Gauge Theories, Higgs Mechanism, Standard Model

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- Antonio Pich: Particle Physics: The Standard Model (lectures)
- W. Hollik: Theory of Electroweak Interactions, Corfu Summer Institute, School and Workshop on Standard Model and Beyond, 2013
- Aitchison I J R & Hey A J G: Gauge Theories In Particle Physics Volume 1: From Relativistic Quantum Mechanics To QED
- Francis Halzen-Alan D.Martin:Quarks and Leptons
- Tai-Pei Cheng,Ling-Fong Li: Gauge Theory of Elementary Particle Physics
- Abdelhak Djouadi: The Anatomy of Electro-Weak Symmetry Breaking, Tome I: The Higgs boson in the Standard Model

The Standard $SU(2)_L \times U(1)_Y$ Model

				$SU(2)_L$	$U(1)_Y$	$SU(3)_c$
$\begin{pmatrix}\nu_e \\ e\end{pmatrix}_L,$	$\begin{pmatrix}\nu_\mu \\ \mu\end{pmatrix}_L,$	$\begin{pmatrix}\nu_\tau \\ \tau\end{pmatrix}_L$	$\equiv \ell_L^{(i)}$	2	-1	1
$e_R,$	$\mu_R,$	$\tau_R,$	$\equiv \ell_R^{(i)}$	1	-2	1
$\begin{pmatrix}u \\ d\end{pmatrix}_L,$	$\begin{pmatrix}c \\ s\end{pmatrix}_L,$	$\begin{pmatrix}t \\ b\end{pmatrix}_L$	$\equiv Q_L^{(i)}$	2	$1/3$	3
$u_R,$	$c_R,$	$t_R,$	$\equiv U_R^{(i)}$	1	$4/3$	3
$d_R,$	$s_R,$	$b_R,$	$\equiv D_R^{(i)}$	1	$-2/3$	3
			$\phi = \begin{pmatrix}\phi^+ \\ \phi^0\end{pmatrix}$	2	1	1

$$Q = \frac{T_3}{2} + \frac{Y}{2}$$

The Weinberg - Salam model $SU(2)_L \times U(1)_Y$

$$\mathcal{L} = \mathcal{L}_{kin}^{gauge} + \mathcal{L}_{kin}^{matter} - V(\phi) + \mathcal{L}_Y, \qquad \text{where:}$$

$$\begin{aligned}
\mathcal{L}_{kin}^{matter} = & \sum_i i\bar{Q}_L^{(i)}\gamma^\mu \left(\partial_\mu - i\frac{g}{2}\vec{\tau}\cdot\vec{W}_\mu - i\frac{g'}{6}B_\mu\right) Q_L^{(i)} \\
& + i\bar{\ell}_L^{(i)}\gamma^\mu \left(\partial_\mu - i\frac{g}{2}\vec{\tau}\cdot\vec{W}_\mu + i\frac{g'}{2}B_\mu\right) \ell_L^{(i)} \\
& + i\bar{\ell}_R^{(i)}\gamma^\mu \left(\partial_\mu + ig'B_\mu\right) \ell_R^{(i)} \\
& + i\bar{U}_R^{(i)}\gamma^\mu \left(\partial_\mu - i\frac{2}{3}g'B_\mu\right) U_R^{(i)} \\
& + i\bar{D}_R^{(i)}\gamma^\mu \left(\partial_\mu + i\frac{g'}{3}B_\mu\right) D_R^{(i)} \\
& + \left|\left(\partial_\mu\phi - i\frac{g}{2}\vec{\tau}\cdot\vec{W}_\mu - i\frac{g'}{2}B_\mu\phi\right)\right|^2
\end{aligned}$$

$$\mathcal{L}_{kin}^{gauge} = -\sum_\alpha \frac{1}{4}\left(\partial_\mu W_\nu^\alpha - \partial_\nu W_\mu^\alpha + \epsilon^{\alpha bc}W_\mu^b W_\nu^c\right)^2 - \frac{1}{4}(\partial_\mu B_\nu - \partial_\nu B_\mu)^2$$

