond
- $\Delta^{(2)} = 0$ for the $N=1$, $SU(5)$ FUTs!

The SU(5) finite model

Kapetanakis, Mondragon, Z

Kobayashi, Kubo, Mondragon, Z

Content

$$3\left(\bar{5}+10\right)+4\left(\overset{H_\alpha}{5}+\overset{\bar{H}_\alpha}{\bar{5}}\right)+24$$

fermion supermultiplets

scalar supermultiplets

Jones-Raby
Hamidi-Schwarz
Quiros et.al
Kazakov
Babu-Enkhba
Gogoladze

$$\rightsquigarrow W=\sum_{i=1}^{3}\left[\frac{1}{2}g_i^u\,10_i\,10_i\,H_i+g_i^d\,10_i\,\bar{5}_i\,\bar{H}_i\right]$$

$$+\,g_{23}^u\,10_2\,10_3\,H_4+g_{23}^d\,10_2\,\bar{5}_3\,\bar{H}_4+g_{32}^d\,10_3\,\bar{5}_2\,\bar{H}_4$$

$$+\sum_{\alpha=1}^{4}g_\alpha^f\,H_\alpha\,24\,\bar{H}_\alpha+g^\lambda/3\,(24)^3$$

(with enhanced discrete symmetry after reduction of couplings)

We find

$$b_g^{(1)} = 0$$

$$b_{i\alpha}^{u(1)} = \frac{1}{16\pi^2}\Big[-\frac{96}{5}g^2 + \sum_{b=1}^{4}\left(g_{ib}^{u}\right)^2 + 3\sum_{j=1}^{3}\left(g_{ja}^{u}\right)^2$$

$$+\frac{24}{5}\left(g_{\alpha}^{f}\right)^2 + 4\sum_{b=1}^{4}\left(g_{ib}^{d}\right)^2\Big]g_{i\alpha}^{u}$$

$$b_{i\alpha}^{d(1)} = \frac{1}{16\pi^2}\Big[-\frac{84}{5}g^2 + 3\sum_{b=1}^{4}\left(g_{ib}^{u}\right)^2 + \frac{24}{5}\left(g_{\alpha}^{f}\right)^2$$

$$+4\sum_{j=1}^{3}\left(g_{j\alpha}^{d}\right)^2 + 6\sum_{b=1}^{4}\left(g_{ib}^{d}\right)^2\Big]g_{i\alpha}^{d}$$

$$b^{\lambda(1)} = \frac{1}{16\pi^2}\Big[-30g^2 + \frac{63}{5}\left(g^{\lambda}\right)^2 + 3\sum_{\alpha=1}^{4}\left(g_{\alpha}^{f}\right)^2\Big]g^{\lambda}$$

$$b_{\alpha}^{f(1)} = \frac{1}{16\pi^2}\Big[-\frac{98}{5}g^2 + 3\sum_{i=1}^{3}\left(g_{i\alpha}^{u}\right)^2$$

$$+4\sum_{i=1}^{3}\left(g_{i\alpha}^{d}\right)^2 + \frac{48}{5}\left(g_{\alpha}^{f}\right)^2$$

$$+\sum_{b=1}^{4}\left(g_{b}^{f}\right)^2 + \frac{21}{5}\left(g^{\lambda}\right)^2\Big]g_{\alpha}^{f}$$

Considering g as the primary coupling, we solve the Red. Eqs.

$$\beta_g = \beta_a \frac{dg}{dg_a}$$

requiring power series ansatz.

$$\Rightarrow (g^u_{ii})^2 = \frac{8}{5} g^2 + \cdots, (g^d_{ii})^2 = \frac{6}{5} g^2 + \cdots$$

$$(g^\lambda)^2 = \frac{15}{7} g^2 + \cdots, (g^f_4)^2 = g^2, (g^f_\alpha)^2 = 0 + \cdots (\alpha = 1,2,3)$$

Higher order terms can be uniquely determined.

$\Rightarrow$ All 1-loop β-functions vanish

Moreover

All 1-loop anomalous dimensions of chiral fields vanish.

$$\gamma^{(1)}_{10i} = \frac{1}{16\pi^2}\left[-\frac{36}{5}g^2 + 3\sum_{b=1}^{4}\left(g^u_{ib}\right)^2 + 2\sum_{b=1}^{4}\left(g^d_{ib}\right)^2\right]$$

$$\gamma^{(1)}_{\bar{5}i} = \frac{1}{16\pi^2}\left[-\frac{24}{5}g^2 + 4\sum_{b=1}^{4}\left(g^d_{ib}\right)^2\right]$$

$$\gamma^{(1)}_{H\alpha} = \frac{1}{16\pi^2}\left[-\frac{24}{5}g^2 + 3\sum_{i=1}^{3}\left(g^u_{i\alpha}\right)^2 + \frac{24}{5}\left(g^f_\alpha\right)^2\right]$$

$$\gamma^{(1)}_{\overline{H}\alpha} = \frac{1}{16\pi^2}\left[-\frac{24}{5}g^2 + 4\sum_{i=1}^{3}\left(g^d_{i\alpha}\right)^2 + \frac{24}{5}\left(g^f_\alpha\right)^2\right]$$

$$\gamma^{(1)}_{24} = \frac{1}{16\pi^2}\left[-\frac{10}{5}g^2 + \sum_{\alpha=1}^{4}\left(g^f_\alpha\right)^2 + \frac{21}{5}\left(g^\lambda\right)^2\right]$$

$\Rightarrow$ Necessary and sufficient condts for finiteness to all orders are satisfied

- SU(5) breaks down to the standard model due to <24>
- Use the freedom in mass parameters to obtain only a pair of Higgs fields light, acquiring v.e.v.
- Get rid of unwanted triplets rotating the Higgs sector (after a fine tuning)

  see Quiros et. al., Kazakov et. al
  Yioshioka
- Adding soft terms we can achieve SUSY breaking.

1) Gauge Couplings Unification

$\sin^2\theta_W$, $\alpha_{em} \rightsquigarrow \alpha_3(M_Z)$

Marciano+Senjanovic
⋮
Amaldi
et. al.

2) Bottom-Tau Yukawa Unif.

SU(5)-type

$\rightsquigarrow m_t \sim 100-200$ GeV

Barger
et. al.
Carena
et. al.

*3) Top-Bottom-Tau Yuk Unif.

$h_t^2 = \frac{4}{3} h_{b,\tau}^2$ in SU(5)-FUT

Similar to SO(10)

Ananthanarayan
et. al.
Barger et. al.

$\rightsquigarrow m_t \sim 160-200$ GeV

Carena et. al.

*4) Gauge-Top-Bottom-Tau Unif.

e.g. FUT-SU(5): $h_t^2 = \frac{8}{5} g_U^2$; $h_{b,\tau}^2 = \frac{6}{5} g_U^2$

M_s [GeV]	$\alpha_{3(5f)}(M_Z)$	$\tan\beta$	$M_{\rm GUT}$ [GeV]	M_b [GeV]	M_t [GeV]
300	0.123	54.1	2.2×10^{16}	5.3	183
500	0.122	54.2	1.9×10^{16}	5.3	183
10^3	0.120	54.3	1.5×10^{16}	5.2	184

FUTA

M_s [GeV]	$\alpha_{3(5f)}(M_Z)$	$\tan\beta$	$M_{\rm GUT}$ [GeV]	M_b [GeV]	M_t [GeV]
800	0.120	48.2	1.5×10^{16}	5.4	174
10^3	0.119	48.2	1.4×10^{16}	5.4	174
1.2×10^3	0.118	48.2	1.3×10^{16}	5.4	174

FUTB

M_s [GeV]	$\alpha_{3(5f)}(M_Z)$	$\tan\beta$	$M_{\rm GUT}$ [GeV]	M_b [GeV]	M_t [GeV]
300	0.123	47.9	2.2×10^{16}	5.5	178
500	0.122	47.8	1.8×10^{16}	5.4	178
1000	0.119	47.7	1.5×10^{16}	5.4	178

MIN SU(5)

The predictions for the three models for different M_s

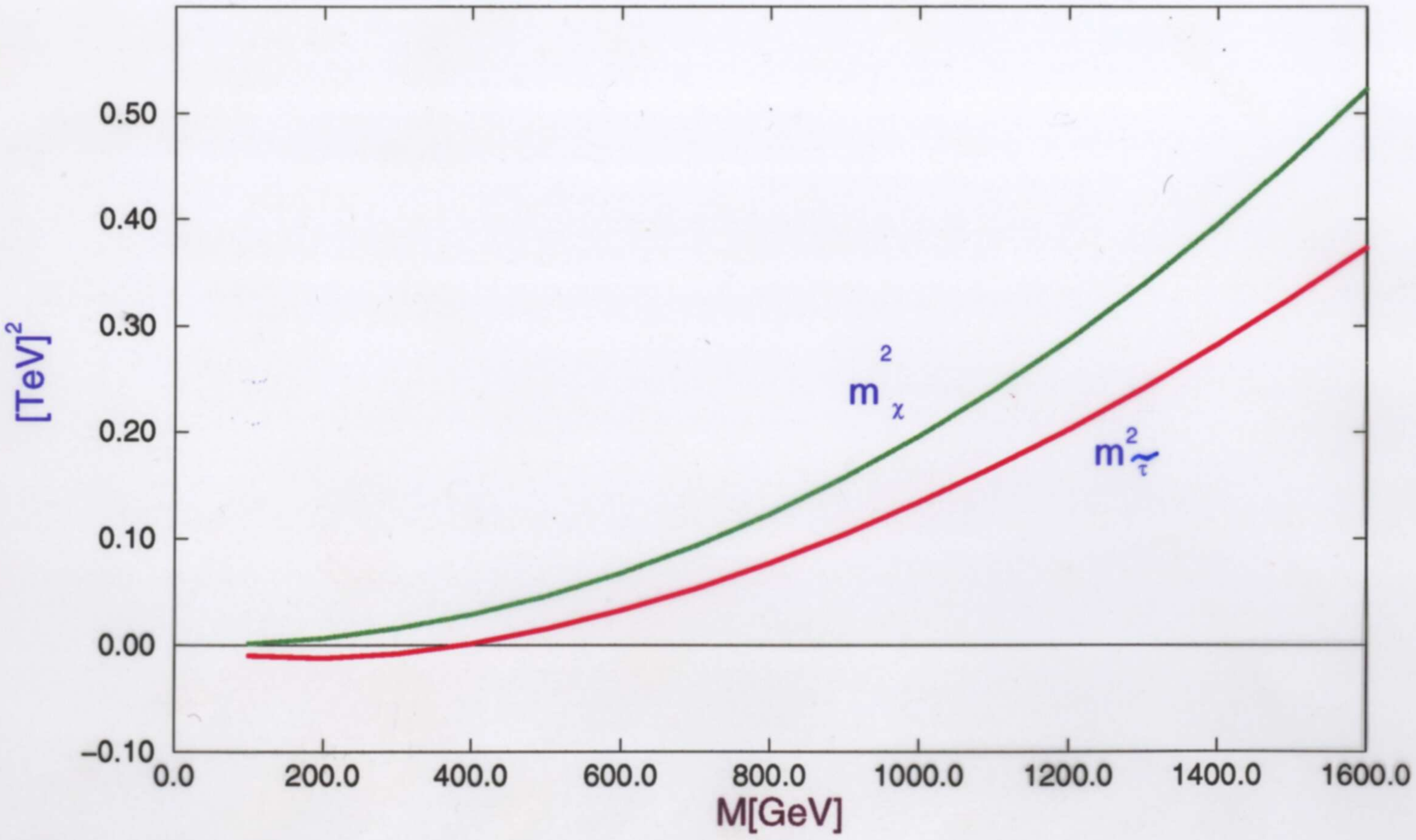
With theoretical corrections and uncertainties

