Introduction to Quantum Gravity II

Abhay Ashtekar

Institute for Gravitation and the Cosmos, Penn State

A broad perspective on the challenges of Quantum Gravity

Focus: Structure & successes of Loop Quantum Gravity; emphasis on BHs & Cosmology.

Organization:

1. Historical & Conceptual Setting
2. Structure of Loop Quantum Gravity
3. Quantum Geometry & Black Hole Entropy

1. Historical and Conceptual Setting

Einstein's resistance to accept quantum mechanics as a fundamental theory is well known. However, he had a deep respect for quantum mechanics and was the first to raise the problem of unifying general relativity with quantum theory.

"Nevertheless, due to the inner-atomic movement of electrons, atoms would have to radiate not only electro-magnetic but also gravitational energy, if only in tiny amounts. As this is hardly true in Nature, it appears that quantum theory would have to modify not only Maxwellian electrodynamics, but also the new theory of gravitation."

(Albert Einstein, Preussische Akademie Sitzungsberichte, 1916)

• Physics has advanced tremendously since 1916 years but the the problem of unification of general relativity and quantum physics still open. Why?

⋆ No experimental data with direct ramifications on the quantum nature of Gravity.

- Physics has advanced tremendously in the last nine decades but the the problem of unification of general relativity and quantum physics is still open. Why?

$\star$ No experimental data with direct ramifications on the quantum nature of Gravity.

$\star$ But then this should be a theorist's haven! Why isn't there a plethora of theories?

$\star$ No experimental data with direct ramifications on quantum Gravity.
$\star$ But then this should be a theorist's haven! Why isn't there a plethora of theories?

• In general relativity, gravity is coded in space-time geometry. Most spectacular predictions —e.g., the Big-Bang, Black Holes & Gravitational Waves— emerge from this encoding. Suggests: Important to respect the Gravity $\sim$ Geometry duality $\Rightarrow$ A satisfactory quantum gravity theory should not pre-suppose a smooth geometry; geometry itself should be treated quantum mechanically. How do you do physics without a space-time continuum in the background?

• Several approaches: Causal sets, twistors, (Causal) Dynamical triangulations, the AdS/CFT conjecture of string theory.
Loop Quantum Gravity grew out of the Hamiltonian approach pioneered by Bergmann, Dirac, and developed by Wheeler, DeWitt, Misner and others.

Contrasting LQG with String theory

Because there are no direct experimental checks, approaches are driven by intellectual prejudices about what the core issues are and what will "take care of itself" once the core issues are resolved.

Particle Physics: 'Unification' Central. (Stelle's lectures)
Extend Perturbative, flat space QFTs; Gravity just another force.

- Higher derivative theories; • Supergravity
- String theory incarnations:

$\star$ Perturbative strings; $\star$ Matrix Models
$\star$ M Theory; $\star$ AdS/CFT Correspondence.

Contrasting LQG with String theory

Particle Physics: ‘Unification’ Central. (Stelle’s lectures)
Extend Perturbative, flat space QFTs; Gravity just another force.

• Higher derivative theories; • Supergravity

• String theory incarnations:

⋆ Perturbative strings; ⋆ Matrix Models; ⋆ M Theory; ⋆ AdS/CFT Correspondence.

• General Relativity: ‘Background independence’ Central: LQG

⋆ Hamiltonian Theory; used for cosmology & BHs),

⋆ Spin-foams; (a bridge to low energy physics via path integrals.)

Issues:

• Unification: Ideas proposed in LQG but strong limitations;
Recall however, QCD versus Grand Unified Theories.

• Background Independence: Progress through AdS/CFT; but a ‘small corner’ of QG; Physics beyond singularities and S-matrices?

A. Ashtekar: LQG: Four Recent Advances and a dozen FAQs;
arXiv:0705.2222

2. Structure of Loop Quantum Gravity

- Geometry: Physical entity, as real as tables and chairs.

Riemann 1854: Göttingen Address; Einstein 1915: General Relativity

Central Lesson of GR: Gravity $\sim$ Geometry. Strongly suggests: Quantum gravity should somehow bring in quantum geometry.

- Matter has constituents. GEOMETRY??

'Atoms of Geometry'? Why then does the continuum picture work so well? Are there physical processes which convert Quanta of Geometry to Quanta of Matter and vice versa?

Quantum Geometry?

