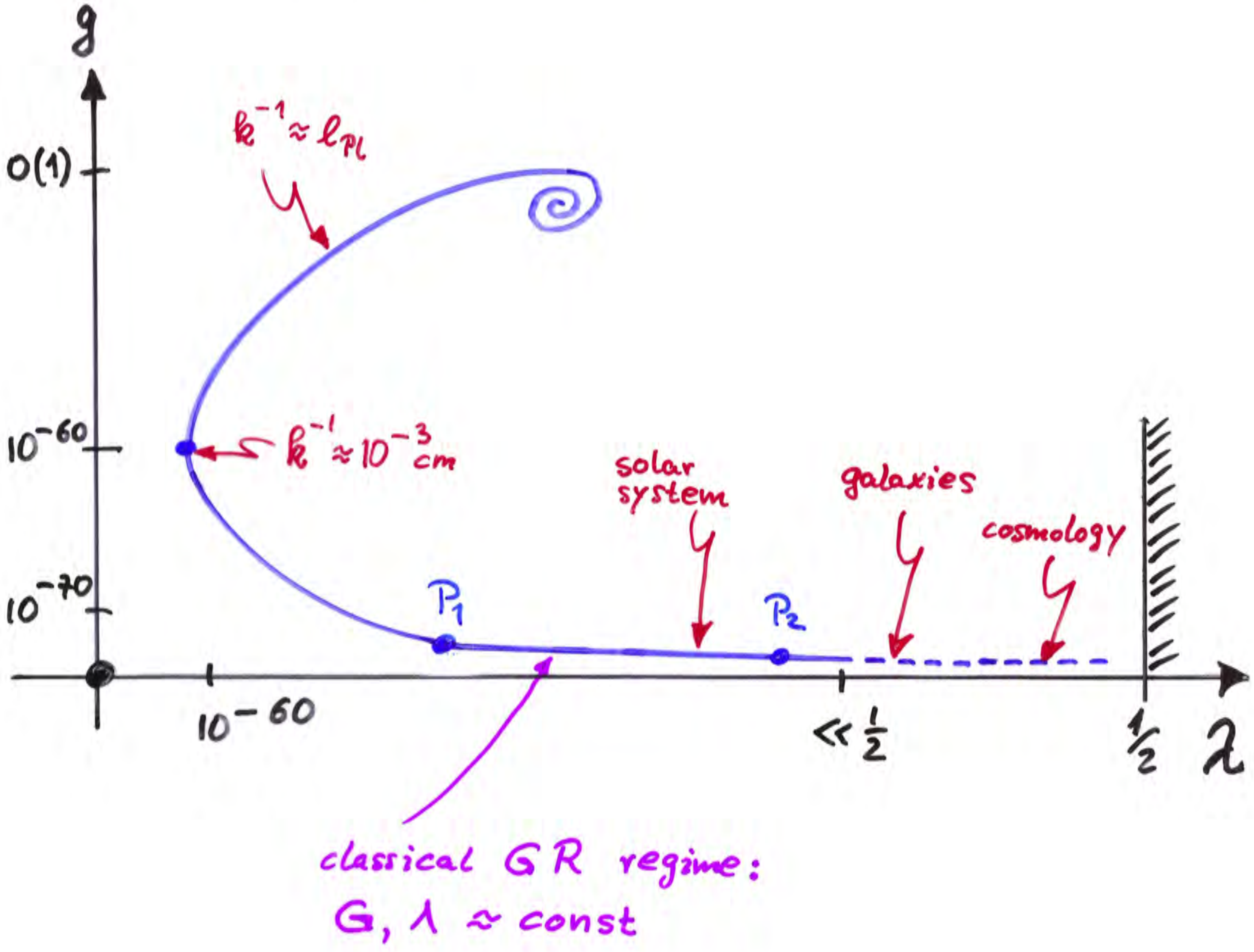
Cosmological

Applications / Implications

A. Bonanno, M.R. (2002)
M. R., H. Weyer (2005)
M. R., F. Saueressig (2005)
A. Bonanno, M.R. (2007)

The RG trajectory "realized in Nature"



"Today" in cosmology:

