Emergent gravity from Group Field Theory

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GFT: Basic ideas

- Group Field Theories: models for quantum spacetime (connected to spinfoams), designed to define a path integral for quantum gravity
- Generalization of matrix models (2D QG) to higher dimensions
- Quantum/statistical field theories on group manifolds (SU(2) etc.): purely pregeometrical framework
- Partition function defined by the specification of a certain action
- Perturbative expansion: the Feynman graphs are "just" simplicial complexes (with geometric data)

Some formulae

- The field for d-dimensional models

$$\phi : \underbrace{G \times G \cdots \times G}_{d} \to \mathbb{C}$$

- The action

$$S_{GFT}[\phi] = \int (dh)^d \phi_{a_1 \ldots a_d} \bar{\phi}_{a_1 \ldots a_d} + \frac{g}{d!} \int (dh)^K V(\{h\}) \phi_{a_1^1 \ldots a_d^1} \ldots \phi_{a_1^{d+1} \ldots a_d^{d+1}} + cc.$$

- The partition function for GFT

$$Z(g) = \int \mathcal{D}\phi \exp\left(-S_{GFT}[\phi]\right)$$

- And the partition function for QG

$$W(g) = \log Z(g)$$

The continuum limit

- Feynman expansion: discrete geometries

$$\langle n \rangle = \frac{1}{W}\frac{\partial W}{\partial \log g}$$

- The continuum limit: phase transitions
- Subtlety: the pure gravity models do possess only a large volume limit (discretization scale is purely combinatoric, no dimensions)
- Matter is required: ratio between correlation length and combinatorial length goes to infinity (or not)
- Key issue: the continuum and/vs semiclassical limit

Scaling assumption

- Obviously the critical behavior has to be computed from the specification of the microscopic action (ongoing work)

Bonzom et al. 1105.3122

- Working hypothesis: there is a singular behavior of the partition function and it has a specific scaling form.

$$W(g) = a(g - g_c)^\gamma$$

- Key point: all the macroscopic coupling constants will be computable functions of the critical exponents (role of universality)

Boundary states

- Evaluation of transition amplitudes: specification of boundary states (necessary to extract the dynamics)
- Correlation functions in GFT

$$\left\langle \int (dh)^6 \phi_{123}\phi_{156}\phi_{453}\phi_{426} \right\rangle$$

- Important point: Schwinger-Dyson equations (and Ward identities) will relate all the correlation functions among themselves

Generating function

- Macroscopic boundary geometries are “superpositions” of different microscopic configurations
- Design a boundary state (how? Coherent states?). Ambiguity of the effective dynamics (see BECs!)
- Idea: use auxiliary GFTs in one dimension less to generate a sum over all the possible random boundary geometries

$$G(\lambda, g) = \int \mathcal{D}\psi \mathcal{D}\phi \exp\left[-\left(\frac{1}{2}\int (dg)^2 \psi_{ab}\psi_{ba} + \frac{\lambda}{3}\int (dg)^3 \psi_{ab}\psi_{bc}\psi_{ca}\phi_{h_1 h_2 h_3}\right) + \right.$$

$$\left. + \left(\frac{1}{2}\int (dg)^3 \phi_{abc}\phi_{cba} + \frac{g}{4}\int \phi_{abc}\phi_{aef}\phi_{dec}\phi_{dbf}\right)\right]$$

Hartle-Hawking

- One can thus compute something like the Hartle-Hawking wavefunction
- Problem: reconstruct the Wheeler-DeWitt equation
- Idea/2: assume a double critical behavior (continuum limit in the boundary AND in the bulk)

$$\boxed{\log G(\lambda, g) \sim a(\lambda - \lambda_c')^{\delta}(g - g_c')^{\gamma}}$$

Towards effective Hamiltonian constraint

- With this profile one can argue that the wavefunction has to have a behavior like

$$\psi(n) \sim n^{\delta} \exp(-\alpha n)$$

large volume only!

- Go backwards: from the solution to the problem
- Approximate method: Hamilton-Jacobi reversed

$$H(x,p) = \Phi\left(p - bx^{b-1} + \frac{\delta}{x}\right)$$

- Ambiguity: one free function

Future directions

- Work out explicitly the Schwinger-Dyson/Ward equations in the critical limit with the boundary state included

Gurau 2011

- Add matter
- Work in Lorentzian signature (help from horizon thermodynamics)
- Compare with other approaches (e.g. coherent states methods)

Oriti, LS 2010

The end

Thank you