Covariant Loop Gravity

carlo rovelli

I. objective

II. history and ideas

III. math

IV. definition of the theory

V. quantum space

VI. extracting physics

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Lectures (``Zakopane Lectures in Loop Gravity") CR: arXiv: 1102.3660
Overall view of field (``LQG, the first 25 years") CR:. arXiv:1012.4707
Main theorem Barrett et al: arXiv: 0907.2440

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cfr: Ashtekar lecture (LQG and LQC)
Bojowald: LQC
Geller: relation hamiltonian-spinfoam
Vidotto: spinfoam cosmology
Wetterich: discrete path integral
Martin-Benito: effective SC
Bonzom: Lessons from topological BF

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Overall view of field (``LQG, the first 25 years") CR:. arXiv:1012.4707
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Aim:

i. Study if there is a quantum theory with GR as classical limit (Lorentzian, 4d, coupled to ordinary matter)

ii. Understand how to extract physics from this theory

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Results:

i. Definition of the dynamics $$Z_{\mathcal{C}} = \sum_{j_f i_e} \prod_d d_{j_f} \prod_v \mathrm{Tr}[\otimes_e f_\gamma i_e]$$

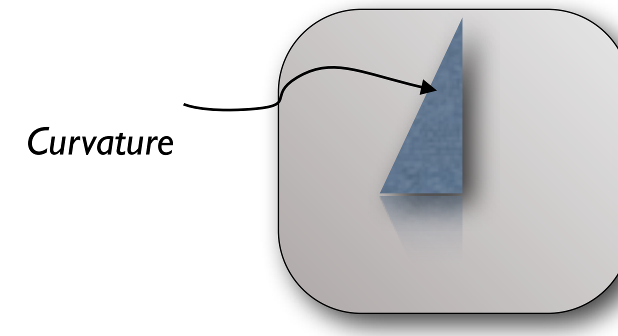
- Theorem I: asymptotic limit
- Theorem II: finiteness

ii. Boundary formalism

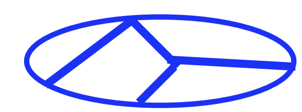
- *n*-point functions
- spinfoam cosmology
- quantum spacetime

II. history of the main ideas

- 1957, Misner Wheeler $Z = \int Dg \;\; e^{iS_{EH}}$

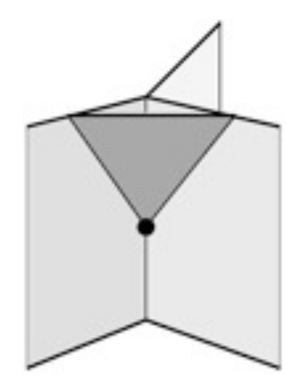


- 1961, Regge Regge calculus → *truncation of GR on a manifold with d-2 defects*
- 1971 Penrose Spin-geometry theorem → *spin network*



a "spin network"

- 1988 - Loop Quantum gravity → *quantum geometry*
- 1994 - Spinfoams
- 2008 Covariant dynamics of LQG (EPRL)
- 2010 Asymptotic theorem



a "spinfoam"

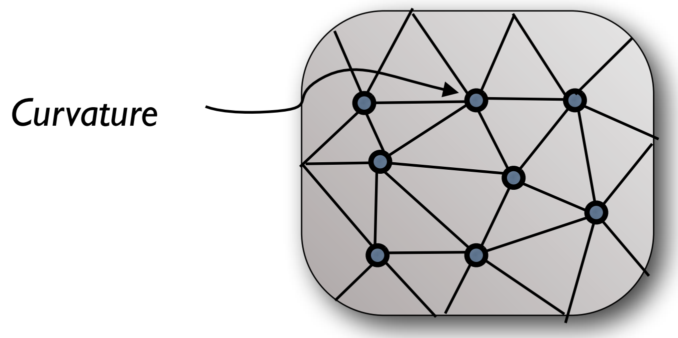
II. summing over geometries

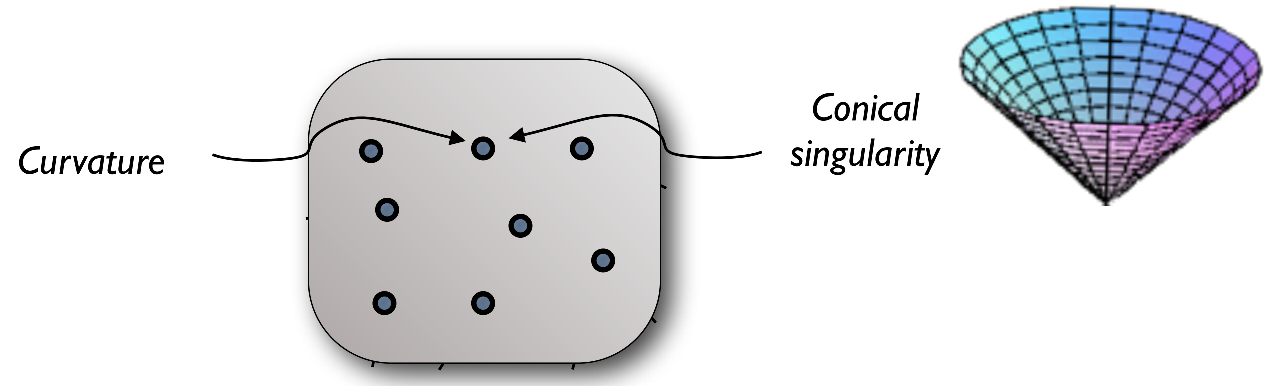
- 1957, Misner Wheeler $$Z = \int Dg \;\; e^{iS_{EH}}$$

  - Formal manipulation of conventional perturbative expansions

  - Limit of a discretization: cfr Lattice QCD. (“vertex expansion”)

2d

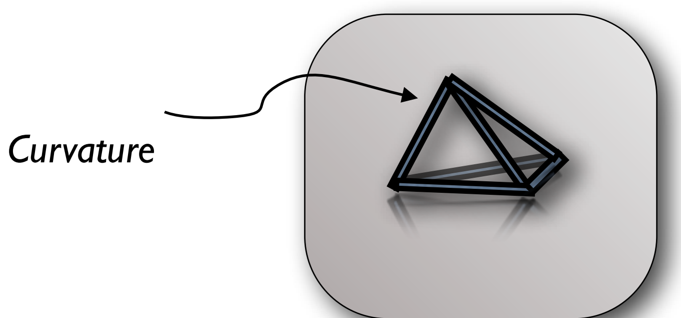




Regge geometry g_R :

Flat except on hinges.

3d



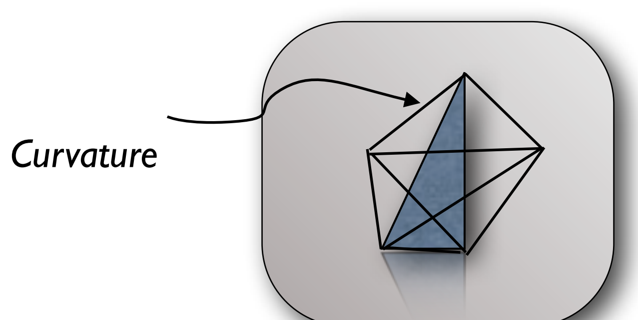
Regge results:

- g_R approximates g
- g_R determined by lengths L_l
- Action:

$$S_R(g_R) = \sum_h \delta_h(L_l)\, Vol_h(L_l)$$

- Lattice distance drops out!

4d



II. 3d

Triads $g_{ab} \to e^i_a$ $g_{ab} = e^i_a\ e^i_b$ $e = e_a dx^a \in R^3$

Spin connection $\omega = \omega_a dx^a \in so(3)$ $\omega(e):\quad de + \omega \wedge e = 0$

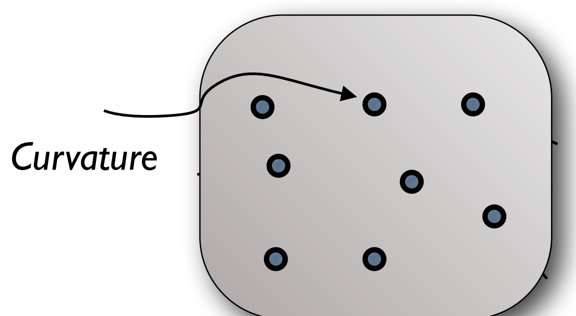
GR action $S[e,\omega] = \int e \wedge F[\omega]$

Regge discretization

Connection: Flat *so(3)* connection modulo gauge on M-D_1

Triad: $e_l = \int_l e \quad \in R^3$ $L_l = |e_l|$

Canonical variables



Connection: Flat *2d so(3)* connection modulo gauge on M-D_0

Phase space: $\Gamma = T_*(SU(2)^P)/Gauge$

Canonical quantization $\mathcal{H} = L_2[SU(2)^P]/Gauge$ $e_l \to$ Left invariant vector field:

Discreteness of length $L_l^2 \to$ Casimir $L_l = \sqrt{j_l(j_l+1)+\frac{1}{4}} = j_l + \frac{1}{2}$

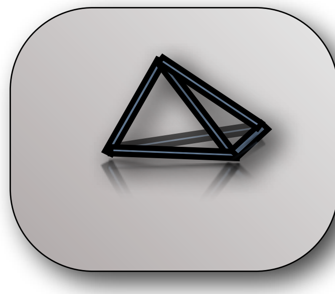
Do not confuse:

- Regge discretization  Truncation of the continuum theory

- Discreteness of length $L_l = j_l + \frac{1}{2}$ Quantum effect

$$Z = \int Dg \;\; e^{iS_{EH}[g]}$$

- Regge discretization



$$\int Dg \to \int dL_l$$

- Ponzano Regge ansatz

$$L_l = j_l + \frac{1}{2} \qquad \int dL_l \to \sum_{j_l}$$

- Define

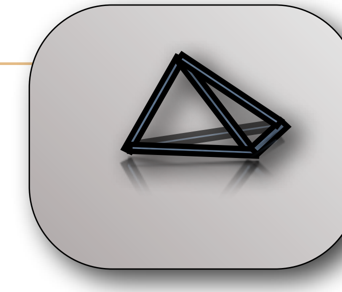
$$Z = \sum_{j_l} \; \prod_l d_{j_l} \; \prod_v \{6j\}$$

$$d_j = 2j+1$$
$$\{6j\} = \mathrm{Tr}[\otimes_e i_e]$$

- Theorem (PR, Roberts)

$$\{6j\} \sim \frac{1}{12\pi V}\left(e^{iS_{Regge}+\frac{\pi}{4}} + e^{-iS_{Regge}-\frac{\pi}{4}}\right)$$

II. moral from 3d



i. The Misner-Wheeler Feynman-integral over geometries can be realized by a strikingly simple algebraic expression based on $SU(2)$ representation theory.

