Lorentzian Quantum Einstein Gravity

Stefan Rechenberger

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

Uni Mainz

12.09.2011

Phys.Rev.Lett. 106 (2011) 251302
with Elisa Manrique and Frank Saueressig

Outline

1. Motivation
2. CDT, Horava Gravity and Asymptotic Safety
3. Causal functional RG equation
4. Results
5. Conclusion

Motivation

Classical GR reaches its limits close to space-time singularities

- Black Holes
- Big Bang

Solution probably lies within a theory of Quantum Gravity

Motivation

Classical GR reaches its limits close to space-time singularities

- Black Holes
- Big Bang

Solution probably lies within a theory of Quantum Gravity

different approaches to a theory of QG

- String Theory
- Loop Quantum Gravity
- Causal Dynamical Triangulations
- Horava Gravity
- **Asymptotic Safety**
- ...

Motivation

Classical GR reaches its limits close to space-time singularities

- Black Holes
- Big Bang

Solution probably lies within a theory of Quantum Gravity

different approaches to a theory of QG

- String Theory
- Loop Quantum Gravity
- Causal Dynamical Triangulations
- Horava Gravity
- **Asymptotic Safety**
- ...

} which one is correct?

lack of experimental data

$\Rightarrow$ nobody helps us to decide which is the best approach

lack of experimental data
$\Rightarrow$ nobody helps us to decide which is the best approach

Best thing to do: compare different approaches

Causal dynamical triangulations (arXiv:1004.0352v1 [hep-th])

- discretization of gravitational path integral $\int \mathcal{D}g_{\mu\nu} e^{iS_{\rm grav}}$
- summing over piecewise flat geometries
- modeling space-time geometries by gluing together simplices (higher dimensional generalizations of triangles)
- important: causal structure

Causal dynamical triangulations (arXiv:1004.0352v1 [hep-th])

- discretization of gravitational path integral $\int \mathcal{D}g_{\mu\nu} e^{iS_{\mathrm{grav}}}$
- summing over piecewise flat geometries
- modeling space-time geometries by gluing together simplices (higher dimensional generalizations of triangles)
- important: causal structure

Horava Gravity (arXiv:0901.3775v2 [hep-th])

- different scaling of space and time
- UV: Lorentz invariance is broken
- IR: Lorentz invariance reestablished
- maybe connection to CDT due to global time foliation (arXiv:1002.3298v2 [hep-th])

Asymptotic Safety (Living Rev. Relativity 9, (2006))

- non-trivial fixed point (for UV completion)
- finite dimensional critical surface (for predictability)

Asymptotic Safety (Living Rev. Relativity 9, (2006))

- non-trivial fixed point (for UV completion)
- finite dimensional critical surface (for predictability)
- already found for
  - pure Einstein Hilbert action
  - f(R) gravity
  - gravity coupled to a scalar field
  - gravity with extra dimensions
  - bimetric truncations
  - ...

Asymptotic Safety (Living Rev. Relativity 9, (2006))

- non-trivial fixed point (for UV completion)
- finite dimensional critical surface (for predictability)
- already found for
  - pure Einstein Hilbert action
  - f(R) gravity
  - gravity coupled to a scalar field
  - gravity with extra dimensions
  - bimetric truncations
  - ...

} strong evidence that nature might be asymptotically safe

Asymptotic Safety (Living Rev. Relativity 9, (2006))

- non-trivial fixed point (for UV completion)
- finite dimensional critical surface (for predictability)
- already found for
  - pure Einstein Hilbert action
  - f(R) gravity
  - gravity coupled to a scalar field
  - gravity with extra dimensions
  - bimetric truncations
  - ...

  } strong evidence that nature might be asymptotically safe

- so far only Euclidean space-time has been studied
- Lorentzian space-times are necessary for comparison with CDT and HG

Causal functional RG equation

Starting point: Einstein Hilbert action

$$S_{\mathrm{EH}} = \frac{1}{16\pi G_{\mathrm{N}}} \int d^D x \sqrt{\gamma} \left(-R + 2\Lambda\right)$$

