$\mathcal{Q}$uantum backreaction in string theory

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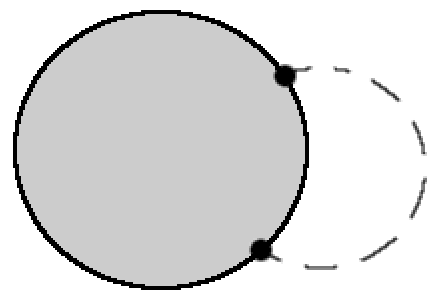
Preliminaries

- There are situations in string theory when a finite number of quanta exert a substantial backreaction upon the background in which the theory is formulated.
- These settings invalidate the conventional separation between classical background and stringy excitations.

- Example 1: D0-brane recoil under the impact of closed strings (solved a few years ago at next-to-leading order in string coupling).
- Example 2: String cosmology in a totally compact space (work in progress!).

D0-brane recoil

- Conventional Dirichlet CFT description of D0-branes fails at next-to-leading order due to IR divergences indicative of recoil (see next slide).
- Modification needed $\longrightarrow$ worldline formalism in which the D0-brane center-of-mass motion is explicitly quantized.
- If successfully implemented, IR-divergences are cancelled in a manner of the Fischler-Susskind mechanism.



The annular divergence

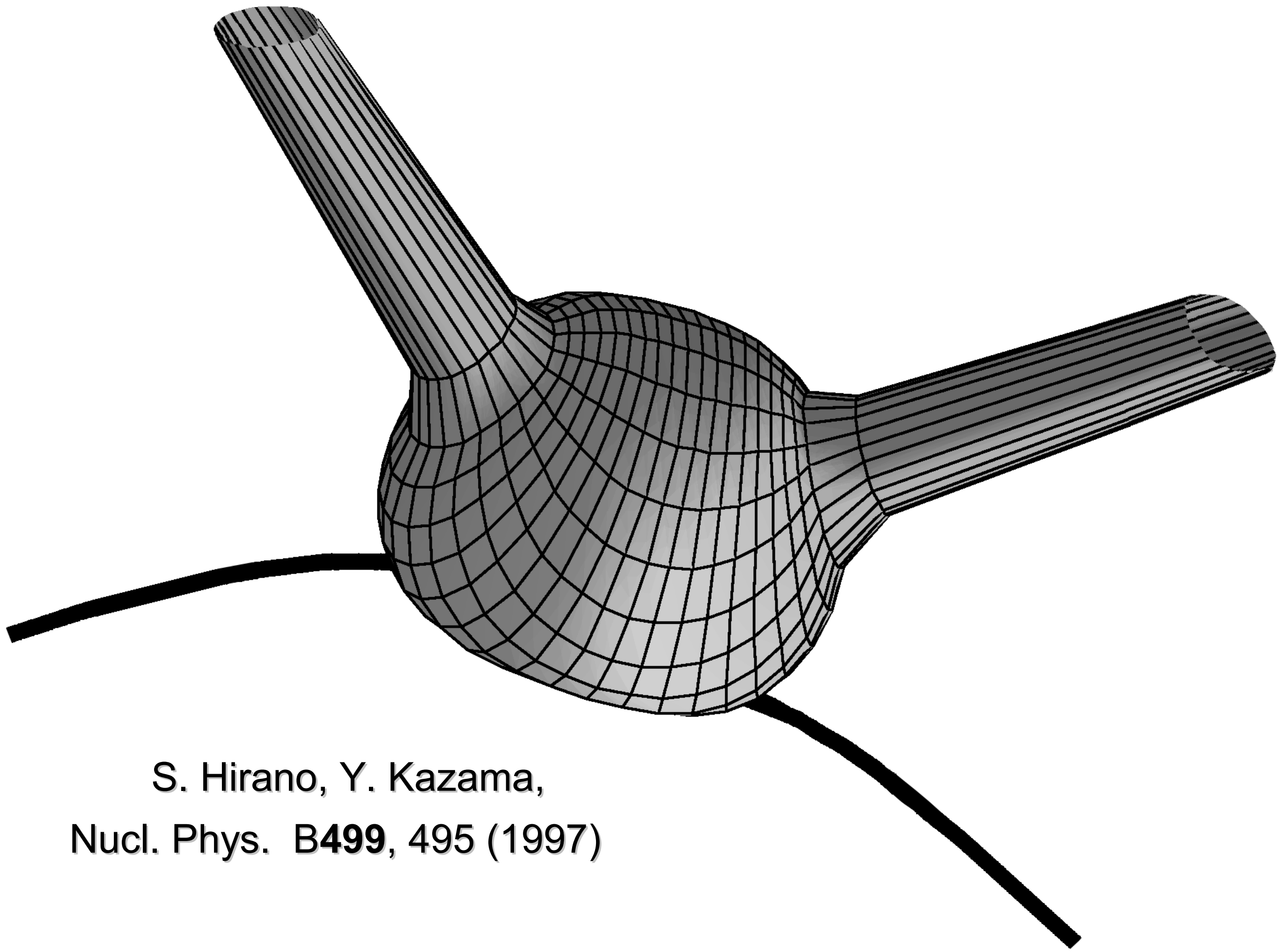
$$\left\langle V^{(1)} \ldots V^{(n)} \right\rangle_{\text{annulus}}$$

$$\sim \int_0^\infty dq \int d\kappa^i q^{-1+\alpha'\kappa^2/4} \int d\theta d\theta' \left\langle V^i(\theta, \kappa^i, 0) V^i(\theta', \kappa^i, 0) V^{(1)} \ldots V^{(n)} \right\rangle_{D_2}$$