$$V(\phi) = -\mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2, \quad \mu^2 > 0$$

$$\mathcal{L}_Y = \sum f_\ell^{(ij)}\bar{\ell}_L^{(i)}\phi\ell_R^{(j)} + f_u^{(ij)}\bar{Q}_L^{(i)}\tilde{\phi}U_R^{(j)} + f_D^{(ij)}\bar{Q}_L^{(i)}\phi D_R^{(j)} + h.c.$$

In $SU(2)_L \times U(1)_Y$, the scalar field is:

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \quad \tilde{\phi} = i\tau_2 \phi^*, \qquad \phi^+ = \phi_1 + i\phi_2, \qquad \phi^0 = \phi_3 + i\phi_4$$

$\mathcal{L}_{scalar}$ has greater symmetry $SO(4) \times U_1$, $SO(4) \approx SU(2) \times SU(2)$

Without the kinetic term of scalar bosons, using the notation $\gamma^\mu A_\mu = \not{A}$, the $\mathcal{L}_{kin}^{matter}$ turns into:

$$\begin{aligned}
\mathcal{L}_{kin}^{matter} \quad &= \quad \bar{\ell}_L^{(i)} \left(\frac{g}{2} \vec{\tau} \cdot \vec{\not{W}} - \frac{g'}{2} \not{B} \right) \ell_L^{(i)} + \bar{Q}_L^{(i)} \left(\frac{g}{2} \vec{\tau} \cdot \vec{\not{W}} + \frac{g'}{2} \not{B} \right) Q_L^{(i)} \\
&\quad - \bar{\ell}_R^{(i)} g' \not{B} \ell_R^{(i)} + \bar{U}_R^{(i)} \frac{2g'}{3} \not{B} U_R^{(i)} - \bar{D}_R^{(i)} \frac{g'}{3} \not{B} D_R^{(i)} \\
&\overset{i=1}{=} \quad g \left(\frac{1}{2} \bar{\ell}_L^1 \vec{\tau} \gamma^\mu \ell_L^1 + \frac{1}{2} \bar{q}_L^1 \vec{\tau} \gamma^\mu q_L^1 \right) \vec{W}_\mu \\
&\quad + \frac{1}{2} g \left(-\bar{\ell}_L^1 \gamma^\mu \ell_L^1 + \frac{1}{3} \bar{q}_L^1 \gamma^\mu q_L^1 - 2 \bar{e}_R \gamma^\mu e_R \right. \\
&\quad \left. + \frac{4}{3} \bar{u}_R \gamma^\mu u_R - \frac{2}{3} \bar{d}_R \gamma^\mu d_R \right) B_\mu + \ldots (i = 2, 3)
\end{aligned}$$

$$\begin{aligned} &\equiv \quad (gJ^{1\mu}W^1_\mu + gJ^{2\mu}W^2_\mu) + (gJ^{3\mu}W^3_\mu + \frac{1}{2}g'J^{Y\mu}B_\mu) + (i=2,3) \\ &= \quad \frac{g}{2}(J^+_\mu W^{+\mu} + J^-_\mu W^{-\mu}) \quad + \quad \text{neutral int.} \quad + \quad (i=2,3), \quad (\textit{Homework}) \end{aligned}$$

where

$$J^+_\mu = J^1_\mu + iJ^2_\mu = \bar{\nu}_L\gamma_\mu e_L + \bar{u}_L\gamma_\mu d_L$$

$$\left(\{\tau_1,\tau_2,\tau_3\} \quad \rightarrow \quad \{\tau_+,\tau_-,\tau_3\}, \quad \tau_\pm = \frac{1}{\sqrt{2}}(\tau_1 \pm \tau_2)\right)$$