$\sim 4\%$

$M_t = 173.8 \pm 5$ GeV ; 178.0 ± 4.3 GeV
CDF + D0

Model A

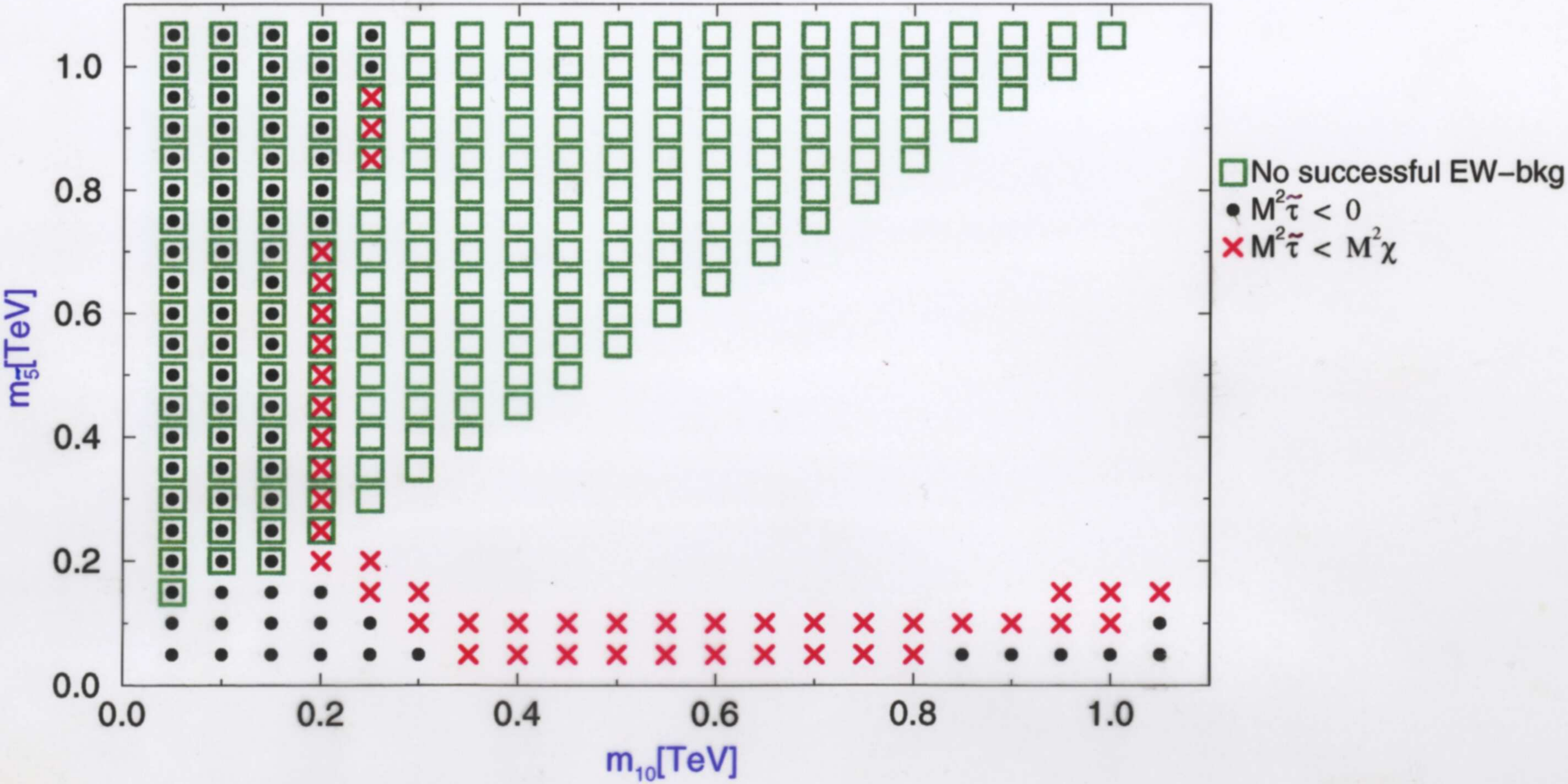
Similar behaviour holds for Model B too



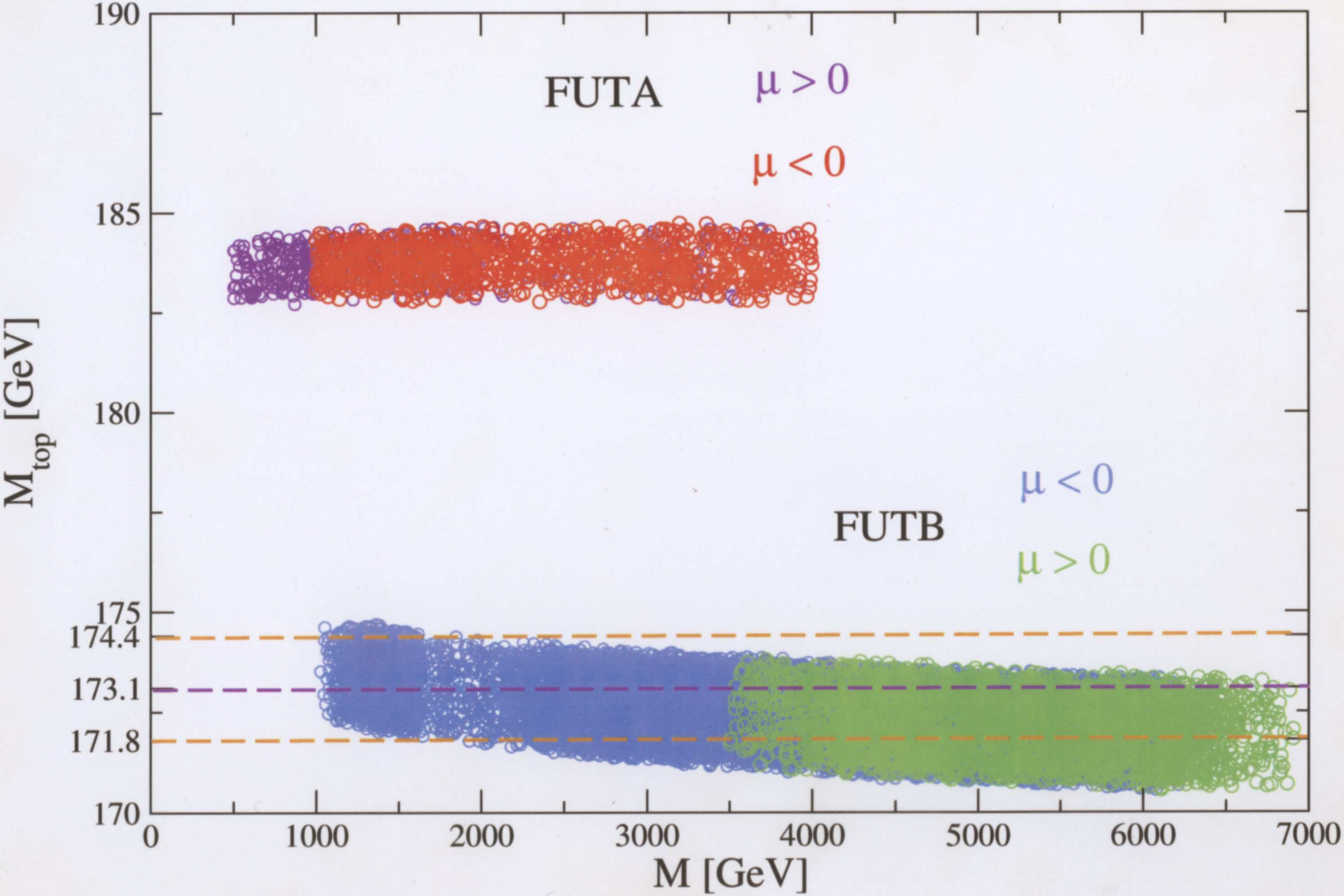
$m^2_{\tilde{\tau}}$ and m^2_χ for the universal choice of soft scalar masses

