Gauge Theories, Higgs Mechanism, Standard Model

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- Antonio Pich: Particle Physics: The Standard Model (lectures)
- W. Hollik: Theory of Electroweak Interactions, Corfu Summer Institute, School and Workshop on Standard Model and Beyond, 2013
- Aitchison I J R & Hey A J G: Gauge Theories In Particle Physics Volume 1: From Relativistic Quantum Mechanics To QED
- Francis Halzen-Alan D.Martin:Quarks and Leptons
- Tai-Pei Cheng,Ling-Fong Li: Gauge Theory of Elementary Particle Physics
- Abdelhak Djouadi: The Anatomy of Electro-Weak Symmetry Breaking, Tome I: The Higgs boson in the Standard Model

Spontaneous Symmetry Breaking

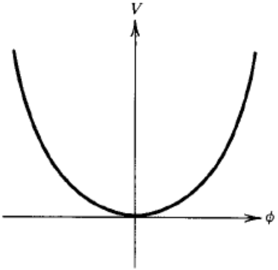
Let's consider the simple system that is described by the Lagrangian density:

$$\mathcal{L} = T - V = \frac{1}{2}(\partial_\mu \phi)^2 - \underbrace{\left(\frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda\phi^4\right)}_{\equiv V(\phi)}, \quad \text{where } \lambda > 0$$

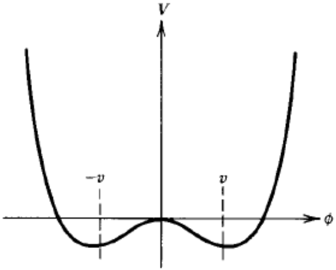
$\mathcal{L}$ is invariant under $\phi \to -\phi$

The form of the function $V(\phi)$ depends on the sign of μ^2:

$\mu^2 > 0$



$\mu^2 < 0$



We find the minima of $V(\phi)$ considering the condition:

$$\frac{\partial V}{\partial \phi} = 0$$

- For $\mu^2 > 0$: Lowest-energy state corresponds to:

$$\phi\left(\mu^2 + \lambda\phi^4\right) \Rightarrow \phi = 0$$

- For $\mu^2 < 0$: The minima of energy do not correspond to $\phi = 0$, but to:

$$\phi\left(\mu^2 + \lambda\phi^4\right) \Rightarrow \phi = \pm\upsilon, \quad \text{with} \quad \upsilon = \sqrt{-\frac{\mu^2}{\lambda}}$$

The perturbative calculations must include expansions around the classical minima:

$$\phi = \upsilon \quad \text{or} \quad \phi = -\upsilon$$

Therefore, we write:

$$\phi(x) = \upsilon + \eta(x), \qquad \eta(x) : \text{quantum fluctuations around the min}$$

Equivalently, we could have picked $\phi = -\upsilon$. Nature has to choose too!

We substitute $\phi(x) = \upsilon + \eta(x)$ in the given Lagrangian density. Thus we obtain:

$$\mathcal{L}' = \frac{1}{2}(\partial_\mu \eta)^2 - \underline{\lambda \upsilon^2 \eta^2} - \lambda \upsilon \eta^3 - \frac{1}{4}\lambda \eta^4$$

The underlined term is the mass term and it has the correct sign (!), while the last term are the self-interactions. So:

$$m_\eta = \sqrt{2\lambda\upsilon^2} = \sqrt{-2\mu^2}$$

However, there is a snag here: $\mathcal{L}$ and $\mathcal{L}'$ are completely equivalent.

The transformation $\phi(x) = \upsilon + \eta(x)$ cannot change the Physics! If we could solve $\mathcal{L}, \mathcal{L}'$ they would give the same physics. But in purturbation theory we only calculate fluctuations around the minimum energy.

Using $\mathcal{L}$ we will always find that the perturbative series does not converge, because we try to expand around the unstable $\phi = 0$

What we should do is to use $\mathcal{L}'$ and expand as to η around the stable vacuum $\phi = \upsilon$.

$\Longrightarrow$ the scalar field has mass!

This way of revealing or generation of mass is called Spontaneous Symmetry Breaking.