$$\lambda_{cosmo} \equiv \frac{\Lambda_{cosmo}}{k^2_{cosmo}} = O(1) \;!!!$$

$\approx H_0^2$

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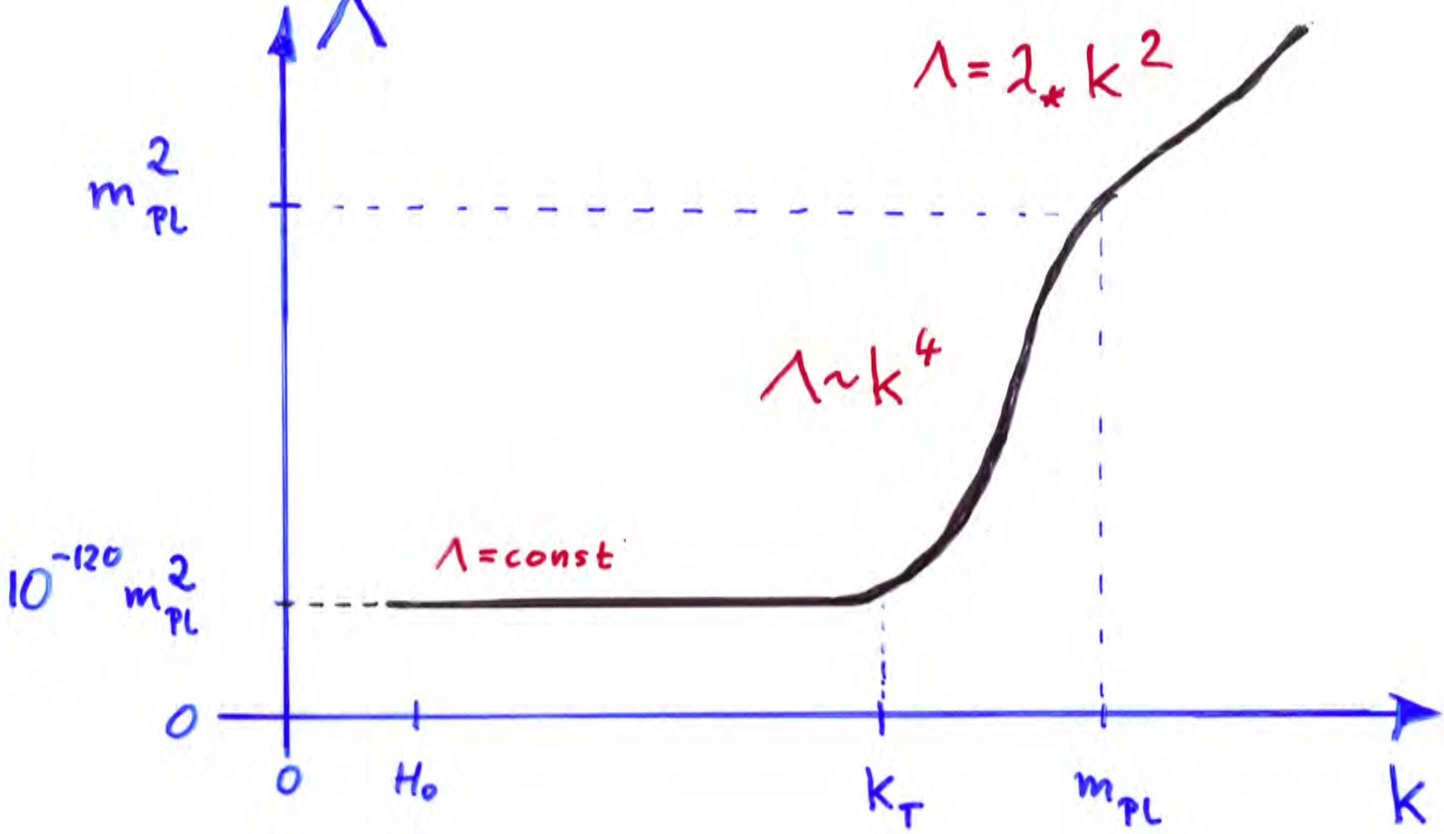
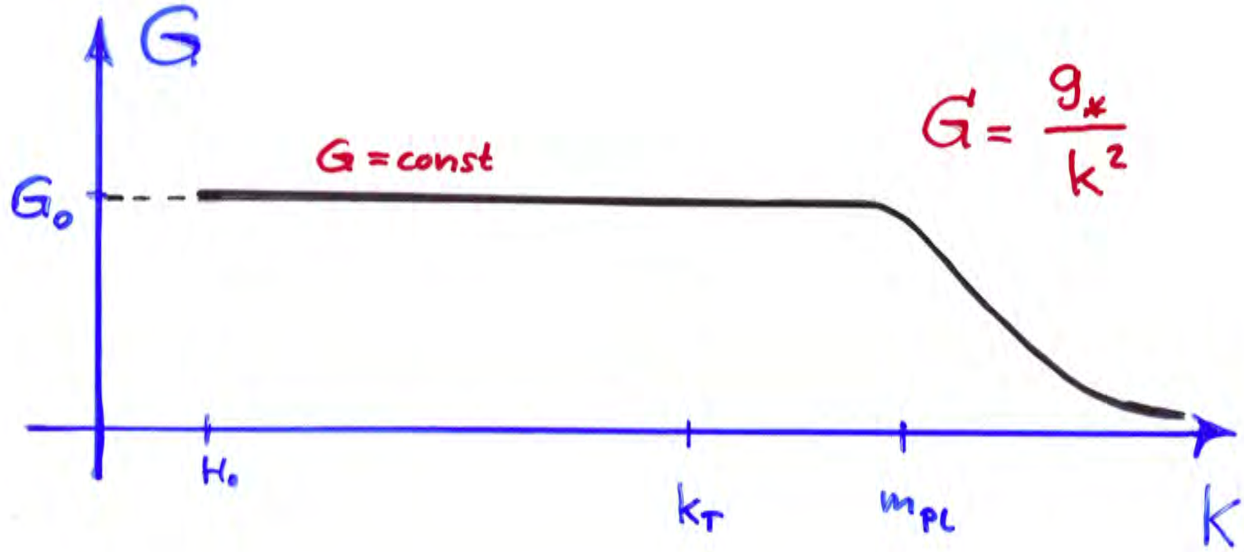




