Unconventional Cosmology

Robert Brandenberger
McGill University

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Outline

Current Paradigm for Early Universe Cosmology

The Inflationary Universe Scenario is the current paradigm of early universe cosmology.

Successes:

- Solves horizon problem
- Solves flatness problem
- Solves size/entropy problem
- Provides a causal mechanism of generating primordial cosmological perturbations (Chibisov & Mukhanov, 1981).

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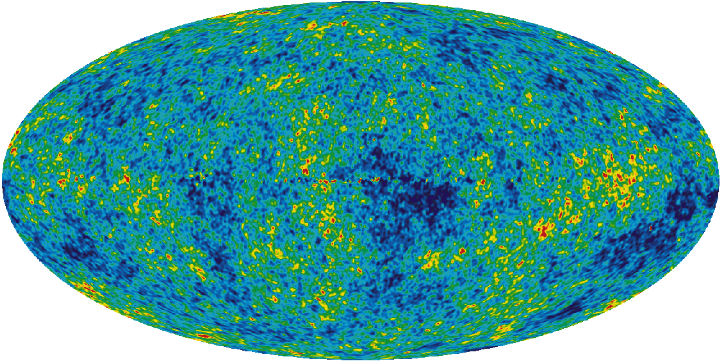
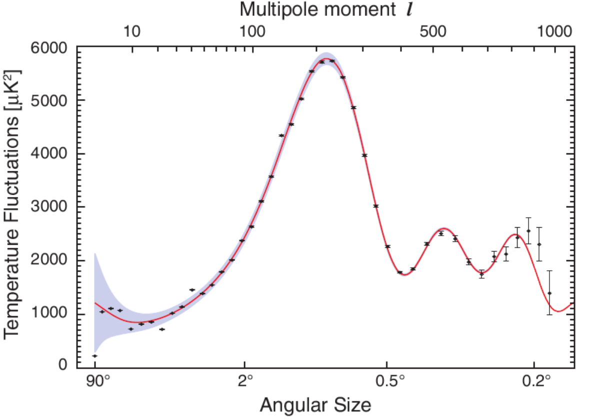


Credit: NASA/WMAP Science Team



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Challenges for the Current Paradigm

- In spite of the phenomenological successes, the inflationary scenario suffers from important conceptual problems.
- Alternatives to the inflationary universe scenario are thus needed.
- Question: Can input from new fundamental physics can help construct new paradigms which can overcome the problems of inflation.
- Question: Can these new paradigms be tested in cosmological observations?

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Review of Inflationary Cosmology

Context:

- General Relativity
- Scalar Field Matter

$$\text{Metric}: \; ds^2 = dt^2 - a(t)^2 d\mathbf{x}^2 \tag{1}$$

Inflation:

- phase with $a(t) \sim e^{tH}$
- requires matter with $p \sim -\rho$
- requires a slowly rolling scalar field φ
- - in order to have a potential energy term
- - in order that the potential energy term dominates sufficiently long

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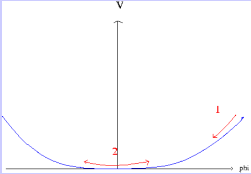
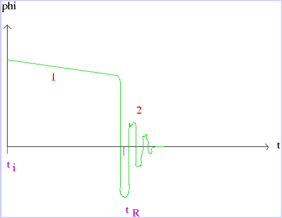
Time line of inflationary cosmology:

- t_i: inflation begins
- t_R: inflation ends, reheating

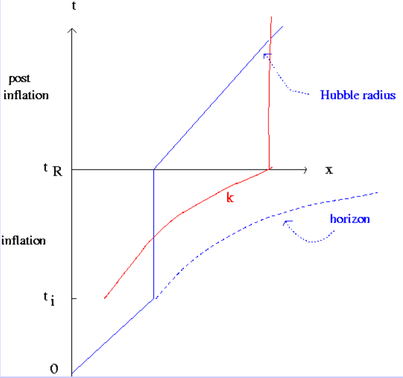
Matter scalar field:



Scalar field evolution:



Space-time sketch of inflationary cosmology:



Note:

- $H = \frac{\dot{a}}{a}$
- curve labelled by k: wavelength of a fluctuation

- inflation renders the universe large, homogeneous and spatially flat
- horizon expands exponentially $\rightarrow$ horizon problem of Standard Big Bang cosmology solved
- classical matter redshifts $\rightarrow$ matter vacuum remains
- quantum vacuum fluctuations: seeds for the observed structure [Chibisov & Mukhanov, 1981]
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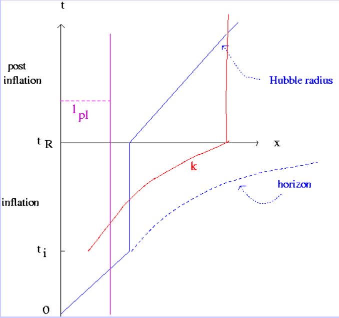
Conceptual Problems of Inflationary Cosmology

- Nature of the scalar field φ (the “inflaton")
- Conditions to obtain inflation (initial conditions, slow-roll conditions, graceful exit and reheating)
- Amplitude problem
- Trans-Planckian problem
- Singularity problem
- Cosmological constant problem
- Applicability of General Relativity

Trans-Planckian Problem



- **Success of inflation**: At early times scales are inside the Hubble radius → causal generation mechanism is possible.
- **Problem:** If time period of inflation is more than $70H^{-1}$, then $\lambda_p(t) < l_{pl}$ at the beginning of inflation
- → new physics MUST enter into the calculation of the fluctuations.

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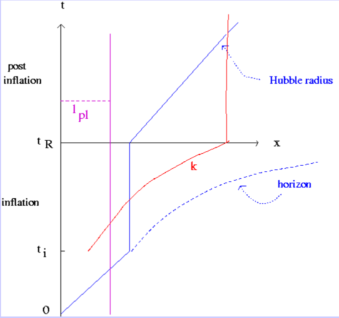

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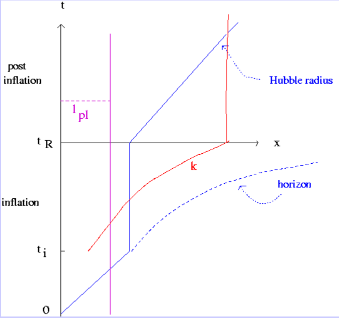

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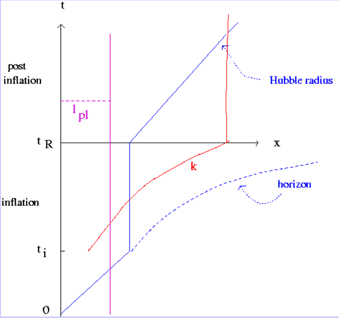

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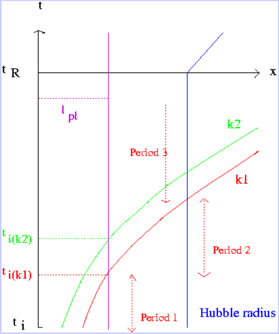
Trans-Planckian Window of Opportunity



- If evolution in Period I is non-adiabatic, then scale-invariance of the power spectrum will be lost [J. Martin and RB, 2000]
- $\rightarrow$ Planck scale physics testable with cosmological observations!