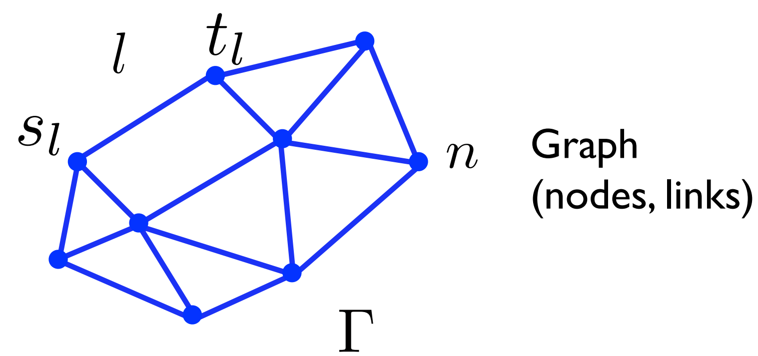
$$Z = \sum_{j_l} \prod_l d_{j_l} \prod_v \mathrm{Tr}[\otimes_e i_e]$$

ii. It is UV finite

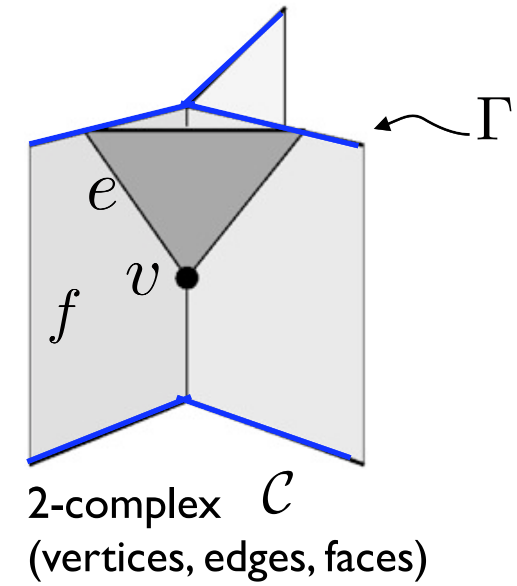
iii. (It is also IR finite Turaev-Viro $SU(2) \to SU(2)_q$ = cosmological constant)

iv. Length is quantized $L_l = j_l + \frac{1}{2}$

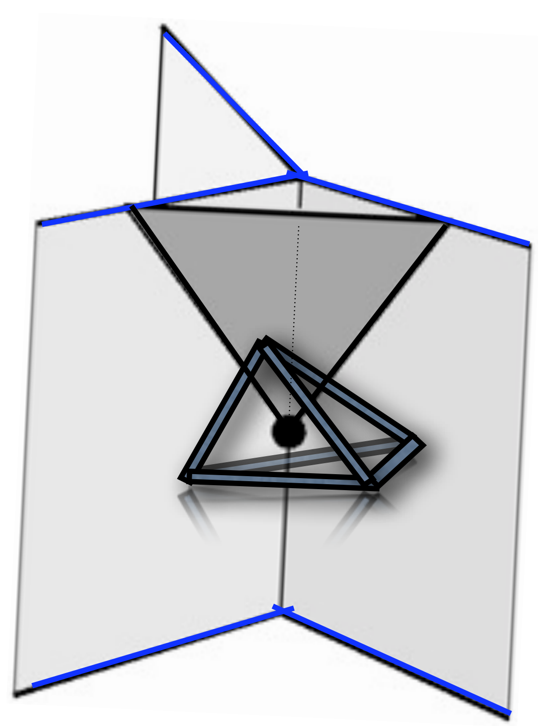
Graph: $\Gamma = \{N, L\}, \quad L \subset N \times N$



Two-complex: $\mathcal{C} = \{V, E, F\}, \quad E \subset V \times V, \quad F \subset P(V)$



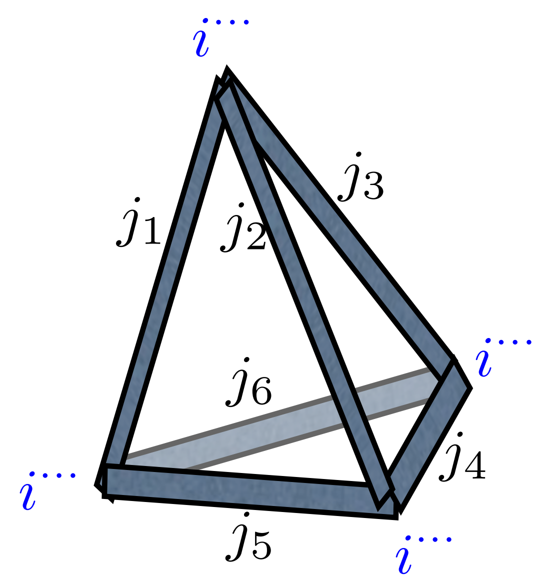
Two-complex as the 2-skeleton
of a cellular decomposition
(any dimension):



SU(2) unitary representations: $|j;m\rangle \in \mathcal{H}_j, \qquad 2j \in N, \;\; m=-j,...,j, \qquad v^m \in \mathcal{H}_j$

Intertwiner space: $\mathcal{K}_{j_1,...,j_n} = \mathrm{Inv}[\mathcal{H}_{j_1} \otimes \ldots \otimes \mathcal{H}_{j_n}] \;\; \ni i^{m_1 ... m_n}$

$$\{6j\} = i^{abc}\; i^{ade}\; i^{bdf}\; i^{cef}$$
$$= \mathrm{Tr}[\otimes_e i_e]$$

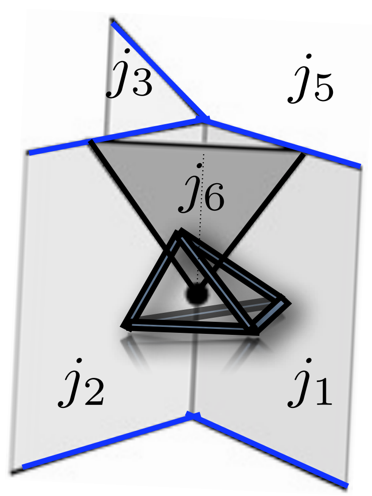


Spin foam:

Spins on faces

Intertwiners on edges

Amplitude at vertex



SU(2) unitary representations: $|j;m\rangle \in \mathcal{H}_j$

SL(2,C) unitary representations: $|k,\nu;j,m\rangle \in \mathcal{H}_{k,\nu} = \bigoplus_{j=k,\infty} \mathcal{H}^j_{k,\nu},$ $\quad 2k \in N, \quad \nu \in R$

SU(2) → SL(2,C) map:

$$f_\gamma : \quad \mathcal{H}_j \quad \to \mathcal{H}_{j,\gamma j}$$
$$|j;m\rangle \mapsto |j,\gamma j; j,m\rangle$$

$$\nu = \gamma j, \quad k = j' = j$$

Main property: $\vec{K} + \gamma \vec{L} = 0$ weakly on the image of f_γ

Boost generator ($\vec{K}$) Rotation generator ($\vec{L}$)

Extend to intertwiner space:

$$f_\gamma : \quad \mathcal{K}_{j_1 \ldots j_n} \quad \to \quad \mathcal{K}^{SL(2,C)}_{(j_1,\gamma j_1)\ldots(j_n,\gamma j_n)}$$

IV. 4d theory

Tetrads $g_{ab} \to e^i_a$ $g_{ab} = e^i_a \; e^i_b$ $e = e_a dx^a \in R^{(1,3)}$

Spin connection $\omega = \omega_a dx^a \in sl(2,C)$ $\omega(e):$ $de + \omega \wedge e = 0$

GR action $$S[e,\omega] = \int e \wedge e \wedge F^*[\omega]$$

GR Holst action $$S[e,\omega] = \int e \wedge e \wedge F^*[\omega] + \frac{1}{\gamma}\int e \wedge e \wedge F[\omega]$$

Canonical variables $\omega, \quad B = (e \wedge e)^* + \frac{1}{\gamma}(e \wedge e)$

Gauge $n_i e^i = 0$, $n_i = (1,0,0,0)$

$$B \to (K = nB, L = nB^*)$$

$$\vec{K} + \gamma \vec{L} = 0$$

"Linear simplicity constraint"

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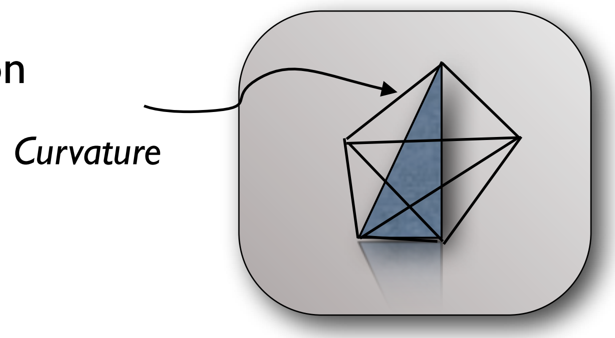
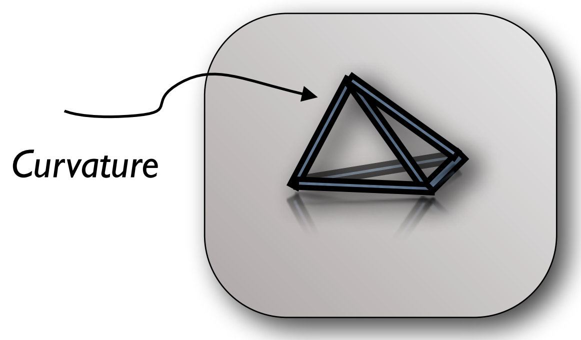


Connection: Flat *so(3)* connection modulo gauge on M-D2

On faces: $\Sigma_f = \int_f e \wedge e \quad \in sl(2,C) \qquad A_f = |\Sigma_f|$