Definition of Planck scale: $m_{Pl} \equiv \ell_{Pl}^{-1} \equiv G_0^{-\frac{1}{2}}$

Are there observable / observed physical phenomena related to the RG-running implied by QEG ?

Candidates in cosmology:

a) Entropy carried by cosmological matter (CMBR photons, ...)

$$\left[S_{CMBR} / \text{Hubble volume}\right]_{\text{today}} \approx 10^{88} \gg 1$$

most plausible initial value $= O(1)$!

b) Automatic inflation in the NGFP regime

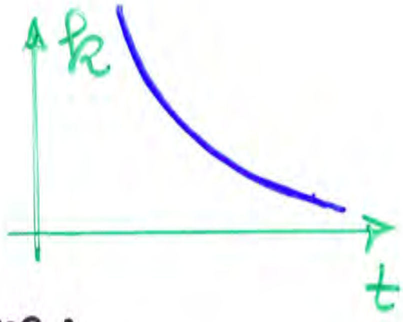
no inflaton needed,
no reheating necessary;
$\Lambda(k)$ large: cosmolog. const. drives inflation
$\Lambda(k)$ small: inflation stops automatically

c) Generation of primordial density perturbations

spectrum scale free:

big bang = "critical phenomenon" governed by the NGFP

For monotonic cosmological cutoff identification $k = k(t)$



and  below the NGFP regime:

$\boxed{\Lambda(t) \equiv \Lambda(k = k(t))$ is a positive and decreasing function of time.$}$

$\Longrightarrow$

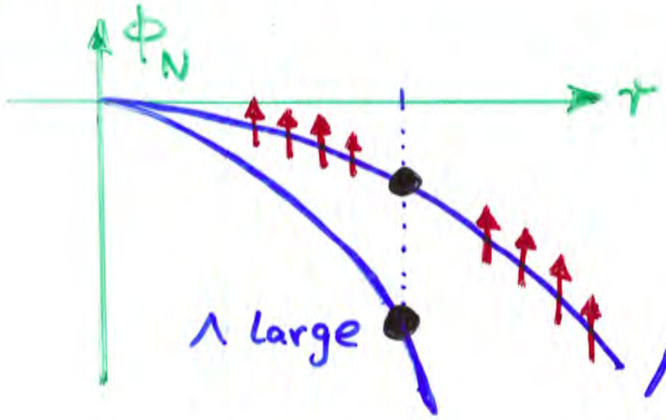
$\boxed{\text{Energy transfer into the matter system ("heating up").}}$

Example: de Sitter plus test particle

$$ds^2 = -\left[1 + 2\phi_N(r)\right] dt^2 + \left[1 + 2\phi_N(r)\right]^{-1} dr^2 + r^2 d\Omega^2$$

$$\phi_N(r) = -\frac{1}{6}\Lambda\, r^2$$

$\Lambda > 0$:



Newtonian interpretation:

Decreasing Λ increases the test particle's potential energy!

RG-Improved Cosmology

Spatially flat RW geometry:

$$ds^2 = -dt^2 + a(t)^2 \left[dr^2 + r^2 (d\theta^2 + \sin^2\theta \, d\varphi^2)\right]$$

$$T_\mu{}^\nu = \mathrm{diag}\left[-\rho(t), p(t), p(t), p(t)\right]$$

The scale factor is determined by

$$\boxed{R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\Lambda(t)\, g_{\mu\nu} + 8\pi G(t)\, T_{\mu\nu}}$$

where $\Lambda(t) \equiv \Lambda(k = k(t))$, $G(t) \equiv G(k = k(t))$

"Improved" Einstein's eq. $\Longleftrightarrow$

modified Friedmann equation:

$$H^2 \equiv \left(\frac{\dot a}{a}\right)^2 = \frac{8\pi}{3} G(t)\, \rho + \frac{1}{3}\Lambda(t)$$

modified conservation law $D^\mu \left[-\Lambda(t) g_{\mu\nu} + 8\pi G(t) T_{\mu\nu}\right] = 0$

$$\underbrace{\dot\rho + 3H(\rho + p)}_{\sim D^\mu T_{\mu 0}} = -\frac{\dot\Lambda + 8\pi\rho\, \dot G}{8\pi G}$$

Describes energy exchange between matter and fields Λ, G ("vacuum").

