Asymptotic Safety and the Gibbons-Hawking Term

A status report

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Work in Progress

Outline

Classical motivation

The Einstein-Hilbert action

The geometrical content of General Relativity is encoded in the **Einstein-Hilbert action**:

$$S_{\mathsf{EH}} = -\frac{1}{16\pi G_N}\int_{\mathcal{M}} \mathsf{d}^d x\sqrt{g}\, R\,, \qquad \partial\mathcal{M} = \emptyset$$

The principle of least action ($\delta S_{\mathsf{EH}}=0$) requires that

$$0 = \frac{1}{16\pi G_N}\left(\int_{\mathcal{M}} \mathsf{d}^d x\sqrt{g}\left[R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R\right]\delta g_{\mu\nu}\right)$$

for **all** $\delta g_{\mu\nu}$.

Hence, one deduces the Einstein field equations in vacuum:

$$G^{\mu\nu} \equiv \quad R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R = 0$$

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$$0 = \frac{1}{16\pi G_N}\left(\int_{\mathcal{M}} \mathsf{d}^d x \sqrt{g}\, G^{\mu\nu}\delta g_{\mu\nu} + \int_{\partial\mathcal{M}} \mathsf{d}^{d-1} x \sqrt{H}\, H^{\alpha\beta} n^{\mu} D_{\mu}\delta g_{\alpha\beta}\right)$$

for **all** $\delta g_{\mu\nu}$, satisfying **Dirichlet boundary conditions** $\delta g_{\mu\nu}|_{\partial\mathcal{M}} = 0$.

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- $H_{\mu\nu} = g_{\mu\nu}|_{\partial\mathcal{M}}$ induced metric on $\partial\mathcal{M}$
- n^{μ} normal vector on $\partial\mathcal{M}$

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for **all** $\delta g_{\mu\nu}$, satisfying **Dirichlet boundary conditions** $\delta g_{\mu\nu}|_{\partial\mathcal{M}} = 0$.

Hence, the boundary contribution obstructs the derivation of field equations as stationary points of S_{EH}.

The Gibbons-Hawking (York) term

Einstein field equation can be recovered by adding a boundary term to S_{EH} which has the variation:

$$\delta S_{\mathsf{GHY}} = -\frac{1}{16\pi G_N}\int_{\partial\mathcal{M}} \mathrm{d}^{d-1}y\sqrt{H}\, H^{\alpha\beta} n^{\mu} D_{\mu}\delta g_{\alpha\beta}$$

Note the identity

$$+2 \cdot \delta K\big|_{\mathsf{Dirichlet}} = H^{\alpha\beta} n^{\mu} D_{\mu} \delta g_{\alpha\beta}\big|_{\mathsf{Dirichlet}}\,, \qquad \delta H_{\mu\nu} = 0\,.$$

where

- $K_{\mu\nu} = D_{\mu} n_{\nu}$ is the extrinsic curvature of $\partial\mathcal{M}$
- and $K = H^{\mu\nu} K_{\mu\nu}$ is its trace

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yields the Einstein equation as a stationary point in case of

- non-empty boundary $\partial\mathcal{M} \neq \emptyset$
- **Dirichlet boundary condition** $\delta g_{\mu\nu}\big|_{\partial\mathcal{M}} = 0$
- a relative coefficient of exactly $+2$

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Functional renormalization group

FRGE for the gravitational average action

…acts on theory space:

$$\left\{ \Gamma_k \left[\qquad \qquad \right] \Big| \qquad \qquad \qquad \qquad \right\}$$

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Functional renormalization group equation

$k\partial_k \Gamma_k$

exact, closed functional differential equation

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$$\Gamma_k = u_k^{(a)} \int \sqrt{g} + u_k^{(b)} \int \sqrt{g} R + u_k^{(c)} \int \sqrt{g}\, R^{\mu\nu} R_{\mu\nu} + \ldots$$

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$\{u_k^{(n)}\}$ coordinatize the **infinite** dimensional theory space

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Truncations: subspaces of span$\{u_k^{(n)}\}$

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Asymptotic Safety

Restricts the possible evolutions of Γ_k by physical arguments

- **Existence of a Non-Gaussian fixed point** (NG-FP)
  Fundamental (non-trivial) theory in the UV

  $\Rightarrow$

  $\mathscr{S}_{\mathrm{UV}} = \{$actions pulled into the NG-FP under the inverse flow$\}$
  (inverse flow = increasing k)

- **Finite dimensional UV-critical hypersurface** $\mathscr{S}_{\mathrm{UV}}$
  $\dim(\mathscr{S}_{\mathrm{UV}}) \equiv n < \infty$: # of measurements needed to fix initial conditions

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The running Gibbons-Hawking term