Canonical variables



Connection: Flat *3d sl(2,c)* connection modulo gauge on M-D1

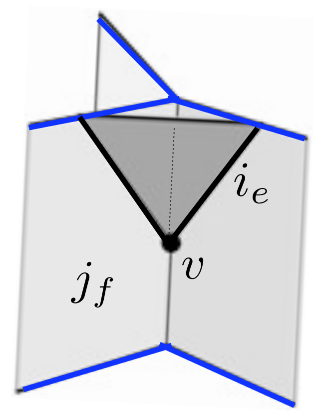
Phase space: $\Gamma = T_*(SL(2,C)^L)/Gauge$

Canonical quantization $\mathcal{H} = L_2[SL(2,C)^L]/Gauge$ $B_l \to$ Left invariant vector field:

Linear simplicity constraint $\vec{K} + \gamma \vec{L} = 0$ Restrict $\mathcal{H} \to L_2[SU(2)^L]/Gauge$

Discreteness of area $A^2_l \to$ Casimir $A_f = \sqrt{j_l(j_l+1)}$

Two-complex $\mathcal{C}$
(dual to a cellular decomposition)



Define

$$Z = \sum_{j_f, i_e} \prod_f d_{j_f} \prod_v A(j_f, i_e)$$

$$d_j = 2j + 1$$
$$A(j_l, i_e) = \mathrm{Tr}[\otimes_e (f_\gamma i_e)]$$

Theorem :
[Barrett, Pereira, Hellmann, Gomes, Dowdall, Fairbairn 2010]

$$A(j_f, i_e) \sim N\left(e^{iS_{Regge}} + e^{-iS_{Regge}}\right)$$

[Freidel Conrady 2008, Bianchi, Satz 2006, Magliaro Perini, 2011]

$$Z_{\mathcal{C}} \to \int Dg \;\; e^{iS_{EH}[g]}$$

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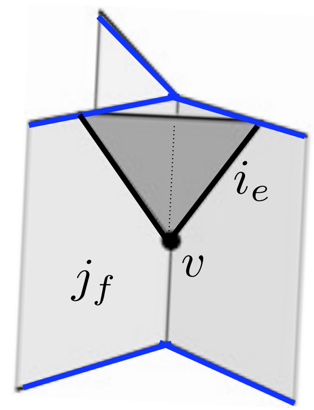

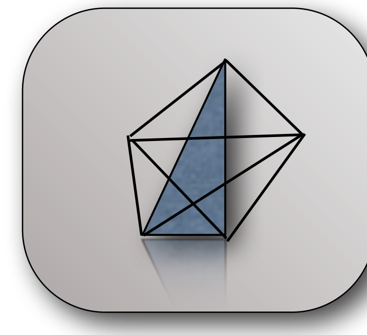

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" Not to take this striking result as a sign we are on the right track would be a bit like believing that God put fossils into the rocks in order to mislead Darwin about the evolution of life." — Stefano Auchino

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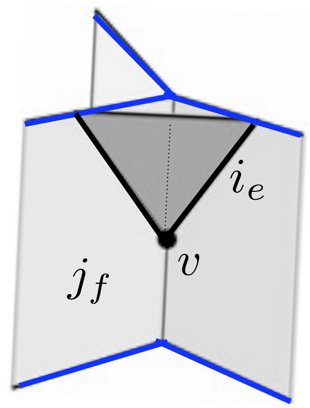

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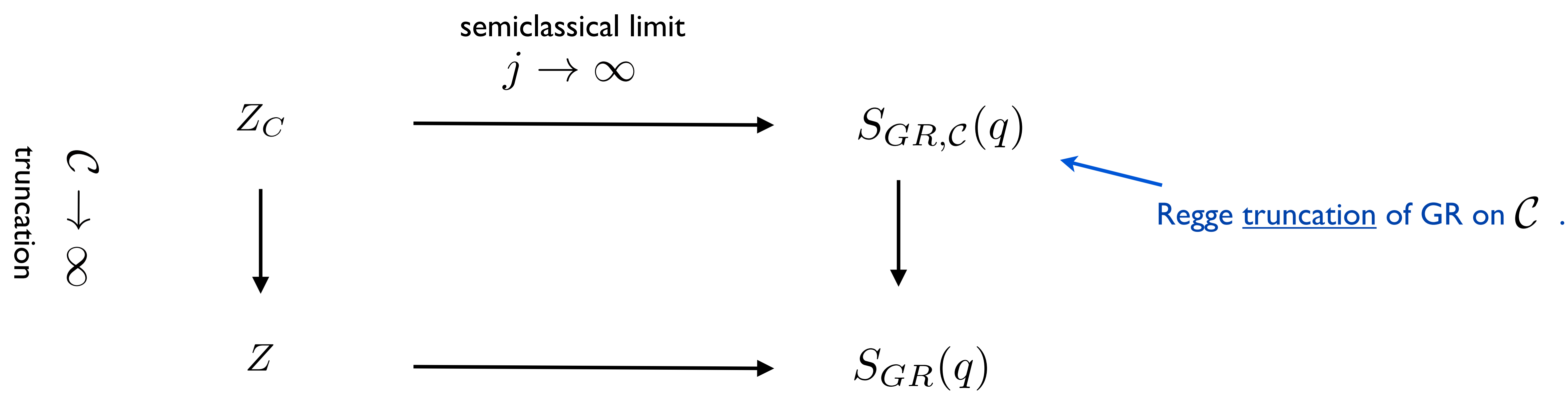
Theorem : $A(j_f, i_e) \sim N\left(e^{iS_{Regge}} + e^{-iS_{Regge}}\right)$ $\qquad Z_{\mathcal{C}} \to \int Dg \;\; e^{iS_{EH}[g]}$

IV. limits

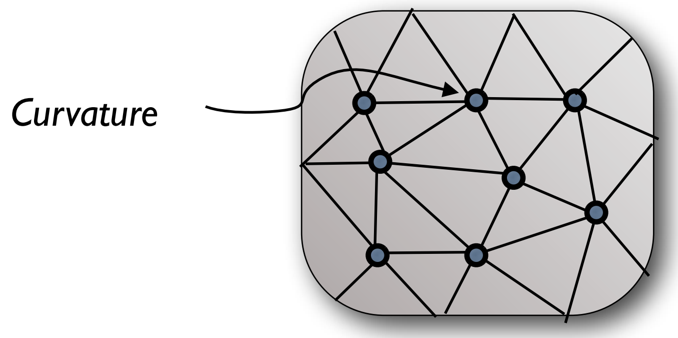
→ Infinite dof limit	*Recovery of all degrees of freedom*	$\mathcal{C} \to \infty$	*Very different from QCD: no lattice spacing a, no critical parameter.*
→ Semiclassical limit	*High quantum numbers* → Large distance limit	$j \to \infty$	*Fix the truncation, disregard Planck scale effects*

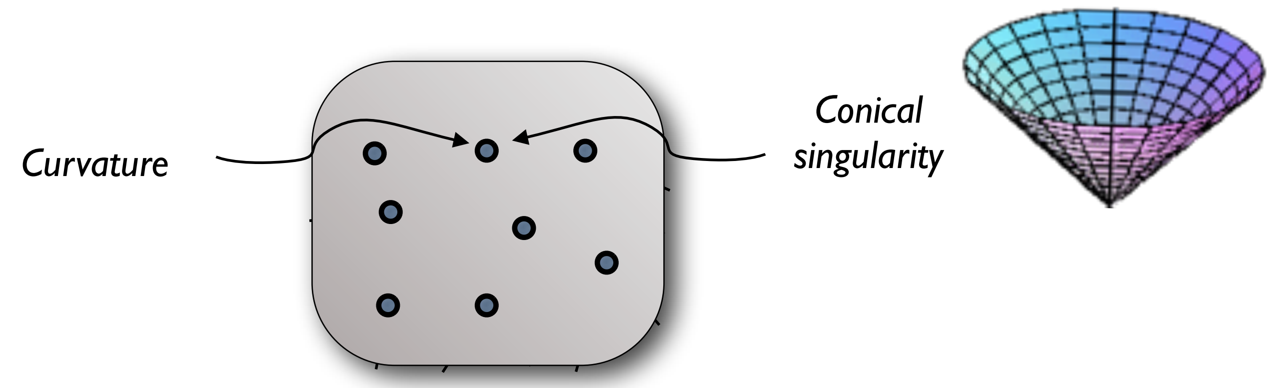


Regime where small -$\mathcal{C}$ it is good: $$\frac{1}{\sqrt{R}} \gg \ell \gg L_{\text{Planck}}$$

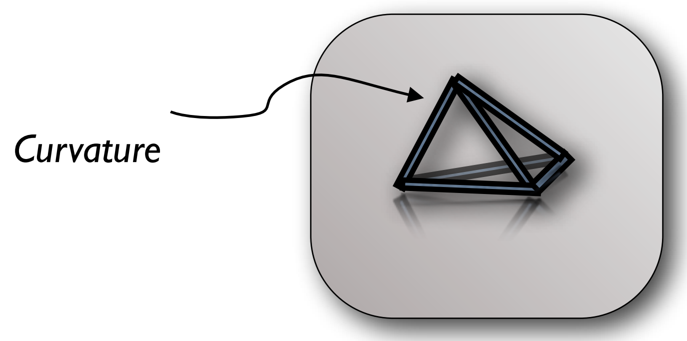
*Recovering the continuum limit is **not** taking a short distance scale cut off to zero.*

2d





3d



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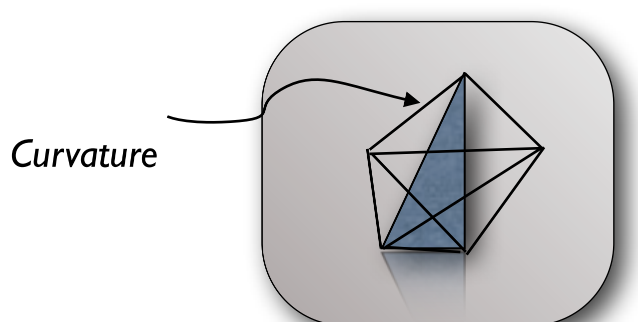

V. How to extract physics? the boundary formalism

Suppose this is defined : $$Z = \int Dg \ \ e^{iS_{EH}}$$

Is physics in these quantities? : $$W(x_1, \dots, x_n) = Z^{-1} \int Dg \ g(x_1) \dots g(x_n) \ e^{iS_{EH}}$$

No, because of the gauge invariance of the theory.

