Modified Gravity from the Standard Model

T.M. Janssen, T. Prokopec
S. P. Miao & R. P. Woodard
gr-qc/0602110 & gr-qc/0707.0847
gr-qc/0904.1151

We all know “the problem”

$G_{\mu\nu} \neq 8\pi G\, (T_{\mu\nu})_{\text{known}}$ on galaxy scales & larger

We also know the possible solutions:

1. Invent more $T_{\mu\nu}$
   - $\rho_{\text{DM}} \sim 6\times\rho_{\text{known}}$, $\rho_{\text{DE}} \sim 18\times\rho_{\text{known}}$
   - This works . . . but it’s epicyclic
2. Change gravity
   - $R \Rightarrow F(R)$ is the only metric-based, stable way
   - This works . . . but it’s ALSO epicyclic
   - UNLESS you can DERIVE F(R) from 1^{st} Principles

V_{eff} in de Sitter

- $dS^2 = -dt^2 + a^2 dx_i dx^i$, $a = e^{Ht}$
- General Form : $V_{eff} = H^4 f(\varphi^2/H^2)$
- Cf. Flat Space : $V_{eff} \longrightarrow \varphi^4 \ln(\varphi)$

Results for Yukawa & SQED

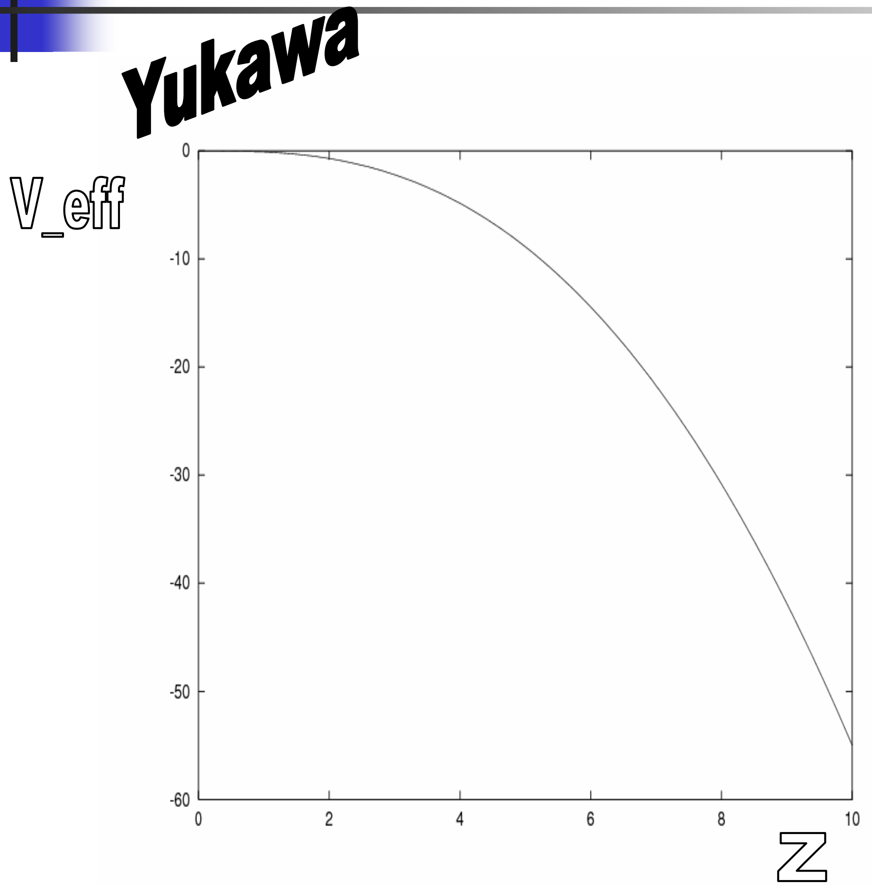
Yukawa: $\Delta\mathcal{L} = -f\varphi\overline{\psi}\psi\sqrt{-g}$