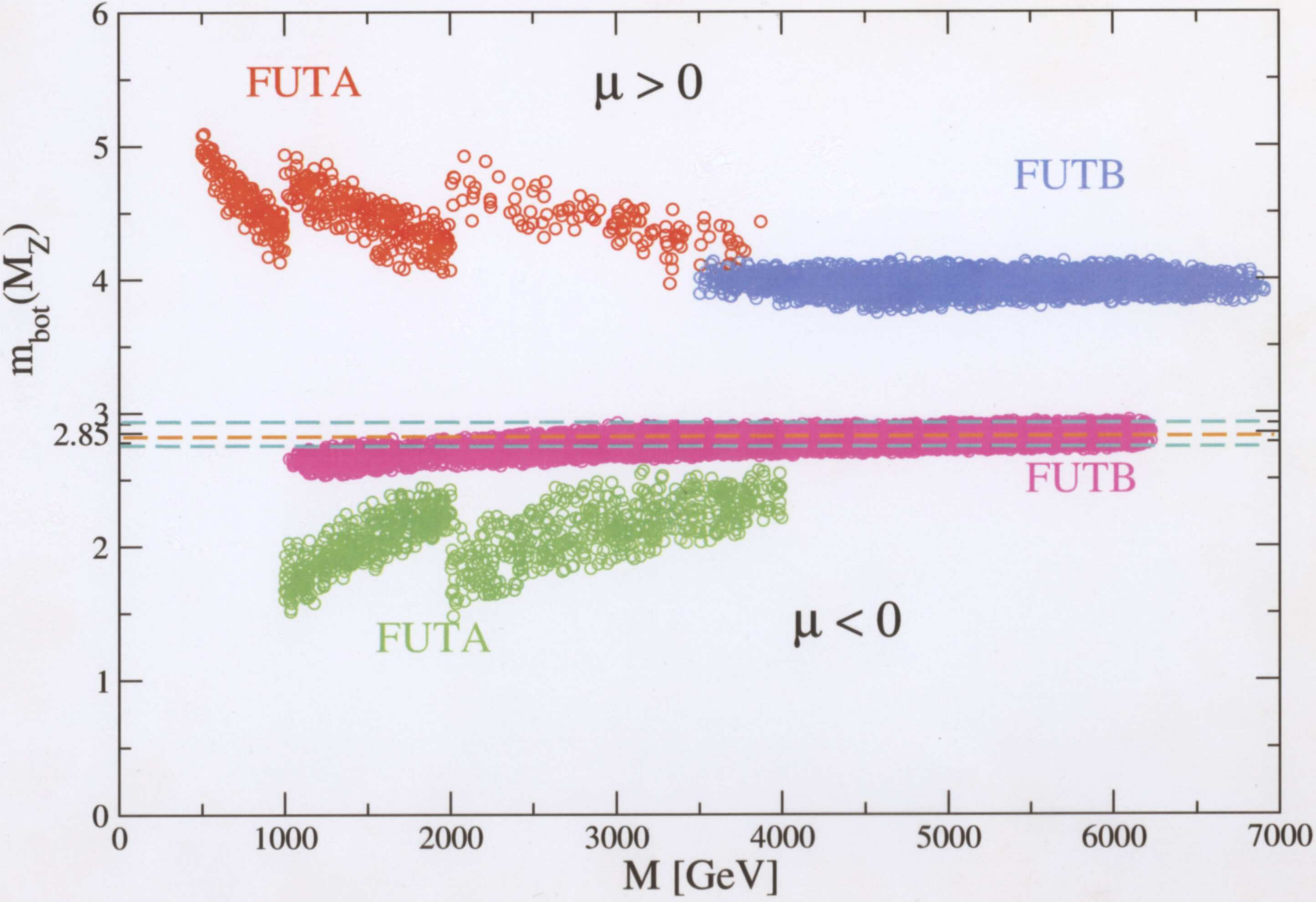
Model A

$M_{susy} = 0.3$ TeV

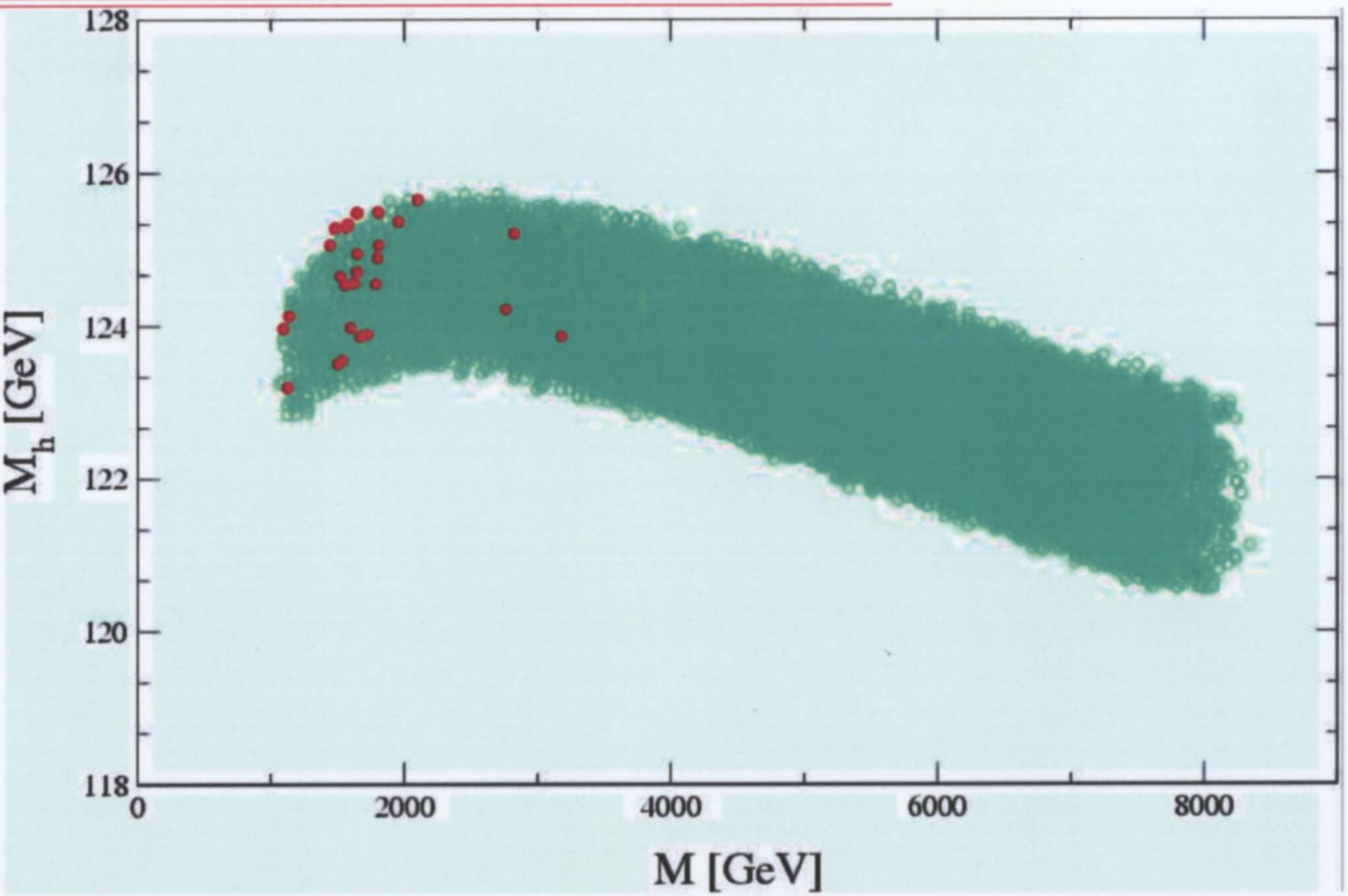


The empty region yields a neutralino as LSP


FUTA
$\mu > 0$
$\mu < 0$
FUTB
$\mu < 0$
$\mu > 0$
M_{top} [GeV]
M [GeV]
190
185
180
175
174.4
173.1
171.8
170
0
1000
2000
3000
4000
5000
6000
7000


FUTA
$\mu > 0$
FUTB
FUTB
FUTA
$\mu < 0$
$m_{bot}(M_Z)$
M [GeV]
2.85
0
1
2
3
4
5
6
0
1000
2000
3000
4000
5000
6000
7000

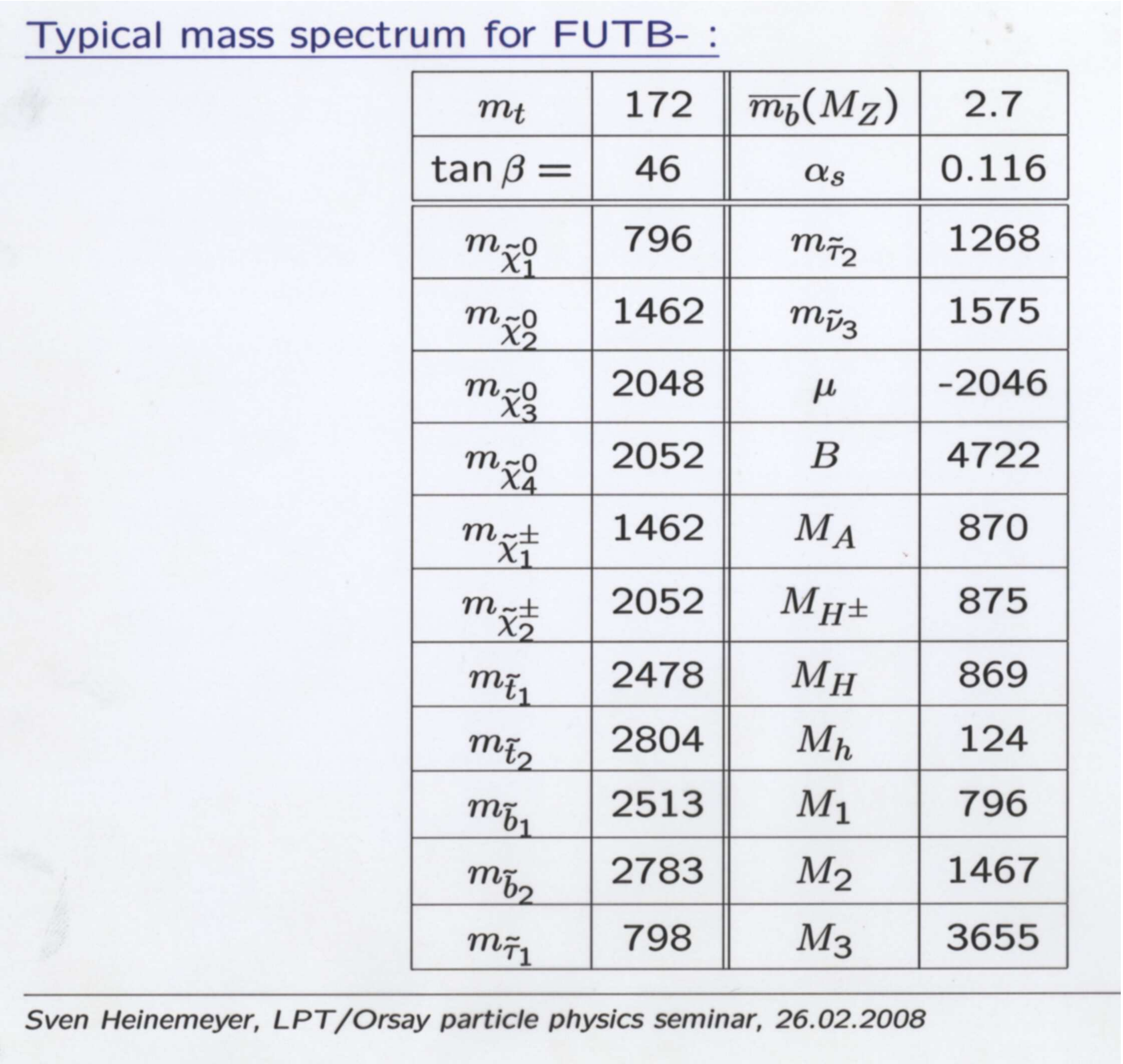
3D) Predictions for the light Higgs boson



green: consistent with B physics constraints
red: agreement with (loose) CDM bound

118 GeV $\leq M_h \leq$ 129 GeV (incl. theor. unc.)

⇒ "easy" to find for LHC (but "only" SM-like . . .)