Spontaneous Breaking of Gauge Symmetries

Now, let's study the Lagrangian density:

$$\mathcal{L} = (D_\mu \phi)^\dagger (D^\mu \phi) + \mu^2 \phi^\dagger \phi - \lambda(\phi^\dagger \phi)^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu},$$

where:

$$\begin{aligned} D_\mu \phi &= (\partial_\mu - igA_\mu)\phi \\ F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu \end{aligned}$$

$\mathcal{L}$ is invariant under the local $U(1)$:

$$\begin{aligned} \phi(x) &\rightarrow \phi'(x) = e^{-i\alpha(x)} \phi(x) \\ A_\mu(x) &\rightarrow A'_\mu(x) = A_\mu(x) - \frac{1}{g} \partial_\mu \alpha(x) \end{aligned}$$

For $\mu^2 > 0$, the potential $V(\phi) = -\mu^2 \phi^\dagger \phi + \lambda(\phi^\dagger \phi)^2$ has its minimum at:

$$|\phi| = \frac{\upsilon}{\sqrt{2}}, \qquad \upsilon = \left(\frac{\mu^2}{\lambda}\right)^{\frac{1}{2}}$$

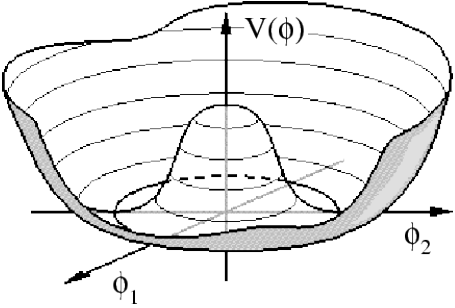
If $\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2)$, then:

$$\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \to \begin{pmatrix} \phi_1' \\ \phi_2' \end{pmatrix} = \begin{pmatrix} \cos\alpha(x) & \sin\alpha(x) \\ -\sin\alpha(x) & \cos\alpha(x) \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$$

There exist infinite degenerate vacua and we can choose:

$$\langle\phi_1\rangle = \upsilon \quad , \quad \langle\phi_2\rangle = 0$$

We consider small oscillations around the minimum defining $\phi_1' = \phi_1 - \upsilon$, $\phi_2' = \phi_2$



- For $g \to 0$:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi_1')^2 + \frac{1}{2}(\partial_\mu \phi_2')^2 - \mu^2 \phi_1'^2 - \lambda \upsilon \phi_1' (\phi_1'^2 + \phi_2'^2) - \frac{\lambda}{4}(\phi_1'^2 + \phi_2'^2)^2$$

There is no mass term for ϕ_2'. Therefore, $\phi_2' = \phi_2$ is the massless Goldstone boson

- For finite g:

$$\begin{aligned} |D_\mu \phi|^2 &= |(\partial_\mu - igA_\mu)\phi|^2 \\ &= \frac{1}{2}(\partial_\mu \phi_1' + gA_\mu \phi_2')^2 + \frac{1}{2}(\partial_\mu \phi_2' - gA_\mu \phi_1')^2 - g\upsilon A^\mu (\partial_\mu \phi_2' - gA_\mu \phi_1') \\ &+ \underline{\frac{g^2 \upsilon^2}{2}} A^\mu A_\mu , \end{aligned}$$

where the underlined term is the mass of A.

(The Goldstone boson disappears completely at the Unitary gauge)

All the above can be generalized without problems to non-abelian theories.

Unitary Gauge

Observing that, in lower order in the fields ϕ_1', ϕ_2'

$$\phi' = \frac{1}{\sqrt{2}}(\upsilon + \phi_1' + i\phi_2') \simeq \frac{1}{\sqrt{2}}(\upsilon + \phi_1')e^{i\phi_2'/\upsilon}$$

We make use of an appropriate gauge transformation:

$$\begin{aligned} \phi(x) &\rightarrow \phi'(x) = \exp(-i\xi(x)/\upsilon)\phi(x) = \frac{1}{\sqrt{2}}(\upsilon + \eta(x)), \\ A_\mu(x) &\rightarrow A'_\mu(x) = A_\mu(x) - \frac{1}{g\upsilon}\partial_\mu \xi(x) \end{aligned}$$

This is a special selection of gauge with $\xi(x)$ such that $\eta(x)$ is real. We can thus predict that the theory is independent of ξ.