- A Paradigm shift to address these issues

"The major question for anyone doing research in this field is: Of which mathematical type *are the variables . . . which permit the expression of physical properties of space. . . Only after that, which* equations *are satisfied by these variables?" Albert Einstein (1946); Autobiographical Notes.*

- Choice in General Relativity: Metric, $g_{\mu\nu}$. Directly determines Riemannian geometry; Geometrodynamics.

In all other interactions, by contrast, the basic variable is a Connection, i.e., a matrix valued vector potential A_a^i;
Gauge theories: Connection-dynamics

- Key new idea: 'Kinematic unification.' Cast GR also as a theory of connections. Import into GR techniques from gauge theories. Naturally lead to a specific Quantum Theory of Riemannian Geometry.

Holonomies/Wilson Lines and Electric Fields/Triads

- Connections: Vehicles for parallel transport.

In QED: parallel transport of the state of an electron
Recall: $\partial\psi \to (\partial - ieA)\psi$
In QCD: parallel transport of the state of a quark
In gravity: parallel transport a spinor

$$p \bullet - - - > - - - \bullet\ q \quad \psi(q) = [\mathcal{P}\ \exp \int_p^q\ A \cdot dS]\, \psi(p) = \hat{h}_e\, \Psi(p)$$

$\hat{h}_e$ is called the holonomy along the edge e from p to q.

- In Gravity: the (canonically conjugate) non-Abelian electric fields E_i^a interpreted as *orthonormal frames/triads*. They determine the physical, curved geometry. Structure group: Rotations of triads SO(3) or, in presence of spinors, its double covering SU(2). This is why, unlike in Yang-Mills theory, in non-perturbative quantum gravity we are led to quantum Riemannina geometry.

- The 'elementary,' abstractly defined operators $\hat{h}_e$ and fluxes $\hat{E}(S)$ of electric fields/triads across 2-surfaces S determine an abstract $\star$-algebra $\mathfrak{a}$. task: Find its representations.

Uniqueness of Canonical Quantization?

- Quantum Mechanics: von Neumann's uniqueness theorem:

There is a unique IRR of the Weyl operators $\hat{U}(\lambda), \hat{V}(\mu)$ by 1-parameter unitary groups on a Hilbert space satisfying:

i) $\hat{U}(\lambda)\,\hat{V}(\mu) = e^{i\lambda\mu}\,\hat{V}(\mu)\,\hat{U}(\lambda)$; and ii) Weak continuity in λ, μ.

This is the standard Schrödinger representation: $\mathcal{H} = L^2(\mathbb{R}, \mathrm{d}x)$;

$\hat{x}\Psi(x) = x\Psi(x)$; $\hat{p}\Psi(x) = -i\mathrm{d}\Psi(x)/\mathrm{d}x$, and $U(\lambda) = e^{i\lambda\hat{x}}$, $V(\mu) = e^{i\mu\hat{p}}$

- Strategy for more general systems: Consider the analog $\mathfrak{a}$ of the Weyl algebra. Look for cyclic representations (where the full Hilbert space is generated by operating on a 'vacuum' state by operators in $\mathfrak{a}$).

In the algebraic approach a state is just a positive linear functional on the algebra $\mathfrak{a}$.

Realized as expectation values in a Hilbert space only after we have a representation.

The 'VEVs' i.e. expectation values in the cyclic state determine the representation through an explicit (GNS) construction. If the VEVs are invariant under a group, the group is unitarily implemented in the representation.

The Issue of Uniqueness

- Uniqueness does not hold for systems with an infinite number of degrees of freedom even after imposing additional symmetry requirements such as Poincaré invariance.

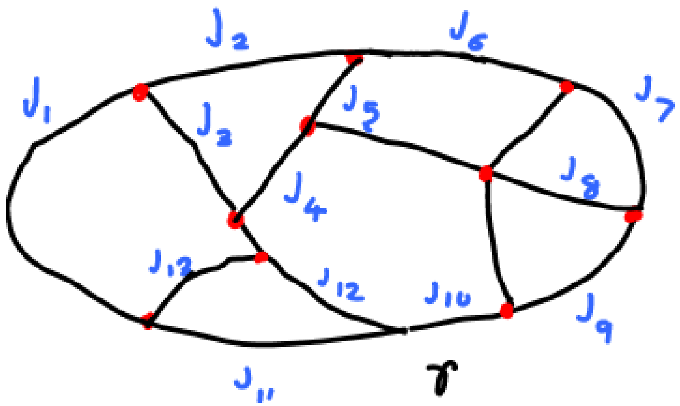
- Surprise: Quantum algebra $\mathfrak{a}$ generated by holonomies and triad-fluxes. It admits a unique cyclic representation in which the diffeomorphism group is unitarily implemented. $\Rightarrow$. Thanks to background independence, quantum kinematics is unique in LQG!
(Lewandowski, Okolow, Sahlmann, Thiemann; Fleischhack)