Entropy Generation

modified conservation law $\Rightarrow$

$$\frac{d}{dt}\left(\rho a^3\right) + p\,\frac{d}{dt}\left(a^3\right) = \widetilde{\mathcal{P}}(t)$$

with $$\widetilde{\mathcal{P}}(t) \equiv -\left(\frac{\dot{\Lambda} + 8\pi\rho\dot{G}}{8\pi G}\right)a^3$$

Cosmological fluid within unit comoving volume:

proper volume: $V = a^3$

contains energy $U = \rho a^3$

and entropy $S = s\,a^3$. $\Rightarrow$

$$\boxed{\frac{dU}{dt} + p\,\frac{dV}{dt} = \widetilde{\mathcal{P}}}$$

$$dU + p\,dV = T\,dS \quad \Rightarrow \quad \boxed{\widetilde{\mathcal{P}} = T\,\frac{dS}{dt}}$$

Classical FRW: $\widetilde{\mathcal{P}} = 0$, $\frac{d}{dt}S = 0$, adiabat. expans.

Improved cosmology: Comoving entropy changes as a consequence of the RG effects

$$\boxed{\frac{d}{dt}S \equiv \frac{d}{dt}\left(s\,a^3\right) = \mathcal{P}} \quad \text{with} \quad \boxed{\mathcal{P} \equiv \frac{\widetilde{\mathcal{P}}}{T}}$$

Rate of entropy production:

$$\mathcal{P}(t) = -\left(\frac{\dot{\lambda} + 8\pi\varrho\dot{G}}{8\pi G}\right) a^3 \cdot \frac{1}{T}$$

Need <u>non-equilibrium</u> relationship $T \leftrightarrow p, \varrho$!

The Postulate:

Entropy generation disturbs equilibrium as little as possible, expansion is "almost adiabatic".

→ Lima (1997)

Concrete assumption about the non-equilibrium thermodynamics of the cosmolog. fluid:

The matter system consists of

$n_{eff} \equiv n_b + \frac{7}{8} n_f$ species of massless d.o.f.,

all at the same temperature T, with

equation of state $p = \frac{1}{3}\varrho$ and

$$\varrho = \varrho(T) = \varkappa^4 T^4, \quad \varkappa^4 \equiv \frac{\pi}{30} n_{eff}.$$

No assumption is made about $s = s(T)$.

 ... as in equilibrium !

The postulate implies:

$$\mathcal{P} \equiv \frac{1}{T}\,\tilde{\mathcal{P}} = \frac{1}{T}\left[\frac{d}{dt}(\rho a^3) + p\,\frac{d}{dt}(a^3)\right]$$

$T = \rho^{1/4}/\varkappa$ $\qquad$ $p = \rho/3$

$$= \frac{d}{dt}\Big[\underbrace{\tfrac{4}{3}\,\varkappa\, a^3 \rho^{3/4}}_{=\,S\,=\,s a^3}\Big]$$

integrate:

$$\boxed{S(t) = \frac{4}{3}\,\varkappa\, a^3\, \rho^{3/4} + S_c}$$

the relevant solutions have $S(t=0) = S_c$

$$\boxed{s(t) = \frac{2\pi^2}{45}\, n_{\mathrm{eff}}\, T(t)^3 + \frac{S_c}{a^3}}$$

For $S_c = 0$ exactly the proper entropy density of radiation in equilibrium! $\Rightarrow$

The "heat transfer" into the matter system caused by the RG running can account for the <u>entire</u> entropy carried by the massless fields today.

In particular $S_{\mathrm{CMBR}}/\text{Hubble vol.} \approx 10^{88}$ can evolve from $S(\text{big bang}) = 0$ without any standard dissipative process.

Solutions to the improved Einstein eq.

(1) Solve RG eqs. for EH-truncation with realistic parameter values $g_T = 10^{-60}$, $k_T = 10^{-30}\, m_{Pl}$.

(2) Solve imp. Einstein eq. with $k(t) \sim H(t)$ and $p = w\rho$.