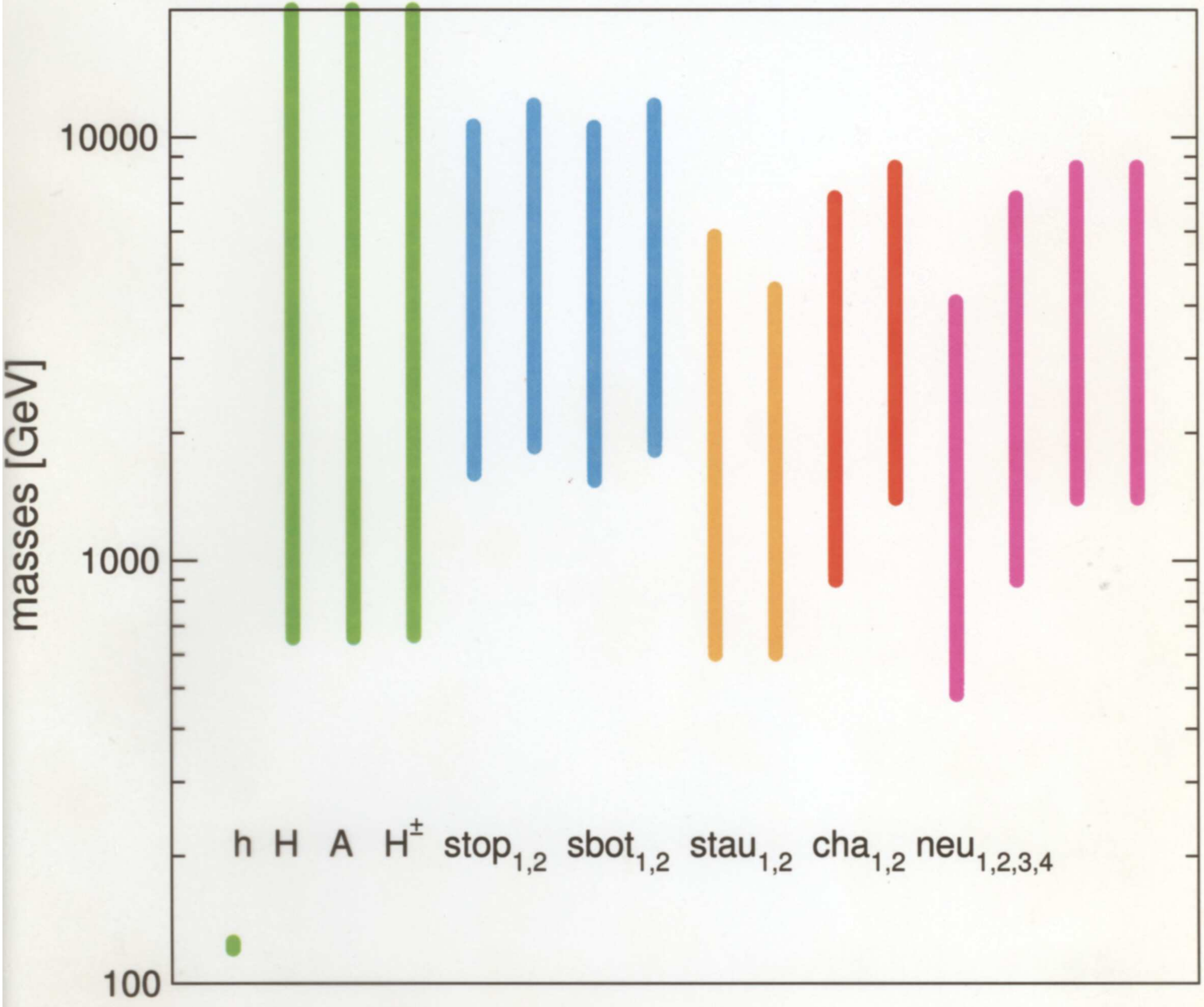


Typical mass spectrum for FUTB- :

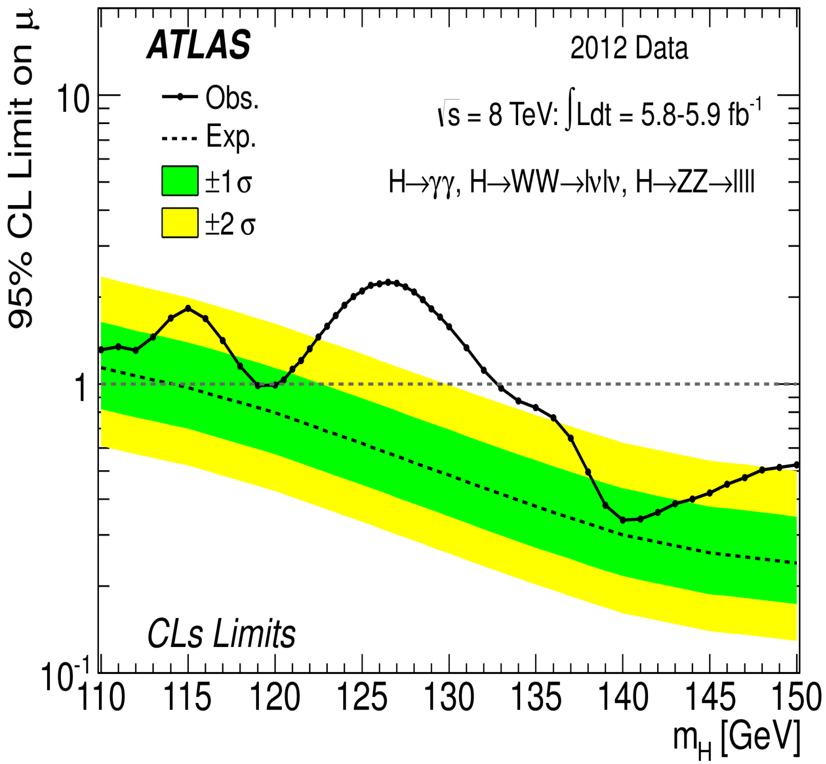
m_t	172	$\overline{m_b}(M_Z)$	2.7
$\tan\beta =$	46	α_s	0.116
$m_{\tilde{\chi}_1^0}$	796	$m_{\tilde{\tau}_2}$	1268
$m_{\tilde{\chi}_2^0}$	1462	$m_{\tilde{\nu}_3}$	1575
$m_{\tilde{\chi}_3^0}$	2048	μ	-2046
$m_{\tilde{\chi}_4^0}$	2052	B	4722
$m_{\tilde{\chi}_1^\pm}$	1462	M_A	870
$m_{\tilde{\chi}_2^\pm}$	2052	$M_{H^\pm}$	875
$m_{\tilde{t}_1}$	2478	M_H	869
$m_{\tilde{t}_2}$	2804	M_h	124
$m_{\tilde{b}_1}$	2513	M_1	796
$m_{\tilde{b}_2}$	2783	M_2	1467
$m_{\tilde{\tau}_1}$	798	M_3	3655

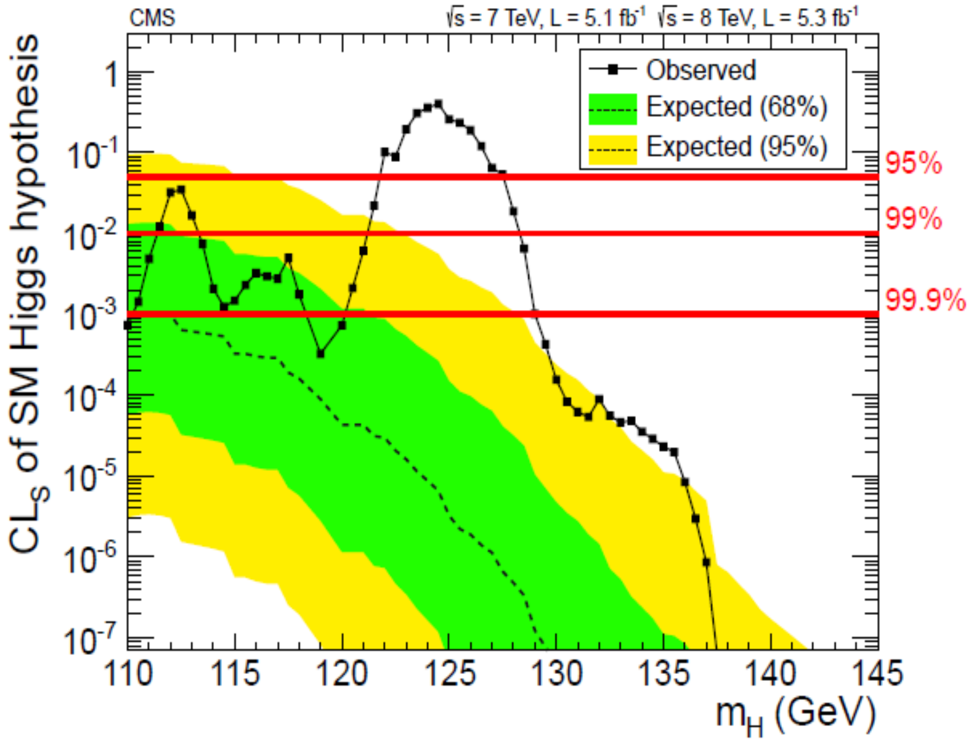
M1	580 GeV
M2	1077 GeV
Mgluino	2754 GeV
Stop1	1876 GeV
Stop2	2146 GeV
Sbot1	1849 GeV
Sbot2	2117 GeV
Mstau1	635 GeV
Mstau2	867 GeV
Char1	1072 GeV
Char2	1597 GeV
Neu1	579 GeV
Neu2	1072 GeV
Neu3	1591 GeV
Neu4	1596 GeV
Mh	123.1 GeV
MH	679 GeV
MA	680 GeV
$MH^{\pm}$	685 GeV
Mtop	172.2 GeV
Mbot(M_Z)	2.71 GeV