Observability is tricky already in classical General relativity !

V. observables

$$ds^2 = dt^2 - (1 + a\cos(\omega(t-z))dx^2 - (1 - a\cos(\omega(t-z))dy^2 - dz^2$$

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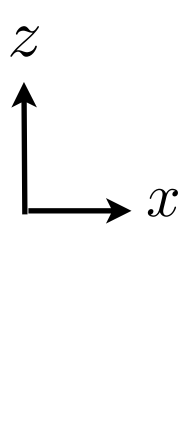

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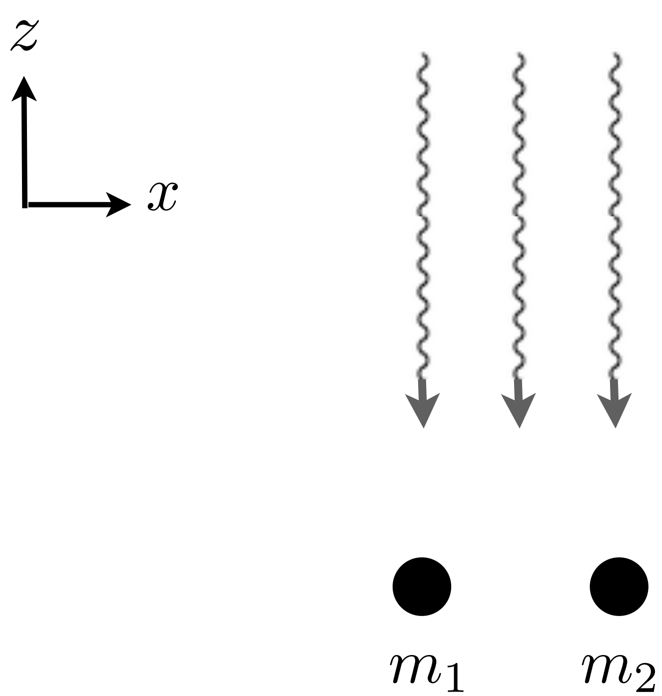

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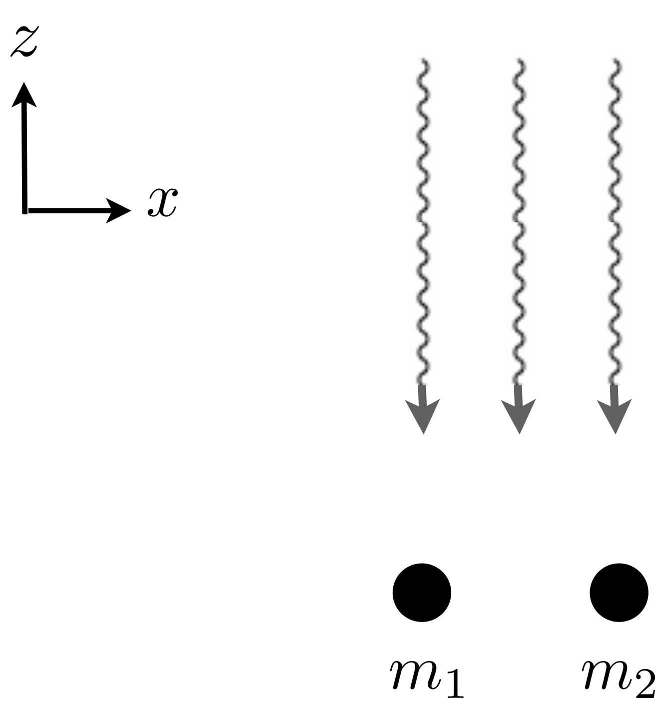

$$\vec{x}_1(t) = const,$$
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No observable consequence

V. observables

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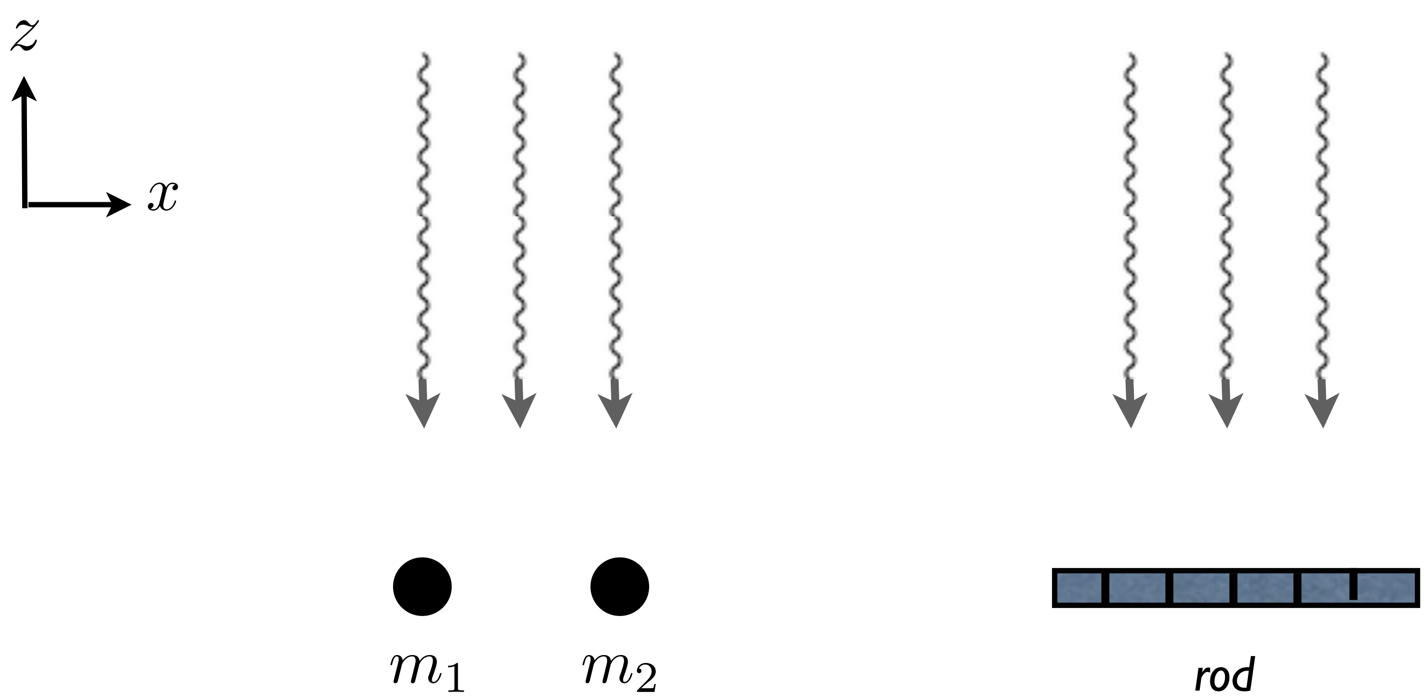

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$L(t) = const$

No observable consequence

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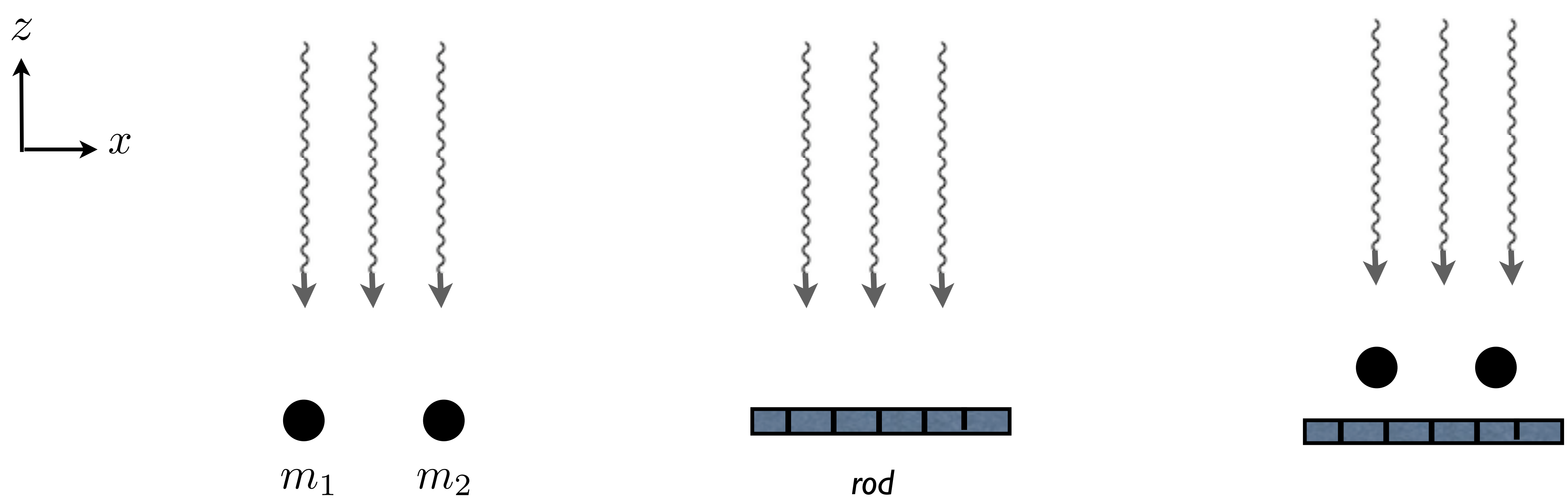

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$$L(t) = const$$

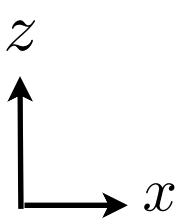
No observable consequence

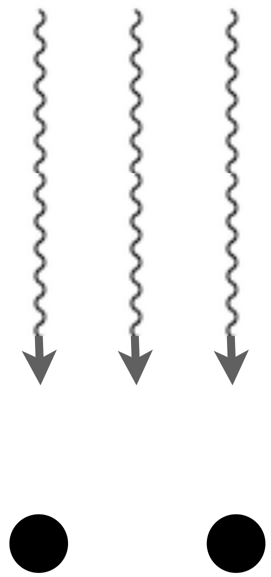
No observable consequence

Observable relative motion

V. observables

$$ds^2 = dt^2 - (1 + a\cos(\omega(t-z))dx^2 - (1 - a\cos(\omega(t-z))dy^2 - dz^2$$