Singularity Problem

- Standard cosmology: Penrose-Hawking theorems $\rightarrow$ initial singularity $\rightarrow$ incompleteness of the theory.
- Inflationary cosmology: In scalar field-driven inflationary models the initial singularity persists [Borde and Vilenkin] $\rightarrow$ incompleteness of the theory.

Penrose-Hawking theorems:

- Ass: i) Einstein action, 2) weak energy conditions $\rho > 0, \quad \rho + 3p \geq 0$
- $\rightarrow$ space-time is geodesically incomplete.

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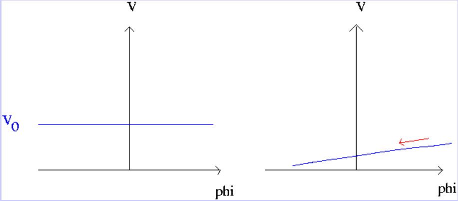
Cosmological Constant Problem



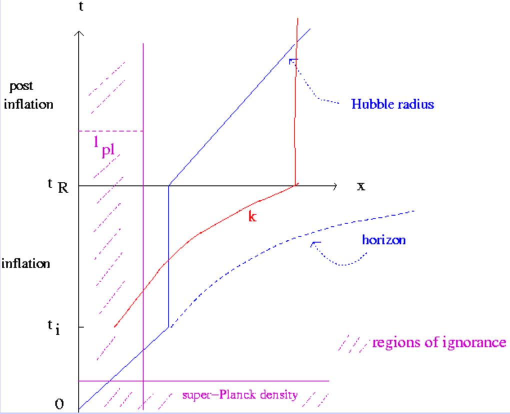
- Quantum vacuum energy does not gravitate.
- Why should the almost constant $V(\varphi)$ gravitate?

$$\frac{V_0}{\Lambda_{obs}} \sim 10^{120} \qquad (2)$$

Applicability of GR

- In all approaches to quantum gravity, the Einstein action is only the leading term in a low curvature expansion.
- Correction terms may become dominant at much lower energies than the Planck scale.
- Correction terms will dominate the dynamics at high curvatures.
- The energy scale of inflation models is typically $\eta \sim 10^{16}$GeV.
- $\rightarrow \eta$ too close to m_{pl} to trust predictions made using GR.

Zones of Ignorance



Message

- Current realizations of inflation have serious conceptual problems.
- We need a new paradigm of very early universe cosmology based on new fundamental physics.

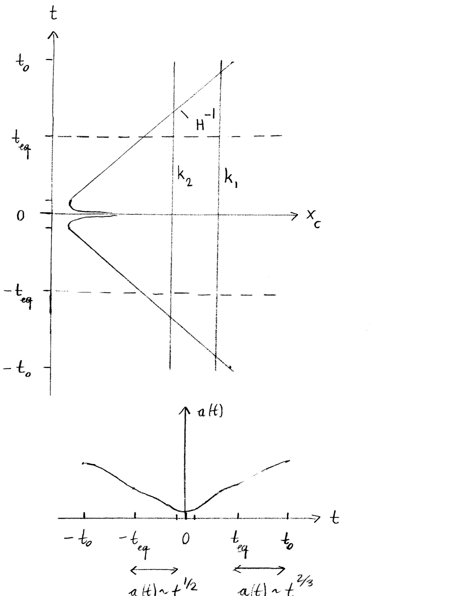
Alternative Scenarios

- Nonsingular cosmologies with a matter-dominated phase of contraction
- Emergent universe scenario [e.g. string gas cosmology]
- Pre-Big-Bang scenario [Gasperini and Veneziano]
- Ekpyrotic universe scenario [Khoury, Ovrut, Steinhardt and Turok]
- Conformal cosmology [Rubakov et al.]
- ....

Plan

Matter Bounce Scenario

F. Finelli and R.B., *Phys. Rev. D65, 103522 (2002)*, D. Wands, *Phys. Rev. D60 (1999)*



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- No horizon problem [horizon $\neq$ Hubble radius]
- Flatness problem mitigated
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Matter Bounce

Emergent Universe

Perturbations

Structure 1

Structure 2
Overview
Analysis
Signatures in CMB anisotropy maps

Structure 3
Basics
Predictions
HL fluctuations

Conclusions

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Overview

In order to obtain a bouncing cosmology it is necessary to:

- either modify the gravitational action
- or introduce a new form of matter which violates the NEC (null energy condition).

It is well motivated to consider models which go beyond the standard coupling of General Relativity to matter obeying the NEC - any approach to quantizing gravity yields terms in the effective action for the metric and matter fields which contain higher derivatives.

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Warmup: Quintom Bounce Model

Y. Cai, T. Qiu, R.B., Y. Piao and X. Zhang, *JCAP 0803:013 (2008)*

Idea:

- Begin with a regular scalar matter field ϕ.
- Introduce a second scalar field $\tilde{\phi}$ with the wrong sign kinetic term in the action.

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{1}{2}\partial_\mu\tilde{\phi}\partial^\mu\tilde{\phi} + \frac{1}{2}M^2\tilde{\phi}^2$$

with $M \gg m$.

$\rightarrow$ it is possible to get a nonsingular bounce.

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Quintom Bounce Scenario

- Begin in a contracting universe, both ϕ and $\tilde{\phi}$ oscillating.
- Choose initial amplitudes such that the regular field ϕ dominates the total energy density $\rightarrow$ amplitude A of ϕ much larger than the amplitude $\tilde{A}$ of $\tilde{\phi}$.
- $\rightarrow$ matter dominated phase of contraction.
- When $A \gg m_{pl}$ the oscillations in ϕ freeze out.
- But $\tilde{\phi}$ is still oscillating $\rightarrow$ energy density of $\tilde{\phi}$ catches up.
- There will be a time $t = 0$ at which $\rho_{total} = 0$.
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Problems for the Quintom Bounce

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Ghost Condensate Bounce

C. Lin, R.B. and L. Perreault Levasseur, 2010

Idea: Instead of a ghost field use a ghost condensate.

Ghost condensate: Take a field ϕ which when expanded about $\phi = 0$ has ghost-like excitations. Construct a Lagrangian such that there is a stable condensate which breaks local Lorentz invariance and about which the model is perturbatively ghost free.

$$\mathcal{L} = M^4 P(X) - V(\phi)\,, \quad X \equiv -g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi \tag{3}$$

$$P(X) = \frac{1}{8}(X - c^2)^2\,, \tag{4}$$

Ghost condensate for cosmology:

$$\phi = ct\,. \tag{5}$$

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Ghost Condensate Bounce II

Friedmann Equations for ghost condensate matter:

$$3M_p^2H^2 = M^4(2XP' - P) + V + \rho_m \, , \quad (6)$$

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Consider fluctuations of the ghost condensate field:

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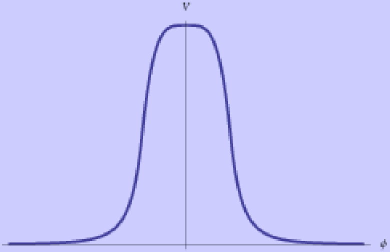
If $\dot{\pi} < 0$ then the ghost condensate carrier negative gravit. energy.

