Effective Dynamics from Spinfoam Cosmology

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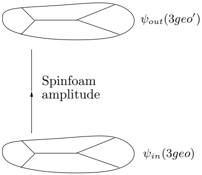
Introduction

- Extraction of effective (classical) dynamics from Spin foams
  - transition amplitudes between spin networks, $\psi_{in} \to \psi_{out}$
  - Fixed graph: ψ_{in} and ψ_{out} same underlying graph



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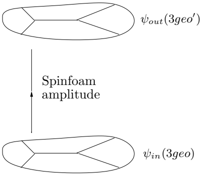

finite graph $\to$ finite number of dof $\to$ minisuperspaces ??

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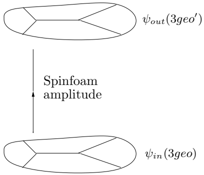

finite graph $\to$ finite number of dof $\to$ minisuperspaces ??

- Simple setting $\to$ homogenous and isotropic configuration. Trying to modelize effective FRW models

Spinfoam cosmology

Bianchi, Rovelli, Vidotto

Spinfoam cosmology approach

Kinematics

- We choose a suitable graph and attach to it appropriate classical data: 3-geometry
- We reduce to the homogeneous and isotropic sector
  - Finite graph $\rightarrow$ finite region of the homogeneous geometry
- We define suitable coherent spin networks peaked on above symmetric configurations

Spinfoam cosmology approach

Dynamics

- We calculate the transition amplitude between two such states at first order in a vertex expansion by using the Spinfoam ansatz (evaluation of the boundary spin network on the identity)
  - Renormalization/ coarse grain procedure in GFT would give an expansion in effective contributions. We assume such a one-vertex contribution is the leading order.
- We look for symmetries of the transition amplitude
  $\rightarrow$ discrete diffeomorphisms

Spinfoam cosmology approach

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  - Homogeneity $\rightarrow$ Hamiltonian constraint

(effective) FRW model ??

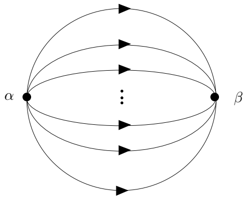
Loop gravity on a fixed graph

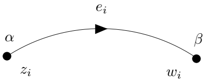
- In LQG, phase space of gravity parametrized by holonomy-flux variables $(g_e, X_e) \longrightarrow$ classical data attached to graphs Γ
- Quantization of the holonomy-flux algebra: spin networks
- Given Γ, quantum states are gauge-invariant functions $\psi_\Gamma(g_e)$ of the group elements g_e living on the edges $e \in \Gamma$
  - Spin network basis: irreps. of $SU(2)$ attached to edges, intertwiners [$SU(2)$ invariant tensors] attached to vertices

Recent approach

- Phase space of loop gravity parameterized by spinors
  - Twisted geometries — Freidel, Speziale, Livine, Tambornino
- Spin networks are the quantization of classical spinor networks
  - **U(N) formalism for intertwiners** — Girelli, Livine, Freidel, Dupuis...

Choice of graph: two-vertex graph with N edges





$$|z\rangle = \begin{pmatrix} z_0 \\ z_1 \end{pmatrix}, \quad |z] = \begin{pmatrix} -\bar{z}_1 \\ \bar{z}_0 \end{pmatrix}$$

- Two spinors $z_i, w_i \in \mathbf{C}^2$ per edge, attached to α and β resp.
- Poisson bracket: $\{z_a, \bar{z}_b\} = -i\delta_{ab} = \{w_a, \bar{w}_b\}$, $a, b = 0, 1$
- Closure contraints: $SU(2)$ invariance in every vertex
- Matching constraints: $U(1)$ invariance in every edge

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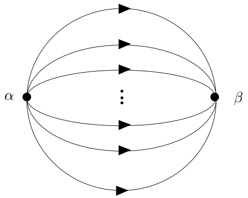