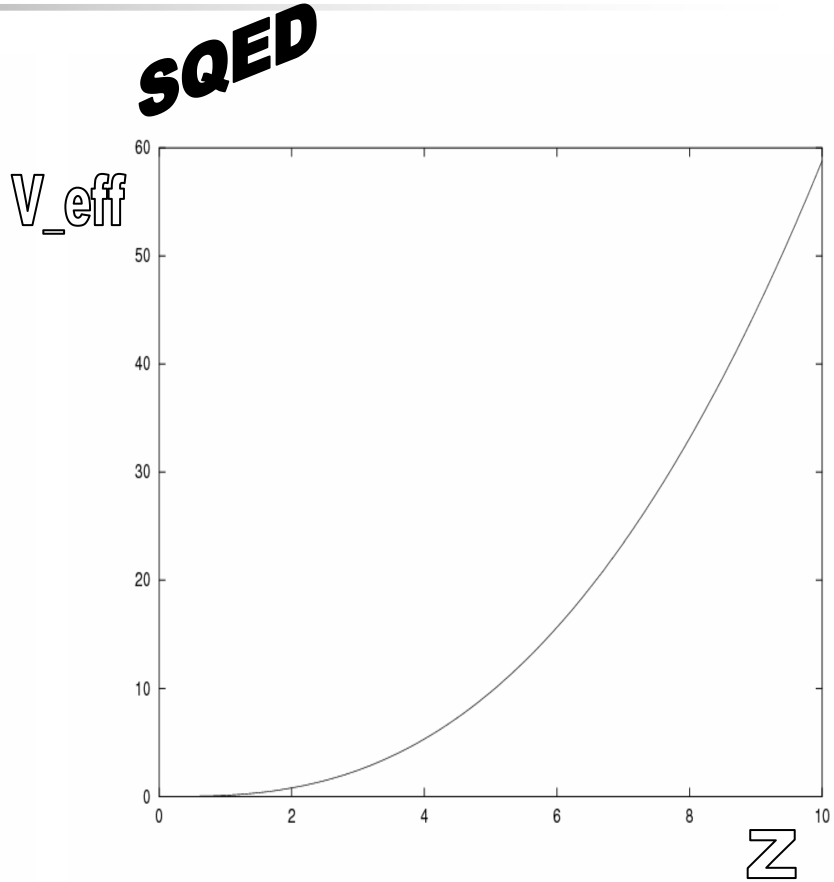
- $V_{\text{eff}} \equiv -\frac{H^4}{8\pi^2} \times F(z)$ with $z \equiv \frac{f^2\varphi^2}{H^2}$
- $$F(z) = 2\gamma z - [\zeta(3) - \gamma]z^2 + \int_0^z dx(1+x)\Big[\psi(1+i\sqrt{x}) + \psi(1-i\sqrt{x})\Big] \longrightarrow \frac{1}{2}z^2\ln(z) + O(z^2)$$

SQED: $\Delta\mathcal{L} = ieA_\mu\varphi^*\partial_\nu\varphi g^{\mu\nu}\sqrt{-g} + \ldots$

- $V_{\text{eff}} \equiv +\frac{3H^4}{8\pi^2} \times F(z)$ with $z \equiv \frac{e^2\varphi^*\varphi}{H^2}$
- $$F(z) = [2\gamma - 1]z - [\frac{3}{2} - \gamma]z^2 + \int_0^z dx(1+x)\Big[\psi(\frac{3}{2} + \frac{1}{2}\sqrt{1-8x}) + \psi(\frac{3}{2} - \frac{1}{2}\sqrt{1-8x})\Big] \longrightarrow \frac{1}{2}z^2\ln(z) + O(z^2)$$

V_{eff} falls for Yukawa and grows for SQED





What is "H" for general $g_{\mu\nu}$

- Our Goals :
  - $\delta / \delta g^{\mu\nu} \longrightarrow$ how V_{eff} changes gravity
  - $\delta / \delta \varphi \longrightarrow$ how V_{eff} changes φ

Generalizing "H" in V_{eff}

Recall Yukawa

- $V_{eff} = -H^4/8\pi^2 \, F(f^2\varphi^2/H^2)$

In de Sitter

- $R^{\rho}{}_{\sigma\mu\nu} = H^2 \, (\delta^{\rho}{}_{\mu} g_{\sigma\nu} - \delta^{\rho}{}_{\nu} g_{\sigma\mu})$

Possibilities for H^2

- $H^2 \Rightarrow R/12$
- $H^2 \Rightarrow (R^{\mu\nu} R_{\mu\nu}/36)^{1/2}$
- $H^2 \Rightarrow (R^{\rho\sigma\mu\nu} R_{\rho\sigma\mu\nu}/24)^{1/2}$

Strategies for generalizing "H"

- 1. $<T_{\mu\nu}>_{dS}=-2\,(-g)^{-1/2}\,\delta\Gamma/\delta g^{\mu\nu}\,|_{dS}$
- 2. V_{eff} & $<T_{\mu\nu}>$ for const $\varepsilon=-H^{-2}dH/dt$
- 3. $\varepsilon(t)=\varepsilon_1 \longrightarrow \varepsilon_2$ Suddenly

Spacetime Exp. Strengthens QFT

WHY?