Equation of motion for the ghost condensate field (leading order in $\dot{\pi}$):

$$c^2 a^{-3} \partial_t \left(a^3 \dot{\pi} \right) = -2M^{-4} \frac{\partial V}{\partial \phi} . \tag{11}$$

Potential:



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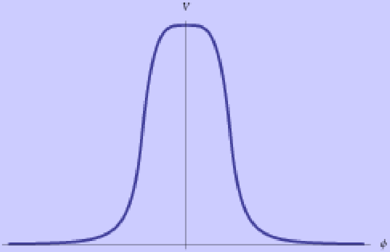
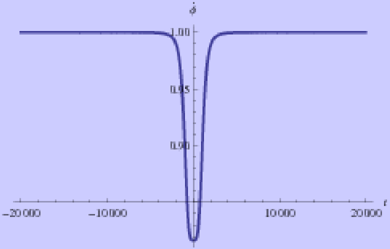


Ghost Condensate Bounce IV

Condition for the bounce point:

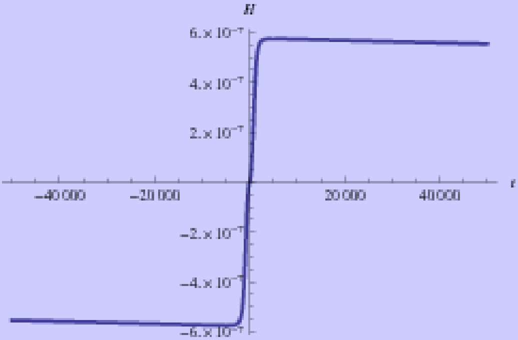
$$M^4 c^3 \dot{\pi} = -V \, . \tag{12}$$

Numerical results:



Ghost Condensate Bounce V

Numerical results (ctd.):



Ghost Condensate Bounce VI

Stability towards anisotropic stress:

$$V(\phi) = V_0 M^{-\alpha}\phi^{-\alpha}, \tag{13}$$

$$c^2\partial_t(a^3\dot{\pi}) = -2a^3 M^{-4-\alpha}\frac{\partial V}{\partial \phi}. \tag{14}$$

$$\dot{\pi} \sim t^{-\alpha}. \tag{15}$$

If $\alpha \geq 6$ then the model is stable against anisotropic stress.

Warmup: Nonsingular Universe Construction

R.B., V. Mukhanov and A. Sornborger., *Phys. Rev.* **D48**, *1629 (1993)*

Motivation: Find gravitational action which forces all solutions to tend to de Sitter at high curvatures.

- Find invariant I which has the property that $I = 0 \rightarrow g_{\mu\nu} = g_{\mu\nu}^{dS}$
- Result: $I = 4R_{\mu\nu}R^{\mu\nu} - R^2$.
- Lagrange multiplier construction of a higher derivative gravity action $\mathcal{L} = R + \varphi I - V(\varphi)$
- $V(\varphi)$ constructed such that i) $I \rightarrow 0$ at large values of R, ii) Einsteinian low R limit.
- Phase space analysis of homogeneous solutions $\rightarrow$ all solutions tend to de Sitter at high curvatures.

Warmup II: Ghost-Free Higher Derivative Model

T. Biswas, A. Mazumdar and W. Siegel, *JCAP 0603:009 (2006)*

Motivation: Find $F(R)$ action which is ghost-free about Minkowski space-time.

- Ghost-freeness $\rightarrow F(R)$ must contain all powers of $\nabla^2 R$.
- $F(R) = R + \sum \frac{c_n}{M^{2n}} R \nabla^{2n} R$
- Resulting theory is asymptotically free.
- Cosmological bouncing solutions result.

Hořava-Lifshitz Gravity

P. Hořava, *Phys. Rev.* **D79**, *084008 (2009)*

Power-counting renormalizable quantum theory of gravity in 4d based on anisotropic scaling between space and time:

$$t \to l^z t \, , \quad x^i \to l x^i \, .$$

Usual metric degrees of freedom:

$$ds^2 = -N^2 dt^2 + g_{ij}(dx^i + N^i dt)(dx^j + N^j dt) \, .$$

Most general action consistent with residual symmetries and power-counting renormalizability:

$$S^g = S^g_K + S^g_V \, .$$

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Hořava-Lifshitz Gravity II

Kinetic piece of the action:

$$S_K^g = \frac{2}{\kappa^2} \int dt d^3x \sqrt{g} N \left(K_{ij} K^{ij} - \lambda K^2 \right) ,$$

where

$$K_{ij} = \frac{1}{2N} [\dot{g}_{ij} - \nabla_i N_j - \nabla_j N_i] ,$$

Potential piece of the action (special case - detailed balance):

$$\begin{aligned} S_V^g &= \int dt d^3x \sqrt{g} N [- \frac{\kappa^2}{2w^4} C_{ij} C^{ij} + \frac{\kappa^2 \mu}{2w^2} \epsilon^{ijk} R_{il} \nabla_j R^l_k \\ &- \frac{\kappa^2 \mu^2}{8} R_{ij} R^{ij} + \frac{\kappa^2 \mu^2}{8(1-3\lambda)} \left(\frac{1-4\lambda}{4} R^2 + \Lambda R - 3\Lambda^2 \right)] \end{aligned}$$

Hořava-Lifshitz Bounce I

R.B., *Phys. Rev.* **D80, 043516 (2009)**

In the presence of nonvanishing spatial curvature, the higher spatial derivative terms in the geometrical action act as ghost matter.

The FRWL equations become:

$$\frac{6(3\lambda - 1)}{\kappa^2} H^2 = \rho - \frac{3\kappa^2\mu^2}{8(3\lambda - 1)} \left(\frac{\bar{k}}{a^2} - \Lambda\right)^2 ,$$

where $\bar{k}$ is the spatial curvature constant;

$$\frac{2(3\lambda - 1)}{\kappa^2} \dot{H} = -\frac{(1+w)\rho}{2} + \frac{\kappa^2\mu^2}{4(3\lambda - 1)} \left(\frac{\bar{k}}{a^2} - \Lambda\right) \frac{\bar{k}}{a^2} .$$

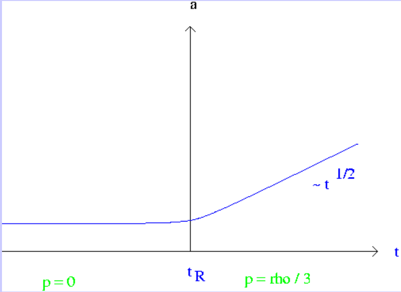
Hořava-Lifshitz Bounce II

- $\bar{k}/a^4$ term acts as ghost radiation!
- For a general potential there is also ghost anisotropic stress.
- $\rightarrow$ in the presence of spatial curvature a cosmological bounce will occur.
- The bounce is stable against the presence of radiation.
- The bounce is marginally stable against the presence of anisotropic stress.