Example: NGFP regime $\begin{cases} \Lambda(k) = \lambda_* k^2 \\ G(k) = g_* / k^2 \end{cases}$

1-parameter family of solutions labeled by $\Omega_\Lambda^* \in (0,1)$:

$$\boxed{\begin{aligned} & a(t) \sim t^\alpha, \quad \alpha = \frac{2}{(3+3w)(1-\Omega_\Lambda^*)} \\ & \rho(t) = \frac{2\,\Omega_\Lambda^*}{9\pi\, g_* \lambda_* (1+w)^4 (1-\Omega_\Lambda^*)^3} \cdot \frac{1}{t^4} \\ & G(t) = \frac{3\, g_* \lambda_* (1+w)^2 (1-\Omega_\Lambda^*)^2}{4\,\Omega_\Lambda^*} \cdot t^2 \\ & \Lambda(t) = \frac{4\,\Omega_\Lambda^*}{3(1+w)^2 (1-\Omega_\Lambda^*)^2} \cdot \frac{1}{t^2} \end{aligned}}$$

$$\rightsquigarrow \Omega_\Lambda \equiv \frac{\rho_\Lambda}{\rho_{crit}} = \text{const} = \Omega_\Lambda^*, \quad \Omega_M = 1 - \Omega_\Lambda$$

Properties of the NGFP solutions:

$\Omega_\Lambda^* \in (\frac{1}{2}, 1)$:

$$\boxed{a(t) \sim t^\alpha, \quad \alpha > 1}$$

- accelerated expansion, "power law inflation"
- Λ-dominated: $\Omega_\Lambda > \Omega_M = 1 - \Omega_\Lambda$
- $\mathcal{P} > 0$, $\quad S(t \to 0) = S_c \quad (= 0)$
- no particle horizon
- $\alpha \to \infty$ for $\Omega_\Lambda^* \nearrow 1$: $\approx$ de Sitter

$\Omega_\Lambda^* = \frac{1}{2}$:

$$\boxed{a(t) \sim t}$$

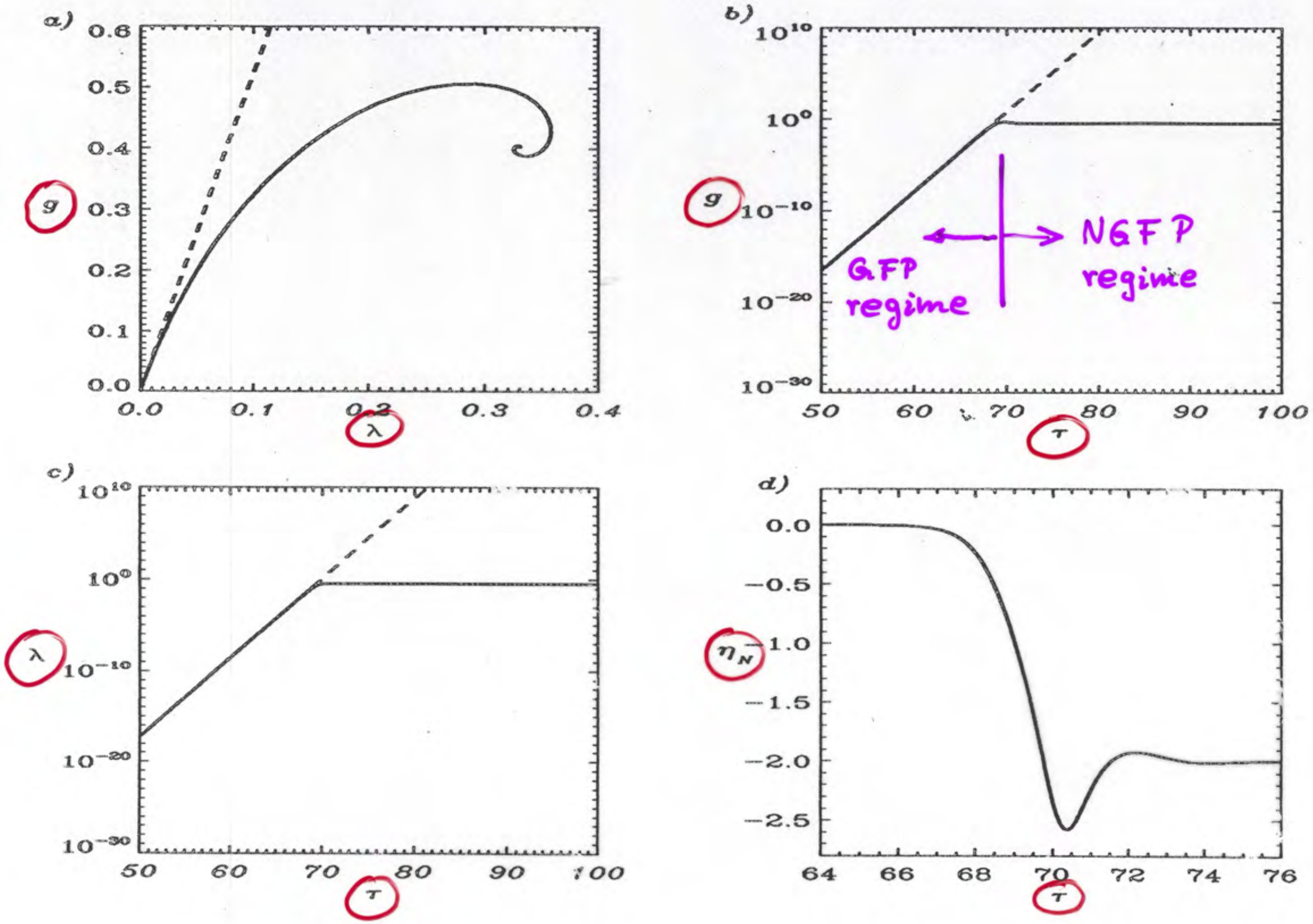
- $\Omega_M = \Omega_\Lambda$
- $\mathcal{P} = 0$; $D^\mu T_{\mu\nu} = 0$, $\dot{\Lambda} + 8\pi \rho \dot{G} = 0$
- no particle horizon