FUTB, $\mu < 0$


masses [GeV]
10000
1000
100
h
H
A
H±
stop1,2
sbot1,2
stau1,2
cha1,2
neu1,2,3,4

It is where the SM predicts it should be



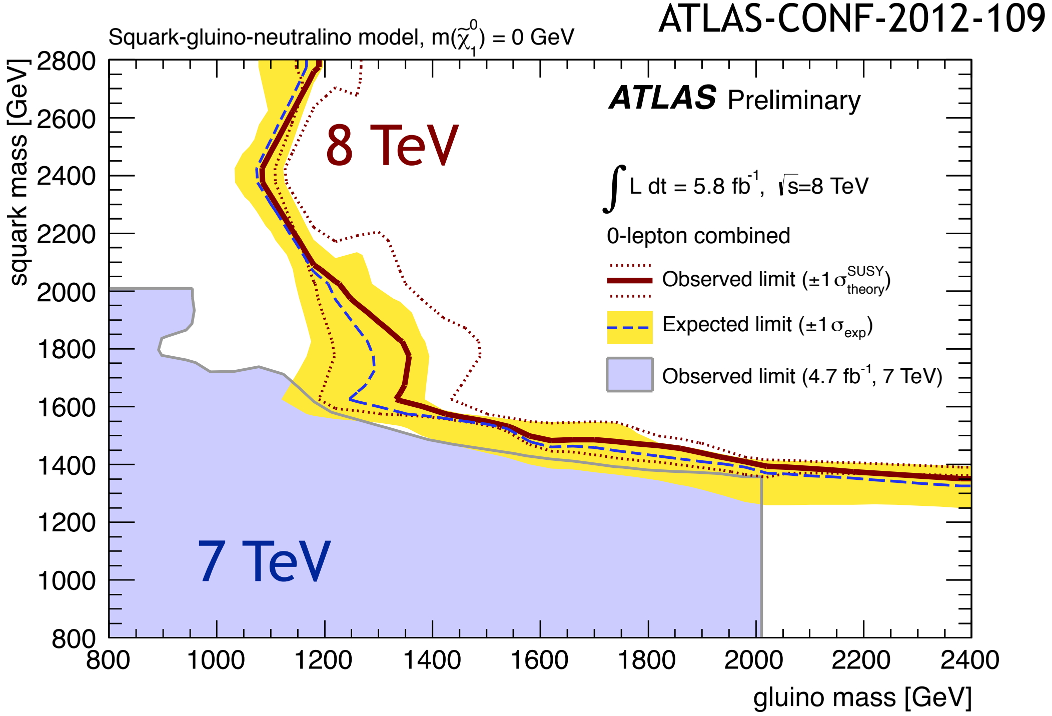


$$M_H = 126 \pm 0.4\,(stat.) \pm 0.4\,(syst.)\,\mathrm{GeV}$$

$$M_H = 125.3 \pm 0.4\,(stat.) \pm 0.5\,(syst.)\,\mathrm{GeV}$$

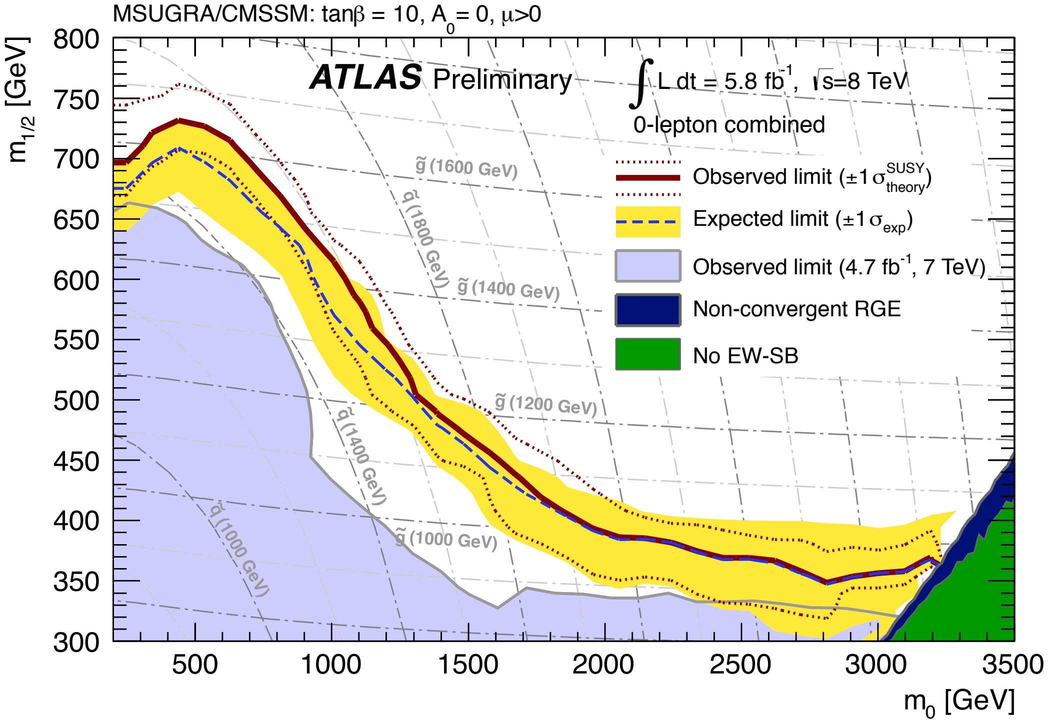
Jets+MET results

- Exclusions in the squark-gluino mass plane for a simplified SUSY model



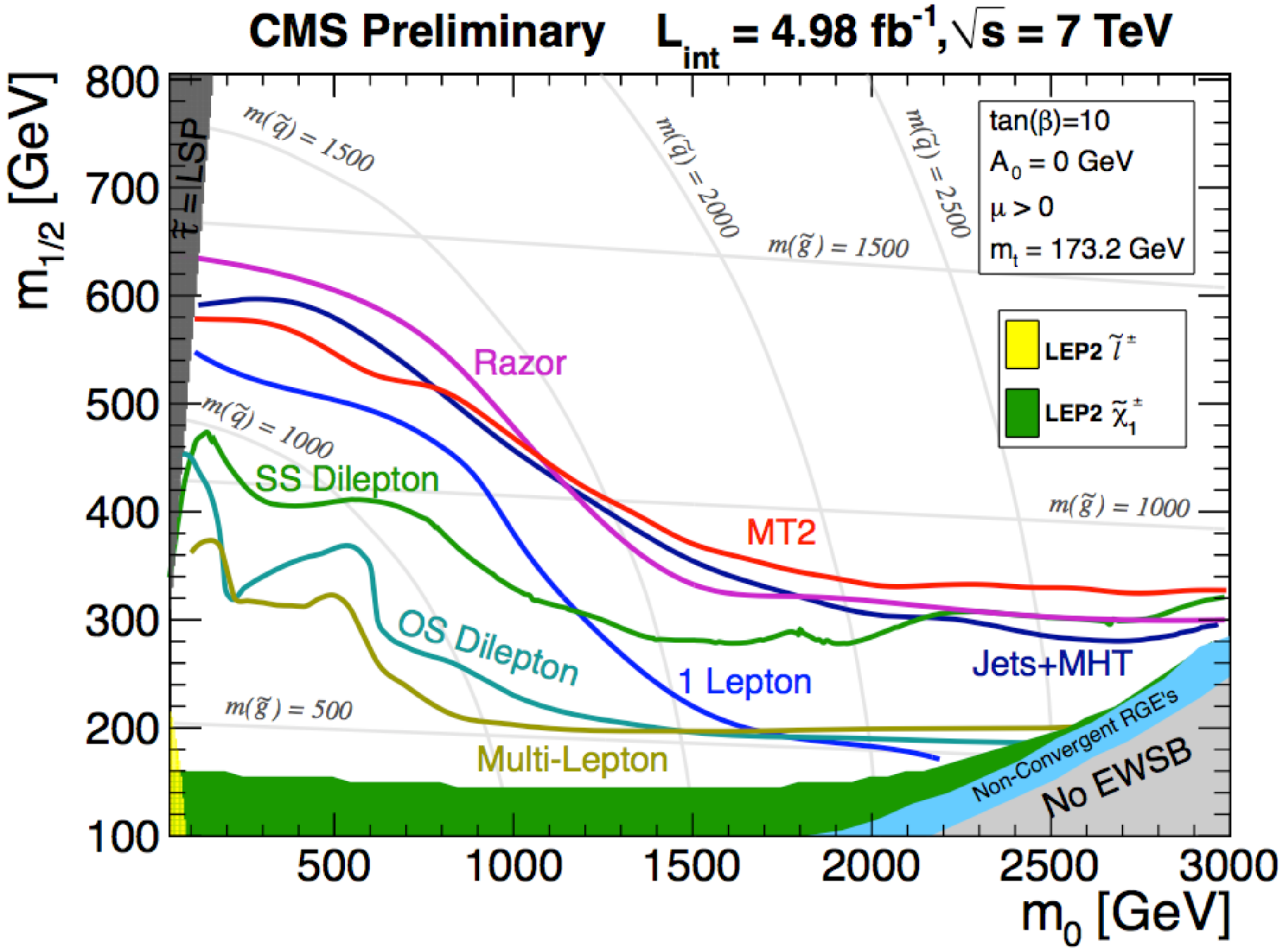
ATLAS-CONF-2012-109

Limits stable up to ~200 GeV mass LSP



- CMSSM ($m_{1/2}$, m_0) plane: equal mass squarks and gluinos excluded below 1500 GeV

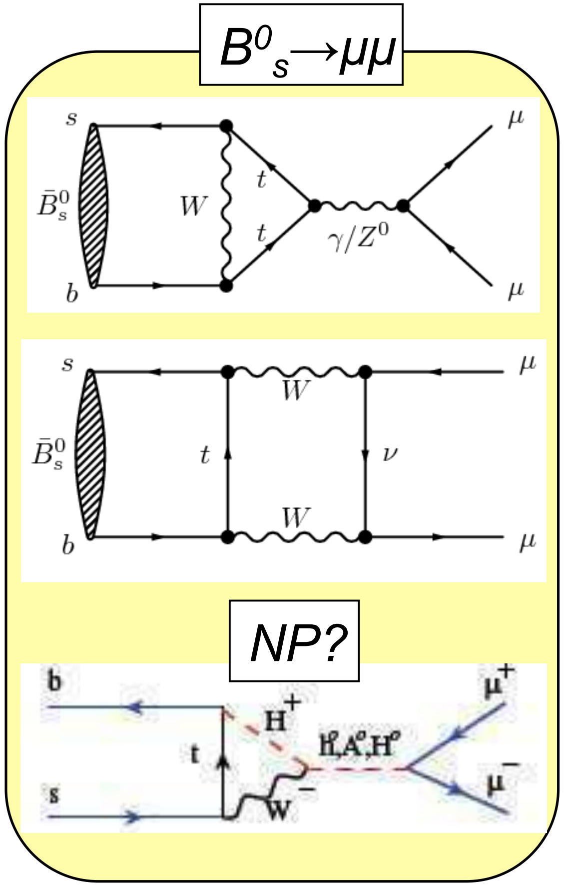
No SUSY (so far).