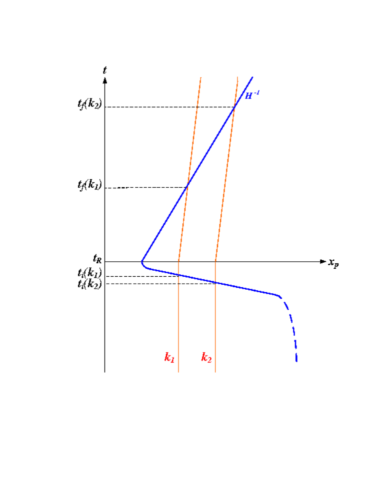
Plan

Dynamics

We consider the following background dynamics for the scale factor $a(t)$:



Space-time sketch



Features

- No horizon problem [horizon $\neq$ Hubble radius]
- Flatness problem mitigated
- No structure formation problem
- No trans-Planckian problem for fluctuations
- Entropy (size) problem not solved

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Principles of String Gas Cosmology

R.B. and C. Vafa, *Nucl. Phys. B316:391 (1989)*

Idea: make use of the new symmetries and new degrees of freedom which string theory provides to construct a new theory of the very early universe.

Assumption: Matter is a gas of fundamental strings

Assumption: Space is compact, e.g. a torus.

Key points:

- New degrees of freedom: string oscillatory modes
- Leads to a maximal temperature for a gas of strings, the Hagedorn temperature
- New degrees of freedom: string winding modes
- Leads to a new symmetry: physics at large R is equivalent to physics at small R

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T-Duality

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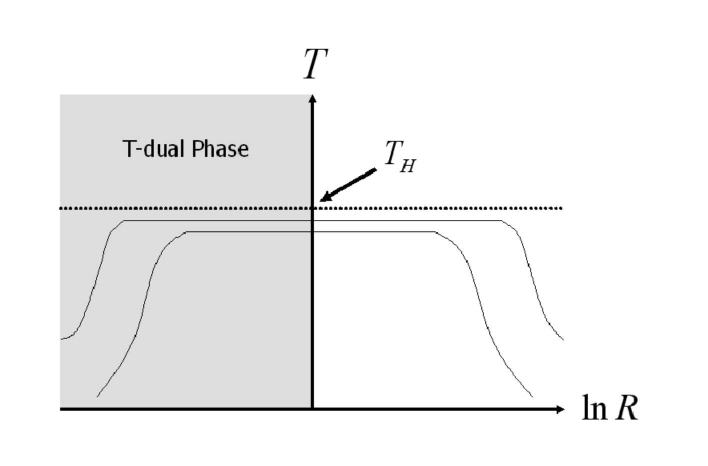
- Momentum modes: $E_n = n/R$
- Winding modes: $E_m = mR$
- Duality: $R \rightarrow 1/R \quad (n, m) \rightarrow (m, n)$
- Mass spectrum of string states unchanged
- Symmetry of vertex operators
- Symmetry at non-perturbative level $\rightarrow$ existence of D-branes

Adiabatic Considerations

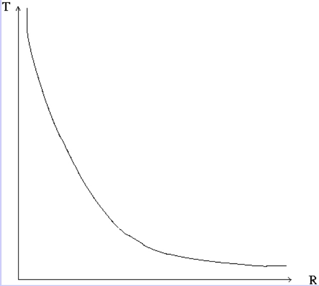
R.B. and C. Vafa, *Nucl. Phys. B316:391 (1989)*

Temperature-size relation in string gas cosmology



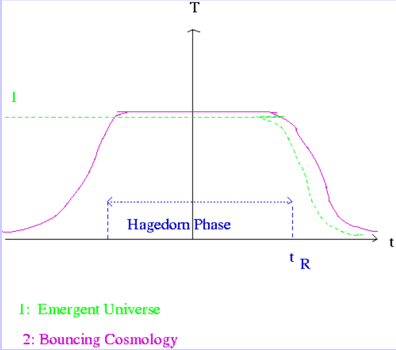
Singularity Problem in Standard and Inflationary Cosmology

Temperature-size relation in standard cosmology



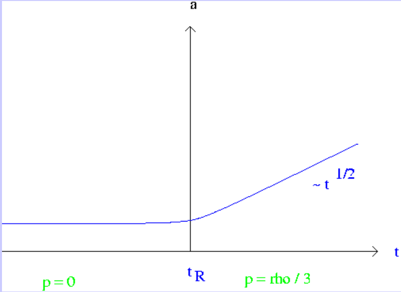
Dynamics

Assume some action gives us $R(t)$



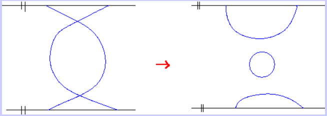
Dynamics II

We will thus consider the following background dynamics for the scale factor $a(t)$:



Dimensionality of Space in SGC

- Begin with all 9 spatial dimensions small, initial temperature close to T_H $\rightarrow$ winding modes about all spatial sections are excited.
- Expansion of any one spatial dimension requires the annihilation of the winding modes in that dimension.



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- $\rightarrow$ dynamical explanation of why there are exactly three large spatial dimensions.

Note: this argument assumes constant dilaton [R. Danos, A. Frey and A. Mazumdar]

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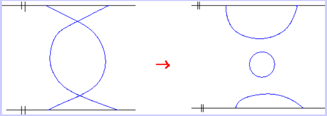

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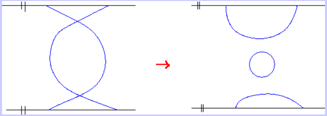

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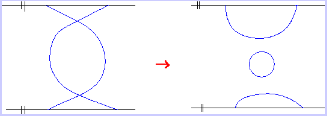

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Moduli Stabilization in SGC

Size Moduli [S. Watson, 2004; S. Patil and R.B., 2004, 2005]

- winding modes prevent expansion
- momentum modes prevent contraction
- $\rightarrow$ $V_{eff}(R)$ has a minimum at a finite value of R, $\rightarrow$ R_{min}
- in heterotic string theory there are enhanced symmetry states containing both momentum and winding which are massless at R_{min}
- $\rightarrow V_{eff}(R_{min}) = 0$
- $\rightarrow$ size moduli stabilized in Einstein gravity background

Shape Moduli [E. Cheung, S. Watson and R.B., 2005]

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Dilaton stabilization in SGC

- The only remaining modulus is the dilaton
- Make use of gaugino condensation to give the dilaton a potential with a unique minimum
- $\rightarrow$ diltaton is stabilized
- Dilaton stabilization is consistent with size stabilization [R. Danos, A. Frey and R.B., 2008]

Preliminary Conclusions

- Inflationary cosmology suffers from conceptual problems, e.g. singularity problem and trans-Planckian problem for fluctuations.
- Alternative scenarios exist which do not suffer from these problems.
- Matter bounce: non-singular bouncing cosmology with a matter-dominated phase of contraction.
- Emergent cosmology, e.g. string gas cosmology.
- Preview: Both alternative scenarios yield a scale-invariant spectrum of cosmological perturbations and are thus compatible with all current observations.
- Preview: Each of these two scenarios makes specific predictions for future observations with which it can be distinguished from inflationary cosmology.