Truncation ansatz

EH-GHY-truncation

$$\int_{\mathcal{M}} \sqrt{g} \qquad \int_{\mathcal{M}} \sqrt{g} R$$

$$\int_{\partial\mathcal{M}} \sqrt{H} \qquad \int_{\partial\mathcal{M}} \sqrt{H} K$$

Truncation ansatz

EH-GHY-truncation

$$\Gamma_k = \; + u_k^{(a)} \int_{\mathcal{M}} \sqrt{g} \quad + u_k^{(b)} \int_{\mathcal{M}} \sqrt{g} R$$
$$\quad + u_k^{(c)} \int_{\partial\mathcal{M}} \sqrt{H} \quad + u_k^{(d)} \int_{\partial\mathcal{M}} \sqrt{H} K$$

Truncation ansatz

EH-GHY-truncation

$$\Gamma_{k} = \quad + \frac{2\lambda_k k^d}{16\pi g_k} \int_{\mathcal{M}} \sqrt{g} \qquad - \frac{k^{d-2}}{16\pi g_k} \int_{\mathcal{M}} \sqrt{g} R$$
$$+ \frac{2\lambda_k^{\partial} k^{d-1}}{16\pi g_k^{\partial}} \int_{\partial\mathcal{M}} \sqrt{H} \quad - \frac{2k^{d-2}}{16\pi g_k^{\partial}} \int_{\partial\mathcal{M}} \sqrt{H} K$$

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where g_k, g_k^{∂} dimensionless Newton type couplings on $\mathcal{M}$, $\partial\mathcal{M}$

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where λ_k, λ_k^∂ dimensionless cosmological type couplings on $\mathcal{M}$, $\partial\mathcal{M}$

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Dirichlet boundary conditions $\delta g_{\mu\nu}|_{\partial\mathcal{M}} =$ fixed, i.e.

$$(\text{metric fluctuations})|_{\partial\mathcal{M}} = 0 \,.$$

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Results

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Results

Hierarchy	NG fixed point
g_k	0.707
λ_k	0.193
$\Downarrow$	
g_k^{∂}	-2.292
$\Downarrow$	
λ_k^{∂}	1.201

Truncation ansatz

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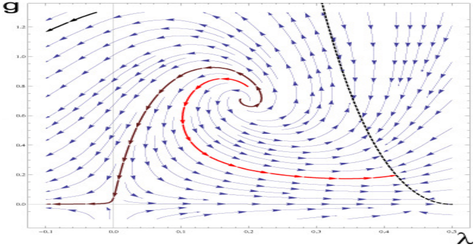
$$\quad + \frac{2\lambda_k^\partial k^{d-1}}{16\pi g_k^\partial} \int_{\partial\mathcal{M}} \sqrt{H} \qquad - \frac{2k^{d-2}}{16\pi g_k^\partial} \int_{\partial\mathcal{M}} \sqrt{H} K$$

Results

Hierarchy	NG fixed point
g_k	0.707
λ_k	0.193
⇓	
g_k^∂	−2.292
⇓	
λ_k^∂	1.201



The running Gibbons-Hawking term

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Running action in EH-GHY truncation:

$$\Gamma_k = \frac{k^2}{16\pi}\left(\frac{1}{g_k}\int_{\mathcal{M}}\sqrt{g}\,R + \frac{2}{g_k^{\partial}}\int_{\partial\mathcal{M}}\sqrt{H}\,K\right) + \dots$$

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First approximation to the scale dependence of g_k, g_k^∂:

$$g_k = g_0\left(1 - \frac{11}{6\pi}g_0 \cdot k^2\right), \qquad g_k^\partial = g_0^\partial\left(1 + \frac{1}{6\pi}g_0^\partial \cdot k^2\right)$$

The running Gibbons-Hawking term

Running action in EH-GHY truncation: mismatch, $\partial_k g_k \neq \partial_k g_k^\partial$

$$\Gamma_k = \frac{k^2}{16\pi g_0}\left(\frac{1}{1-ck^2}\int_{\mathcal{M}}\sqrt{g}\,R + \frac{2}{1+\tilde{c}k^2}\int_{\partial\mathcal{M}}\sqrt{H}\,K\right) + \dots \quad c, \tilde{c} > 0$$

'Correct' relative coefficient at most at one scale , $k = 0$, say. $(g_0 = g_0^\partial)$

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Conclusion

Classical general relativity

- Gibbons-Hawking-(York) term needed for $\partial\mathcal{M} \neq \emptyset$
- Relative coefficient has to be $+2$

FRGE results for EH-GHY truncation

- Einstein-Hilbert subsystem is uneffected by boundary contribution
- Truncation shows a Non-Gaussian fixed point
- Couplings for EH and GHY show **different** scale dependence
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