We already know the form of the scalar field's vev:

$$\langle\phi\rangle = \frac{\upsilon}{\sqrt{2}}\begin{pmatrix}0\\1\end{pmatrix}, \qquad \upsilon = \sqrt{\frac{2\mu^2}{\lambda}}$$

Substituting $\phi = \langle\phi\rangle + \hat{\phi}$ in $\mathcal{L}^{\phi}_{kin}$ ($\mathcal{L}^{matter}_{kin}$'s last term), we obtain:

$$\begin{aligned} &-\frac{1}{4}\left(-g^2\frac{\upsilon^2}{2}W^\alpha_\mu W^{\mu\alpha} + gg'\upsilon^2 B_\mu W^{\mu 3} - g'\frac{\upsilon^2}{2}B_\mu B^\mu\right) \\ &= \quad \frac{\upsilon^2}{8}\left(g^2(W^1_\mu)^2 + g^2(W^2_\mu)^2 + g^2(W^3_\mu)^2 + g'^2(B_\mu)^2 - 2gg'W^3_\mu B^\mu\right) \quad (\textit{Homework}) \end{aligned}$$

- The blue term is a mass term for the physical fields $W_\mu^\pm \equiv (W_\mu^1 \mp iW_\mu^2)/\sqrt{2}$ (W_μ^1, W_μ^2 are the original gauge fields):

$$M_W^2 = \frac{g^2 \upsilon^2}{4}$$

- The red term becomes:

$$\frac{\upsilon^2}{8} \begin{pmatrix} W_\mu^3 & B_\mu \end{pmatrix} \begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix} \begin{pmatrix} W^{3\mu} \\ B^\mu \end{pmatrix}$$

After diagonalization (e.g. by rotating by an angle θ_W, where $\tan\theta_W = g'/g$) we obtain the physical fields , which are the mass eigenstates: (Homework)

$$\begin{aligned} Z_\mu &= \cos\theta_W W_\mu^3 - \sin\theta_W B_\mu && \rightarrow \text{(physical } Z^0\text{)} \\ A_\mu &= \sin\theta_W W_\mu^3 + \cos\theta_W B_\mu && \rightarrow \text{(physical photon)} \end{aligned}$$

with masses:

$$M_Z^2 = \frac{\upsilon^2}{4}(g^2 + g'^2) \qquad\qquad m_A = 0$$

$$\Rightarrow \rho = \frac{M_W^2}{M_Z^2 \cos^2\theta_W} = 1$$

Symmetry Broken: $SU(2)_L \times U(1)_Y \rightarrow U(1)_{e/m}$

The combinations

$$\left(\phi^+, \quad \frac{i}{\sqrt{2}}(\phi_0 - \bar{\phi}^0), \quad \phi^-\right)$$

of the initial Higgs have been "eaten" by the $W^\pm, Z^0$ and became their longitudinal polarization states, thus giving them mass.

Counting the degrees of freedom we conclude that the theory also contains a physical Higgs field: .

$$\Longrightarrow \ H \equiv \frac{\phi^0 + \bar{\phi}^0}{\sqrt{2}} - \upsilon$$

For the perturbed system around the vacuum, the scalar field (in the unitary gauge) is:

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \upsilon + H(x) \end{pmatrix}$$

Finally, if we substitute the ϕ in the potential term $V(\phi)$ of the total $\mathcal{L}$ we find the mass of the physical Higgs field:

$$V(\phi) \Longrightarrow \quad m_H^2 = 2\mu^2 = 2\lambda\upsilon^2 \qquad (Homework)$$

- Charged Currents:

The weak isospin currents of $SU(2)$ are:

$$J^{\mu}_{\alpha} \equiv \sum_{SU(2)\ doublets} \bar{f}_L \gamma^{\mu} \frac{\tau_{\alpha}}{2} f_L, \qquad \alpha = 1, 2, 3$$

and $W^{\pm}$ are connected with the charged combinations $J^{\mu}_{\pm} \equiv (J^{1\mu} \pm iJ^{2\mu})$ with the interaction:

$$\mathcal{L}^{cc}_{int} \equiv \frac{g}{\sqrt{2}} (W^{+}_{\mu} J^{\mu +} + W^{-}_{\mu} J^{\mu -})$$

$$\hookrightarrow \frac{1}{4}\mathcal{L}_{eff} \equiv \frac{G_F}{\sqrt{2}} (J^{+}_{\mu} J^{\mu -}), \qquad G_F/\sqrt{2} = g^2/8M^2_W$$

$$\hookrightarrow \upsilon = (\sqrt{2} G_F)^{-1/2} \simeq 250 GeV \qquad \upsilon = \sqrt{\frac{2\mu^2}{\lambda}}$$

- Neutral Currents:

$$\begin{aligned} \mathcal{L}^{nc} &= g J^3_{\mu} W^{3\mu} + \frac{1}{2} g' J^{Y}_{\mu} B_{\mu} \quad \Rightarrow \\ \mathcal{L}^{nc}_{ph.b.} &= e J^{em}_{\mu} A^{\mu} + \frac{g}{\cos\theta_W} J^0_{\mu} Z^{\mu} \quad \text{(In the physical basis)} \qquad \textit{(Homework)} \end{aligned}$$

where $e = g \sin\theta_W$, $\quad J^0_{\mu} = J^3_{\mu} - \sin^2\theta_W J^{em}_{\mu}$

$$\hookrightarrow \frac{1}{4}\mathcal{L}^{nc}_{eff} = \frac{g^2}{2\cos^2\theta_W M^2_Z} J^0_{\mu} J^{0\mu} = -\frac{g^2}{2M^2_W} J^0_{\mu} J^{0\mu} \qquad \hookrightarrow \sin^2\theta_W \simeq 0.23$$

Fermion Masses from the Yukawa Terms

The Yukawa terms in the Lagrangian density that refer to the leptons (writing explicitly only the first generation, the rest lie in the ...) is:

$$\mathcal{L}_{Yuk}^{\ell ept} = \ldots + f_\ell^{(11)} \left[\begin{pmatrix} \bar{\nu}_e & \bar{e} \end{pmatrix}_L \begin{pmatrix} \phi^+ \\ \phi^o \end{pmatrix} e_R + \bar{e}_R \begin{pmatrix} \phi^- & \bar{\phi}^o \end{pmatrix} \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \right] + \ldots$$

After the Spontaneous Symmetry Breaking (SSB):

$$\hookrightarrow \phi = \sqrt{\frac{1}{2}} \begin{pmatrix} 0 \\ \upsilon + H(x) \end{pmatrix}$$

Substituting this ϕ into $\mathcal{L}_{Yuk}^{\ell ept}$ we obtain:

$$\mathcal{L}_{Yuk}^{\ell ept} = \ldots + \frac{f_\ell^{(11)}}{\sqrt{2}} \upsilon \left(\bar{e}_L e_R + \bar{e}_R e_L \right) + \frac{f_\ell^{(11)}}{\sqrt{2}} \left(\bar{e}_L e_R + \bar{e}_R e_L \right) H + \ldots$$

The first term is a mass term: $m_e = f_\ell^{(11)} \upsilon / \sqrt{2}$. Therefore, $\mathcal{L}_{Yuk}^{\ell ept}$ becomes:

$$\mathcal{L}_{Yuk}^{\ell ept} = \ldots + m_e \bar{e} e + \frac{m_e}{\upsilon} \bar{e} e H + \ldots$$

The second term is the coupling of the electron to the scalar Higgs field.
Note that the interaction of the Higgs to leptons is proportional to the mass of the lepton involved, i.e. the Higgs couples more strongly to the heaviest lepton τ.

