Massive Gravity Theories in (Anti)-de Sitter Spacetime

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Outline

Based On

This talk is based on I. G., B. Tekin, “Massive Higher Derivative Gravity in D-dimensional Anti-de Sitter Spacetimes”, P. R. D **80**, 064033 (2009).

Introduction

- Experience from quantum field theory implies that at high energies Einstein's gravity should be replaced with: Einstein-Hilbert term+Higher Curvature terms,
- Higher curvature terms are motivated by the quantum gravity scenarios such as string theory and asymptotic safety.
- To have a better IR behaviour a mass term added to the theory.
- Mass can be given to graviton by adding Pauli-Fierz mass term.

Tree Level and Boundary Unitarity

- To have a physically meaningfull theory it must be unitary.
- Tree level unitarity is tachyon and ghost freedom.
  - Ghost is characterized by negative kinetic energy,
  - Tachyon is characterized by negative mass square.
- Thus, unitarity analysis is basically a check of proper signs in the graviton propagator $(-,+,+,\dots)$ that is $\frac{1}{p^2-m^2}$.

The Lineer Equation of Motion

- The most general quadratic gravity model augmented with Pauli-Fieerz mass term is

$$I = \int d^D x \sqrt{-g} \left\{ \frac{1}{\kappa} (R - 2\Lambda_0) + \alpha R^2 + \beta R^2_{\mu\nu} + \gamma \left(R^2_{\mu\nu\sigma\rho} - 4R^2_{\mu\nu} + R^2 \right) \right\}$$
$$+ \int d^D x \sqrt{-g} \left\{ -\frac{M^2}{4\kappa} \left(h^2_{\mu\nu} - h^2 \right) + \mathcal{L}_{\text{matter}} \right\}, \qquad (1)$$

- To get the one-particle exchange aplitude we need the lineer equations of motion:
  - we take tha variation of (1) with respect to the metric $g_{\mu\nu}$, $(-,+,+,\dots)$ to get the equations of motion,
  - then we linearize the equations of motion around a constant curvature background $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$
- The linearized equations of motion are

$$T_{\mu\nu}(h) = a\mathcal{G}^L_{\mu\nu} + (2\alpha + \beta) \left(\bar{g}_{\mu\nu} \bar{\Box} - \bar{\nabla}_\mu \bar{\nabla}_\nu + \frac{2\Lambda}{D-2} \bar{g}_{\mu\nu} \right) R^L$$
$$+ \beta \left(\bar{\Box} \mathcal{G}^L_{\mu\nu} - \frac{2\Lambda}{D-1} \bar{g}_{\mu\nu} R^L \right) + \frac{M^2}{2\kappa} \left(h_{\mu\nu} - \bar{g}_{\mu\nu} h \right), \qquad (2)$$

where we have defined $a \equiv \frac{1}{\kappa} + \frac{4\Lambda D}{D-2}\alpha + \frac{4\Lambda}{D-1}\beta + \frac{4\Lambda(D-3)(D-4)}{(D-1)(D-2)}\gamma$.

Tree-Level Scattering Amplitude

- To get the physical parts of $h_{\mu\nu}$ we decompose it as

$$h_{\mu\nu} \equiv h_{\mu\nu}^{TT} + \bar{\nabla}_{(\mu} V_{\nu)} + \bar{\nabla}_{\mu} \bar{\nabla}_{\nu} \phi + \bar{g}_{\mu\nu} \psi, \tag{3}$$

- Taking divergence and double divergence of (3)

$$h = \bar{\Box}\phi + D\psi, \qquad \bar{\Box}h = \bar{\Box}^2\phi + \frac{2\Lambda}{(D-2)}\bar{\Box}\phi + \bar{\Box}\psi, \tag{4}$$

where we used $\bar{\nabla}^{\nu}\bar{\nabla}^{\mu}h_{\mu\nu} = \bar{\Box}h$, which is *not* a gauge condition but imposed on us as a result of the nonzero mass term.

- Using (4)

$$\psi = \left\{ \frac{\Lambda}{\kappa} + 4\Lambda f - c\Lambda\bar{\Box} - \frac{M^2}{2\kappa}(D-1) \right\}^{-1} \left(\frac{(D-1)(D-2)}{2\Lambda}\bar{\Box} + D \right)^{-1} T, \tag{5}$$

where $c \equiv \frac{4(D-1)\alpha}{D-2} + \frac{D\beta}{D-2}$.

- Decomposing the energy-momentum tensor one can write the one-particle exchange amplitude between two covariantly conserved sources as

$$A = \frac{1}{4}\int d^D x\sqrt{-\bar{g}}\, T'_{\mu\nu}(x)\, h^{\mu\nu}(x) = \frac{1}{4}\int d^D x\sqrt{-\bar{g}} \left(T'_{\mu\nu} h^{TT\mu\nu} + T'\psi \right). \tag{6}$$

Tree-Level Scattering Amplitude

- Finally,

$$\begin{aligned}
4A &= 2T'_{\mu\nu}\left\{(\beta\bar{\Box}+a)(\triangle_L^{(2)}-\frac{4\Lambda}{D-2})+\frac{M^2}{\kappa}\right\}^{-1}T^{\mu\nu} \\
&+ \frac{2}{D-1}T'\left\{(\beta\bar{\Box}+a)(\bar{\Box}+\frac{4\Lambda}{D-2})-\frac{M^2}{\kappa}\right\}^{-1}T \\
&- \frac{4\Lambda}{(D-2)(D-1)^2}T'\left\{(\beta\bar{\Box}+a)(\bar{\Box}+\frac{4\Lambda}{D-2})-\frac{M^2}{\kappa}\right\}^{-1}\left\{\bar{\Box}+\frac{2\Lambda D}{(D-2)(D-1)}\right\}^{-1}T \\
&+ \frac{2}{(D-2)(D-1)}T'\left\{\frac{1}{\kappa}+4\Lambda f-c\bar{\Box}-\frac{M^2}{2\kappa\Lambda}(D-1)\right\}^{-1}\left\{\bar{\Box}+\frac{2\Lambda D}{(D-2)(D-1)}\right\}^{-1}T.
\end{aligned} \tag{7}$$

where $f \equiv (\alpha D+\beta)\,\frac{(D-4)}{(D-2)^2}+\gamma\frac{(D-3)(D-4)}{(D-1)(D-2)}$.