- Loops $\Rightarrow$ classical physics of virtuals
- Expansion $\Rightarrow$ holds virtuals apart longer

MAXIMUM EFFECT FOR:

- Inflation
- Massless and NOT conformally invariant

TWO PARTICLES:

- m=0 and ξ=0 Scalars
- Gravitons

Infrared Logarithms

WHAT: factors of ln(a) = Ht in QFT loops

FOR EXAMPLE: $\lambda\varphi^4$

- $p=(\lambda H^4/16\pi^2)[-2\ln^2(a) - 7/2\ln(a)] + O(\lambda^3)$

WHY:

- $i\Delta(x;x) = UV + (H^2/4\pi^2)\ln(a)$
- $\int_0^t dt'\, 1 = \ln(a)/H$

- NB: ln(a) even in Power Spectrum
  - Weinberg, hep-th/0605244

Leading Log Approximation

- General Expansion for $\Delta\mathcal{L} = -f\varphi\overline{\psi}\psi\sqrt{-g}$

$$\sum_n f^{2n}\{\alpha_n[\ln(a)]^n + \beta_n[\ln(a)]^{n-1} + \ldots\ldots\}$$

$$\Rightarrow \sum_n \alpha_n[f^2 \ln(a)]^n$$

- Only $\varphi's$ gives $\ln(a)$

$$\Rightarrow \quad \text{Integrate out } \psi's \text{ and drop } \partial's$$

- Starobinskiĭ gets α_n (not β_n etc.)

$$\Big\langle A(\varphi(t))\Big\rangle = \int dx A(x)\rho(t,x)$$

- Even late time limits

$$\rho(t,x) \quad \Rightarrow \quad N \exp\Big[-\frac{8\pi^2}{H^4}V_{eff}(x)\Big]$$