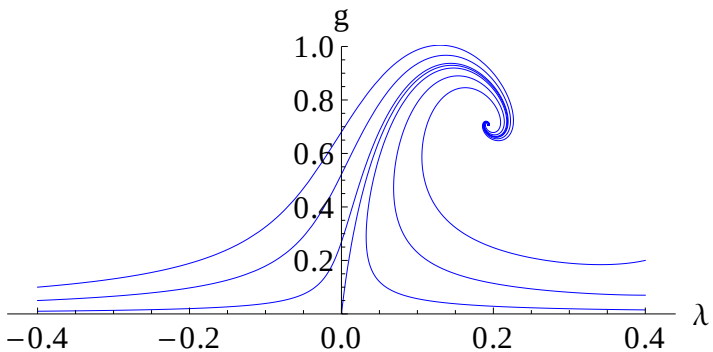
- G_{N} ... Newton constant
- D ... space-time dimension ($D = d + 1$)
- γ ... metric
- R ... curvature scalar of space-time
- Λ ... cosmological constant

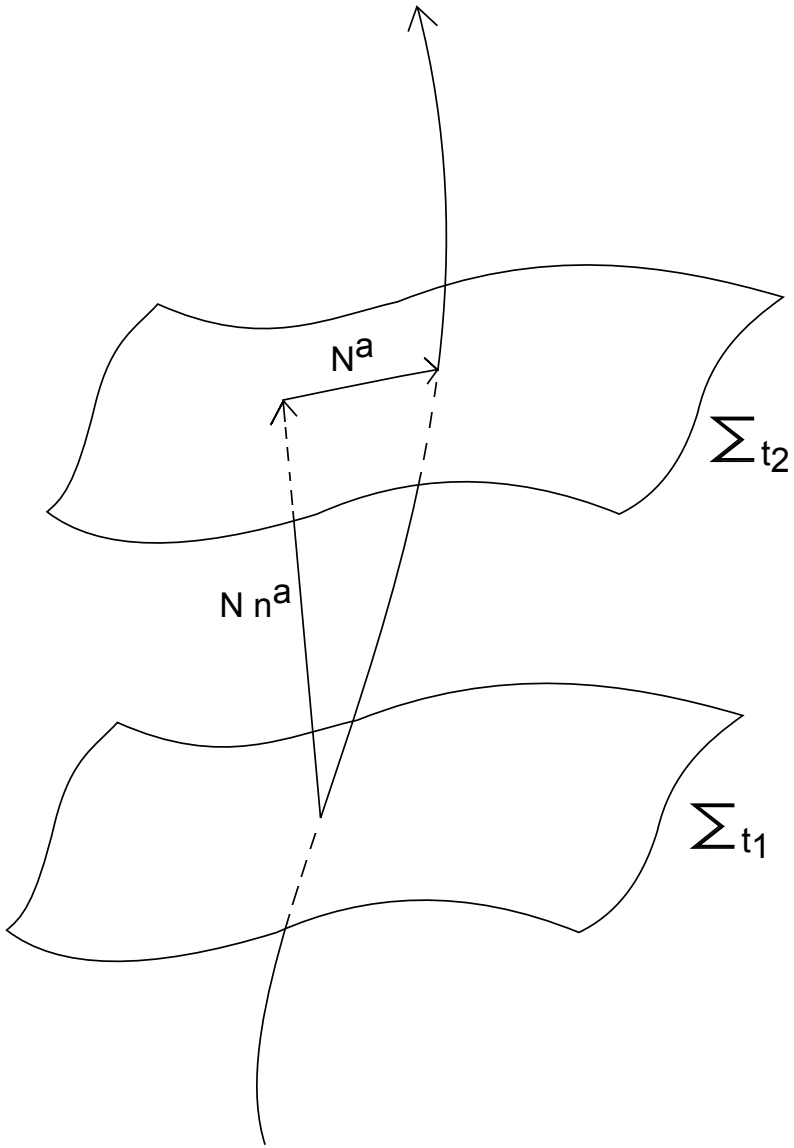
Causal functional RG equation

Starting point: Einstein Hilbert action

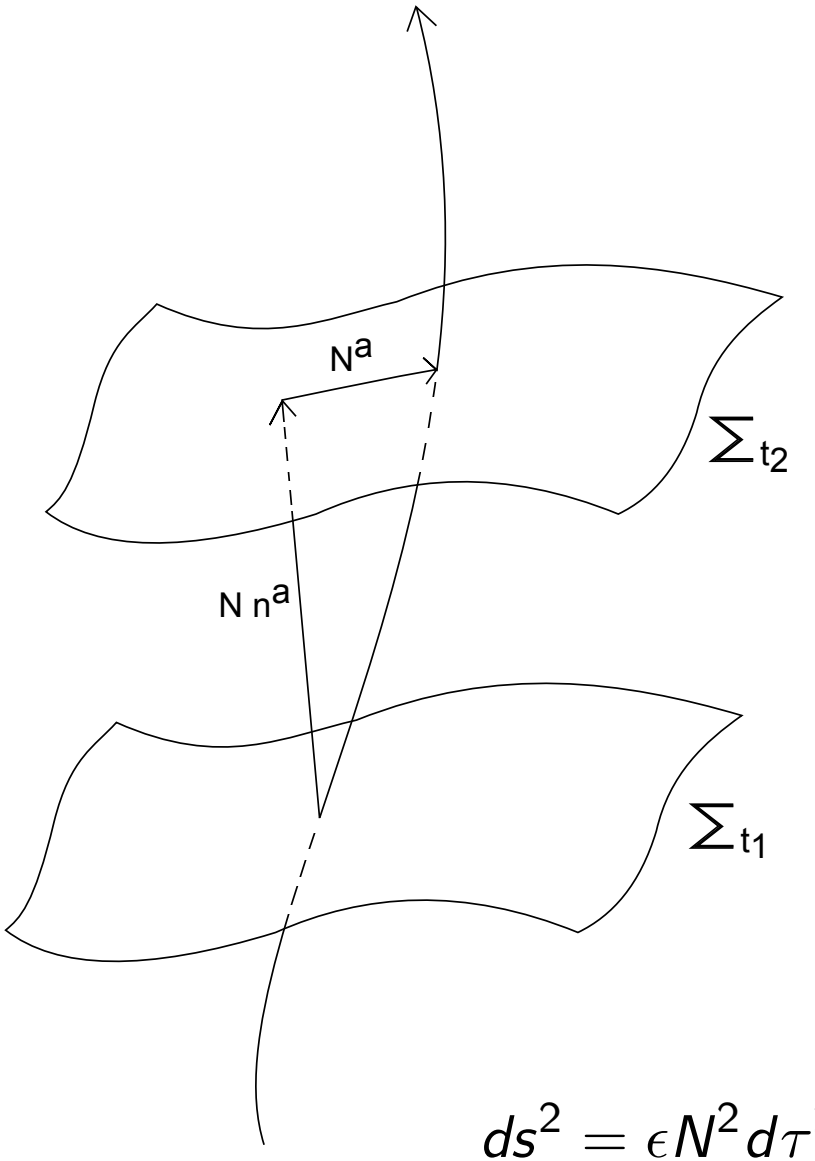
$$S_{\mathrm{EH}} = \frac{1}{16\pi G_{\mathrm{N}}} \int d^D x \sqrt{\gamma} \left(-R + 2\Lambda\right)$$

- G_{N} ... Newton constant
- D ... space-time dimension ($D = d + 1$)
- γ ... metric
- R ... curvature scalar of space-time
- Λ ... cosmological constant





- split of space-time $M^D = M^d \times S^1$
- Σ ... spatial slices
- n^a ... vector orthonormal to Σ
- N ... lapse function
- N^a ... shift vector



- split of space-time $M^D = M^d \times S^1$
- Σ ... spatial slices
- n^a ... vector orthonormal to Σ
- N ... lapse function
- N^a ... shift vector
- σ_{ij} ... spatial metric

$$\gamma_{\mu\nu} = \begin{pmatrix} \epsilon N^2 + N_i N^i & N_j \\ N_i & \sigma_{ij} \end{pmatrix}$$

$$ds^2 = \epsilon N^2 d\tau^2 + \sigma_{ij} \left(dx^i + N^i d\tau\right) \left(dx^j + N^j d\tau\right)$$

- geometric cutoff in time direction
- standard cutoff in spatial direction

$$k\partial_k g_k = \beta_g(g, \lambda; m), \qquad k\partial_k \lambda_k = \beta_\lambda(g, \lambda; m)$$

- dim.less Newton constant: g
- dim.less cosmological constant: λ
- dim.less Kaluza-Klein mass: $m = \frac{2\pi}{Tk}$
- circumference of time circle: T