Tree-Level Amplitude

- From (7) one can figure out the particle spectrum,
- One can also compute the Newtonia potentials in flat spacetime,
- In curved background the Green's function (matrix) of Lichnerowicz operator must be handled,
- We can see the $M^2 \to 0$ and $\Lambda \to 0$ limits does not commute,
- First taking flatspace limit we encounter the van Dam-Veltman-Zakharov (vDVZ) discontinuity,
- First taking the massless limit take us to New Massive Gravity (NMG) theory.

vDVZ Discontinuity

- For this case the amplitude become

$$4A = -2T'_{\mu\nu}\left\{\beta\partial^4 + \frac{1}{\kappa}\partial^2 - \frac{M^2}{\kappa}\right\}^{-1} T^{\mu\nu} + \frac{2}{D-1}T'\left\{\beta\partial^4 + \frac{1}{\kappa}\partial^2 - \frac{M^2}{\kappa}\right\}^{-1} T \quad (8)$$

Unless $\beta = 0$, we have a massive ghost.

- The Newtonian potential energy between $T'_{00} \equiv m_1\delta(x-x_1)$, $T^{00} \equiv m_2\delta(x-x_2)$ in three and four dimensions can be obtained as

$$\begin{aligned} U &= \frac{1}{2\beta(m_+^2 - m_-^2)}\frac{m_1 m_2}{4\pi}[K_0(m_- r) - K_0(m_+ r)] \; D = 3, \\ U &= \frac{m_1 m_2}{3\beta(m_+^2 - m_-^2)}\frac{1}{4\pi r}[e^{-m_- r} - e^{-m_+ r}] \; D = 4. \end{aligned} \quad (9)$$

where $r \equiv |\vec{x_1} - \vec{x_2}|$.

- As $\beta \to 0$, the potential energies become

$$U = -\frac{\kappa}{8\pi} m_1 m_2 K_0(Mr) \qquad D = 3, \quad (10)$$

$$U = -\frac{4}{3}\frac{Gm_1 m_2}{r} e^{-Mr} \qquad D = 4 \quad (11)$$

M. Porrati, P. L. B **498**, 92 (2001).

New Massive Gravity

- For $M^2 = 0$ then taking $\Lambda \to 0$ limit

$$4A = -2T'_{\mu\nu}\left\{\beta\partial^4 + \frac{1}{\kappa}\partial^2\right\}^{-1} T^{\mu\nu} + \frac{2}{(D-1)}T'\left\{\beta\partial^4 + \frac{1}{\kappa}\partial^2\right\}^{-1} T$$
$$-\frac{2}{(D-1)(D-2)}T'\left\{c\partial^4 - \frac{1}{\kappa}\partial^2\right\}^{-1} T \qquad (12)$$

- Generically there are three poles :

$$\partial_1^2 = 0, \qquad \partial_2^2 = -\frac{1}{\kappa\beta} \qquad , \partial_3^2 = \frac{1}{\kappa c}. \qquad (13)$$

$$Res(\partial_1^2) = \frac{2\kappa(3-D)}{(D-2)}, \; Res(\partial_2^2) = \frac{2\kappa(D-2)}{(D-1)}, \; Res\,(\partial_3^2) = -\frac{2\kappa}{(D-1)(D-2)}$$

- From the second poleand its residue; $\kappa\beta < 0$ and $\kappa < 0$,
- From the residue massless pole; $D > 3$
- The residue of the third pole becomes positive for negative κ. To eliminate this residue $c = 8\alpha + 3\beta = 0$. E. A. Bergshoeff, O. Hohm and P. K. Townsend, P. R. L. **102**, 201301 (2009); P. R. D **79**, 124042 (2009).

New Massive Gravity

- Newtonian potential

$$U = \frac{\kappa}{8\pi} m_1 m_2 \left(K_0(m_g r) - K_0(m_0 r) \right) \qquad D = 3, \qquad (14)$$

where $m_g^2 \equiv -\frac{1}{\kappa\beta}$ and $m_0^2 \equiv \frac{1}{\kappa(8\alpha+3\beta)}$. Clearly, m_0 is a massive ghost that gives a repulsive component.

- This result also confirms that, at this level, NGM has the same Newtonian limit as the usual massive gravity (10), if the Pauli-Fierz mass term is chosen as $M = m_g$.
- Beyond three dimensions, in flat space, massive ghost does not decouple unless $\beta = 0$. As an example, let us look at $D = 4$:

$$U = -\frac{G m_1 m_2}{r} \left(1 - \frac{4}{3} e^{-m_g r} + \frac{1}{3} e^{-m_a r} \right), \qquad (15)$$

where $m_a^2 \equiv \frac{1}{2\kappa(3\alpha+\beta)}$. The middle, repulsive term signals the ghost problem. K.S. Stelle, P. R. D **16**, 953 (1977).

Conclusion

- We compute the one-partical scattering amplitude of the most general quadratic curvature gravity augmented with PF mass term,
- For the flatspace and massless limit we encounter with the vDVZ discontinuity and NMG,
- NMG is non-ghost and non-tachyonic theory at tree-level,
- The unitarity of NMG must be checked for loop levels.

Thank you!