Unforeseen power of Diffeomorphism Invariance or, equivalently, Background Independence! (AA)
The unique representation has a technical feature that distinguishes it from the Fock-type representations used in perturbative treatment of gauge theories. Only the holonomy operators $\hat{h}$ are well defined; not the connection operators $\hat{A}$ themselves. (QM Analogy: $U(\lambda)$ well defined but not x)
Key technical reason reason why LQG has spin nets and discrete eigenvalues of geometric operators.

Polymer Geometry

• This unique kinematics was first constructed explicitly in the early nineties. High mathematical precision. Provides a Quantum Geometry which replaces the Riemannian geometry used in classical gravity theories. (AA, Baez, Lewandowski, Marolf, Mourão, Rovelli, Smolin, Thiemann,...) Details: Review by AA & Lewandowski; monographs by Rovelli; Thiemann.

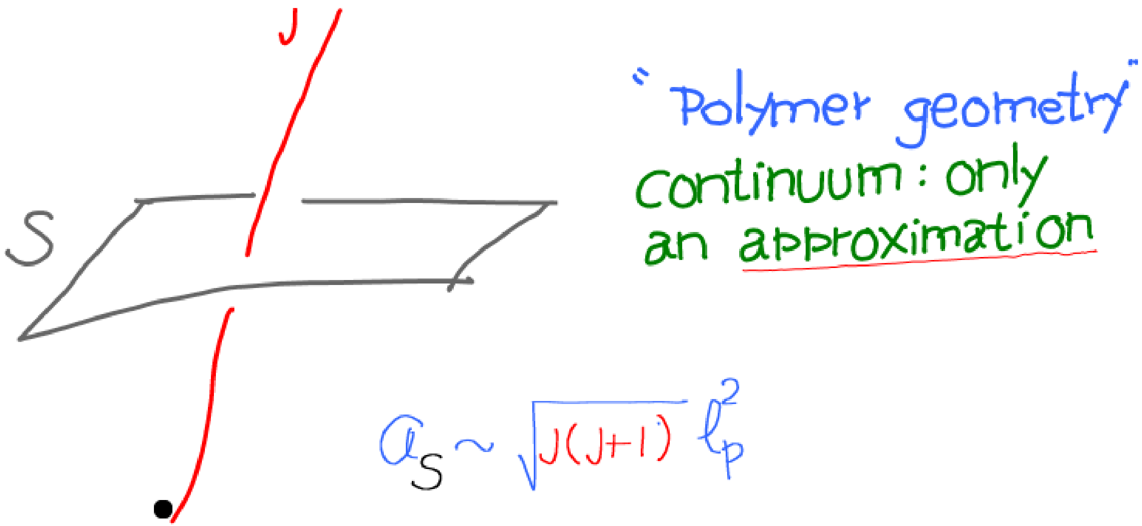
• Quantum States: $\Psi \in \mathcal{H} = L^2(\bar{\mathcal{A}}, d\mu_o)$
μ_o a diffeomorphism invariant, regular measure on the space $\bar{\mathcal{A}}$ of (generalized) connections.



$$\mathcal{H} = \bigoplus_{\gamma, \vec{J}} \underbrace{\mathcal{H}_{\gamma, \vec{J}}}_{\text{Finite-dim. H.S. of a Spin-system}}$$

• Fundamental excitations of geometry 1-dimensional. Polymer geometry at the Planck scale. Continuum arises only in the coarse rained approximation.

- Flux lines of area. Background independence!



- Examples of Novel features:

⋆ All eigenvalues of geometric operators discrete. Area gap. Eigenvalues not just equally-spaced; crowd in a rather sophisticated way. Geometry quantized in a very specific manner. (Recall Hydrogen atom.)

⋆ Inherent non-commutativity: Areas of intersecting surfaces don't commute. Inequivalent to the Wheeler-DeWitt theory (quantum geometrodynamics).

Summary: AA & Lewandowski, Encyclopedia of Mathematical Physics

Applications of this Quantum Geometry

• This unique kinematical arena provides a Quantum Riemannian Geometry to formulate dynamics, i.e. the quantum version of Einstein's equations. Main challenge of LQG.