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Theory of Cosmological Perturbations: Basics

Cosmological fluctuations connect early universe theories with observations

- Fluctuations of matter $\rightarrow$ large-scale structure
- Fluctuations of metric $\rightarrow$ CMB anisotropies
- N.B.: Matter and metric fluctuations are coupled

Key facts:

- 1. Fluctuations are small today on large scales
- $\rightarrow$ fluctuations were very small in the early universe
- $\rightarrow$ can use linear perturbation theory
- 2. Sub-Hubble scales: matter fluctuations dominate
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Quantum Theory of Linearized Fluctuations

V. Mukhanov, H. Feldman and R.B., *Phys. Rep. 215:203 (1992)*

Step 1: Metric including fluctuations

$$ds^2 = a^2[(1+2\Phi)d\eta^2 - (1-2\Phi)d\mathbf{x}^2] \qquad (16)$$

$$\varphi = \varphi_0 + \delta\varphi \qquad (17)$$

Note: Φ and $\delta\varphi$ related by Einstein constraint equations

Step 2: Expand the action for matter and gravity to second order about the cosmological background:

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where

$$v \sim a\zeta$$

where ζ is the curvature fluctuation in co-moving coordinates.

Step 3: Resulting equation of motion (Fourier space)

$$v_k'' + (k^2 - \frac{z''}{z})v_k = 0 \tag{21}$$

Features:

- oscillations on sub-Hubble scales
- squeezing on super-Hubble scales $v_k \sim z$

Quantum vacuum initial conditions:

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- squeezing on super-Hubble scales $v_k \sim z$

Quantum vacuum initial conditions:

$$v_k(\eta_i) = (\sqrt{2k})^{-1} \qquad (22)$$

Scale Invariance

Power spectrum:

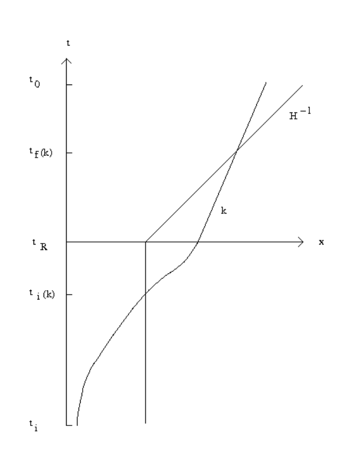
$$\mathcal{P}_v(k,t) \equiv k^3 |v_k(t)|^2$$

Scale invariance:

$$\mathcal{P}_\zeta(k,t) \sim k^{n-1} \sim k^0 \, .$$

Plan

Structure formation in inflationary cosmology



N.B. Perturbations originate as quantum vacuum fluctuations.

Origin of Scale-Invariance

Heuristic analysis [W. Press, 1980]: time-translation symmetry of de Sitter phase $\rightarrow$ scale-invariance of spectrum.

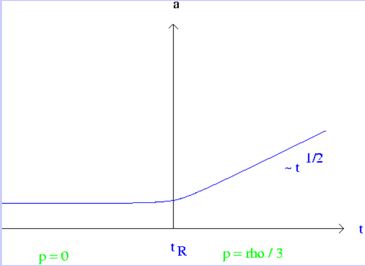
Mathematical analysis [Mukhanov and Chibisov, 1982]:

$$\begin{aligned}
\mathcal{P}_\zeta(k,t) &\propto \mathcal{P}_v(k,t) \\
&\sim k^3\left(\frac{a(t)}{a(t_H(k))}\right)^2 |v_k(t_H(k))|^2 \\
&\sim k^3 \eta_H(k)^2 |v_k(t_H(k))|^2 \\
&\sim k^0
\end{aligned}$$

using $a(\eta) \sim \eta^{-1}$ in the de Sitter phase and $\eta_H(k) \sim k^{-1}$.

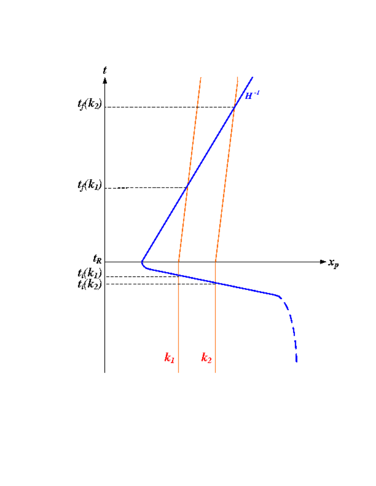
Plan

Background for string gas cosmology



Structure formation in string gas cosmology

A. Nayeri, R.B. and C. Vafa, *Phys. Rev. Lett. 97:021302 (2006)*



N.B. Perturbations originate as thermal string gas fluctuations.

Method

- Calculate matter correlation functions in the Hagedorn phase (neglecting the metric fluctuations)
- For fixed k, convert the matter fluctuations to metric fluctuations at Hubble radius crossing $t = t_i(k)$
- Evolve the metric fluctuations for $t > t_i(k)$ using the usual theory of cosmological perturbations

Extracting the Metric Fluctuations

Ansatz for the metric including cosmological perturbations and gravitational waves:

$$ds^2 = a^2(\eta)\big((1+2\Phi)d\eta^2 - [(1-2\Phi)\delta_{ij} + h_{ij}]dx^i dx^j\big) \, . \quad (23)$$

Inserting into the perturbed Einstein equations yields

$$\langle|\Phi(k)|^2\rangle = 16\pi^2 G^2 k^{-4} \langle \delta T^0{}_0(k) \delta T^0{}_0(k) \rangle \, , \quad (24)$$

$$\langle|h(k)|^2\rangle = 16\pi^2 G^2 k^{-4} \langle \delta T^i{}_j(k) \delta T^i{}_j(k) \rangle \, . \quad (25)$$

Power Spectrum of Cosmological Perturbations

Key ingredient: For thermal fluctuations:

$$\langle \delta\rho^2 \rangle = \frac{T^2}{R^6} C_V \,. \qquad (26)$$

Key ingredient: For string thermodynamics in a compact space

$$C_V \approx 2 \frac{R^2/\ell_s^3}{T(1 - T/T_H)} \,. \qquad (27)$$

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Power spectrum of cosmological fluctuations

$$P_\Phi(k) = 8G^2 k^{-1} < |\delta\rho(k)|^2 > \quad (28)$$
$$= 8G^2 k^2 < (\delta M)^2 >_R \quad (29)$$
$$= 8G^2 k^{-4} < (\delta\rho)^2 >_R \quad (30)$$
$$= 8G^2 \frac{T}{\ell_s^3} \frac{1}{1 - T/T_H} \quad (31)$$