Results

$$m = const.(e.g.2\pi) \quad \Rightarrow \quad T \propto \frac{1}{k}$$

flow eq. provide a fixed point in Euclidean and Lorentzian signature

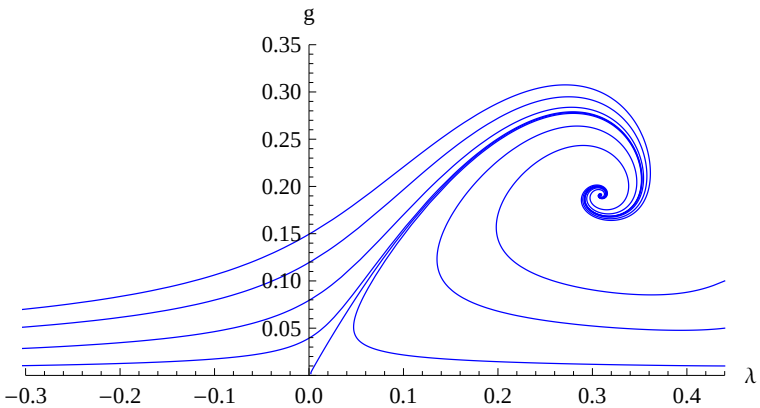
ϵ	g_*	λ_*	$g_*\lambda_*$	$\theta_{1,2}$
$+1$	0.19	0.31	0.059	$1.07 \pm 3.31i$
-1	0.21	0.30	0.063	$0.94 \pm 3.10i$

Results

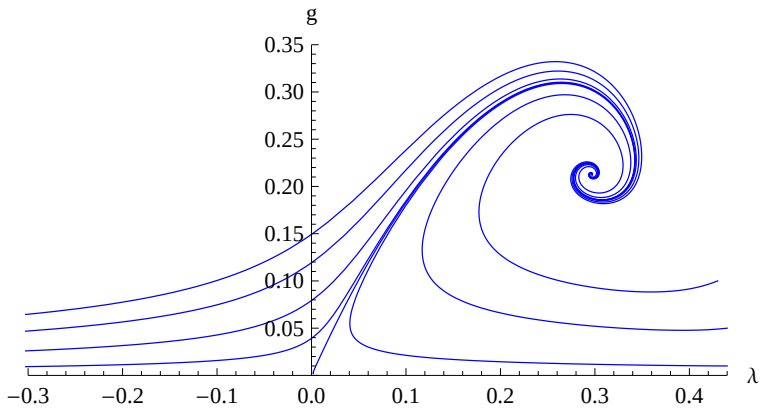
$$m = const.(e.g.2\pi) \quad \Rightarrow \quad T \propto \frac{1}{k}$$

flow eq. provide a fixed point in Euclidean and Lorentzian signature

ϵ	g_*	λ_*	$g_*\lambda_*$	$\theta_{1,2}$
$+1$	0.19	0.31	0.059	$1.07 \pm 3.31i$
-1	0.21	0.30	0.063	$0.94 \pm 3.10i$