• Steady progress has been made especially over the last five years by applying these background independent methods to various "sectors" of general relativity. Examples:

⋆ Space-times admitting an isolated horizon as an inner boundary. Analysis covers black holes in equilibrium and cosmological horizons in one go.

⋆ Spatially homogeneous space-times. Covers important cosmological models.

⋆ Gowdy Models two 2 spatial Killing fields (Mena's talk): Great interest to mathematical relativists; Admit inhomogeneity and gravitational waves (with certain symmetries).

⋆ Inhomogeneous cosmological perturbations (e.g., using quantum fields on quantum cosmological space-times).

⋆ Piecewise linear truncations of general relativity: "Regge calculus" but based on quantum geometries used in spin foam models (Rovelli's lectures).

• Although starting point is GR, fundamental DOF quite different from those of classical GR which is based on a continuum. Radical departure from GR in the Planck regime.

3. Black Holes: Zooming in on Quantum Geometry

- First law of BH Mechanics + Hawking's discovery that $T_{\rm BH} = \kappa\hbar/2\pi \;\; \Rightarrow$ for large BHs, $S_{\rm BH} = a_{\rm hor}/4{\ell_{\rm pl}}^2$ (Bekenstein 1973)

- Entropy: Why is the entropy proportional to area? For a $M_\odot$ black hole, we must have $\exp 10^{76}$ micro-states, a HUGE number even by standards of statistical mechanics. Where do these micro-states come from?

For gas in box, the microstates come from molecules; for a ferromagnet, from Heisenberg spins; Black hole ?
Cannot be gravitons: gravitational fields stationary.

- To answer these questions, must go beyond the classical space-time approximation used in the Hawking effect. Must take into account the quantum nature of gravity.

- Distinct approaches. In Loop Quantum Gravity, this entropy arises from the huge number of microstates of the quantum horizon geometry. 'Atoms' of geometry itself!

