Gauge Theories, Higgs Mechanism, Standard Model

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- Antonio Pich: Particle Physics: The Standard Model (lectures)
- W. Hollik: Theory of Electroweak Interactions, Corfu Summer Institute, School and Workshop on Standard Model and Beyond, 2013
- Aitchison I J R & Hey A J G: Gauge Theories In Particle Physics Volume 1: From Relativistic Quantum Mechanics To QED
- Francis Halzen-Alan D.Martin:Quarks and Leptons
- Tai-Pei Cheng,Ling-Fong Li: Gauge Theory of Elementary Particle Physics
- Abdelhak Djouadi: The Anatomy of Electro-Weak Symmetry Breaking, Tome I: The Higgs boson in the Standard Model

C Charge Conjugation

The characteristic feature of the transformation C is that it transforms a particle to its antiparticle (opposite quantum numbers).
However, the spin and state of motion remain unchanged

$$\psi^c(-Q, -B, -L, -s, \ldots) = U(C)\psi(Q, B, L, s)U^{-1}(C),$$

where $U(C)$ unitary and $U(C) = U^{-1}(C)$.

We shall see the form of the CC transformation in the case of a $\psi(x)$ Dirac field.

The Dirac equation in an e/m field is $(i\gamma^\mu \partial_\mu - q\gamma^\mu A_\mu(x) - m)\psi(x) = 0$. An electron (with $q = -e$) is transformed into a positron under the conjugation transformation $\psi^c(x) = U(C)\psi(x)U(C)$ which must also satisfy the Dirac equation.

$$\begin{aligned} (i\gamma^\mu \partial_\mu + e\gamma^\mu A_\mu(x) - m)\psi(x) &= 0 \\ (i\gamma^\mu \partial_\mu - e\gamma^\mu A_\mu(x) - m)\psi^c(x) &= 0 \end{aligned}$$

that happens indeed if: (Homework)

$$\psi^c(x) = \underbrace{i\gamma^2\gamma^0}_{\equiv C}\bar{\psi}^T(x) = i\gamma^2(\psi^\dagger(x))^T = i\gamma^2\psi^*(x)$$

Neutrino Masses

The absence of ν_R in the SM prevents us from writing a Dirac mass term, as we did for the rest of the fermions.

It is possible though to write a mass term using only the left - handed field ψ_L and give non-zero mass to neutrinos.
We observe that:

$$\begin{aligned}(\psi^c)_R &= \frac{1}{2}(1+\gamma_5)\psi^c = \frac{1}{2}(1+\gamma_5)i\gamma^2\psi^* \\ &= i\gamma^2\frac{1}{2}(1-\gamma_5)\psi^* = i\gamma^2\psi_L^* = (\psi_L)^c = \psi_L^c\end{aligned}$$

Therefore, $(\psi^c)_R$ behaves as left-handed under Lorentz transformations.

Thus, we can construct a Majorana field only from ψ_L:

$$\chi = \psi_L + \psi_L^c$$

The Majorana fermion field is defined by its property to be identical to its conjugate under C charge conjugation: $\chi = \chi^c$, which is obviously true.

Thus, the particle and the antiparticle are identical in this case. Of course, this property holds only for a neutral, colour-less fermion like a neutrino.

The mass term for a Majorana field is written as:

$$m_L \bar{\chi}\chi = m_L(\bar{\psi}_L^c \psi_L + \bar{\psi}_L \psi_L^c)$$

Such a term changes the leptonic number by 2 $\Delta L = 2$ (and is not allowed in the SM (Homework))

In the same way as above, we can write a mass term for a Majorana field that is constructed only from ψ_R:

$$\omega = \psi_R + \psi_R^c = \omega^c \qquad m_R \bar{\omega}\omega = m_R(\bar{\psi}_R^c \psi_R + \bar{\psi}_R \psi_R^c)$$

In extensions of the SM there are more possibilities.