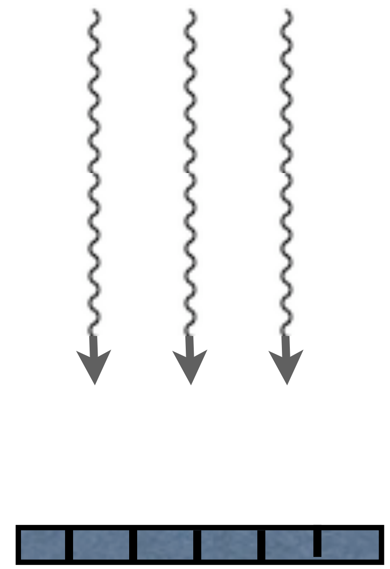




m_1 m_2

$$\vec{x}_1(t) = const,$$
$$\vec{x}_1(t) = const.$$

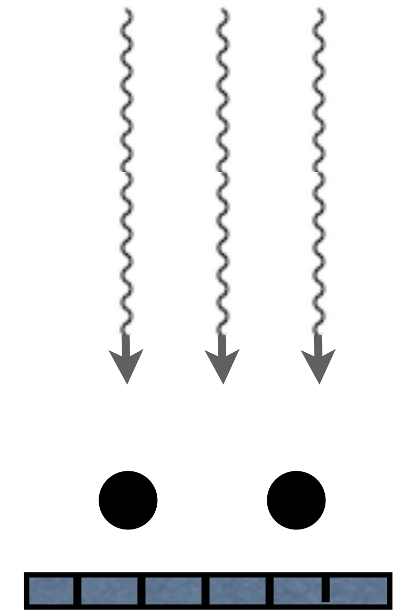
No observable consequence



rod

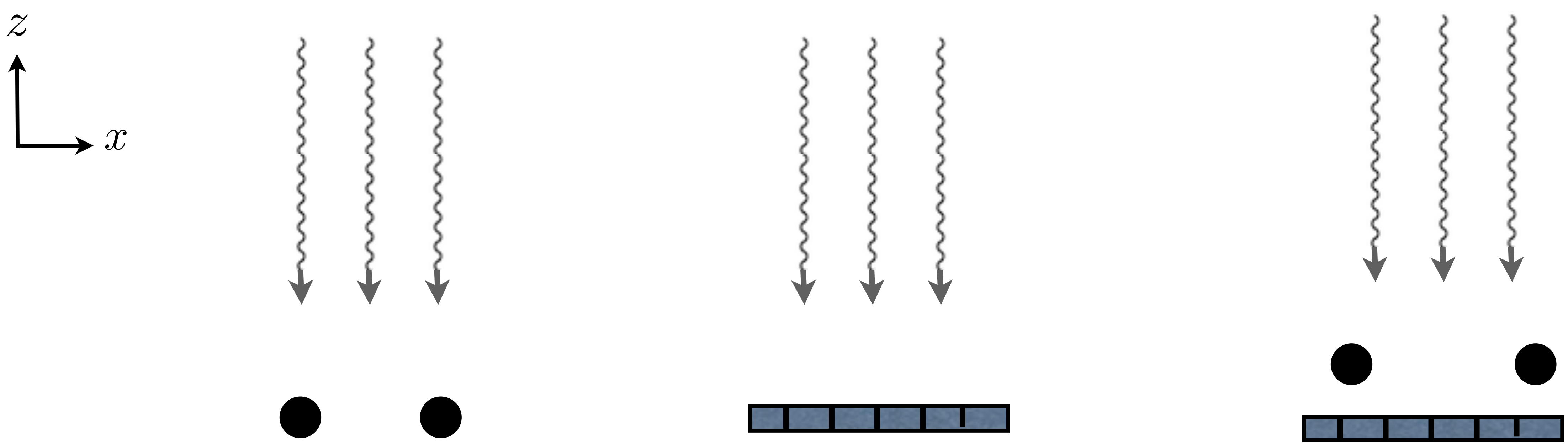
$$L(t) = const$$

No observable consequence



Observable relative motion

$$ds^2 = dt^2 - (1 + a\cos(\omega(t-z))dx^2 - (1 - a\cos(\omega(t-z))dy^2 - dz^2$$



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rod

$\vec{x}_1(t) = const,$

$\vec{x}_1(t) = const.$

$L(t) = const$

No observable consequence

No observable consequence

Observable relative motion

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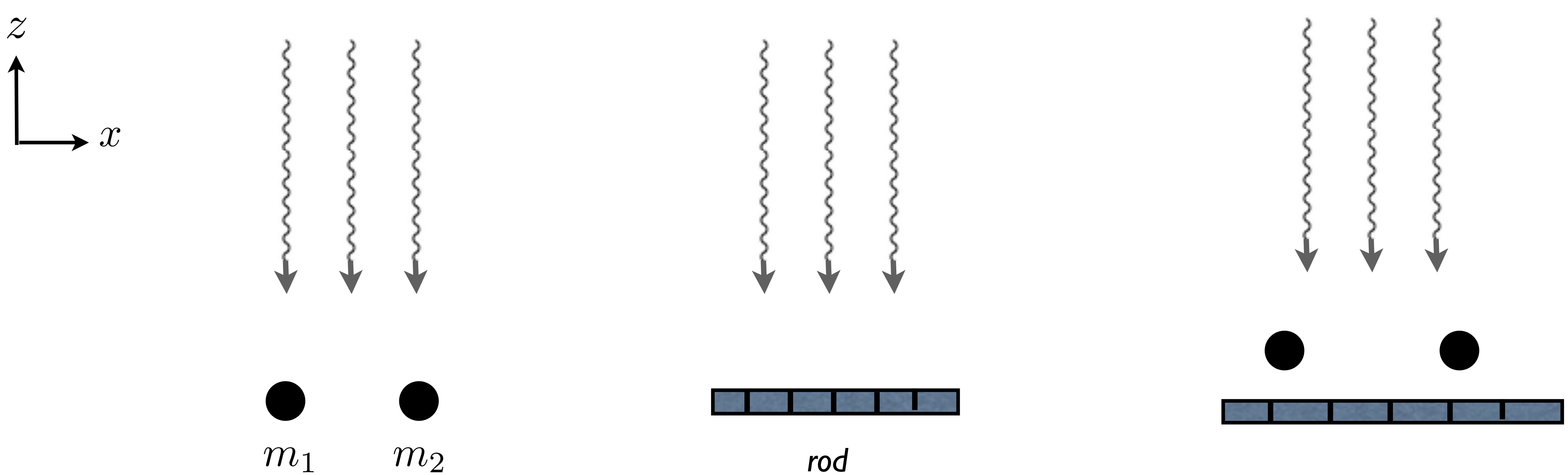

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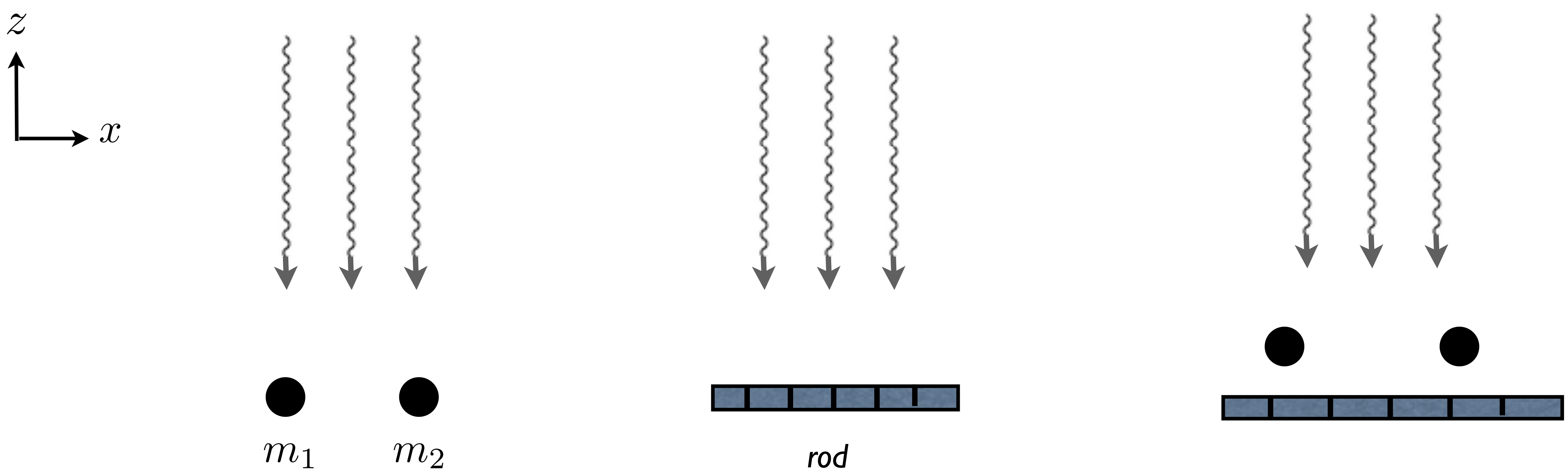