The quark masses are generated in the same way. The only new point is related to the generation of the u, c and t masses. In this case we should either introduce a new doublet $\tilde{\phi}$, which transforms as $(2,-1)$ under the $SU(2) \times U(1)$, or form a Higgs doublet from ϕ

$$\tilde{\phi} \equiv i\tau_2 \phi^* = \begin{pmatrix} \bar{\phi}^0 \\ -\phi^- \end{pmatrix} \quad \overset{SSB}{\longrightarrow} \quad \sqrt{\frac{1}{2}} \begin{pmatrix} \upsilon + H(x) \\ 0 \end{pmatrix}$$

which transforms as $\tilde{\phi} : (2,-1)$ under $SU(2) \times U(1)$ (transforms in the same way as ϕ, but has opposite hyper-charge).

$$\mathcal{L}_{Yuk}^{ferm} = \ldots + f_u^{(11)} \left[\begin{pmatrix} \bar{u} & \bar{d} \end{pmatrix}_L \begin{pmatrix} \bar{\phi}^0 \\ -\phi^- \end{pmatrix} u_R + \bar{u}_R \begin{pmatrix} \phi^0 & -\phi^+ \end{pmatrix} \begin{pmatrix} u \\ d \end{pmatrix}_L \right] + \ldots$$

After the SSB, it becomes:

$$\mathcal{L}_{Yuk}^{ferm} = \ldots + \frac{f_u^{(11)}}{\sqrt{2}} \upsilon \left[\bar{u}_L u_R + \bar{u}_R u_L \right] + \frac{f_u^{(11)}}{\sqrt{2}} \left[\bar{u}_L u_R + \bar{u}_R u_L \right] H + \ldots = m_u \bar{u} u + \frac{m_u}{\upsilon} \bar{u} u H + \ldots$$

The d, s and b quarks acquire their masses exactly as charged leptons do:

$$\mathcal{L}_{Yuk} = \ldots f_d^{(11)} \left[\begin{pmatrix} \bar{u} & \bar{d} \end{pmatrix}_L \begin{pmatrix} \phi^+ \\ \phi^o \end{pmatrix} d_R + \bar{d}_R \begin{pmatrix} \phi^- & \bar{\phi}^o \end{pmatrix} \begin{pmatrix} u \\ d \end{pmatrix}_L \right] + \ldots$$

$$= m_d \bar{d} d + \frac{m_d}{\upsilon} \bar{d} d H + \ldots$$

Fermion Masses and Generation Mixing

Here we shall analyze further the Yukawa terms. We consider the coupling of the neutral Higgs

$$\frac{1}{\sqrt{2}}(\phi^0 + \bar{\phi}^0) = H + \upsilon$$

to the fermion masses:

$$\frac{H+\upsilon}{\upsilon}(\bar{P}_L H_P P_R + \bar{N}_L H_N N_R + h.c.) \quad \epsilon\, \mathcal{L}_{Yuk}$$

Then we diagonalize with appropriate unitary transformations U and V:

$$P_L = U_P\, p_L, \qquad P_R = V_P\, p_R, \qquad N_L = U_N\, n_L, \qquad N_R = V_N\, n_R$$

at the basis of the mass eigenstates p and n.

$$\hookrightarrow \frac{H+\upsilon}{\upsilon}\left[\bar{p}_L(U_P^\dagger H_P V_P)p_R + \bar{n}_L(U_N^\dagger H_N V_N)n_R + h.c.\right]$$

Now the diagonalized mass matrices are:

$$m_P \equiv U_P^\dagger H_P V_P, \qquad m_N \equiv U_N^\dagger H_N V_N$$

- It is obvious that the coupling of the physical Higgs to the fermions, $H_{P,N}/\upsilon$ are diagonalized together with the quark masses:

$$g_{H\bar{q}q} \equiv U^\dagger_{P,N} \frac{H_{P,N}}{\upsilon} V_{P,N} = \frac{m_f}{\upsilon} = (2^{1/4}\sqrt{G_F})m_f = \frac{g m_f}{2M_W}$$

- The Neutral Currents that were flavour-diagonal at the basis of the interactions remain so at the physical basis (p, n), too:

$$\begin{aligned} \bar{P}_L\gamma_\mu P_L &= \bar{p}_L U^\dagger_P \gamma_\mu U_P p_L = \bar{p}_L\gamma_\mu p_L \\ \bar{N}_L\gamma_\mu N_L &= \bar{n}_L U^\dagger_N \gamma_\mu U_N n_L = \bar{n}_L\gamma_\mu n_L \end{aligned}$$

  because $U_{P,N}$ are unitary in flavour-space.