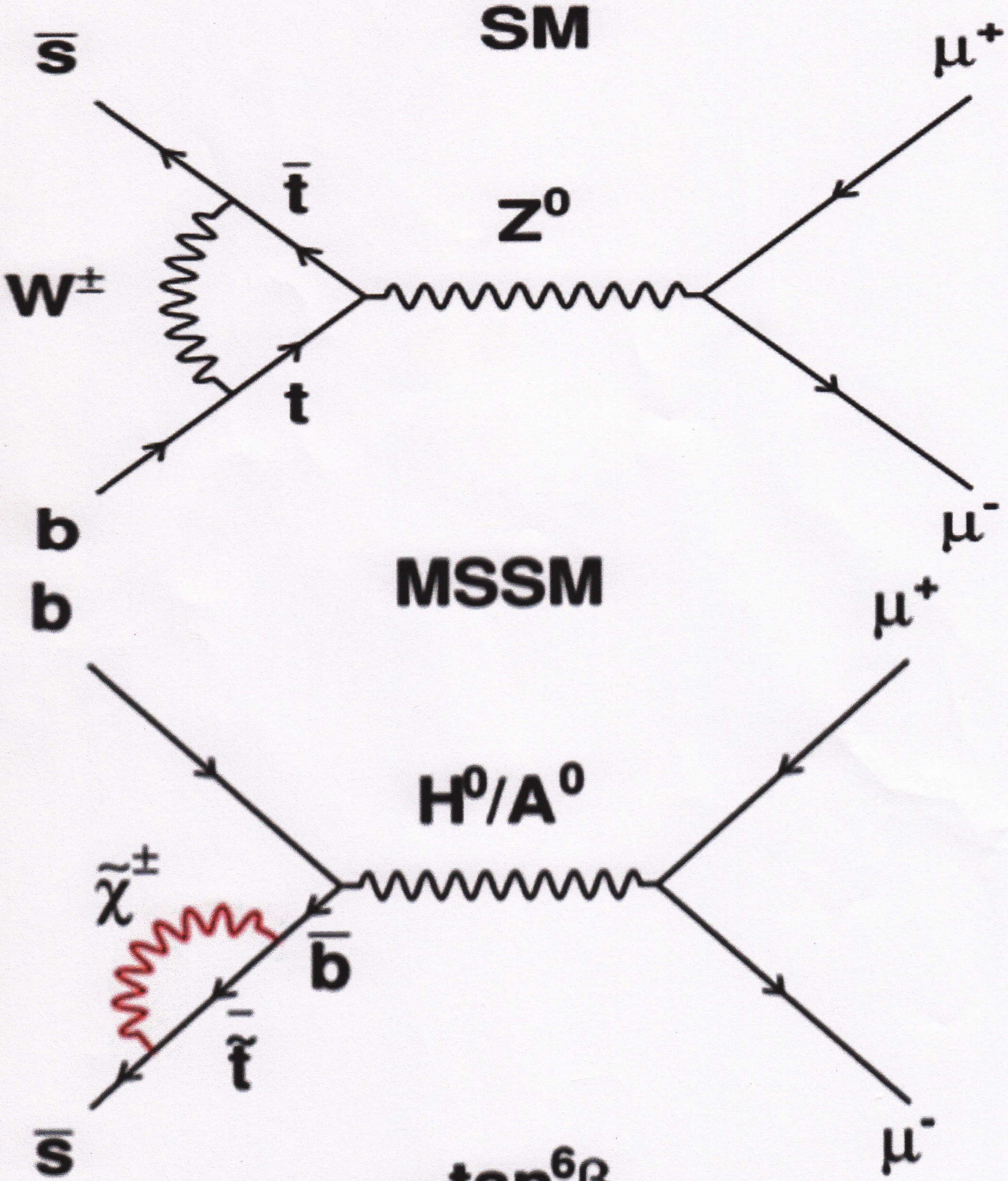


At least within constrained MSS models.

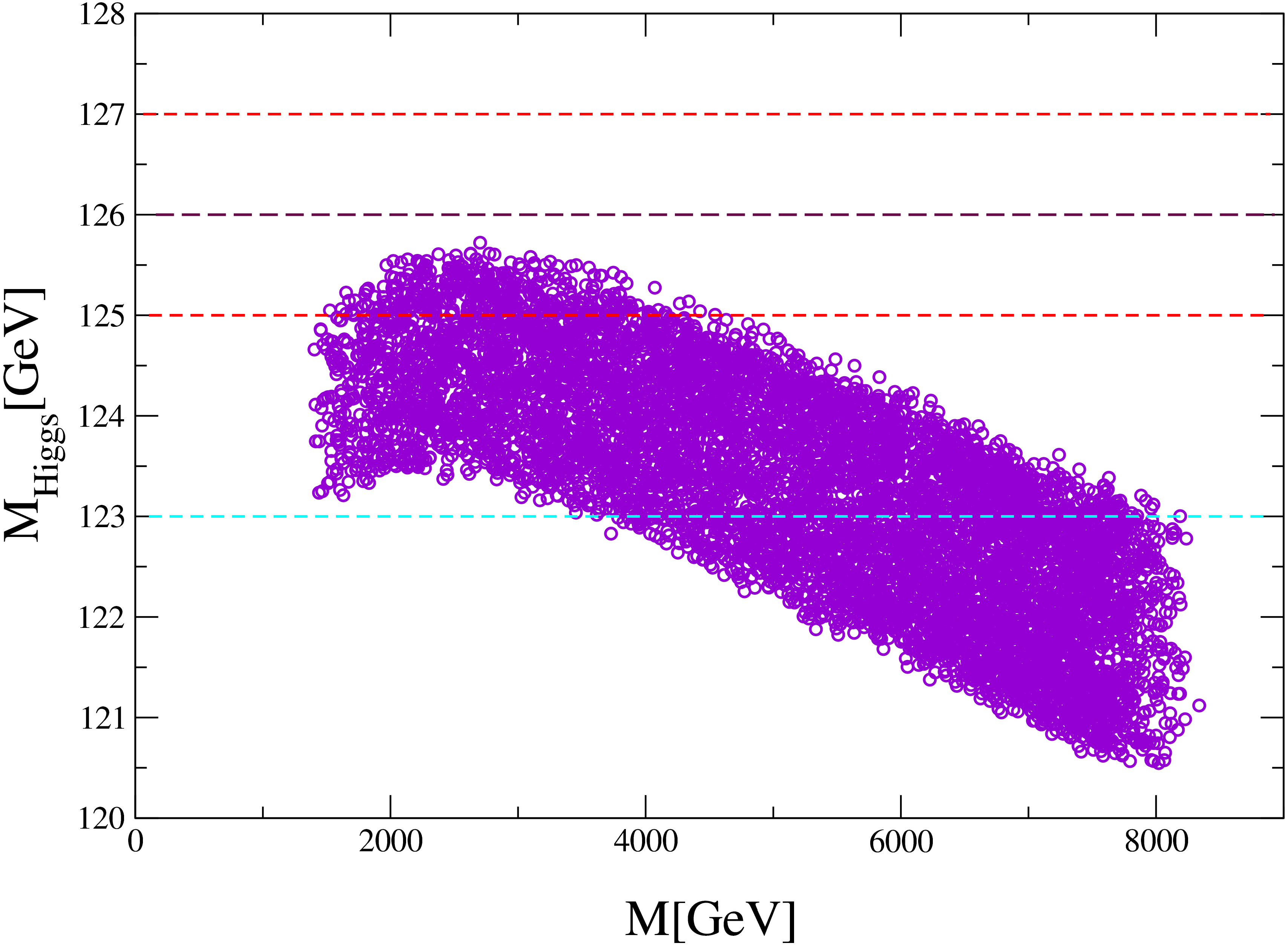
Search for NP in $B_{s(d)}\rightarrow\mu^+\mu^-$

- Highly suppressed in SM - FCNC plus helicity $(m_\mu/M_B)^2$ - and well predicted
  - $BR(B_s\rightarrow\mu^+\mu^-) = 3.2\pm0.03\ \ 10^{-9}$
  - $BR(B_d\rightarrow\mu^+\mu^-) = 0.11\pm0.01\ 10^{-9}$
    - A.J.Buras et al: arXiv: 1208.0934
- Sensitive to NP
  - Could be strongly enhanced in SUSY
  - In MSSM scales like ~$\tan^6\beta$ →
- Limit or measurement of $B_{s,d}\rightarrow\mu^+\mu^-$ will strongly constraint parameter space