Key features:

- scale-invariant like for inflation
- slight red tilt like for inflation

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Comments

- Evolution for $t > t_i(k)$: $\Phi \simeq \mathrm{const}$ since the equation of state parameter $1 + w$ stays the same order of magnitude unlike in inflationary cosmology.
- Squeezing of the fluctuation modes takes place on super-Hubble scales like in inflationary cosmology $\rightarrow$ acoustic oscillations in the CMB angular power spectrum
- In a dilaton gravity background the dilaton fluctuations dominate $\rightarrow$ different spectrum [R.B. et al, 2006; Kaloper, Kofman, Linde and Mukhanov, 2006]

Spectrum of Gravitational Waves

R.B., A. Nayeri, S. Patil and C. Vafa, *Phys. Rev. Lett. (2007)*

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\begin{aligned}
P_h(k) &= 16\pi^2 G^2 k^{-1} < |T_{ij}(k)|^2 > && (32)\\
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Requirements

- static Hagedorn phase (including static dilaton) $\rightarrow$ new physics required.
- $C_V(R) \sim R^2$ obtained from a thermal gas of strings provided there are winding modes which dominate.
- Cosmological fluctuations in the IR are described by Einstein gravity.

Note: Specific higher derivative toy model: T. Biswas, R.B., A. Mazumdar and W. Siegel, 2006

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Network of cosmic superstrings

- Remnant of the Hagedorn phase: network of cosmic superstrings
- This string network will be present at all times and will achieve a scaling solution like cosmic strings forming during a phase transition.
- Scaling Solution: The network of strings looks statistically the same at all times when scaled to the Hubble radius.

Kaiser-Stebbins Effect

Space perpendicular to a string is conical with deficit angle

$$\alpha = 8\pi G\mu , \tag{36}$$

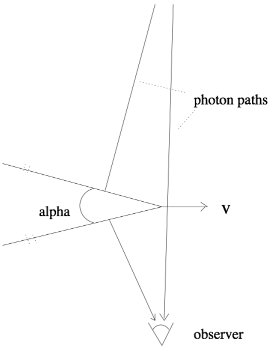
Photons passing by the string undergo a relative Doppler shift

$$\frac{\delta T}{T} = 8\pi\gamma(v)vG\mu , \tag{37}$$

$\rightarrow$ network of line discontinuities in CMB anisotropy maps

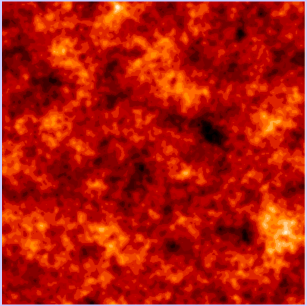
.

N.B. characteristic scale: comoving Hubble radius at the time of recombination $\rightarrow$ need good angular resolution to detect these edges.


photon paths
alpha
V
observer

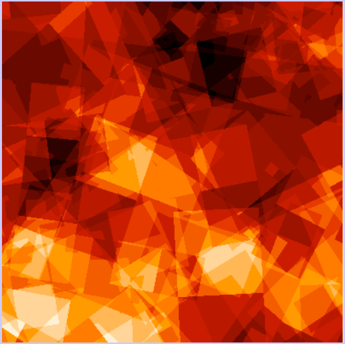
Gaussian temperature map

$10'^{0}$ x 10^{0} map of the sky at 1.5' resolution (South Pole Telescope specifications)



Cosmic string temperature map

10^0 x 10^0 map of the sky at 1.5' resolution

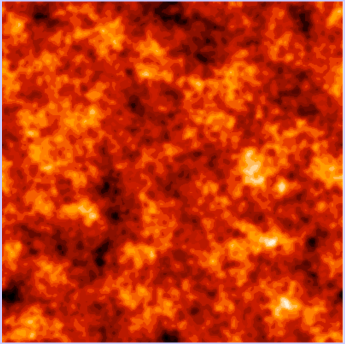


This signal is superimposed on the Gaussian map. The relative power of the string signature depends on $G\mu$ and is bound to contribute less than 10% of the power.

CANNY edge detection algorithm

- Challenge: pick out the string signature from the Gaussian "noise" which has a much larger amplitude
- New technique: use CANNY edge detection algorithm [Canny, 1986]
- Idea: find edges across which the gradient is in the correct range to correspond to a Kaiser-Stebbins signal from a string
- Step 1: generate "Gaussian" and "Gaussian plus strings" CMB anisotropy maps: size and angular resolution of the maps are free parameters, flat sky approximation, cosmic string toy model in which a fixed number of straight string segments is laid down at random in each Hubble volume in each Hubble time step between t_{rec} and t_0.

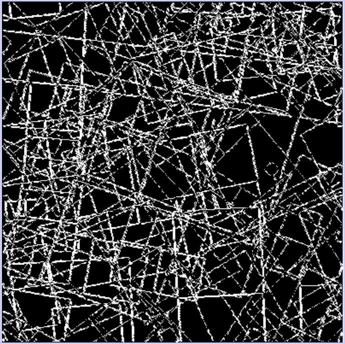
Temperature map Gaussian + strings



CANNY algorithm II

- Step 2: run the CANNY algorithm on the temperature maps to produce edge maps.
- Step 3: Generate histogram of edge lengths
- Step 4: Use Fisher combined probability test to check for difference compared to a Gaussian distribution.

Edge map



Preliminary results

- For South Pole Telescope (SPT) specification: limit $G\mu < 2 \times 10^{-8}$ can be set [A. Stewart and R.B., 2008, R. Danos and R.B., 2008]
- Anticipated SPT instrumental noise only insignficantly effects the limits [A. Stewart and R.B., 2008]
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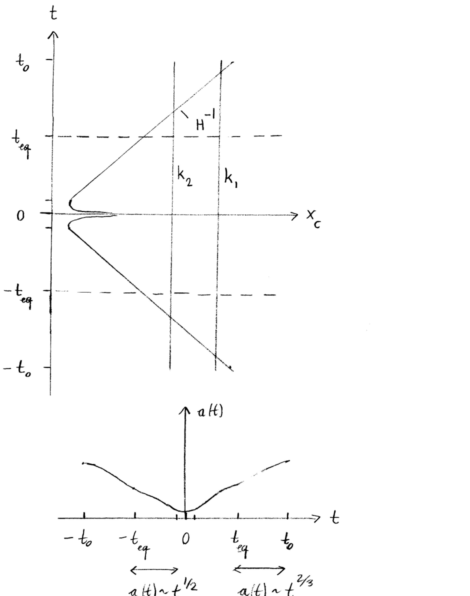
Plan