Indeed: (Homework)

$$\mathcal{L}_0'' = \frac{1}{2}(\partial_\mu \eta)^2 - \frac{1}{2}\mu^2\eta^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}(gv)^2 A_\mu A^\mu$$
$$\mathcal{L}_I'' = \frac{1}{2}g^2 A_\mu A^\mu \eta(2v+\eta) - \lambda v \eta^3 - \frac{1}{4}\lambda\eta^4$$

$\mathcal{L}_0''$ is the free Lagrangian for a vector boson of mass $M = gv$ and for a scalar meson of mass $m = \sqrt{2}\mu$

The Goldstone boson disappeared from $\mathcal{L}'' = \mathcal{L}_0'' + \mathcal{L}_I''$. The seeming degree of freedom is fake, because it only corresponds to the freedom of gauge selection.

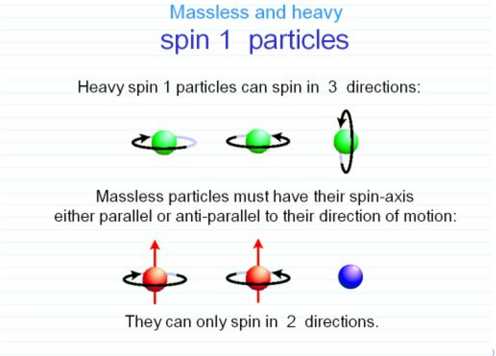
So:

Massless A_μ (2 degrees of freedom)

$+\ \xi$ (1 degree of freedom)

$\downarrow$

Massive A_μ (3 degrees of freedom)



$\rightarrow$ Before the spontaneous symmetry breaking we had:

- 2 scalar fields ϕ_1, ϕ_2 (2 degrees of freedom)
- A massless gauge boson A_μ (2 degrees of freedom)

$\hookrightarrow$ Total: 4

$\rightarrow$ After the spontaneous symmetry breaking we have:

- 1 scalar field η (1 degree of freedom)
- A massive gauge field A_μ (3 polarization states $\equiv$ d.o.f)

$\hookrightarrow$ Total: 4

This is the Higgs mechanism. The field $\xi(x)$ is called a would-be Goldstone boson.

Non-Abelian Case

$SU(2)$: gauge theory with a complex doublet of scalar fields: $\phi = \begin{pmatrix} \phi_\alpha \\ \phi_\beta \end{pmatrix}$

$$\mathcal{L} = (D_\mu \phi)^\dagger (D_\mu \phi) - V(\phi) - \frac{1}{4} F^\alpha_{\mu\nu} F^{\alpha\mu\nu},$$

where:

$$\begin{aligned} D_\mu \phi &= \left(\partial_\mu - ig \frac{\vec{\tau} \cdot \vec{A}_\mu}{2} \right) \phi \\ F^\alpha_{\mu\nu} &= \partial_\mu A^\alpha_\nu - \partial_\nu A^\alpha_\mu + g \epsilon^{\alpha bc} A^b_\mu A^c_\nu \\ V(\phi) &= -\mu^2 (\phi^\dagger \phi) + \lambda (\phi^\dagger \phi)^2 \\ \phi &= \begin{pmatrix} \phi_\alpha \\ \phi_\beta \end{pmatrix} = \sqrt{\frac{1}{2}} \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix} \end{aligned}$$

For $g \to 0$: $\mathcal{L}$ is invariant under $\phi \to \phi' = e^{i\alpha_\alpha \tau_\alpha / 2} \phi$.

If we demand $\alpha_\alpha \to \alpha_\alpha(x)$, then we have the complete $\mathcal{L}$.

For $\mu^2 > 0$, $\lambda > 0$, the $V(\phi)$ has minimum at $\phi^\dagger\phi = \frac{1}{2}(\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2) = \mu^2/2\lambda$

$$\rightarrow SO(4) \approx SU(2) \times SU(2)$$

We have to expand $\phi(x)$ around a special minimum. We can choose $\phi_1 = \phi_2 = \phi_4 = 0$, $\phi_3^2 = \mu^2/\lambda = \upsilon^2$

$\rightarrow$ Spontaneous symmetry breaking of $SU(2)$ symmetry

$$\phi = \phi' + \phi_0 \quad , \quad \phi_0 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \upsilon \end{pmatrix}$$

$$\Longrightarrow \quad \phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \upsilon + \eta(x) \end{pmatrix}$$

$$(D_\mu\phi)^\dagger(D_\mu\phi) = ...(Homework)... + \frac{1}{2}\left(\frac{g\upsilon}{2}\right)^2 (A_\mu^1 A^{1\mu} + A_\mu^2 A^{2\mu} + A_\mu^3 A^{3\mu})$$