$\Omega_\Lambda^* \in (0, \frac{1}{2})$:

$$\boxed{a(t) \sim t^\alpha, \quad \alpha < 1}$$

- decelerated expansion
- matter dominated: $\Omega_M > \Omega_\Lambda$
- $\mathcal{P} < 0$, entropy decreases,
  $S(t \to 0) = +\infty$

The RG-trajectory with realistic parameter values



$$g_T = 10^{-60}$$

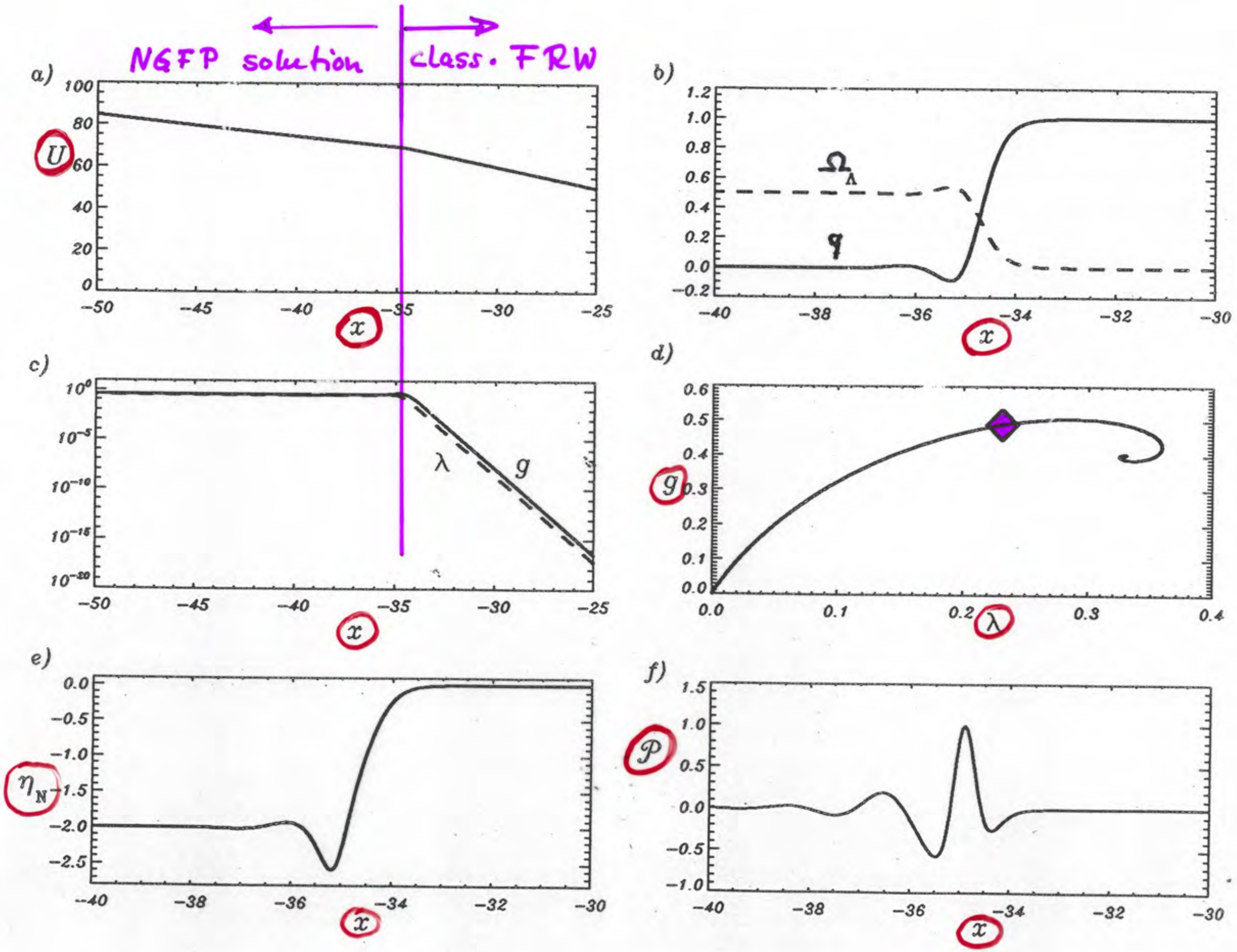
$$k_T = 10^{-30}\, m_{Pl}$$

$$\tau \equiv \ln\left(\frac{k}{k_T}\right)$$

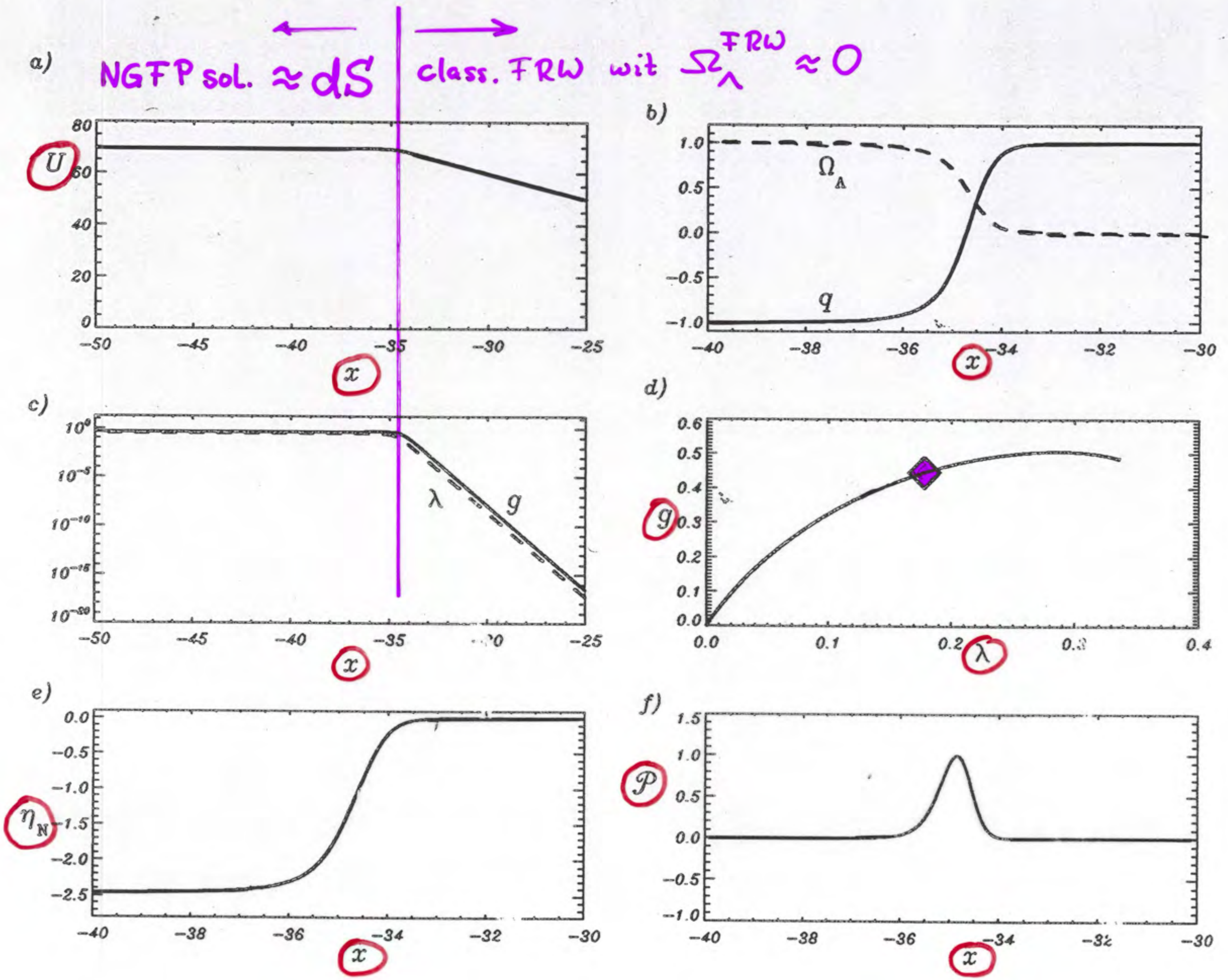
Complete Cosmology: $\boxed{\Omega_\Lambda^* = \frac{1}{2}}$ $(\alpha = 1)$