Matter Bounce Scenario

F. Finelli and R.B., *Phys. Rev. D65, 103522 (2002)*, D. Wands, *Phys. Rev. D60 (1999)*



Origin of Scale-Invariant Spectrum

- The initial vacuum spectrum is blue:

$$P_\zeta(k) \, = \, k^3 |\zeta(k)|^2 \, \sim \, k^2 \tag{38}$$

- The curvature fluctuations grow on super-Hubble scales in the contracting phase:

$$v_k(\eta) \, = \, c_1 \eta^2 + c_2 \eta^{-1} \, , \tag{39}$$

- For modes which exit the Hubble radius in the matter phase the resulting spectrum is scale-invariant:

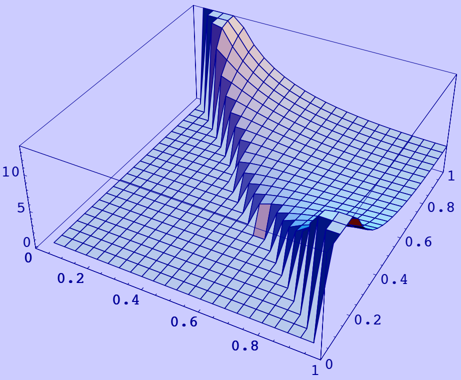
$$\begin{aligned} P_\zeta(k, \eta) \quad & \sim \quad k^3 |v_k(\eta)|^2 a^{-2}(\eta) \qquad (40) \\ & \sim \quad k^3 |v_k(\eta_H(k))|^2 \left(\frac{\eta_H(k)}{\eta}\right)^2 \sim k^{3-1-2} \\ & \sim \quad \text{const} \, , \end{aligned}$$

Transfer of the Spectrum through the Bounce

- In a nonsingular background the fluctuations can be tracked through the bounce explicitly (both numerically in an exact manner and analytically using matching conditions at times when the equation of state changes).
- Explicit computations have been performed in the case of quintom matter (Y. Cai et al, 2008), mirage cosmology (R.B. et al, 2007), Horava-Lifshitz bounce (X. Gao et al, 2010).
- Result: On length scales larger than the duration of the bounce the spectrum of v goes through the bounce unchanged.

Bispectrum of the Matter Bounce Scenario

Y. Cai, W. Xue, R.B. and X. Zhang, *JCAP 0905:011 (2009)*



Large tensor to scalar ratio

- The amplitude of the gravitational waves is squeezed with the same factor as that of the scalar modes.
- Thus, a large tensor to scalar ratio is generated.
- To render a matter bounce model consistent with observations which indicate $r < 0.2$ a mechanism which enhances the scalar modes around the bounce point is required.
- One solution: matter bounce curvaton (Y. Cai, R.B. and X. Zhang, 1101.0822).

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Matter Bounce Curvaton

Y. Cai, R.B. and X. Zhang, arXiv:1101.0822

- Add light scalar field ψ to the model.
- It acquires a scale-invariant spectrum of entropy fluctuations in the contracting phase.
- If ψ is coupled to the field which dominates at early times and if the equation of state changes during the bounce, then the entropy fluctuations seed an adiabatic mode via

$$\dot{\zeta} = -\frac{H}{\dot{H}}\nabla^2\Phi - 4\pi GH\sum_i \frac{Q_i}{\dot{\phi}_i}\left(\frac{\dot{\phi}_i^2}{\dot{H}}\right)^{\cdot} .$$

where

$$Q_i \equiv \delta\phi_i + \frac{\dot{\phi}_i}{H}\Phi .$$

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$$Q_i \equiv \delta\phi_i + \frac{\dot{\phi}_i}{H}\Phi \,.$$

- The new contribution to ζ inherits the scale invariance from that of the entropy mode.
- Depending on the model of the bounce, the contribution to ζ induced by the entropy mode may dominate.
- This leads to a suppression of r.

Issue: Extra scalar metric degree of freedom.

- GR: 10 + 1 degrees of freedom for metric and matter fluctuations.
- 4 + 1 scalar modes, 4 vector modes, 2 tensor modes.
- 4 gauge degrees of freedom: 2 scalar and 2 vector.
- Hamiltonian constraint, no ansotropic stress $\rightarrow$ 1 scalar degree of freedom.
- HL gravity: less symmetry $\rightarrow$ only 1 scalar gauge degree of freedom.
- $\rightarrow$ extra scalar metric degree of freedom.

Fluctuations in Hořava-Lifshitz Gravity

X. Gao, Y. Wang, R.B. and A. Riotto, *Phys. Rev.* **D81**, *083508 (2010)*

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Fluctuations in Hořava-Lifshitz Gravity II

Results of detailed analyses:

- In the non-projectable version of HL gravity the extra scalar metric degree of freedom is non-dynamical at linear order [X. Gao, Y. Wang, R.B. and A. Riotto (2010)].
- In the projectable version of HL gravity the extra scalar mode is present and either tachyonic or a ghost [A. Cerioni and R.B., 2010].
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X. Gao, Y. Wang, W. Xue and R.B., *JCAP 1002, 020 (2010)*

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- Since modes of interest are always in the extreme IR, the effects of the higher spatial derivative terms are highly suppressed.
- $\rightarrow$ Scale invariance of the spectrum survives the bounce phase.

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Plan

Conclusions

- Conventional (inflationary) cosmology has conceptual problems.
- Some of these problems are solved in un-conventional (alternative) scenarios which are in agreement with the current data on inhomogeneities.
- Emergent universe: universe begins in a quasi-static phase.
- Specific realization: string gas cosmology, predicts a slight blue tilt in the spectrum of gravitational radiation.
- Matter bounce: non-singular bouncing cosmology with a matter-dominated phase of contraction.
- Specific prediction: special shape of the three point function.

Message

- Maybe un-conventional cosmology is more conventional than what is now considered as conventional cosmology.
- Maybe experts on fundamental physics (both string theory and canonical quantum gravity) should not force inflation into their scenarios if inflation does not emerge naturally. Maybe one of the alternative scenarios will emerge much more naturally.

Action

Action: Dilaton gravity plus string gas matter

$$S = \frac{1}{\kappa}\left(S_g + S_\phi\right) + S_{SG}\,, \tag{41}$$

$$S_{SG} = -\int d^{10}x\sqrt{-g}\sum_\alpha \mu_\alpha \epsilon_\alpha\,, \tag{42}$$

where

- μ_α: number density of strings in the state α
- ϵ_α: energy of the state α.