$\rightarrow M_A = g\upsilon/2 \rightarrow$ 3 massive vector bosons

For the scalar part:

$$\begin{aligned} \phi^\dagger\phi &= \phi'^\dagger\phi' + \phi_0^\dagger\phi' + \phi'^\dagger\phi_0 + \phi_0^\dagger\phi_0 \\ (\phi^\dagger\phi)^2 &= \upsilon^2\phi'^\dagger\phi' + (\phi_0^\dagger\phi' + \phi'^\dagger\phi_0)^2 + ... \end{aligned}$$

If $\phi' \equiv \begin{pmatrix} \phi'_\alpha \\ \phi'_\beta \end{pmatrix}$, then the quadratic term in ϕ'

$$\rightarrow \frac{\lambda v^2}{2}(\phi'_\beta + \phi'^{\dagger}_\beta)^2 \qquad (\textit{Homework})$$

This means that only the combination

$$\frac{(\phi'_\beta + \phi'^{\dagger}_\beta)}{\sqrt{2}}$$

obtains a mass. This is the physical Higgs particle.

The states

$$\phi'_\alpha \quad , \quad \phi'^{\dagger}_\alpha \quad , \quad \frac{(\phi'_\beta - \phi'^{\dagger}_\beta)}{\sqrt{2}}$$

remain massless and thus are identified as the would-be Goldstone bosons.

Unitary Gauge

By writing $\phi(x) = e^{i\vec{\tau}\vec{\theta}(x)/\upsilon}\begin{pmatrix} 0 \\ \frac{\upsilon+\eta(x)}{\sqrt{2}} \end{pmatrix}$ and by making small perturbations:

$$\phi(x) \simeq \begin{pmatrix} 1+i\theta_3/\upsilon & i(\theta_1 - i\theta_2)/\upsilon \\ i(\theta_1 + i\theta_2)/\upsilon & 1 - i\theta_3/\upsilon \end{pmatrix}\begin{pmatrix} 0 \\ \upsilon+\eta(x) \end{pmatrix} \simeq \frac{1}{\sqrt{2}}\begin{pmatrix} \theta_2 + i\theta_1 \\ \upsilon + \eta - i\theta_3 \end{pmatrix}$$

We see that the 4 fields are indeed independent and completely parametrize the deviations from the vacuum ϕ_0.

Then we make a suitable gauge transformation

$$\begin{aligned} \phi(x) \quad &\to \quad \phi'(x) = U(x)\phi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \upsilon+\eta \end{pmatrix}, \\ \frac{\vec{\tau}\cdot\vec{A}_\mu}{2} \quad &\to \quad U(x)\frac{\vec{\tau}\cdot\vec{A}_\mu}{2}U^{-1}(x) - \frac{i}{g}[\partial_\mu U(x)]U^{-1}(x), \end{aligned}$$

where $U(x) = \exp(-i\frac{\vec{\tau}\cdot\vec{\theta}}{\upsilon})$, and get rid of the Goldstone bosons.

Indeed, from the properties of gauge transformations we have:

$$\begin{aligned} D_\mu \phi &= U^{-1}(x) D'_\mu \phi' \\ F^\alpha_{\mu\nu} F^{\alpha\mu\nu} &- \text{invariant} \end{aligned}$$

Therefore, the Lagrangian density becomes:

$$\mathcal{L} = (D'_\mu \phi')^\dagger (D'^\mu \phi') + \frac{\mu^2}{2}(\upsilon + \eta)^2 - \frac{\lambda}{4}(\upsilon + \eta)^4 - \frac{1}{4} F'^\alpha_{\mu\nu} F'^{\alpha\mu\nu},$$

where the first term yields:

$$\begin{aligned} (D'_\mu \phi')^\dagger (D'^\mu \phi') &= \ldots + \frac{g^2}{8} \begin{pmatrix} 0 & \upsilon \end{pmatrix} \begin{pmatrix} \vec{\tau} \cdot \vec{A'}_\mu & \vec{\tau} \cdot \vec{A'}^\mu \end{pmatrix} \begin{pmatrix} 0 \\ \upsilon \end{pmatrix} \quad \text{(Homework)} \\ &= \ldots + \frac{g^2 \upsilon^2}{8} \left[(A'^1_\mu)^2 + (A'^2_\mu)^2 + (A'^3_\mu)^2 \right], \end{aligned}$$

where we identify the mass of the 3 bosons:

$$M_{A'_1} = M_{A'_2} = M_{A'_3} = \frac{g\upsilon}{2}$$