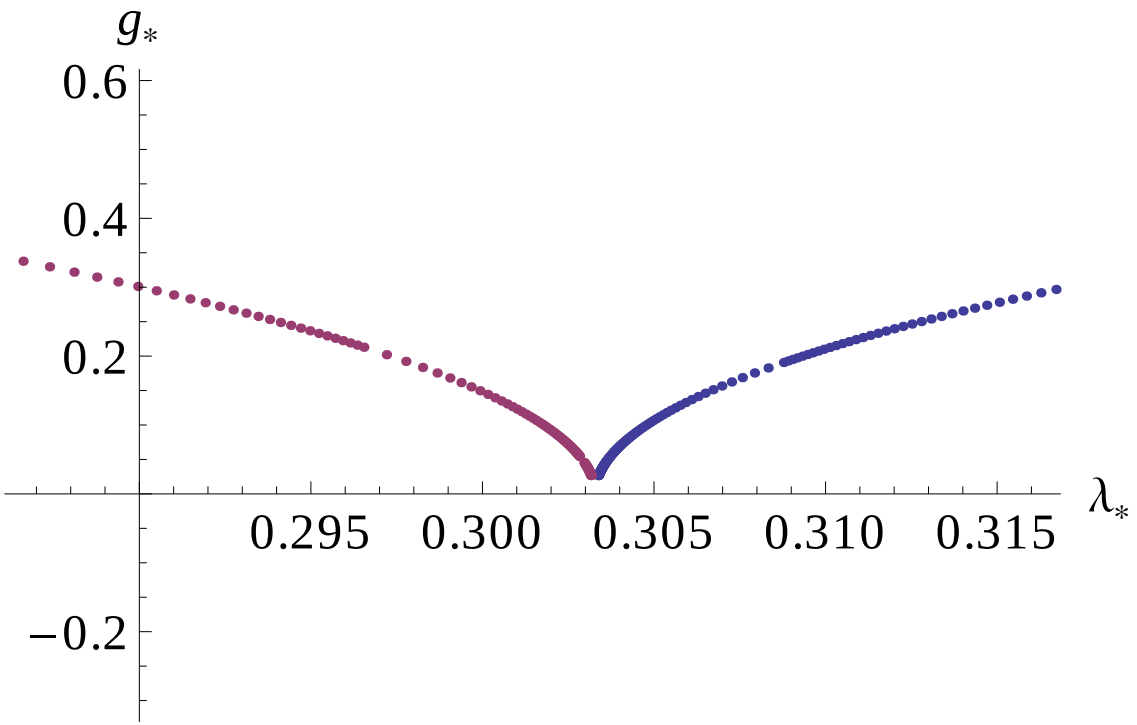
Euclidean



Lorentzian

m dependence of the fixed point values

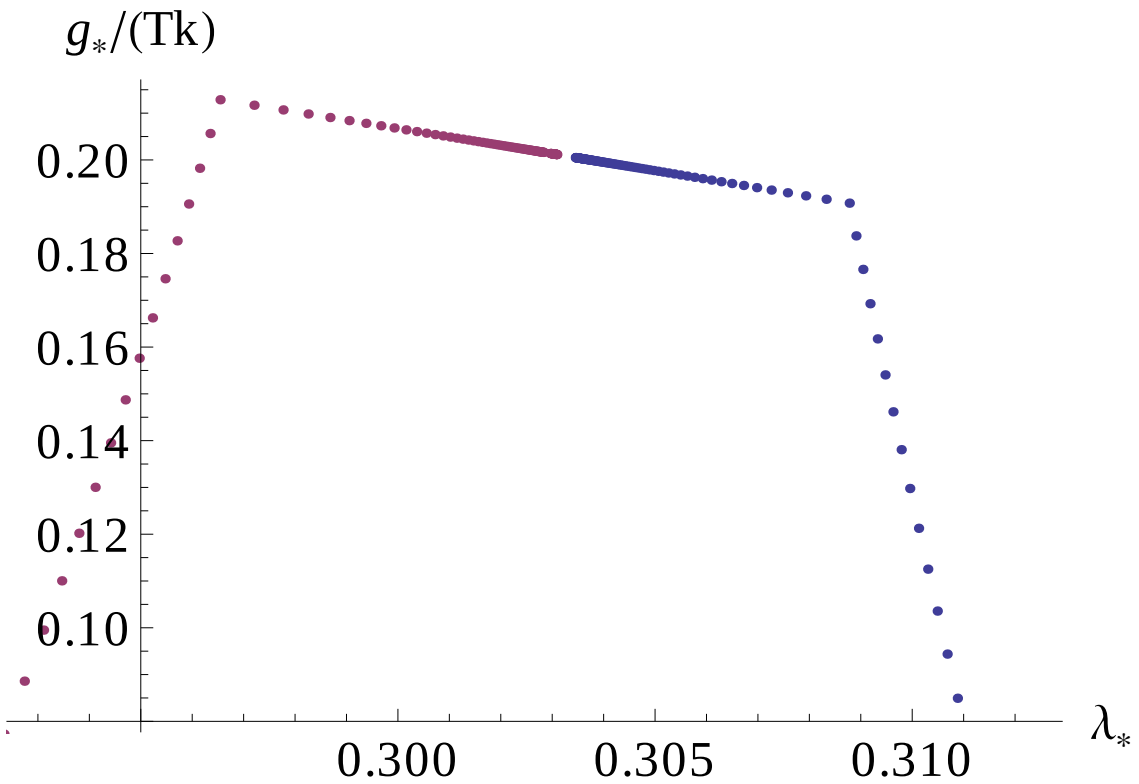
Lorentzian (red) and Euclidean (blue)



$$\lim_{m\to\infty} g_* = 0$$

dimensionless Newton constant in 3D: $g_3 = \frac{g}{Tk}$

dimensionless Newton constant in 3D: $g_3 = \frac{g}{Tk}$



Conclusion

- FP for Euclidean and Lorentzian signature
- characteristics are similar
- also similar to covariant formulation
- time circle collapses toward UV
- signature does NOT matter in UV
- formulation prepares ground for comparison to other theories

Thank you for your attention!

Questions?