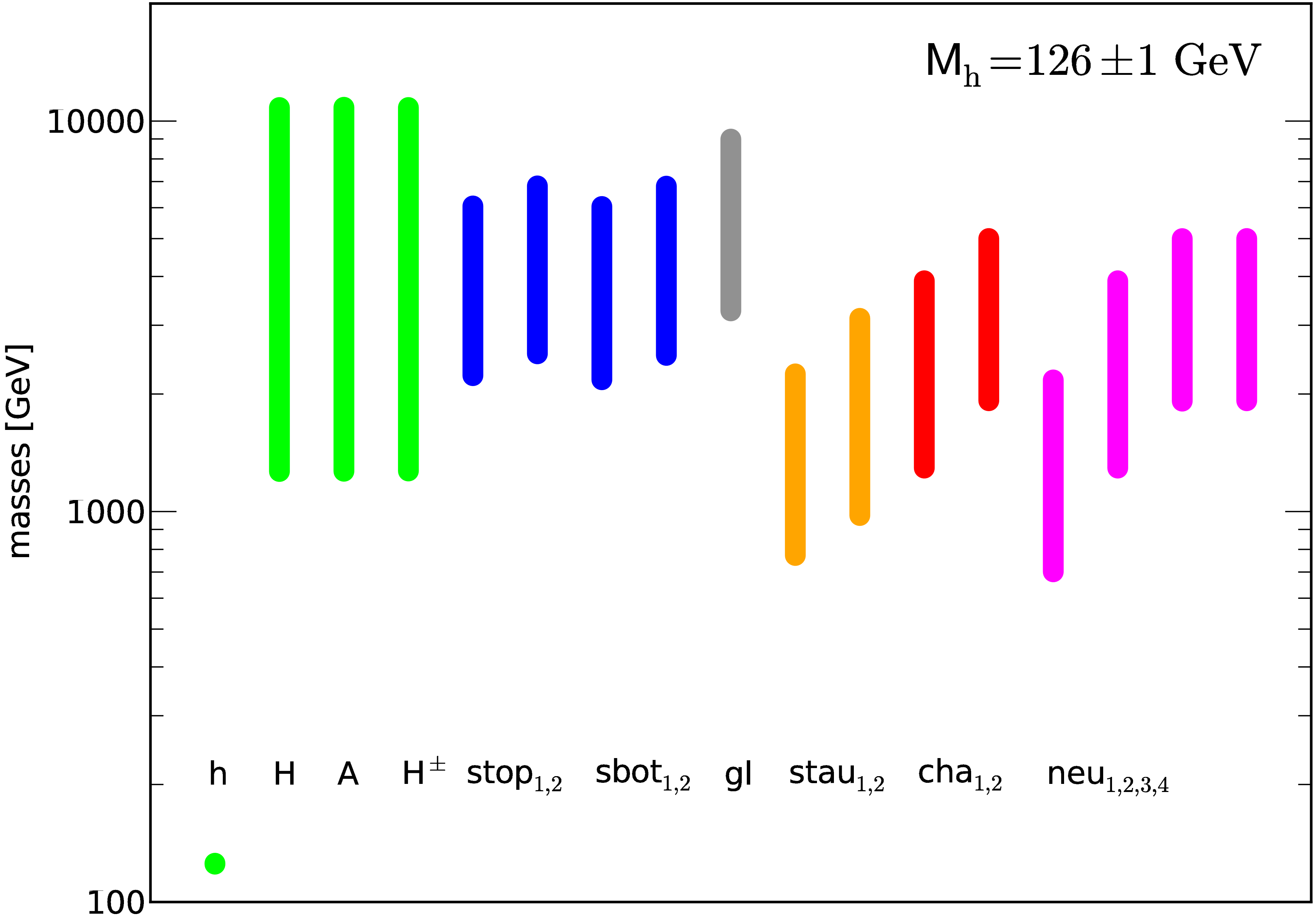




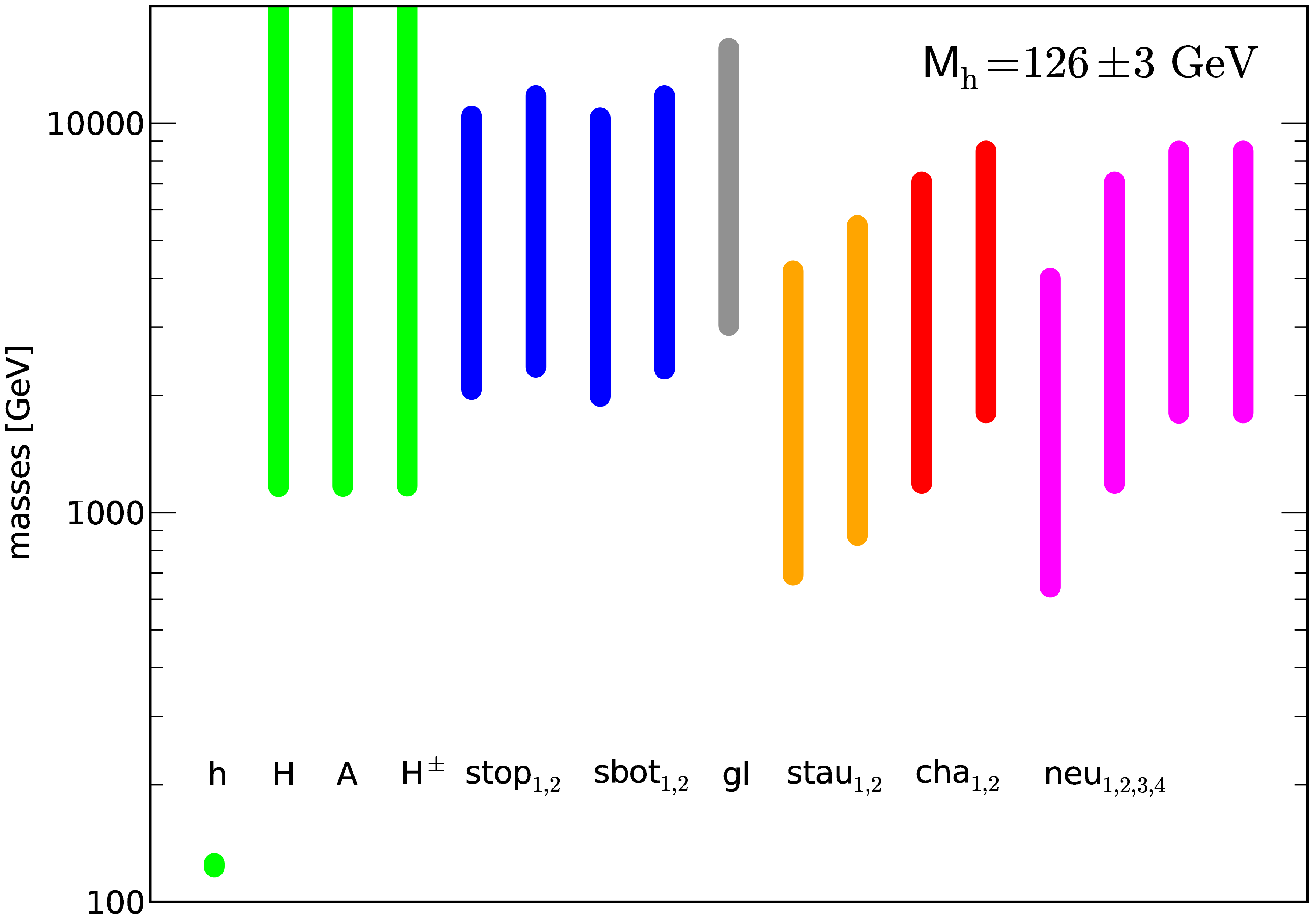
$\sim \tan^6\beta$



128
127
126
125
124
123
122
121
120
M_{Higgs}[GeV]
0
2000
4000
6000
8000
M[GeV]



$M_h = 126 \pm 1$ GeV
masses [GeV]
10000
1000
100
h
H
A
$H^\pm$
$\text{stop}_{1,2}$
$\text{sbot}_{1,2}$
gl
$\text{stau}_{1,2}$
$\text{cha}_{1,2}$
$\text{neu}_{1,2,3,4}$



$M_h = 126 \pm 3$ GeV
masses [GeV]
10000
1000
100
h
H
A
$H^\pm$
$\text{stop}_{1,2}$
$\text{sbot}_{1,2}$
gl
$\text{stau}_{1,2}$
$\text{cha}_{1,2}$
$\text{neu}_{1,2,3,4}$

MSSM

Mondragon
Tracas, Z

- Based on the new observation that top, bottom Yukawa couplings and α_s satisfy RGI relations, i.e. are successfully (theoretically and experimentally!) reduced

•• Assuming in addition a RGI relation among the trilinear couplings in the superpotential and in the corresponding ones in the soft supersymmetry breaking (scalar) sector

→ Prediction of the Higgs masses and s-spectrum

All-loop relations among SSB β-functions

Yamada
Hisano-Shifman
Kazakov
Jack-Jones-Pickering

$$\beta_M = 2\mathcal{O}\left(\frac{\beta_g}{g}\right)$$

$$\beta_h^{ijk} = \gamma^i{}_l h^{ljk} + \gamma^j{}_l h^{ilk} + \gamma^k{}_l h^{ijl} - 2\gamma_1{}^i{}_l C^{ljk} - 2\gamma_1{}^j{}_l C^{ilk} - 2\gamma_1{}^k{}_l C^{ijl}$$

$$(\beta_{m^2})^i_j = \left[\Delta + X\frac{\partial}{\partial g}\right]\gamma^i_j$$

where $\mathcal{O} = \left(Mg^2\frac{\partial}{\partial g^2} - h^{lmn}\frac{\partial}{\partial C^{lmn}}\right)$

$$\Delta = 2\mathcal{O}\mathcal{O}^* + 2|M|^2 g^2\frac{\partial}{\partial g^2} + \tilde{C}_{lmn}\frac{\partial}{\partial C_{lmn}} + \tilde{C}^{lmn}\frac{\partial}{\partial C^{lmn}}$$

$$(\gamma_1)^i_j = \mathcal{O}\gamma^i_j, \quad C_{lmn} = (C^{lmn})^*$$

$$\tilde{C}^{ijk} = (m^2)^i{}_l C^{ljk} + (m^2)^j{}_l C^{ilk} + (m^2)^k{}_l C^{ijl}$$

Assuming the existence of RGI surfaces on which

a) $C = C(g)$ or equivalently

$$\frac{dC^{ijk}}{dg} = \frac{\beta_C^{ijk}}{\beta_g}$$

i.e. reduction of couplings

b) $$h^{ijk} = -M \frac{dC^{ijk}(g)}{d\ln g}$$

$$\Rightarrow \begin{cases} M = \frac{\beta_g}{g} M_0 \\ h^{ijk} = -M_0 \beta_C^{ijk} \\ b^{ij} = -M_0 \beta_\mu^{ij} \\ m_i^2 + m_j^2 + m_k^2 = |M|^2 \frac{d\ln C^{ijk}}{d\ln g} \end{cases}$$

Jack-Jones

Kobayashi-Kubo-Z

are RGI to all-loops.

In supergravity framework,

$M_0 = m_{3/2}$ gravitino mass

Sketch of proof

Assuming $C\frac{\partial}{\partial C} = C^{*}\frac{\partial}{\partial C^{*}}$

for a RGI surface $F(g, C^{ijk}, C^{ijk*})$

$$\rightsquigarrow \frac{d}{dg} = \left(\frac{\partial}{\partial g} + 2\,\frac{\beta_C}{\beta_g}\,\frac{\partial}{\partial C}\right)$$

Consider

$$O = \left(M g^2 \frac{\partial}{\partial g^2} - h\frac{\partial}{\partial C}\right)$$

$$\xrightarrow{(b)} O = \frac{1}{2} M \frac{d}{d\ln g}$$

and $\beta_M = M \frac{d}{d\ln g}\left(\frac{\beta_g}{g}\right)$

$$\Longrightarrow M = \frac{\beta_g}{g} M_0$$ || Generalised Hisano - Shifma

Similarly we obtain the rest relations

Application of the RGI relations in MSSM

$$W = Y_t Q H_2 t^c + Y_b Q H_1 b^c + \mu H_1 H_2 + \cdots$$

$$\mathcal{L}_{SSB} = \sum_{\phi} m_{\phi}^2 \phi^* \phi + \left[m_3^2 H_1 H_2 + \sum_{i=1}^{3} \frac{1}{2} M_i \lambda_i \lambda_i + h.c. \right]$$

$$+ h_t \phi_Q H_2 \phi_{t^c} + h_b \phi_Q H_1 \phi_{b^c} + \cdots + h.c.$$

In MSSM the assumption (a) is a fact! as we have seen, i.e.

$$dY_{t,b} / dg_3 = \beta_{Y_{t,b}} / \beta_{g_3}$$ hold.

Then assuming (b), i.e. that

$$h_{t,b} = -M \, dY_{t,b} / dg$$

is RGI to all-loops,

we obtain that

the following relations are RGI to all-loops

$$M = \beta_{g_3}/g_3 \; M_U$$

$$h_{t,b} = - M_U \, g_3 \, dY_{t,b}/dg_3$$

$$m_3 = - M_U \, g_3 \, d\mu/dg_3$$

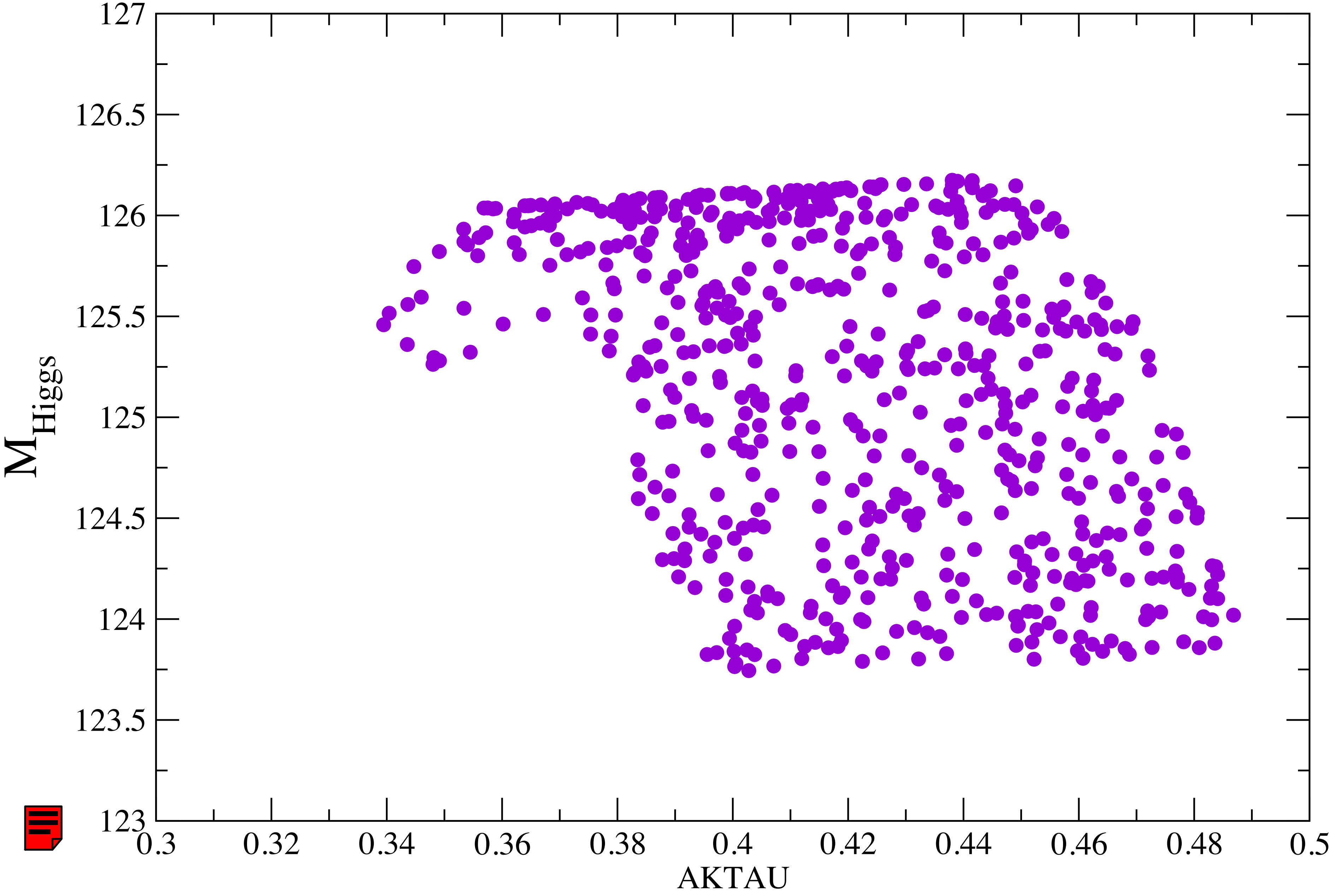
$$m_i^2 + m_j^2 + m_k^2 = |M_U|^2 \underset{\overset{\shortparallel}{1 \left(\substack{\text{at} \\ \text{1-loop}}\right)}}{dY_{t,b}/d\ln g_3}$$