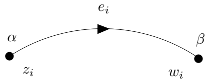

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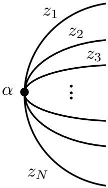
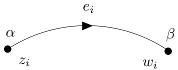
- $|z\rangle\langle z| = \frac{1}{2}\left(\langle z|z\rangle \mathbb{I} + \vec{V}(z)\cdot\vec{\sigma}\right), \quad |z][z| = \frac{1}{2}\left(\langle z|z\rangle \mathbb{I} - \vec{V}(z)\cdot\vec{\sigma}\right)$
  - z_i defined by $\vec{V}(z_i)$ up to a global phase

2-vertex spinor network: Geometrical interpretation



- Dual to a polyhedron with N faces
- $\vec{V}(z_i) = \langle z_i|\vec{\sigma}|z_i\rangle$ vector normal to the i-th face
- $|\vec{V}(z_i)| = \langle z_i|z_i\rangle$ (twice) the area of the i-th face

- Closure constraints: $\mathcal{C}_\alpha \equiv \sum_i \vec{V}(z_i) = 0$, analogously for β.
  - $\vec{V}(z_i)$ generators of $SU(2)$ algebra $\rightarrow$ fluxes
  - $SU(2)$-transf.: $|w_i] = g_e|z_i\rangle \rightarrow$ holonomies



- Matching constraints: $\mathcal{M}_i \equiv |\vec{V}(z_i)| - |\vec{V}(w_i)| = 0$, $\forall i$

$U(N)$-formalism

Girelli, Livine, Freidel

- $U(N)$-action on the set of spinors: $z_k \rightarrow (Uz)_k, \quad U \in U(N)$
  Commutes with closure constraint
- Scalar products between spinors are $SU(2)$-invariant
  - $E_{ij}^v = \langle z_i^v | z_j^v \rangle$
    - ⋆ They generate a $U(N)$-algebra
  - $F_{ij}^v = [z_i^v | z_j^v \rangle, \quad \bar{F}_{ij}^v = \langle z_j^v | z_i^v]$
    - ⋆ They close algebra with the above observables

Symmetry reduction

- Matching constraints: invariance under $U(1)^N$

$$\forall i, \quad \mathcal{M}_i = E_{ii}^{\alpha} - E_{ii}^{\beta} = 0 \longleftrightarrow \langle z_i | z_i \rangle = \langle w_i | w_i \rangle$$

- Symmetry reduction: imposing a larger symmetry

$\rightarrow$ invariance under $U(N)$ Borja, Diaz-Polo, Garay, Livine

$$\forall i, j, \qquad \mathcal{E}_{ij} \equiv E_{ij}^{\alpha} - E_{ji}^{\beta} = 0, \qquad \mathcal{E}_{ii} = \mathcal{M}_i$$

  - Polyhedra dual to α and β are identical $\rightarrow$ homogeneity
  - $\forall i \quad |w_i] = e^{i\phi} |z_i\rangle$. Same phase for all edges $\rightarrow$ isotropy
  - Reduced phase space: $\{A, \phi\} = 1$, $A \equiv \frac{1}{2} \sum_i \langle z_i | z_i \rangle$
    - ⋆ A: global area, ϕ: conjugate angle
    - ⋆ ϕ matches the angle ξ parameterizing twisted geometries
    - ! Individual areas $A_k = \langle z_k | z_k \rangle$ not imposed to be equal

Quantum representation

- $SU(2)$-invariant observables: $\hat{E}_{ij}, \quad \hat{F}_{ij}, \quad \hat{F}^{\dagger}_{ij}$
- Each space of N-valent intertwiners at fixed total area J $\mathcal{R}^J = \bigoplus_{J=\sum_i j_i} \text{Inv}_{SU(2)} \otimes_i V^i$, carries an irrep. of $U(N)$
  - $\hat{E}_{ij} : \mathcal{R}^J \to \mathcal{R}^J$ generator $U(N)$-action
- Whole space of N-valent intertwiners: $\mathcal{H}_N = \bigoplus_J \mathcal{R}^J$ $\to$ Fock structure
  - $\hat{F}_{ij} : \mathcal{R}^J \to \mathcal{R}^{J-1}$ annhilation operator
  - $\hat{F}^{\dagger}_{ij} : \mathcal{R}^J \to \mathcal{R}^{J+1}$ creation operator
- Quantum matching constraints in every edge $\to$ spin networks