Introduce comoving number density:

$$\mu_\alpha = \frac{\mu_{0,\alpha}(t)}{\sqrt{g_s}}\,, \tag{43}$$

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Energy-Momentum Tensor

Ansatz for the metric:

$$ds^2 = -dt^2 + a(t)^2 d\vec{x}^2 + \sum_{a=1}^{6} b_a(t)^2 dy_a^2 \,, \tag{44}$$

Contributions to the energy-momentum tensor

$$\rho_\alpha = \frac{\mu_{0,\alpha}}{\epsilon_\alpha \sqrt{-g}} \epsilon_\alpha^2 \,, \tag{45}$$

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Single string energy

ϵ_α is the energy of the string state α:

$$\epsilon_\alpha = \frac{1}{\sqrt{\alpha'}}\Big[\alpha' p_d^2 + b^{-2}(n,n) + b^2(w,w) + 2(n,w) + 4(N-1)\Big]^{1/2} , \quad (48)$$

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Background equations of motion

Radion equation:

$$\ddot{b} + \dot{b}\left(3\frac{\dot{a}}{a} + 5\frac{\dot{b}}{b}\right) = \frac{8\pi G \mu_{0,\alpha}}{\alpha' \sqrt{\hat{G}_{a}\epsilon_\alpha}} \tag{49}$$

$$\times \left[\frac{n_a^2}{b^2} - w_a^2 b^2 + \frac{2}{(D-1)}\left[b^2(w,w) + (n,w) + 2(N-1)\right]\right]$$

Scale factor equation:

$$\ddot{a} + \dot{a}\left(2\frac{\dot{a}}{a} + 6\frac{\dot{b}}{b}\right) = \frac{8\pi G \mu_{0,\alpha}}{\sqrt{\hat{G}_{i}\epsilon_\alpha}} \tag{50}$$

$$\times \left[\frac{p_d^2}{3} + \frac{2}{\alpha'(D-1)}\left[b^2(w,w) + (n,w) + 2(N-1)\right]\right],$$

Special states

Enhanced symmetry states

$$b^2(w,w) + (n,w) + 2(N-1) = 0 . \tag{51}$$

Stable radion fixed point:

$$\frac{n_a^2}{b^2} - w_a^2 b^2 = 0 . \tag{52}$$

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Gaugino condensation

Add a single non-perturbative ingredient - gaugino condensation - in order to fix the remaining modulus, the dilaton

Kähler potential: (standard)

$$\mathcal{K}(S) = -\ln(S+\bar{S}), \quad S = e^{-\Phi} + ia. \tag{53}$$

where $\Phi = 2\phi - 6\ln b$ is the 4-d dilaton, b is the radion and a is the axion.

Non-perturbative superpotential (from gaugino condensation):

$$W = M_P^3\left(C - Ae^{-a_0 S}\right) \tag{54}$$

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Dilaton potential I

Yields a potential for the dilaton (and radion)

$$V = \frac{M_P^4}{4} b^{-6} e^{-\Phi} \left[\frac{C^2}{4} e^{2\Phi} + ACe^{\Phi} \left(a_0 + \frac{1}{2} e^{\Phi} \right) e^{-a_0 e^{-\Phi}} + A^2 \left(a_0 + \frac{1}{2} e^{\Phi} \right)^2 e^{-2a_0 e^{-\Phi}} \right] . \quad (55)$$

Expand the potential about its minimum:

$$V = \frac{M_P^4}{4} b^{-6} e^{-\Phi_0} a_0^2 A^2 \left(a_0 - \frac{3}{2} e^{\Phi_0} \right)^2 e^{-2a_0 e^{-\Phi_0}} \times \left(e^{-\Phi} - e^{-\Phi_0} \right)^2 . \quad (56)$$

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Dilaton potential II

Lift the potential to 10-d, redefining b to be in the Einstein frame.

$$V(b,\phi) = \frac{M_{10}^{16}\hat{V}}{4} e^{-\Phi_0} a_0^2 A^2 \left(a_0 - \frac{3}{2} e^{\Phi_0}\right)^2 e^{-2a_0 e^{-\Phi_0}} \times e^{-3\phi/2}\left(b^6 e^{-\phi/2} - e^{-\Phi_0}\right)^2 . \qquad (57)$$

Dilaton potential in 10d Einstein frame

$$V \simeq n_1 e^{-3\phi/2}\left(b^6 e^{-\phi/2} - n_2\right)^2 \qquad (58)$$

Analysis including both string matter and dilaton potential I

Worry: adding this potential will mess up radion stablilization

Thus: consider dilaton and radion equations resulting from the action including both the dilaton potential and string gas matter.

Step 1: convert the string gas matter contributions to the 10-d Einstein frame

$$g^E_{\mu\nu} = e^{-\phi/2} g^s_{\mu\nu} \quad (59)$$

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Joint analysis II

Step 2: Consider both dilaton and radion equations:

$$
\begin{aligned}
- & \frac{M_{10}^8}{2}\left(3a^2\dot{a}b^6\dot{\phi} + 6a^3b^5\dot{b}\dot{\phi} + a^3b^6\ddot{\phi}\right) \\
+ & \frac{3}{2}n_1 a^3 b^6 e^{-3\phi/2}\left(b^6 e^{-\phi/2} - n_2\right)^2 \\
+ & a^3 b^{12} n_1 e^{-2\phi}\left(b^6 e^{-\phi/2} - n_2\right) \\
+ & \frac{1}{2\epsilon}e^{\phi/4}\Big(-\mu_0\epsilon^2 + \mu_0|p_d|^2 \\
+ & 6\mu_0\left[\frac{n_a^2}{\alpha'}e^{-\phi/2}b^{-2} - \frac{w^2}{\alpha'}e^{\phi/2}b^2\right]\Big) \\
= & \ 0\,,
\end{aligned}
\tag{62}
$$

Joint analysis III

$$
\begin{aligned}
\ddot{b} \quad + \quad & 3\frac{\dot{a}}{a}\dot{b} + 5\frac{\dot{b}^2}{b} = -\frac{n_1 b}{M_{10}^8} e^{-3\phi/2}\left(b^6 e^{-\phi/2} - n_2\right)^2 \\
& -\frac{2n_1}{M_{10}^8} b^7 e^{-2\phi}\left(b^6 e^{-\phi/2} - n_2\right) \\
& +\frac{1}{2-D}\left[-\frac{10b}{M_{10}^8} n_1 e^{-3\phi/2}\left(b^6 e^{-\phi/2} - n_2\right)^2\right. \\
& \left.-\frac{12n_1}{M_{10}^8} b^7 e^{-2\phi}\left(b^6 e^{-\phi/2} - n_2\right)\right] \\
& +\frac{8\pi G_D \mu_0}{\alpha'\sqrt{\hat{G}_{a\epsilon}}} e^{2\phi}\left[n_a^2 b^{-2} e^{-\phi/2} - w_a^2 b^2 e^{\phi/2}\right. \\
& \left.+\frac{2}{D-1}\left(e^{\phi/2} b^2 w^2 + n \cdot w + 2(N-1)\right)\right]
\end{aligned} \tag{63}
$$

Joint analysis IV

Step 3: Identifying extremum

- Dilaton at the minimum of its potential and
- Radion at the enhanced symmetry state

Step 4: Stability analysis

- Consider small fluctuations about the extremum
- show stability (tedious but straightforward)

Result: Dilaton and radion stabilized simultaneously at the enhanced symmetry point.

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