Quantum Horizon Geometry & Entropy

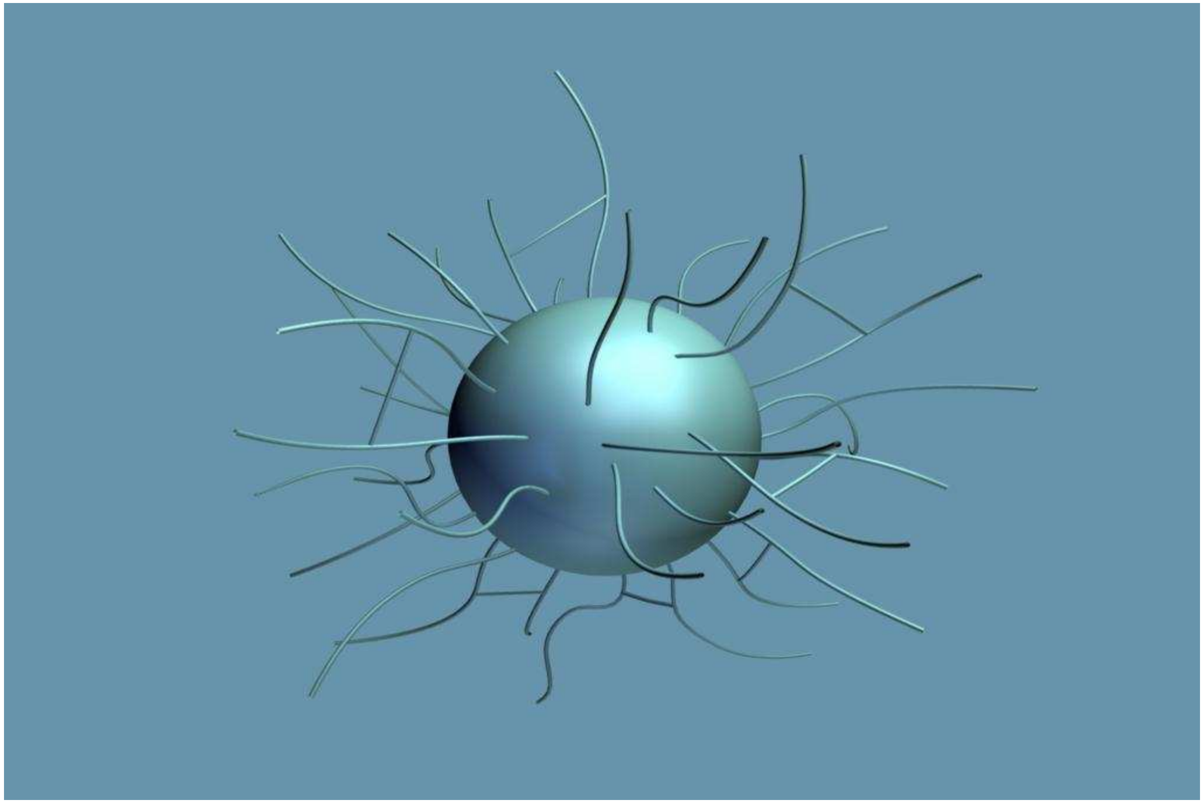
- Heuristics: Wheeler's It from Bit

Divide the horizon into elementary cells, each carrying area ${\ell_{\rm pl}}^2$.
Assign to each cell a 'Bit' i.e. 2 states.
Then, # of cells $n \sim a_o/{\ell_{\rm pl}}^2$; No of states $\mathcal{N} \sim 2^n$;
$S_{\rm hor} \sim \ln \mathcal{N} \sim n \ln 2 \sim a_o/{\ell_{\rm pl}}^2$. Thus, $S_{\rm hor} \propto a_o/{\ell_{\rm pl}}^2$.

- Argument made rigorous in quantum geometry. Many inaccuracies of the heuristic argument have to be overcome: Quanta of area not ${\ell_{\rm pl}}^2$ but $4\pi\gamma \sqrt{j(j+)}\, {\ell_{\rm pl}}^2$; Calculation has to know that the surface is black hole horizon; What is a quantum horizon?

- Interesting mathematical structures $\mathrm{U}(1)$ Chern-Simons theory; non-commutative torus, quantum $\mathrm{U}(1)$, mapping class group, ...(AA, Baez, Corichi, Krasnov; Domagala, Lewandowski; Meissner; AA, Engle, Van Den Broeck, ...)

Quantum Horizon

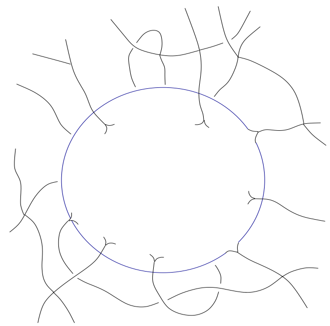


Polymer excitations of geometry in the bulk puncture the horizon.
Quantum horizon geometry described by the $\mathrm{U}(1)$ Chern-Simons theory.

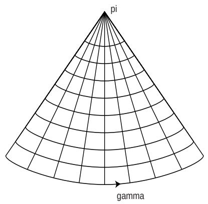
Quantum Horizon Geometry and Entropy

• Horizon geometry flat everywhere except at punctures. At punctures the bulk polymer excitations cause a 'tug' giving rise to quantized deficit angles. They add up to 4π providing a 2-sphere quantum geometry. 'Quantum Gauss-Bonnet Theorem'.

• As in Statistical mechanics,
have to construct a suitable ensemble ρ by specifying macroscopic parameters (multipoles) characterizing the classical horizon geometry. $S_{\rm hor} = {\rm Tr} \rho \ln \rho$ gives the log of the number of quantum horizon geometry states compatible with the classical geometry.



• $S_{\rm hor} = a_{\rm hor}/4{\ell_{\rm pl}}^2 - (1/2)\ln(a_{\rm hor}/{\ell_{\rm pl}}^2) + o(a_{\rm hor}/{\ell_{\rm pl}}^2)$
for a specific value of the parameter γ.
Procedure incorporates all physically interesting BHs and Cosmological horizons in one swoop.



4. Summary

- The interplay between geometry and physics is the deepest feature of general relativity. Loop Quantum Gravity elevates it to the quantum level. Just as classical GR is based on Riemannian geometry, LQG is based on a specific quantum theory of Riemannian geometry.

- Thanks to the LOST-F theorems, background independence leads to a unique quantum kinematics. In it, the basic excitations of Riemannian geometry are 1-dimensional and eigenvalues of geometrical operators are discrete. Continuum Riemannian geometry arises as a coarse grained approximation.

- Quantum geometry has several important physical ramifications. Using the isolated horizon framework to describe black holes in equilibrium, one can account for entropy of physically relevant black holes as well as cosmological horizons in one go.

- In the next lecture (as well as those of Bojowald and Mena) will deal with a complementary application in the canonical theory: Cosmology. Rovelli's lectures: Path integrals by summing over quantum geometries; Spin Foams.