$U(N)$ coherent intertwiners

Freidel, Livine, Dupuis

- Eigenstates of the annihilation operators $\hat{F}_{ij}$

$$
\begin{aligned}
||\{z_i\}\rangle &= \sum_J \frac{1}{J!(J+1)!} \left(\sum_{ij} [z_i|z_j\rangle \hat{F}^\dagger_{ij} \right)^J |0\rangle \\
&= \sum_{\{j_e\}} \frac{1}{\sqrt{\prod_e (2j_e)!}} ||\{j_e, z_e\}\rangle \\
&= \int dg g \triangleright (\text{HO's-coherent state})
\end{aligned}
$$

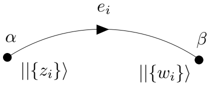
- $U(N)$-coherent: $\hat{U}||\{z_i\}\rangle = ||\{(Uz)_i\}\rangle$

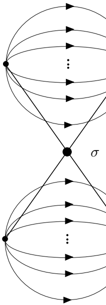
- Scalar product and norm explicitly known

$$
\langle \{z_i\}||\{z_i\}\rangle = \sum_J \frac{A(z_i)^{2J}}{J!(J+1)!} = \frac{I_1(2A(z_i))}{A(z_i)}, \quad A(z_i) \equiv \frac{1}{2} \sum_i \langle z_i|z_i\rangle
$$

Basic transition amplitude

- $SU(2)$-BF theory
- One-vertex transition amplitude





$$\psi_{out}(g_f) = Tr \otimes_f g_f||\{z_f\}\rangle \otimes \langle\{w_f\}||$$

$$\mathcal{A}_\sigma = \psi_{\partial\sigma}(1) = \psi_{in}(1)\psi_{out}(1) \qquad \text{factorized}$$

$$\psi_{in}(g_i) = Tr \otimes_i g_i||\{z_i\}\rangle \otimes \langle\{w_i\}||$$

- Symmetric configuration: $|w_i] = e^{i\phi}\,|z_i\rangle \longrightarrow ||\{w_i\}\rangle = ||\{e^{i\phi} z_i\}\rangle$

Basic transition amplitude

$$\mathcal{A}_\sigma = \langle\{e^{i\phi}z_i\}||\{z_i\}\rangle\langle\{e^{i\phi'}z_f\}||\{z_f\}\rangle$$
$$= \frac{e^{i\phi}I_1(2e^{-i\phi A})}{A}\frac{e^{i\phi'}I_1(2e^{-i\phi' A'})}{A'} = \psi_{in}(A,\phi)\psi_{out}(A',\phi')$$

Symmetries of $\mathcal{A}_\sigma$

- $\hat{C}\psi(A,\phi) = 0, \quad \hat{C} = A^2\partial_A^2 - 2A\partial_A + 2$
  - Differential equation explicitly known
  - No need to take the large area (spin) limit

Basic transition amplitude

$$\mathcal{A}_\sigma = \langle \{e^{i\phi} z_i\} || \{z_i\} \rangle \langle \{e^{i\phi'} z_f\} || \{z_f\} \rangle$$
$$= \frac{e^{i\phi} I_1(2e^{-i\phi A})}{A} \frac{e^{i\phi'} I_1(2e^{-i\phi' A'})}{A'} = \psi_{in}(A, \phi) \psi_{out}(A', \phi')$$

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  - Differential equation explicitly known
  - No need to take the large area (spin) limit
- $\{A, \phi\} = 1 \quad \rightarrow \quad C = -A^2\phi^2 - i2A\phi + 2$
  - Link with FRW (work in progress)

    (using $SL(2, \mathbb{C})$ SF model Dupuis, Freidel, Livine Speziale)

Outlook

- Using coherent intertwiners based on spinors it is possible to compute exactly basic transition amplitudes
- Within a simple setting, explicit realization of the recursion relations on boundary data (Bonzom, Freidel, Livine)
  $\rightarrow$ symmetries of the transition amplitude
  $\rightarrow$ effective classical dynamics
- Goal: to play around with the boundary data and the SF bulk to modelize specific models with physical interest (cosmology)