The simplest thing to do is to add right-handed neutrinos ν_R and introduce a Dirac mass term

$$m_D(\bar{\nu}_R\nu_L + \bar{\nu}_L\nu_R)$$

like we did for the rest of the fermions and expect to be of the the same order of magnitude (as the charged leptons). We could also have Majorana mass terms:

$$m_L(\bar{\nu}_L^c\nu_L + \bar{\nu}_L\nu_L^c) + m_R(\bar{\nu}_R^c\nu_R + \bar{\nu}_R\nu_R^c)$$

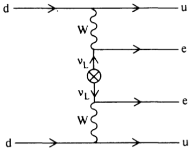
The total of Dirac and Majorana terms can be written in the form:

$$\begin{pmatrix} \bar{\nu}_L & \bar{\nu}_L^c \end{pmatrix} \begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix} \begin{pmatrix} \nu_R^c \\ \nu_R \end{pmatrix} + h.c. \qquad (Homework)$$

Of particular interest is the "seesaw" mechanism, which is realized if $m_L = 0$, $m_R >> m_D$

(Homework): Find in this case the eigenvalues of the neutrino masses matrix. What should be the value of m_R in order to have neutrino masses $\approx 1eV$ for m_D as those of the charged lepton masses?

The existence of non-zero Majorana masses has observable consequences, i.e. the neutrinoless double $\beta-$decay (that has not been observed until now)



$\otimes$ corresponds to a Majorana-mass insertion $\nu_L \nu_L + h.c.$

Of particular interest are the phenomena that are related to neutrino oscillations. If the neutrinos have mass, then we have to distiguish between eigenstates of mass and those of electroweak interactions. Therefore, in analogy with what we said about quark mixing, we will have:

$$J^{\mu}_{charg.} = \begin{pmatrix} \bar{e} & \bar{\mu} & \bar{\tau} \end{pmatrix}_L \gamma^{\mu} \begin{pmatrix} \nu_e \\ \nu_{\mu} \\ \nu_{\tau} \end{pmatrix}_L = \begin{pmatrix} \bar{e} & \bar{\mu} & \bar{\tau} \end{pmatrix}_L \gamma^{\mu} U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix},$$

where ν_i are the mass eigenstates and $\nu_e, \nu_{\mu}, \nu_{\tau}$ the weak eigenstates.

The neutrino-oscillations mechanism becomes clear if we examine only 2 generations. Then we have:

$$\begin{pmatrix} |\nu_e\rangle \\ |\nu_{\mu}\rangle \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \end{pmatrix}$$

The eigenstates of the mass $|\nu_i\rangle$ evolve in time as known:

$$|\nu_i(t)\rangle = |\nu_i(0)\rangle e^{-iE_i t},$$

where energies E_i are

$$E_i = (p^2 + m_i^2)^{1/2} \simeq p + \frac{m_i^2}{p} \quad \textit{for } p >> m_i$$

After time t, a pure ν_e state becomes:

$$\begin{aligned}
|\nu(t)\rangle &= |\nu_1(0)\rangle \cos\theta e^{-iE_1 t} + |\nu_2(0)\rangle \sin\theta e^{-iE_2 t} \\
&= |\nu_e\rangle(\cos^2\theta e^{-iE_1 t} + \sin^2\theta e^{-iE_2 t}) + |\nu_\mu\rangle \sin\theta\cos\theta(-e^{-iE_1 t} + e^{-iE_2 t})
\end{aligned}$$

i.e. a mixture of ν_e and ν_μ.

We can calculate the probabilities of observing a ν_e or a ν_μ:

$$\begin{aligned}
P(\nu_e \to \nu_e, t) &= |\langle\nu_e|\nu(t)\rangle|^2 = 1 - \sin^2(2\theta)\sin^2\frac{(E_1 - E_2)t}{2} \\
P(\nu_e \to \nu_\mu, t) &= |\langle\nu_\mu|\nu(t)\rangle|^2 = \sin^2(2\theta)\sin^2\frac{(E_1 - E_2)t}{2} \quad \textit{(Homework)}
\end{aligned}$$

obviously with sum 1.

Higgs Boson

- The coupling constants of H are completely specified in the W-S model.

$$\begin{aligned} q_{\bar{f}fH} &= m_f/\upsilon = (2^{1/4}G_F^{1/2})m_f \\ q_{VVH} &= 2(2^{1/4}G_F^{1/2})M_V, \quad V = W, Z \end{aligned}$$

- On the other hand, the Higgs mass is actually arbitrary:

$$m_H = \sqrt{2}\mu, \qquad \mu^2 = \lambda(2\sqrt{2}G_F)^{-1}$$

- The Higgs particle profile is completely determined for a given Higgs mass (total decay amplitude, half-life, decay rates in various channels).