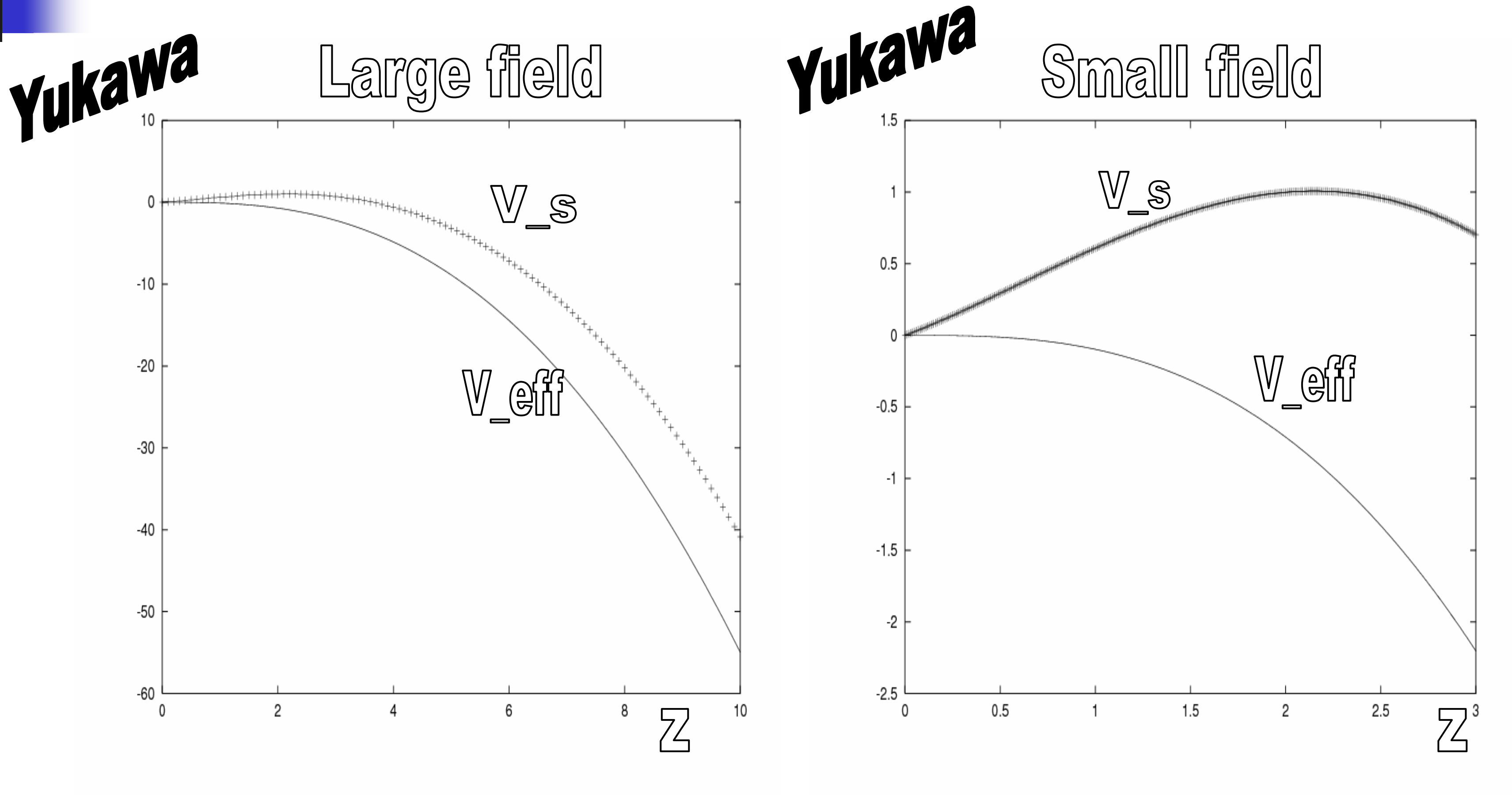
Stress Tensor at Leading Log

- Integrate out ψ's
- Drop derivatives
- $\langle T_{\mu\nu}\rangle = -g_{\mu\nu} \times V_s$
- Yukawa: $V_s = -H^4/8\pi^2 \, F_s(f^2\varphi^2/H^2)$

$$\begin{aligned} F_s(z) &= [\gamma - \frac{1}{2}]z - [\zeta(3) - \gamma + \frac{1}{4}]z^2 \\ &+ \frac{1}{2}(z + z^2)\Big[\psi(1 + i\sqrt{z}) + \psi(1 - i\sqrt{z})\Big] \end{aligned}$$

$$\begin{aligned} F(z) &= 2\gamma z - [\zeta(3) - \gamma]z^2 \\ &+ \int_0^z dx(1 + x)\Big[\psi(1 + i\sqrt{x}) + \psi(1 - i\sqrt{x})\Big] \end{aligned}$$

V_{eff} & V_s Similar for Large φ but Not for Small φ



$V_{eff} \neq V_s$ Distinguishes $H^2(t)$ from $\Lambda/3$

- True: $F_s(z) = \frac{1}{2} z F'(z) - \frac{1}{2} z - \frac{1}{4} z^2$
- For $V_{eff} = -H^4/8\pi^2 F(f^2\varphi^2/H^2)$
  $\rightarrow -(R/12)^2/8\pi^2 F(12 f^2\varphi^2/R)$
- For $\mathcal{L}_{eff} = -\Phi(R)\,(-g)^{1/2}$
  - $\langle T_{\mu\nu}\rangle \rightarrow -g_{\mu\nu}\,\{\Phi(R) - \frac{1}{2} R\,\Phi'(R)\}$ (drop sub-leading terms)
- Would give $F_s(z) = \frac{1}{2} z F'(z)$
- The $-\frac{1}{2} z - \frac{1}{4} z^2$ from counterterms
  - $\delta\xi\varphi^2 R$: $\ln(H^2) \rightarrow \ln(\Lambda/3)$
  - $\delta\lambda\varphi^4$: $\ln(H^2) \rightarrow \ln(\Lambda/3)$