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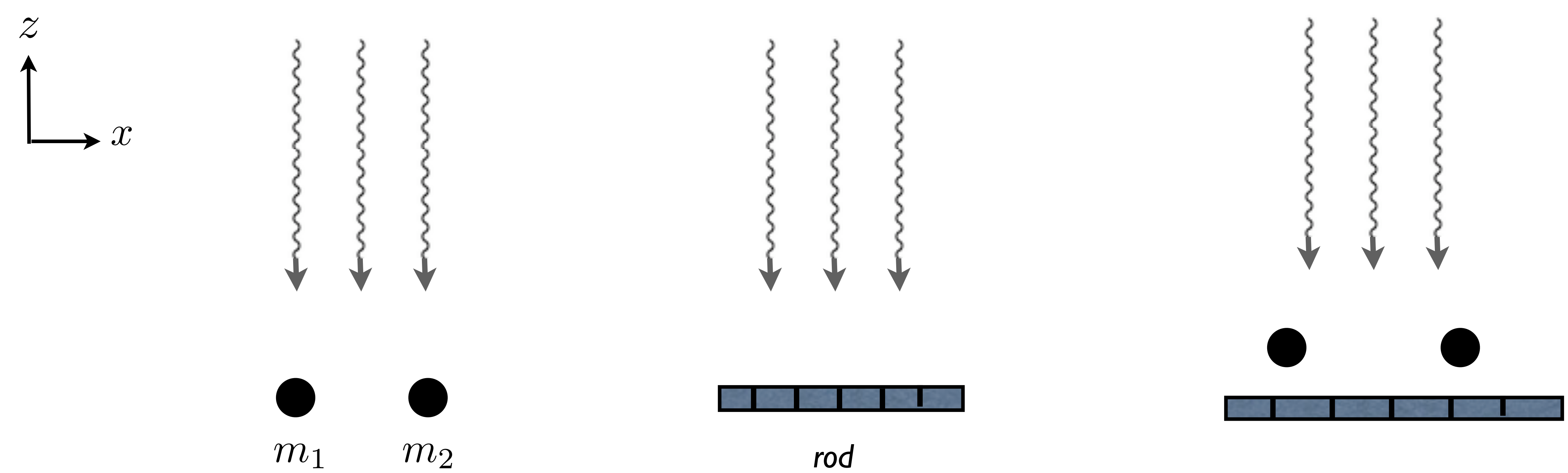

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$$L(t) = const$$

No observable consequence

No observable consequence

Observable relative motion

i. Observability is tricky in gravitational physics

ii. Locality → Relative locality

Hamilton function $$S(q,t,q',t') = \int_t^{t'} dt\ L(q(t),\dot q(t))$$

- Hamilton's "boundary logic": $$p(q,t,q',t') = \frac{\partial S(q,t,q',t')}{\partial q} \qquad (q,q')_{t,t'} \to (p,p')_{t,t'}$$

- Notice also $$E(q,t,q',t') = -\frac{\partial S(q,t,q',t')}{\partial t}$$

(q,t) on equal footing $$\underbrace{(q,t,q',t')}_{q_i} - \underbrace{(p,E,p',E')}_{p_i}$$

- Parametrized systems $$q(t) \to \big(q(\tau),\ t(\tau)\big) \qquad S(q_i,q_i') = \int_\tau^{\tau'} d\tau\ L(q,t,\dot q,\dot t)$$

→ Dynamics is the relative evolution of a set of variables, not the evolution of these variables in time. Hamilton dynamics captures this relational dynamics.

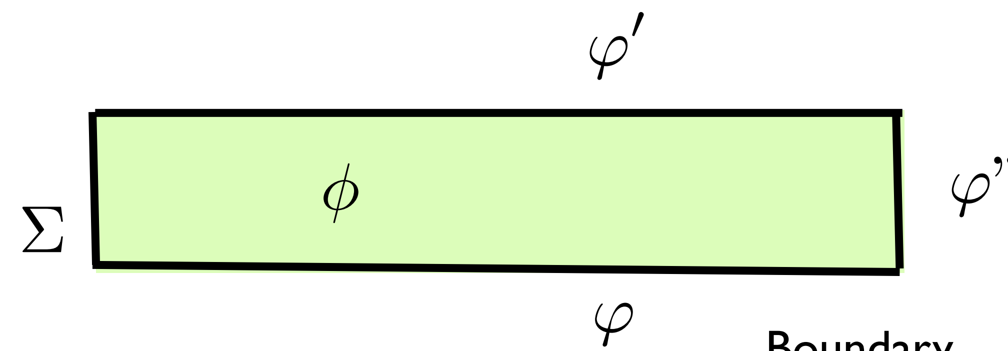
- Quantum theory $W(q,t,q',t') = \langle q|e^{iH(t'-t)}|q'\rangle = \langle q,t|q',t'\rangle \sim e^{\frac{i}{\hbar}S(q,t,q',t')}$

$$= \int_{q,t,q',t'} Dx(t) \;\; e^{\frac{i}{\hbar}S[x(t)]}$$

Evolution operator

Hamilton function !

- Field theory $W[\varphi_b, \Sigma] = \int_{\varphi_b,\Sigma} D\phi \;\; e^{iS[\phi]}$



Boundary conditions

- General covariant field theory $W[\varphi_b, \Sigma] = W[\varphi_b]$

- For the gravitational theory: φ_b gives the geometry of the boundary

Semiclassical limit

$$W[\varphi_b] \;\to\; e^{\frac{i}{\hbar G} S_{GR}[\varphi_b]} + \mathit{correction\ in\ } \hbar G$$

Hamilton function of GR

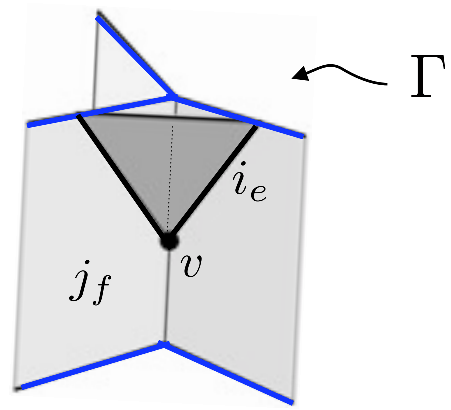
Field propagator → Particle propagator: $\langle 0|\phi(\vec{x}', t')\phi(x,t)|0\rangle = \langle 0|\phi(\vec{x}')\; e^{iH(t'-t)}\; \phi(x)|0\rangle$

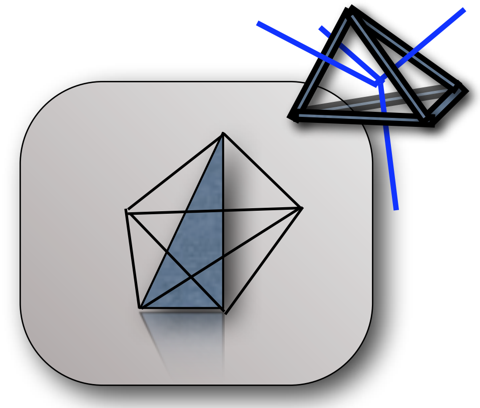
$$= \int d\varphi\, d\varphi'\; W[\varphi, t, \varphi', t']\; \varphi(\vec{x})\, \varphi'(\vec{x}')\; \overline{\Psi_0[\varphi]}\, \Psi_0[\varphi'] \qquad = \langle W \mid \phi(\vec{x})\phi'(\vec{x}') \mid \Psi_0\rangle$$

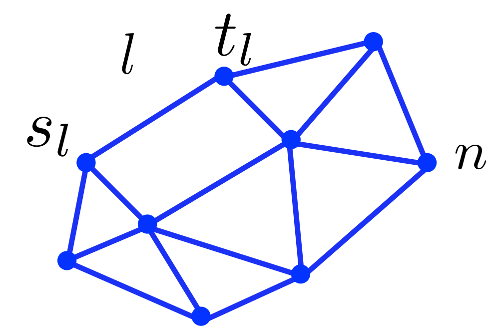
Field propagator: $W[\varphi, t, \varphi', t']$ — Field insertion: $\varphi(\vec{x})\, \varphi'(\vec{x}')$ — Vacuum boundary state: $\overline{\Psi_0[\varphi]}\, \Psi_0[\varphi']$

Two-complex with boundary

$\mathcal{C}$
$\Gamma = \partial\mathcal{C}$







Graph
(nodes, links)

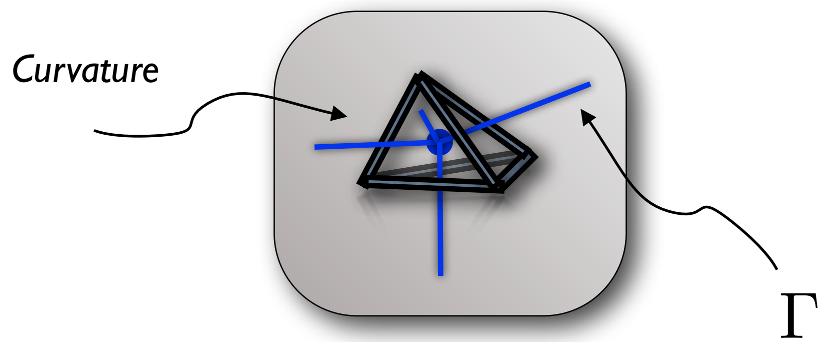
Quantum gravity transition amplitudes

$$Z(j_l, i_e) = \sum_{j_f, i_n} \prod_f d_{j_f} \prod_v A(j_f, i_e)$$

$$d_j = 2j + 1$$
$$A(j_l, i_e) = \mathrm{Tr}[\otimes_e (f_\gamma i_e)]$$

$$Z_{\mathcal{C}}(j_l, i_e) \in \mathcal{H}_{\partial C} = L_2[SU(2)^L]$$

II. 3d quantum geometry

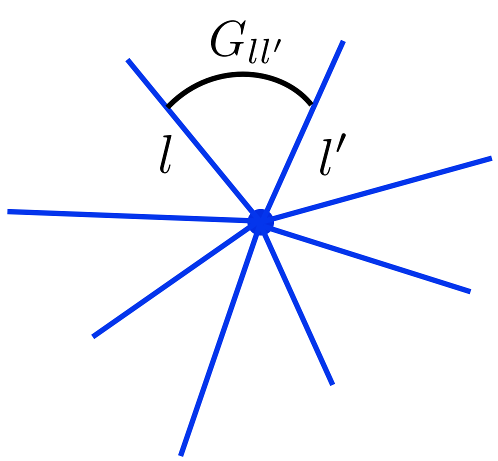


Connection: Flat *3d su(2)* connection modulo gauge on M-D$_l$

Phase space: $\Gamma = T_*(SU(2)^L)/Gauge$

State space $\mathcal{H}_\Gamma = L^2[SU(2)^L]/Gauge$

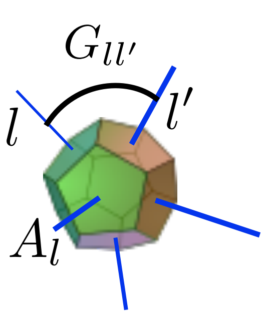
Derivative operator: $\vec{L}_l = \{L_l^i\}, i = 1, 2, 3$ where $L^i\psi(h) \equiv \frac{d}{dt}\psi(he^{t\tau_i})\Big|_{t=0}$ $\sum_{l\in n} \vec{L}_l = 0$