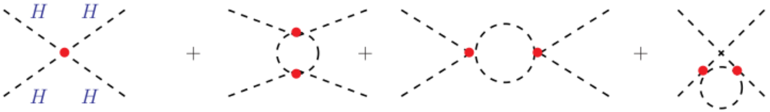
Higgs Boson Mass Limits

It is interesting to see among which limits the Higgs mass could be found

- Limit from the requirement that the theory is not trivial:

First, let's examine the one-loop radiative corrections considering only the Higgs contributions.

The tree Feynmann diagrams and the Higgs self-coupling (λ) one-loop corrections diagrams are:



The change of λ with the energy scale Q is described by the Renormalization Group Equation (RGE):

$$\frac{d}{d\log Q^2}\lambda(Q^2) = \frac{3}{4\pi^2}\lambda^2(Q^2) + higher\ order\ terms$$

Choosing as physical scale $Q_0 = \upsilon$, the solution is:

$$\lambda(Q^2) = \lambda(\upsilon^2)\left[1 - \frac{3}{4\pi^2}\lambda(\upsilon^2)\log\frac{Q^2}{\upsilon^2}\right]^{-1}$$

If $Q^2 << \upsilon^2 \rightarrow \lambda(Q^2) \sim \lambda(\upsilon^2)/\log(\infty) \rightarrow 0_+$

$\hookrightarrow$ then the theory is considered trivial because the coupling vanishes.

On the other hand:
If $Q^2 >> \upsilon^2 \quad \rightarrow \quad \lambda(Q^2) \rightarrow \infty$

The energy scale in which the coupling becomes infinite is called Landau Pole and it is:

$$\Lambda^* = \upsilon \exp\left(\frac{4\pi^2}{3\lambda}\right) = \upsilon \exp\left(\frac{8\pi^2\upsilon^2}{3m_H^2}\right)$$

The general argument is that in order to be able to deal with the scalar part of W-S *perturbatively at all energy scales*, λ must vanish (in W-S it means $m_H = 0$)

Consequently, the theory, since it does not include any interactions, is *trivial*.

It is possible though to use the argument with a different (and more useful) way:

We can use the solution of the RGE for λ, in order to determine the range of the energy scale for which W-S is valid perturbatively,

i.e. to determine the upper energy limit Λ^* below which λ remains finite $\rightarrow Q^2 \leq \Lambda^{*2}$.

Therefore, for every given m_H, we can determine a corresponding energy scale Λ^* (Landau Pole) where the theory is trivial.

The other way around, if we want W-S to make any sense up to a certain energy scale Λ^*, then:

$$m_H^2 \leq \frac{8\pi^2 \upsilon^2}{3\log\frac{\Lambda^{*2}}{\upsilon^2}} \qquad (Homework)$$

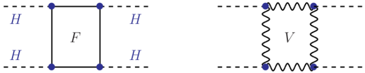
Example 1: For the unification scale, $\Lambda^* \sim 10^{16}\,GeV \quad \rightarrow m_H \leq 200\,GeV$

Example 2: For $\Lambda^* \sim 1\,TeV \rightarrow m_H \leq 750\,GeV$, which is very close to the non-perturbative lattice calculations.

- Stability Limit

In the above discussion we have considered only the Higgs contributions. This is a good approximation for a relatively big λ.

The RGE for λ, including contributions from fermions and vector bosons



is:

$$\frac{d}{d\log Q^2}\lambda \simeq \frac{1}{16\pi^2}\left[12\lambda^2 + 6\lambda\lambda_t^2 - 3\lambda_t^4 - \frac{3}{2}\lambda(3g_2^2 + g_1^2) + \frac{3}{16}(2g_2^4 + (g_2^2 + g_1^2)^2)\right],$$

where $\lambda_t = \sqrt{2}m_t/\upsilon$.

For not too large values of λ, the extra contributions will change only a little the preceding results.