- However, the Charged Currents: $W^+_\mu \bar{P}_L\gamma^\mu N_L = W^+_\mu \bar{p}_L\gamma^\mu (U^\dagger_P U_N) n_L$

  Therefore the coupling is given by the generalized Cabibbo mixing matrix: $U \equiv U^\dagger_P U_N$

  For N_G generations, U contains N_G^2 parameters ($2N_G^2 \to N_G^2$ because of $U^\dagger U = 1$)

But $2N_G - 1$ are the relative phases between the fields of the quark flavours. So we have:

$$N_G^2 - (2N_G - 1) = (N_G - 1)^2$$

real observable parameters.

- For $N_G = 1 \rightarrow (N_G - 1)^2 = 0$: only $\begin{pmatrix} u \\ d \end{pmatrix}$ connection without mixing
- For $N_G = 2 \rightarrow (N_G - 1)^2 = 1$: Cabibbo mixing $\begin{pmatrix} cos\theta_c & sin\theta_c \\ -sin\theta_c & cos\theta_c \end{pmatrix}$
- For $N_G = 3 \rightarrow (N_G - 1)^2 = 4$: 3 orthogonal and 1 complex phase ($n(n-1)/2$ angles in $n \times n$ orthogonal matrix)

Kobayashi - Maskawa $\rightarrow$

$$U = \begin{pmatrix} c_1 & -s_1 c_3 & -s_1 s_3 \\ s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 s_3 + s_2 c_3 e^{i\delta} \\ s_1 s_2 & c_1 s_2 c_3 + c_2 s_3 e^{i\delta} & c_1 s_2 s_3 - c_2 c_3 e^{i\delta} \end{pmatrix},$$

$$\text{where } c_i(s_i) \equiv \cos\theta_i(\sin\theta_i)$$

If neutrinos are massless we can make unitary rotations on neutrino fields in order to make the couplings with $W_\mu^\pm$ diagonal.

More explicitly:

$$\mathcal{L}_Y = f_\ell^{(ij)} \bar{\ell}_L^{(i)} \phi \ell_R^{(j)} + f_u^{(ij)} \bar{Q}_L^{(i)} \tilde{\phi} U_R^{(j)} + f_D^{(ij)} \bar{Q}_L^{(i)} \phi D_R^{(j)} + h.c.$$

After the SSB:

$$\Longrightarrow \mathcal{L}_Y = \frac{H(x)}{\sqrt{2}} \left[f_\ell^{(ij)} \bar{\ell}_L^{(i)} \ell_R^{(j)} + f_u^{(ij)} \bar{Q}_L^{(i)} U_R^{(j)} + f_D^{(ij)} \bar{Q}_L^{(i)} D_R^{(j)} \right] + \frac{\upsilon}{\sqrt{2}} \left[f_\ell^{(ij)} \bar{\ell}_L^{(i)} \ell_R^{(j)} + f_u^{(ij)} \bar{Q}_L^{(i)} U_R^{(j)} + f_D^{(ij)} \bar{Q}_L^{(i)} D_R^{(j)} \right] + h.c.$$

Therefore, the matrices of fermion masses at the basis of currents are:

$$M_A^{(ij)} = -\frac{\upsilon}{\sqrt{2}} f_A^{(ij)}, \qquad A = \ell, U, D$$

An arbitrary complex matrix can take diagonal form with real, non-negative diagonal elements, i.e. there exist non-singular matrices A, B such that:

$$AMB^{-1} = D,$$

where A, B unitary, M arbitrary, D diagonal and $M^\dagger M, MM^\dagger$ hermitian matrices with real, non-negative eigenvalues.