$$\sim \int_0^\infty dq \int d\kappa^i q^{-1+\alpha'\kappa^2/4} \int d\theta d\theta' \left\langle V^i(\theta, 0, 0) V^i(\theta', 0, 0) V^{(1)} \ldots V^{(n)} \right\rangle_{D_2}$$

$$\sim P^2 \left\langle V^{(1)} \ldots V^{(n)} \right\rangle_{D_2} \int_0^\infty dq \int d\kappa^i q^{-1+\alpha'\kappa^2/4}$$

$$\int_0^\infty dq \int d\kappa^i q^{-1+\alpha'\kappa^2/4} \sim \int_0^\infty dq \int_0^\infty d\kappa \kappa^{d-1} q^{-1+\alpha'\kappa^2/4} \sim \int_0^\infty \frac{dq}{q\,(\log q)^{d/2}}$$



S. Hirano, Y. Kazama,

Nucl. Phys. B**499**, 495 (1997)

Worldline formalism

$$G(x_1, x_2 | k_1, \cdots, k_m) = \sum \frac{(g_{st})^\chi}{V_\chi} \int [\mathcal{D}f]_{\text{diff}} \mathcal{D}t\, \mathcal{D}X\, \delta\left(X_\mu(\theta) - f_\mu(t(\theta))\right)$$
$$\times \exp\left[-S_D(f) - S_{st}(X)\right] \prod_{a=1}^{m} \left\{ g_{st} \mathcal{V}_a(k_a) \right\},$$

*** reduction formula for endpoint positions ***

- May seem complicated, but can be systematically analyzed at next-to-leading order.
- The divergence on the sphere due to curved worldlines cancels the (static worldline) divergence on the annulus from the moduli space boundary.
- Subtleties: general principle to define the worldline action $S_D(f)$, generalization of divergence cancellation to higher orders???

String gas cosmology

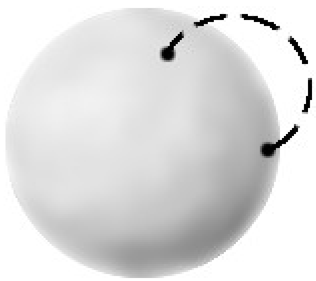
- A spatially compact universe (can be taken as a torus) filled with a gas of quantum closed strings.

- Attractive phenomenology has been suggested (including a possible explanation for the number of macroscopic space-time dimensions, alternatives to inflation, implications of T-duality to cosmological singularity resolution, etc).

- With all this, no exact formulation of string theory in this setting has ever been given.

- The main issue is quantum backreaction (quantum strings sourcing corrections to the metric, destroying a well-defined classical background in which string theory is formulated).

Torus worldsheet divergence



$$\int_{|q|<1} \frac{d^2q}{q\bar{q}}\, q^{\alpha'(k_i^2+m_i^2)/4}\, \bar{q}^{\alpha'(k_i^2+\tilde{m}_i^2)/4}\, \mathscr{G}^{ij} = \frac{8\pi\delta_{m_i^2,\tilde{m}_i^2}\mathscr{G}^{ij}}{\alpha'(k_i^2+m_i^2-i\epsilon)}$$

- Again, a divergence is present when all the spatial dimensions are taken to be compact (indicating a need to modify the theory).
- In analogy to the recoil problem, one can try to look for a Fischler-Susskind cure by introducing an explicit path integral over the long-wavelength modes of the background fields (metric, dilaton).
- Can be seen as a stringy UV-completion of minisuperspace models popular in canonical quantum cosmology.

Quantized background

Issues: enormous ambiguity in quantization of minisuperspace models known from the canonical quantum gravity literature; lack of consensus in defining observables.

Pragmatic approach: take a particular set of conventions regarding minisuperspace (a la Marolf-Halliwell) and examine divergence cancellation.

$$ds^2 = -N^2(t)dt^2 + a^2(t)dx^i dx^i, \qquad \phi = \phi(t)$$

$$\langle a_2, \phi_2 | a_1, \phi_1 \rangle = \int_{-\infty}^{\infty} dT \int_{\substack{(a(0),\phi(0))=(a_1,\phi_1) \\ (a(T),\phi(T))=(a_2,\phi_2)}} \mathcal{D}a(t)\mathcal{D}\phi(t)\mathcal{D}X(\tau,\sigma)\, e^{-S_{\rm bg}(a,\phi) - S_{\rm w.s.}(X)} V_1 \cdots V_n$$

Prospects and challenges

- Fischler-Susskind-like cancellation is expected between divergences on the sphere (due to curved target geometries) and the torus (due to worldsheet modulus integration.
- The emergence of relevant divergence structures (two additional vertex operator insertions) has been seen and relies essentially on the presence of path integrals over infrared modes of the background. Technical details remain to be worked out.
- Implications for minisuperspace quantization? Implications for string gas cosmology? The role of winding modes? Brandenberger-Vafa mechanism? T-duality in a dynamical time-dependent setting, etc...

Conclusions

- Quantum backreaction necessitates a departure from the usual formulation of string theory in a fixed classical background and introducing an explicit quantization of a set of infrared modes of the background.
- For the case of recoil of low-dimensional branes, the situation appears to be under good analytic control at next-to-leading order in string coupling. Generalizations could be illuminating.
- For the case of string gas cosmology, relevant algebraic structures appear to have been identified, but many technical details remain to be settled. Hopefully, interesting phenomenology is forthcoming.