But for $\lambda << \lambda_t, g_1, g_2$ we have approximately:

$$\frac{d}{d\log Q^2}\lambda \simeq \frac{1}{16\pi^2}\left[12\lambda^2 - 12\frac{m_t^4}{\upsilon^4} + \frac{3}{16}(2g_2^4 + (g_2^2 + g_1^2)^2)\right]$$

and its solution, considering again the electroweak scale as point of reference (and λ_t, g_1, g_2 constants), is:

$$\lambda(Q^2) = \lambda(\upsilon^2) + \frac{1}{16\pi^2}\left[-12\frac{m_t^4}{\upsilon^4} + \frac{3}{16}(2g_2^4 + (g_2^2 + g_1^2)^2)\right]\log\frac{Q^2}{\upsilon^2}$$

If λ is too small, then a top quark contribution's domination is possible and could lead to $\lambda(Q^2) < 0$. Then the vacuum will not be stable anymore, because it has no minimum.

Therefore, in order to have a scalar potential with finite minimum, i.e. $\lambda(Q^2) > 0$, it should hold:

$$m_H^2 > \frac{\upsilon^2}{8\pi^2}\left[-12\frac{m_t^4}{\upsilon^4} + \frac{3}{16}(2g_2^4 + (g_2^2 + g_1^2)^2)\right]\log\frac{Q^2}{\upsilon^2} \qquad (\textit{Homework})$$

Thus, we obtain serious limit for the Higgs boson mass, which depends from the upper boundary Λ^* of the theory.

Example 1: for $\Lambda^* \sim 10^{16}GeV \rightarrow m_H \geq 130GeV$
Example 2: for $\Lambda^* \sim 10^{3}GeV \rightarrow m_H \geq 70GeV$

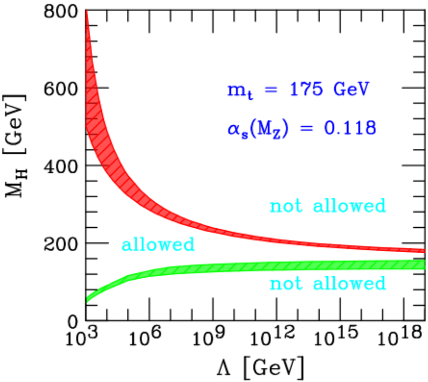
This boundary can change if the vacuum is metastable.

Consequently, the requirements for λ to be positive and finite put limits on m_H.

The condition of absolute stability up to the Planck scale is:

$$m_H > 129.4 \pm 1.8Gev$$

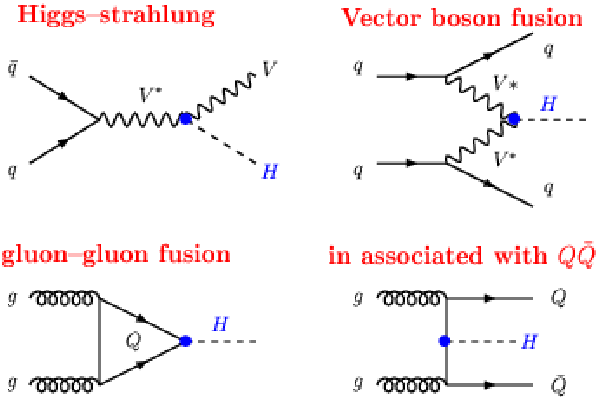
From this result we conclude that vacuum stability of the SM up to the Planck scale is excluded at 2σ for $m_H < 126GeV$.

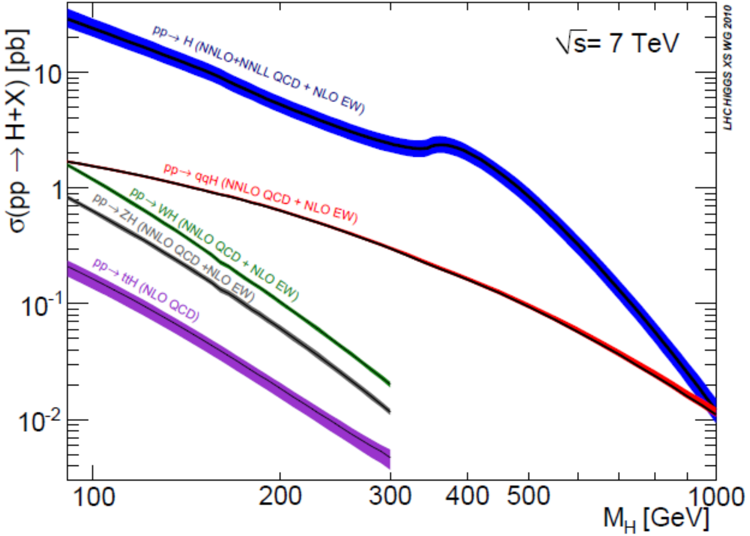


Higgs Production in hadron colliders

In W-S the main Higgs production mechanisms in hadron colliders use the fact that the Higgs boson is coupled "in privilege" with the heavy particles, namely W, Z, t and less with b.