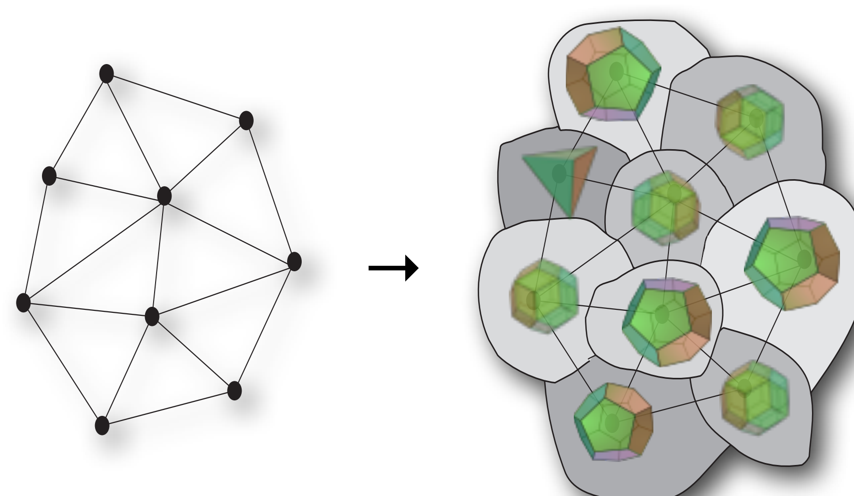
The gauge invariant operator: $G_{ll'} = \vec{L}_l \cdot \vec{L}_{l'}$ satisfies $\sum_{l\in n} G_{ll'} = 0$

Is precisely the Penrose metric operator on the graph

It satisfies 1971 Penrose spin-geometry theorem, and 1897 Minkowski theorem: semiclassical states have a geometrical interpretation as polyhedra.

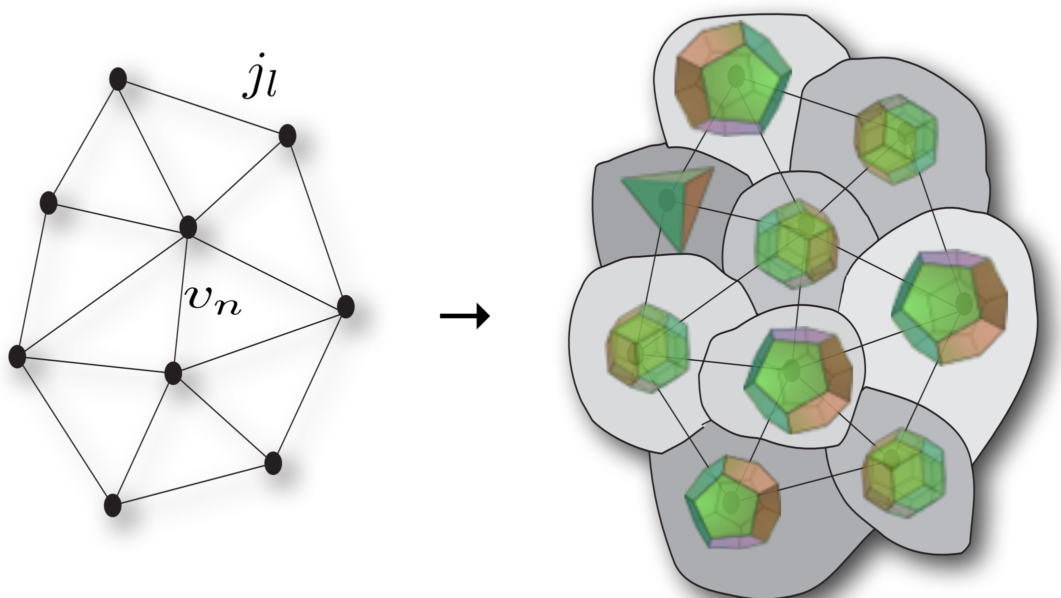


Polyhedron



area $A_l^2 = G_{ll}$ volume $V_n^2 = \frac{2}{9}\, \vec{L}_{l_1} \cdot (\vec{L}_{l_2} \times \vec{L}_{l_3})$

- Area and volume (A_l, V_n) form a complete set of commuting observables → basis $|\Gamma, j_l, v_n\rangle$



Nodes: discrete quanta of volume ("quanta of space") with quantum number v_n.

Links: discrete quanta of area, with quantum number j_l.

Geometry is quantized:

(i) eigenvalues are discrete
(ii) the operators do not commute
(iii) a generic state is a quantum superposition

→ coherent states theory (based on *Perelomov 1986* SU(2) coherent state techniques)

→ States in $\mathcal{H}_\Gamma = L^2[SU(2)^L/SU(2)^N]$ describe quantum geometries:

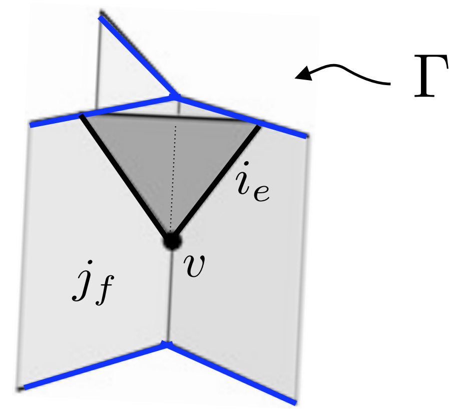
not quantum states <u>in</u> spacetime

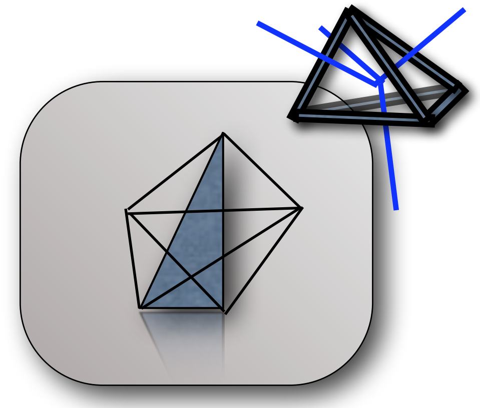
but rather quantum states <u>of</u> spacetime

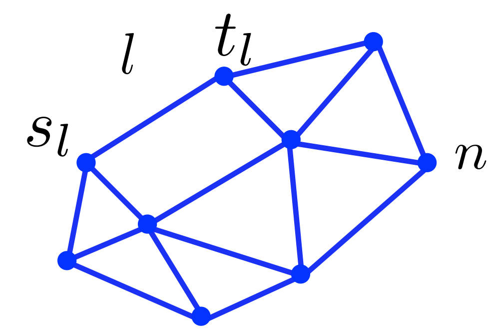
- Area eigenvalues $$A = 8\pi\gamma\hbar G\ \sqrt{j_l(j_l+1)}$$

Two-complex with boundary

$\mathcal{C}$
$\Gamma = \partial\mathcal{C}$







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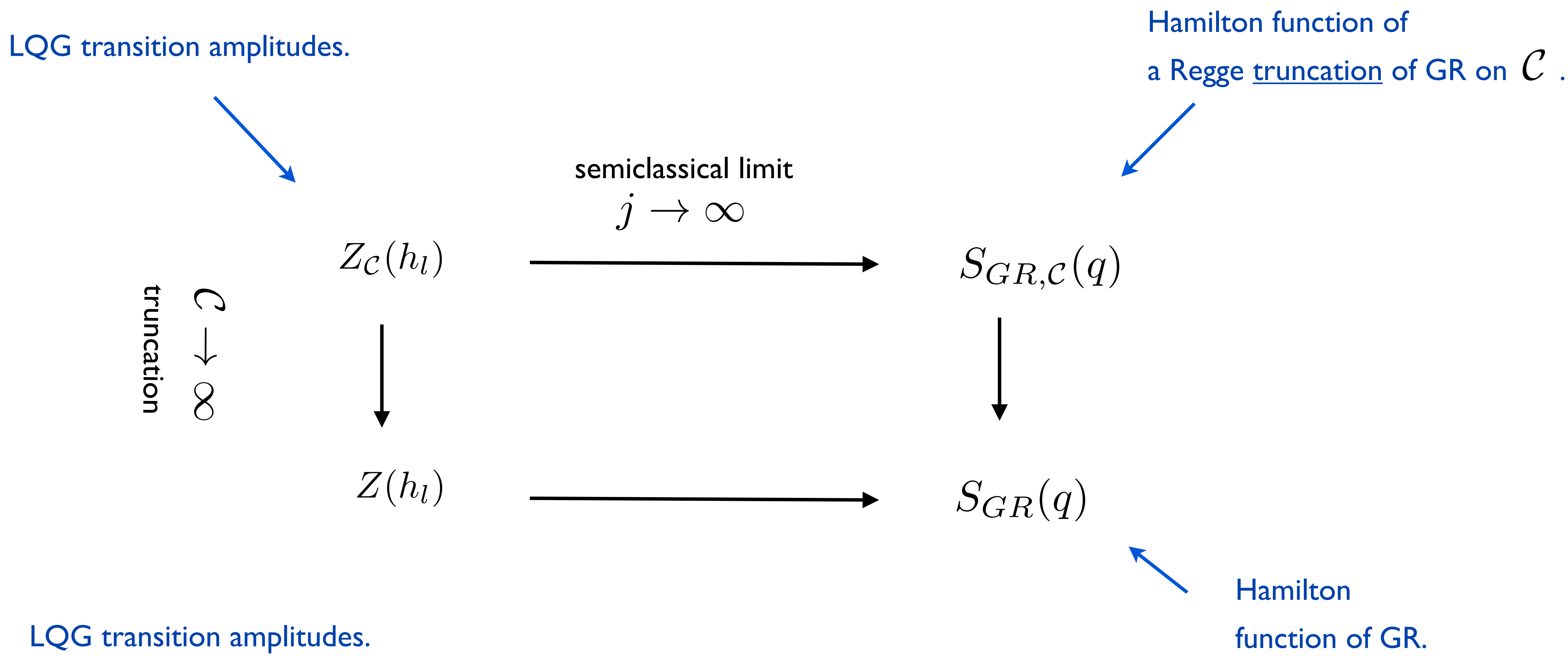
$$Z_{\mathcal{C}}(j_l, i_e) \in \mathcal{H}_{\partial C} = L_2[SU(2)^L]$$

$$Z_{\mathcal{C}}(h_l) \in \mathcal{H}_{\partial C} = L_2[SU(2)^L]$$

Finite in the *q*-deformed model

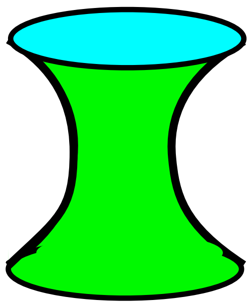
IV. limits



Regime where small -$\mathcal{C}$ it is good: $$\frac{1}{\sqrt{R}} \gg \ell \gg L_{\text{Planck}}$$

