On the Space of Generalized Fluxes for Loop Quantum Gravity

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[to appear soon]

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Motivation

Loop Quantum Gravity

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- (Smeared) variables: holonomies $h_{\mathrm{e}}[A] \in \mathrm{SU}(2)$, fluxes $E(S,f) \in \mathfrak{su}^*(2)$
- Configuration space (projective limit): $\overline{\mathcal{A}} = \varinjlim \mathcal{A}_\gamma$
- Hilbert space (inductive limit):
  $$\mathcal{H}_0 = L^2(\overline{\mathcal{A}}, \mathrm{d}\mu_0) = \varprojlim \mathcal{H}_\gamma = \overline{\left(\cup_\gamma L^2(\mathcal{A}_\gamma, \mathrm{d}\mu_H)\right) / \sim}$$

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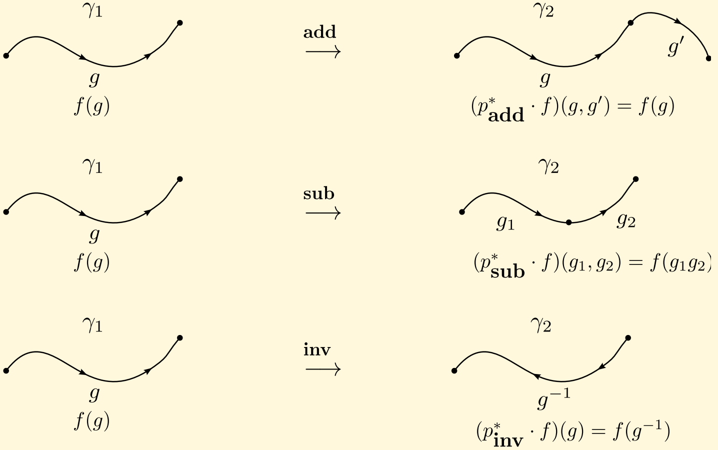
- Group Fourier transform $\mathcal{F}_\gamma$ [hep-th/1004.3450]

$$\begin{array}{ccc} \cup_\gamma \mathcal{H}_\gamma & \xrightarrow{\mathcal{F}_\gamma} & \cup_\gamma \mathcal{H}_{\star,\gamma} \\ \downarrow \pi & & \downarrow \pi_\star \\ \left(\cup_\gamma \mathcal{H}_\gamma\right) / \sim & \xrightarrow{\widetilde{\mathcal{F}}} & \left(\cup_\gamma \mathcal{H}_{\star,\gamma}\right) / \sim \end{array}$$

$\overline{\left(\cup_\gamma \mathcal{H}_{\star,\gamma}\right) / \sim} \simeq L^2_\star(\overline{\mathcal{E}}, \mathrm{d}\mu_{\star,0})$? $\quad \overline{\mathcal{E}}$: space of generalized fluxes

The space of generalized connections



- Cyl is well-defined: norm and (pointwise)-product cylindrical consistent
- $\Delta(\overline{\mathrm{Cyl}}) = \overline{\mathcal{A}}$

Group Fourier transform

- Group Fourier transform (one edge):

$$\mathcal{F}_{\mathrm{e}} : C(G) \to \mathcal{C}_{\star}(\mathfrak{g}^*)$$

$$f(g) \mapsto \widehat{f}(x) := \mathcal{F}_{\mathrm{e}}(f)(x) = \int_G \mathrm{d}g\, f(g)\, \mathrm{e}_g(x)$$

- $\star$-product: $\mathrm{e}_{g_1} \star \mathrm{e}_{g_2} = \mathrm{e}_{g_1 g_2}$
- $\mathcal{F}(f_1) \star \mathcal{F}(f_2) = \mathcal{F}(f_1 * f_2) \longrightarrow$ dual to convolution

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- $\mathrm{SU}(2) \longrightarrow \mathrm{U}(1)^3 \longrightarrow \mathrm{U}(1)$ (quantization of linearized gravity)

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Theorem

Suppose $\mathcal{A}_\gamma$ *are abelian groups, and let* $\overline{\mathcal{A}}$ *be the projective limit with projections* $p_\gamma : \overline{\mathcal{A}} \to \mathcal{A}_\gamma$*. Then the dual group* $\widehat{\overline{\mathcal{A}}}$ *equals the inductive limit of the dual groups* $\widehat{\mathcal{A}}_\gamma$*.*

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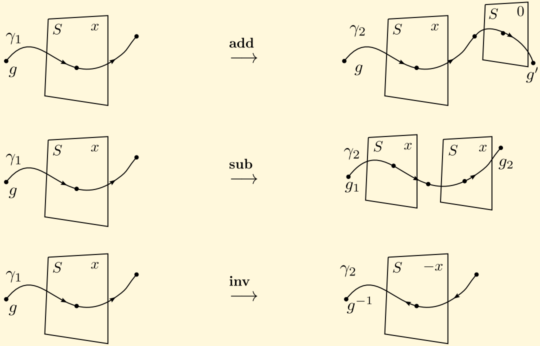
- Consistency conditions:



Open Issues and Outlook

- $\widehat{\overline{\mathcal{A}}} = \mathsf{Hom}(\mathsf{Hom}(\mathcal{P}, \mathrm{U}(1)))$ (inductive limit): better characterization?
- LQG kinematics treats A and E very asymmetrically!
- Start from scratch encoding the conditions tailored to the fluxes: possible?
- Loop Quantum Cosmology
  - Configuration space: $\overline{\mathbb{R}}_{\mathsf{Bohr}}$ (projective limit)
  - Flux interpretation for LQC?
  - Embed LQC into $\mathrm{U}(1)$-LQG?

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Thank you for your attention!