Thus, the 4 main production processes are:

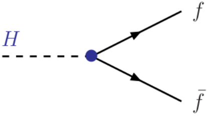




σ(pp → H+X) [pb]
M_H [GeV]
√s= 7 TeV
LHC HIGGS XS WG 2010
pp→ H (NNLO+NNLL QCD + NLO EW)
pp→ qqH (NNLO QCD + NLO EW)
pp→ WH (NNLO QCD + NLO EW)
pp→ ZH (NNLO QCD +NLO EW)
pp→ ttH (NLO QCD)
10
1
10^-1
10^-2
100
200
300
400
500
1000

W-S Higgs Boson Decay

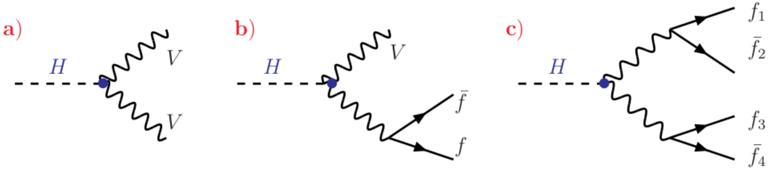
- Decay in quarks and leptons:



$$\Gamma(H \to f\bar{f}) = \frac{G_F N_C}{4\sqrt{2}\pi} m_H m_f^2 \beta_f^3$$

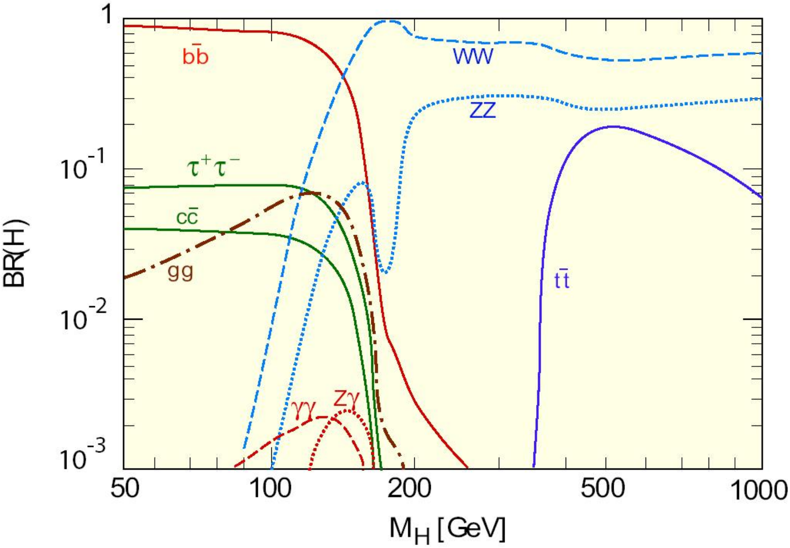
where $\beta = (1 - 4m_f^2/m_H^2)^{1/2}$ the velocity of the fermions in their final state and $N_C = 3(1)$ for quarks (leptons).

- Decay in W, Z bosons:



$$\Gamma(H \to VV) = \frac{G_F m_H^3}{16\sqrt{2}\pi} \delta_V \sqrt{1 - 4x}(1 - 4x + 12x^2),$$

where $x = M_V^2/m_H^2$, $\delta_W = 2$, $\delta_Z = 1$.


1
10^{-1}
10^{-2}
10^{-3}
BR(H)
50
100
200
500
1000
M_H [GeV]
$b\bar{b}$
WW
ZZ
$\tau^+\tau^-$
$c\bar{c}$
gg
$t\bar{t}$
$\gamma\gamma$
$Z\gamma$