*Recovering the continuum limit is **not** taking a short distance scale cut off to zero.*

(i) cosmology. Transition amplitude → Hamilton function



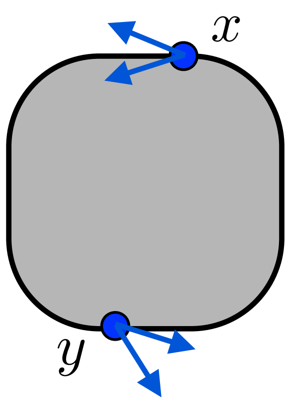
Classical Hamilton function $$S(a, a') = \frac{2}{3}\sqrt{\frac{\Lambda}{3}}(a'^3 - a^3)$$

$$W(a, a') \;\rightarrow\; e^{\frac{i}{\hbar}S(a,a')}$$

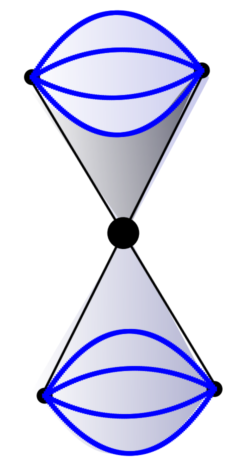
$$\langle Z|\psi_{a\dot{a}} \otimes \psi_{a\dot{a}'}\rangle$$

(ii) n-point functions. The background enters in the choice of a "background" boundary state

$$\frac{\langle Z|G_{l_a l_b} G_{l_c l_d}|\psi_0\rangle}{\langle Z|\psi_0\rangle} \sim \langle 0|g_{ab}(x) g_{cd}(y)|0\rangle$$



In principle this technique allows generic *n*-point functions to be computed, and compared with Effective Quantum GR, and *corrections* to be computed.



(i) Cosmology. Starting from $Z_{\mathcal{C}}(h_l)$, it is possible to compute the transition amplitude between homogeneous isotropic geometries

$$W(z_i, z_f) = \int_{SO(4)^4} dG_1^i\, G_2^i\, dG_1^f\, G_2^f \prod_{l^i} P_t(H_l(z_i)\,,\, G_1^i G_2^{i\,-1}) \prod_{l^f} P_t(H_l(z_f)\,,\, G_1^f G_2^{f\,-1})$$

$$P_t(H,G) = \sum_j (2j+1) e^{-2t\hbar j(j+1)} tr\left[D^{(j)}(H) Y^\dagger D^{(j^+,j^-)}(G) Y\right].$$

$$\rightarrow \qquad e^{\frac{i}{\hbar}\frac{2}{3}\sqrt{\frac{\Lambda}{3}}(a'^3 - a^3)} = e^{\frac{i}{\hbar}S(a,a')}$$

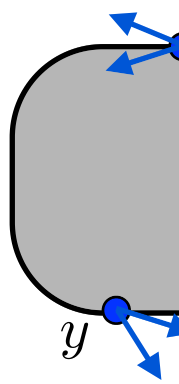
Vidotto's talk

Result: The expanding Friedmann dynamics and the DeSitter Hamilton function are recovered

[Bianchi Vidotto Krajewski CR 2010]

(ii) Gravitational waves. Starting from $Z_{\mathcal{C}}(h_l)$, it is possible to compute the two point function of the metric on a background. The background enters in the choice of a "background" boundary state ψ_0

$$\frac{\langle Z_{\mathcal{C}}|G_{l_a l_b} G_{l_c l_d}|\psi_0\rangle}{\langle Z_{\mathcal{C}}|\psi_0\rangle} \sim \langle 0|g_{ab}(x)g_{cd}(y)|0\rangle$$



This can be computed at first order in the *expansion* in the number of vertices.

$$\langle W|E_n^a \cdot E_n^b \; E_m^c \cdot E_m^d|j_{ab}, \Phi_a(\vec{n})\rangle =$$

$$\int \prod_{a=1}^{5} dg_a^+ dg_a^- \;\; A_i^{na} A_i^{nb} \;\; A_i^{nc} A_i^{nd} \;\; e^{\sum_{ab,\pm} 2j_{ab}^{\pm} \log\langle -\vec{n}_{ab}|(g_a^{\pm})^{-1} g_b^{\pm}|\vec{n}_{ba}\rangle}$$

$$A_i^{na} = \gamma j_{na}^{\pm} \frac{\langle -\vec{n}_{an}|(g_a^{\pm})^{-1} g_n^{\pm} \sigma^i|\vec{n}_{na}\rangle}{\langle -\vec{n}_{an}|(g_a^{\pm})^{-1} g_n^{\pm}|\vec{n}_{na}\rangle}$$

$$\longrightarrow \quad \langle h_{\mu\nu}(x) h_{\rho\sigma}(y)\rangle = \frac{-1}{2|x-y|^2}(\delta_{\mu\rho}\delta_{\nu\sigma} + \delta_{\mu\sigma}\delta_{\nu\rho} - \delta_{\mu\nu}\delta_{\rho\sigma}).$$

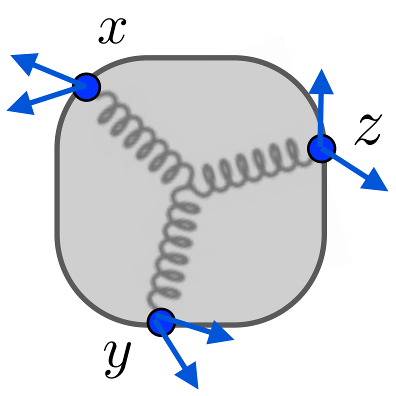
Result: The free graviton propagator is recovered in the Lorentzian theory

[Bianchi Magliaro Perini 2009 , Ding 2011, Zhang 2011,]

(ii) Scattering.

Result: The Regge n-point function is recovered in the large j limit (euclidean theory)

[Zhang, CR 2011]



IV. an overview

i. 4 dimensions, Lorentzian quantum GR:
fundamental formulation clear, fundamental degrees of freedom clear

ii. Classical limit: 4d GR
not a theorem, but strong indications

iii. Couples with Standard Model (fermions, YM)
compatible with observed world

iv. Ultraviolet finite
theorem

v. Includes a positive cosmological constant (quantum group). Finite.
theorem

vi. Lorentz covariant

vii. Quantum space (Planck scale discreteness)
clear picture of quantum geometry

viii. Transition amplitudes
background independent amplitudes

ix. Unification
nothing to say

IV. a comparison

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ii.	Classical limit: 4d GR not a theorem, but strong indications	ii.	Classical limit clear: 10d GR
iii.	Couples with Standard Model (fermions, YM) compatible with observed world	iii.	not yet clearly compatible with observed world
iv.	Ultraviolet finite theorem	iv.	Ultraviolet finite strong indications. not a theorem
v.	Includes a positive cosmological constant (quantum group). Finite. theorem	v.	Problems with positive cosmological constant
vi.	Lorentz covariant	vii.	Lorentz covariant
vii.	Quantum space (Planck scale discreteness) clear picture of quantum geometry	viii.	Quantum space (Planck scale discreteness) unclear picture of quantum geometry
viii.	Transition amplitudes background independent amplitudes	ix.	Transition amplitudes background independent transition amplitudes not clear (except if the world is AdS, which it is not)
ix.	Unification nothing to say	ix.	Unification beautiful picture

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viii. Quantum space (Planck scale discreteness)
unclear picture of quantum geometry

ix. Transition amplitudes
background independent transition amplitudes not clear (except if the world is AdS, which it is not)

ix. Unification
beautiful picture

IV. a comparison

Loops

i. 4 dimensions, Lorentzian quantum GR:
fundamental formulation clear, fundamental degrees of freedom clear

ii. Classical limit: 4d GR
not a theorem, but strong indications

iii. Couples with Standard Model (fermions, YM)
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i.	4 dimensions, Lorentzian quantum GR: fundamental formulation clear, fundamental degrees of freedom clear	✓	i.	fundamental formulation not clear, fundamental degrees of freedom not clear	✗
ii.	Classical limit: 4d GR not a theorem, but strong indications	?	ii.	Classical limit clear: 10d GR	?
iii.	Couples with Standard Model (fermions, YM) compatible with observed world	✓	iii.	not yet clearly compatible with observed world	?
iv.	Ultraviolet finite theorem	✓	iv.	Ultraviolet finite strong indications. not a theorem	?
v.	Includes a positive cosmological constant (quantum group). Finite. theorem	✓	v.	Problems with positive cosmological constant	✗
vi.	Lorentz covariant		vii.	Lorentz covariant	
vii.	Quantum space (Planck scale discreteness) clear picture of quantum geometry	✓	viii.	Quantum space (Planck scale discreteness) unclear picture of quantum geometry	✗
viii.	Transition amplitudes background independent amplitudes	✓	ix.	Transition amplitudes background independent transition amplitudes not clear (except if the world is AdS, which it is not)	✗
ix.	Unification nothing to say		ix.	Unification beautiful picture	

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Loops' main open problems

i. Coupling with fermions and YM not yet studied.

ii. Higher corrections not yet studied.

iii. Does the cosine term in the action disturbs the classical limit?

iv. Does the limit $Z_{\mathcal{C}} \to Z$ (vertex expansion) converges in any useful sense?

v. The absence of IR divergences in the *q*-deformed theory means that there may be cosmological constants size radiative corrections. Do these interfere with (iii)?

vi. Are gauge degrees of freedom sufficiently suppressed at a finite order expansion?

vii. Radiative corrections and scaling.