Since all gauge couplings meet at the unification scale M_U, we have the following boundary conds at M_U

$$Y_{t,b} = c_{t,b} \, g_U \, , \quad h_{t,b} = M_U \, Y_{t,b}$$

$$m_3 = - M_U \, \mu \, ,$$

$$m_{H_2}^2 + m_{\phi_Q}^2 + m_{\phi_{t^c}}^2 = M_U^2 \, , \quad m_{H_1}^2 + m_{\phi_Q}^2 + m_{\phi_{b^c}}^2 = M_U^2$$


127
126.5
126
125.5
125
124.5
124
123.5
123
M_{Higgs}
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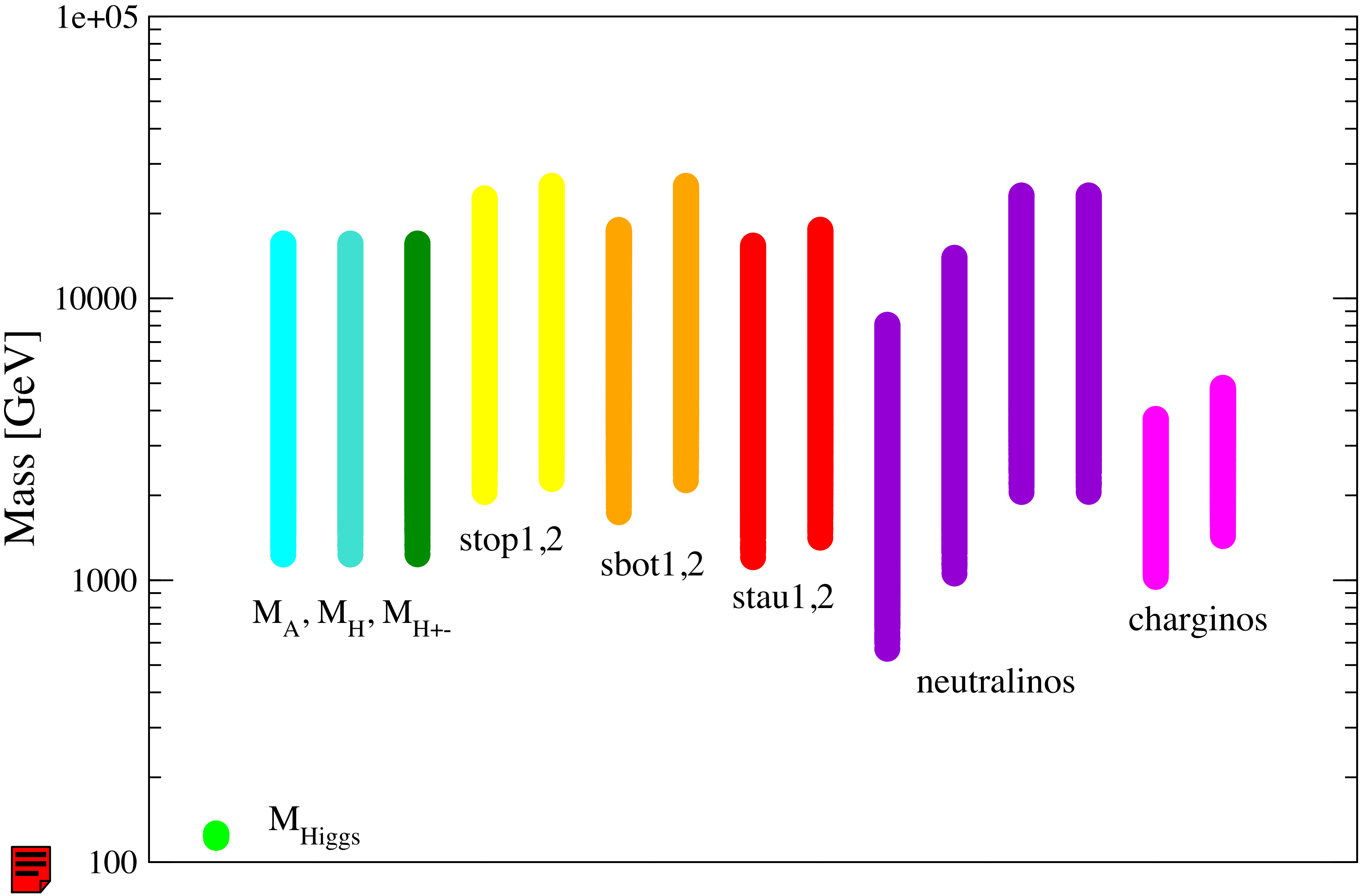
0.32
0.34
0.36
0.38
0.4
0.42
0.44
0.46
0.48
0.5
AKTAU


1e+05
10000
1000
100
Mass [GeV]
M_A, M_H, M_{H+-}
stop1,2
sbot1,2
stau1,2
neutralinos
charginos
M_{Higgs}

Anomaly-mediated ~~SUSY~~

$$\Rightarrow \left\{ \begin{aligned} M &= m_{3/2}\, \beta_g / g \\ h^{ijk} &= -m_{3/2}\, \beta_C^{ijk} \\ b^{ij} &= -m_{3/2}\, \beta_M^{ij} \\ (m^2)^i_j &= \frac{1}{2} |m_{3/2}|^2 \frac{d\gamma^i_j}{dt} \end{aligned} \right.$$

RGI
to all-loops

Assuming

existence of RGI surfaces on which

a) $C = C(g)$ or

$$\frac{dC^{ijk}}{dg} = \frac{\beta_C^{ijk}}{\beta_g}$$

b) $h^{ijk} = -M \dfrac{dC^{ijk}(g)}{d\ln g}$

without relying on specific solutions

$\rightsquigarrow$ consequences of anomaly-med. susy scenario are obtained from the b-functions of SSB parameters.

Assuming $c \frac{\partial}{\partial c} = c^* \frac{\partial}{\partial c^*}$

for a RGI surface $F(g, C^{ijk}, C^{*ijk})$

$$\rightsquigarrow \frac{d}{dg} = \left(\frac{\partial}{\partial g} + 2 \frac{b_c}{b_g} \frac{\partial}{\partial c} \right) \; !!$$

- Consider

$$\mathcal{O} = \left(M \cdot g^2 \frac{\partial}{\partial g^2} - h \frac{\partial}{\partial c} \right)$$

$$\overset{(b)}{\rightsquigarrow} \quad \mathcal{O} = \frac{1}{2} M \frac{d}{d \ln g}$$

and $$b_M = M \frac{d}{d \ln g} \left(\frac{b_g}{g} \right)$$

$$F(g, c, c^*) = \mathrm{const}$$

$$dF = \left(\frac{\partial}{\partial g}\, dg + \frac{\partial}{\partial c}\, dc + \frac{\partial}{\partial c^*}\, dc^*\right) F$$

$$= 0$$

$$\rightsquigarrow \frac{dF}{dg} = \left(\frac{\partial}{\partial g} + \frac{dc}{dg}\frac{\partial}{\partial c} + \frac{dc^*}{dg}\frac{\partial}{\partial c^*}\right) F$$

$$= 0$$

and if $c \dfrac{\partial}{\partial c} = c^* \dfrac{\partial}{\partial c^*}$

$$\rightsquigarrow \frac{dF}{dg} = \left(\frac{\partial}{\partial g} + 2\frac{\partial}{\partial c}\frac{dc}{dg}\right) F = 0$$

$$\rightsquigarrow \frac{d}{dg} = \frac{\partial}{\partial g} + 2\frac{\beta_c}{\beta_g}\frac{\partial}{\partial c} \quad \|$$

$\rightsquigarrow$ $M = \frac{b_g}{g} M_0$ $\|$ Generalized Hisano-Shifman

M_0 - integration const. which in sugra becomes $m_{3/2}$

$\rightsquigarrow$ $b_M = m_{3/2} \frac{d}{dt} \left(b_g / g \right)$

.. Similarly

$(\gamma_1)^i_j = \mathcal{O} \gamma^i_j = \frac{1}{2} m_{3/2} \frac{d \gamma^i_j}{dt}$ $\|$

... From (b) and H-S

$\rightsquigarrow$ $h^{ijk} = - m_{3/2} b_c^{ijk}$

and using $(\gamma_1)^i_j$ above

$\rightsquigarrow$ $b_h^{ijk} = - m_{3/2} \frac{d}{dt} b_c^{ijk}$

$\rightsquigarrow$ $h^{ijk} = - m_{3/2} b_c^{ijk}$

is RGI

.... We have also proved that the sum rule is also RGI to all loops which generalizes the corresponding relation for $(m^2)^i_j$

Remarks

- Differences in assuming **existence** of RGI surfaces in (a) + (b) and considering **specific** solution of REs.
- e.g. at 1st order in g the sum rule in first case

$$m_i^2 + m_j^2 + m_k^2 = |M|^2 \frac{d\ln C^{ijk}}{d\ln g}$$

and $$\frac{d\ln C^{ijk}}{d\ln g} = \frac{g}{C^{ijk}} \frac{dC^{ijk}}{dg} = \frac{g}{C^{ijk}} \frac{\beta_C^{ijk}}{\beta_g}$$

which is clearly model dependent.

but assuming a power series solution

$$\frac{d\, c^{ijk}}{d \ln g} = 1$$

model independent!

•• All-loop sum rule does not depend on specific solution while

$$(m^2)^i{}_j = \frac{1}{2}\, \frac{g^2}{b_g}\, |M|^2\, \frac{d\gamma^i{}_j}{dg}$$

it does!

••• Resolution of the fatal problem of anomaly induced scenario:

Use the Sum Rule!