That means there exist non-singular matrices A, B such that:

$$\begin{aligned} AMM^\dagger A^{-1} &= D^2 \\ BM^\dagger MB^{-1} &= D^2 \end{aligned}$$

The solutions are: $M = A^{-1}DB, \quad M^\dagger = B^{-1}DA$

Even more explicitly:

$$\mathcal{L}_{quarks}^{masses} = \begin{pmatrix} \bar{d}_{0L} & \bar{s}_{0L} & \bar{b}_{0L} \end{pmatrix} M_D \begin{pmatrix} d_{0R} \\ s_{0R} \\ b_{0R} \end{pmatrix} + \begin{pmatrix} \bar{u}_{0L} & \bar{c}_{0L} & \bar{t}_{0L} \end{pmatrix} M_U \begin{pmatrix} u_{0R} \\ c_{0R} \\ t_{0R} \end{pmatrix} + h.c.$$

We diagonalize by making the transformations:

$$\begin{pmatrix} d_0 \\ s_0 \\ b_0 \end{pmatrix}_L \to {U_N^L}^{-1} \begin{pmatrix} d_0 \\ s_0 \\ b_0 \end{pmatrix}_L \quad , \quad \begin{pmatrix} u_0 \\ c_0 \\ t_0 \end{pmatrix}_L \to {U_P^L}^{-1} \begin{pmatrix} u_0 \\ c_0 \\ t_0 \end{pmatrix}_L$$

$$\begin{pmatrix} d_0 \\ s_0 \\ b_0 \end{pmatrix}_R \to {V_N^R}^{-1} \begin{pmatrix} d_0 \\ s_0 \\ b_0 \end{pmatrix}_R \quad , \quad \begin{pmatrix} u_0 \\ c_0 \\ t_0 \end{pmatrix}_R \to {V_P^R}^{-1} \begin{pmatrix} u_0 \\ c_0 \\ t_0 \end{pmatrix}_R ,$$

where $U_N^L, V_N^R, U_P^L, V_P^R$ are such, so:

$$\begin{aligned} {U_N^L}^{-1} M_D V_N^R &= \begin{pmatrix} m_d & & \\ & m_s & \\ & & m_b \end{pmatrix} \\ {U_P^L}^{-1} M_U V_P^R &= \begin{pmatrix} m_u & & \\ & m_c & \\ & & m_t \end{pmatrix} \end{aligned}$$

Now at the new, physical basis we have:

$$\mathcal{L}_{quarks}^{masses} =$$

$$= \begin{pmatrix} \bar{d}_L & \bar{s}_L & \bar{b}_L \end{pmatrix} \begin{pmatrix} m_d & & \\ & m_s & \\ & & m_b \end{pmatrix} \begin{pmatrix} d_R \\ s_R \\ b_R \end{pmatrix} + \begin{pmatrix} \bar{u}_L & \bar{c}_L & \bar{t}_L \end{pmatrix} \begin{pmatrix} m_u & & \\ & m_c & \\ & & m_t \end{pmatrix} \begin{pmatrix} u_R \\ c_R \\ t_R \end{pmatrix}$$

$$= \sum_q m_q \bar{q}_L q_R = \sum_q m_q \bar{q} q$$

The transformation of the fermion fields from the basis of the currents to the physical basis leaves the e/m and neutral currents unchanged, but the charged current becomes:

$$J^\mu = \begin{pmatrix} \bar{u} & \bar{c} & \bar{t} \end{pmatrix}_L \gamma^\mu U \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L ,$$

where $U = {U_P^L}^{-1} V_N^L \quad \leftarrow 3 \times 3$
with the properties:

$$U^\dagger U = 1 \qquad |detU|^2 = 1$$