$U \equiv \ln \frac{H}{H_T}$

$x = \ln \frac{a}{a_T}$



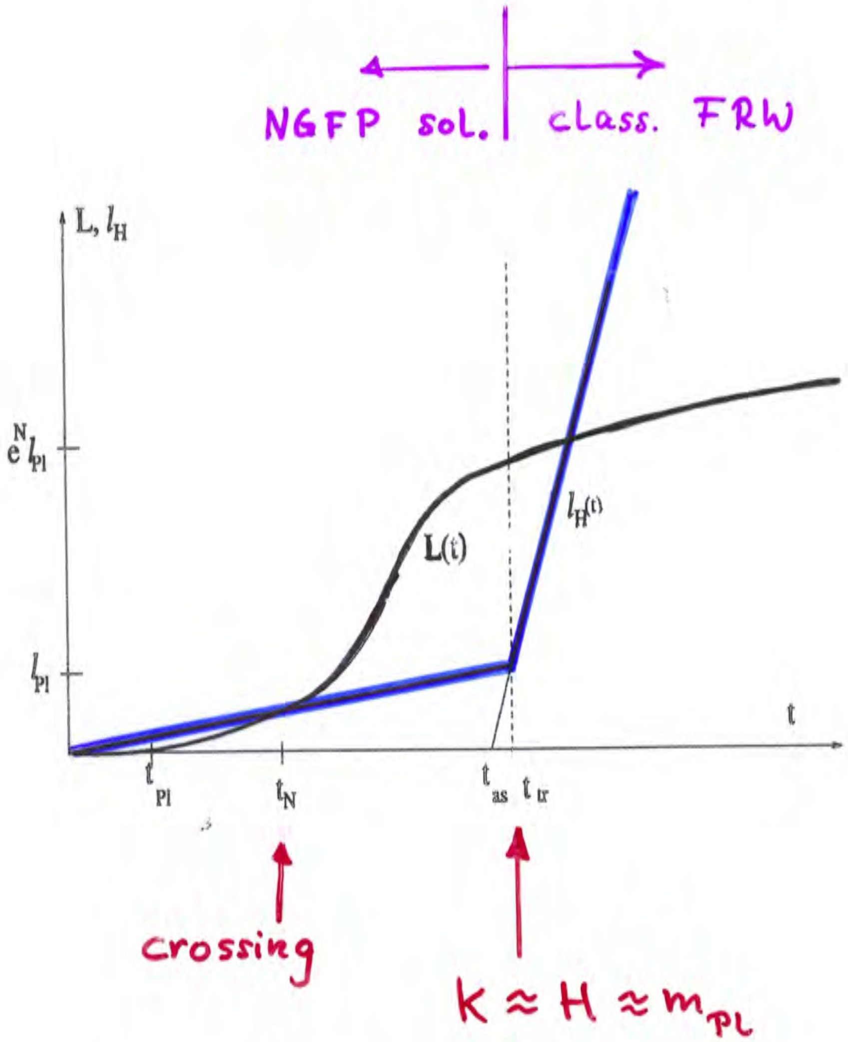
Complete Cosmology : $\boxed{\Omega_\Lambda^* = 0.98}$ $(\alpha = 25)$



$$U \equiv \ln \frac{H}{H_T}$$

$$x \equiv \ln \frac{a}{a_T}$$

$\alpha \gg 1$:



Hubble radius: $\boxed{\ell_H(t) \equiv \frac{1}{H(t)}}$

Proper length of structure with comoving length Δx:

$$\boxed{L(t) = \Delta x \cdot a(t)}$$

example: $\Omega_\Lambda^* = 0.98$, $\alpha = 25$

$t_{tr} = 25\, t_{Pl}$

$N = 60 \rightsquigarrow t_{60} = 2.05\, t_{Pl}$
$= t_{tr} / 12.2$

Generating Primordial Density Perturbations

Decoherence Scenario:

Classical fluctuations $\delta\rho(\vec{x})$ originate from quantum fluctuations of the curvature, $\delta R^{\cdot\cdot}_{\cdots}(\vec{x})$:

$$\langle\!\langle \delta\rho(\vec{x})\, \delta\rho(0) \rangle\!\rangle$$

$$\propto \langle \delta R^{\cdot\cdot}_{\cdots}(\vec{x})\, \delta R^{\cdot\cdot}_{\cdots}(0) \rangle \propto \frac{1}{|\vec{x}|^4}$$

dictated by properties of the UV-FP $(\eta_* = -2)$

for $a|\vec{x}| \ll \ell_H$

$\Rightarrow$ $\delta\rho$ - power spectrum for <u>sub</u>-Hubble scale modes is scale invariant !

($n = 1$, "Harrison - Zeldovich")

These modes cross the Hubble radius and become "super-Hubble" in the NGFP cosmologies with $\alpha > 1$.

They can act as seeds for structure formation.

Conclusion

Cosmology is a natural laboratory for confronting the predictions of asymptotically safe QEG with observations.

Candidates for observable / observed phenomena possibly explained by QEG :

- entropy of matter
- primordial density perturbations
  - NGFP regime is a "critical phenomenon"
  - Λ-driven inflation
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