Geometry of const $\varepsilon \equiv -H^{-2}dH/dt$

- $\varepsilon = dH^{-1}/dt \longrightarrow H(t) = H_1/[1+\varepsilon H_1 \Delta t]$ where $\Delta t \equiv t - t_1$
- $H(t) = d\ln(a)/dt \longrightarrow a(t) = a_1[1+\varepsilon H_1 \Delta t]^{1/\varepsilon}$
  - $R = 6(2-\varepsilon)H^2$
  - $R^{\mu\nu} R_{\mu\nu} = 12(3-3\varepsilon+\varepsilon^2)H^4$
  - $R^{\rho\sigma\mu\nu} R_{\rho\sigma\mu\nu} = 12(2-2\varepsilon+\varepsilon^2)H^4$

Hubble Effective Potential

- Usual : $\Gamma[\Phi=\text{const}] \equiv -\int d^4x V_{eff}$
- Hubble : $\Gamma[\Phi=\text{const}\ H(t)] \equiv -\int d^3x \int dt\ a^3 V_{eff}$
- $\nu^2 \equiv [(3-\varepsilon^2)/4 - \lambda\Phi^2/2H^2]/(1-\varepsilon)^2$

$$
\begin{aligned}
V'_{\rm hub}(\Phi) = & \frac{\lambda}{96\pi^2}(1-5\epsilon+3\epsilon^2)H^2\Phi \\
& + \left\{\xi + \frac{\lambda(\xi-\frac{1}{6})}{32\pi^2}\left[\ln\left[\frac{(1-\epsilon)^2H^2}{\mu_1^2}\right] + \psi\left(\frac{1}{2}+\nu\right) + \psi\left(N+\frac{5}{2}-\nu\right)\right]\right\}R\Phi \\
& + \left\{\frac{\lambda}{6} + \frac{\lambda^2}{64\pi^2}\left[\ln\left[\frac{(1-\epsilon)^2H^2}{\mu_2^2}\right] + \psi\left(\frac{1}{2}+\nu\right) + \psi\left(N+\frac{5}{2}-\nu\right)\right]\right\}\Phi^3 \\
& + \frac{(1-\epsilon)^2H^2}{32\pi^2}\sum_{M=-1}^{N}\frac{1}{\nu-M-\frac{3}{2}}\left\{\frac{\Gamma(2\nu-M)\Gamma(2\nu-2M)\,(4k_0^2\eta^2)^{M+\frac{3}{2}-\nu}}{2\Gamma(M+1)\Gamma^2(\nu+\frac{1}{2}-M)}\right. \\
& \left. -\left(\nu^2-\frac{1}{4}\right)\right\}\lambda\Phi + O(\hbar^2)\,.
\end{aligned}
$$

Generalizing $V_{eff}=H^4 f(\varepsilon)$

- One Possibility :
  - $A \equiv R^2 = 36(2-\varepsilon)^2 H^4$
  - $B \equiv \Box R = 36\varepsilon(1-\varepsilon)(2-\varepsilon)H^4$

  $\longrightarrow$ $\varepsilon = [A+B+(A^2-6AB+B^2)^{1/2}]/2A$

  $\longrightarrow$ $H^4 = A^3/\ 9[3A-B-\ (A^2-\ 6AB+B^2)^{1/2}]^2$
- But is inevitably unstable!

  (Ostrogradskian Instability)

Sudden Transitions in ε

- $\varepsilon(t)=0\ \forall t$ (de Sitter) $\longrightarrow V_{eff} \neq 0$
- $\varepsilon(t)=+2\ \forall t$ (Radiation) $\longrightarrow V_{eff}=0$
- $\varepsilon(t)=\varepsilon_1 \longrightarrow \varepsilon_2 \longrightarrow$ u(t,k) & du/dt continuous

  $\longrightarrow V_{eff}$ continuous, so $V_{eff} \neq 0$ for $0 \longrightarrow 2$

  $\longrightarrow \varepsilon(t)=2$ knows about $\varepsilon(t)=0$ epoch

  $\longrightarrow$ cannot be local
- Can build from R & $\square^{-1}$R

Summary

F(R) models can give a(t)

- epicyclic
- Not from fundamental theory

de Sitter $\longrightarrow V_{eff} = H^4\, F(\varphi^2/H^2)$

Modifications of gravity

Modifications of matter, too

Constant $\varepsilon \longrightarrow V_{eff}=H^4F(\varepsilon,\varphi^2/H^2)$

Could get using R & $\Box$R

$\varepsilon_1 \longrightarrow \varepsilon_2 \longrightarrow V_{eff}$ not local

- Could get using R & $\Box^{-1}$R