Matter perturbations in Galileon cosmology

Phys.Rev.D83:043515,2011

Tokyo University of Science
Antonio De Felice, Ryotaro Kase,
and Shinji Tsujikawa

1. Motivation

The source for the late-time cosmic acceleration is named Dark Energy.
The simplest candidate for dark energy is the cosmological constant.

Cosmological constant (Λ)

However if it originates from a vacuum energy of particle physics, its energy scale is too much larger than the dark energy density today.

➡ **Dark energy problem may imply some modification of gravity on large scales.**

Modified gravitational theories

- **There must be a stable accelerating solution which explains cosmic acceleration.**
- **These models need to recover General Relativistic (GR) behavior at short distances to satisfy solar system constraints.**

f(R) gravity,
Scalar–tensor theories

➡ with a potential term

DGP braneworld,
Galileon gravity

➡ without a potential term

- Recovery of GR behavior at short distances

1. Chameleon mechanism

f(R) gravity,
Scalar–tensor theories

If a scalar field has a potential with a large effective mass in the region high density, then the model can recover GR.

→ **However a fine tuning of initial conditions is required to realize a viable cosmology.**

2. Vainshtein mechanism

DGP braneworld,
Galileon gravity

In the DGP braneworld the non–linear effect of the field self–interaction term $\Box\phi(\partial_\mu\phi\partial^\mu\phi)$ allows the possibility to recover GR at short distances.

→ **However the DGP model suffers from a ghost problem, in addition to the difficulty for consistency with the combined data analysis.**

Galileon gravity

A. Nicolis, R. Rattazzi,
E. Tricherini
Phys. Rev. D 79, 064036 (2009)

C. Deffayet, G. Esposito-Farese,
A. Vikman
Phys. Rev. D 79, 084003 (2009)

The field self-interaction $\Box\phi(\partial_\mu\phi\partial^\mu\phi)$ appearing in the DGP model satisfies the Galilean symmetry in the flat space-time: $\partial_\mu\phi \to \partial_\mu\phi + b_\mu$
Imposing this symmetry in the flat space-time one can show that the field Lagrangian consists of five terms, and they were extended to covariant forms in the curved space-time as the following.

$$\mathcal{L}_1 = M^3\phi\,, \quad \mathcal{L}_2 = (\nabla\phi)^2\,, \quad \mathcal{L}_3 = (\Box\phi)(\nabla\phi)^2/M^3\,,$$

$$\mathcal{L}_4 = (\nabla\phi)^2\left[2(\Box\phi)^2 - 2\phi_{;\mu\nu}\phi^{;\mu\nu} - R(\nabla\phi)^2/2\right]/M^6,$$

$$\begin{aligned}\mathcal{L}_5 =& (\nabla\phi)^2[(\Box\phi)^3 - 3(\Box\phi)\,\phi_{;\mu\nu}\phi^{;\mu\nu} \\ &+ 2\phi_{;\mu}{}^{\nu}\phi_{;\nu}{}^{\rho}\phi_{;\rho}{}^{\mu} - 6\phi_{;\mu}\phi^{;\mu\nu}\phi^{;\rho}G_{\nu\rho}]/M^9\end{aligned}$$

The field equations of motion are kept up to the second-order. This property is welcome to avoid the appearance of an extra degree of freedom associated with ghosts.

2. Background cosmology

Since we are interested in the evolution of matter density perturbations long after the radiation-domination, let us consider the following action;

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{M_{\rm pl}^2}{2}R + \frac{1}{2}\sum_{i=1}^{5} c_i\mathcal{L}_i + \mathcal{L}_m\right]$$

$c_1 = 0$

In the FLRW background one can have a de Sitter solutions with $H = H_{\rm dS} = \text{constant},\ \dot{\phi} = \dot{\phi}_{\rm dS} = \text{constant}.$

The background equations

$$3M_{\rm pl}^2H^2 = \rho_{\rm DE} + \rho_m\,,$$
$$3M_{\rm pl}^2H^2 + 2M_{\rm pl}^2\dot{H} = -p_{\rm DE}$$

at the de Sitter (dS) point.

$$\begin{cases}\alpha & \equiv c_4 x_{\rm dS}^4\\ \beta & \equiv c_5 x_{\rm dS}^5\end{cases}$$

$$x_{\rm dS} \equiv \frac{\dot{\phi}_{\rm dS}}{H_{\rm dS}M_{\rm pl}}$$

$$c_2 x_{\rm dS}^2 = 6 + 9\alpha - 12\beta\,,$$
$$c_3 x_{\rm dS}^3 = 2 + 9\alpha - 9\beta\,.$$

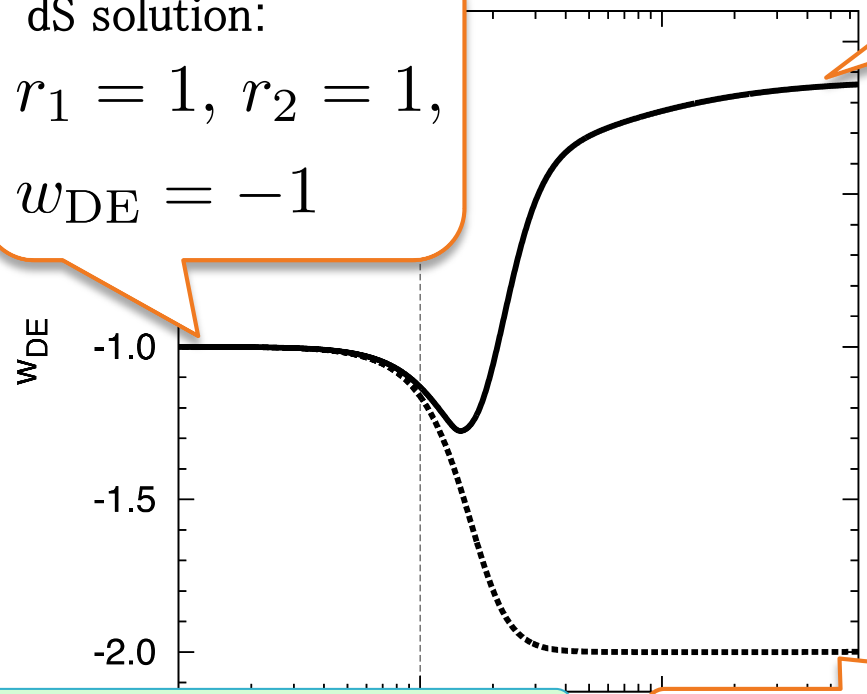
In order to discuss the cosmological dynamics, it is convenient to introduce the following variables

$$r_1 \equiv \frac{\dot{\phi}_{\rm dS} H_{\rm dS}}{\dot{\phi} H}, \quad r_2 \equiv \frac{1}{r_1}\left(\frac{\dot{\phi}}{\dot{\phi}_{\rm dS}}\right)^4$$

At the dS point, one has

$$r_1 = 1, \; r_2 = 1$$

There are three distinct fixed points $(r_1, r_2) = (0, 0), (1, 0), (1, 1)$



dS solution:

$$r_1 = 1, \; r_2 = 1, \; w_{\rm DE} = -1$$

late time tracking solution:

$$r_1 \simeq 0, \; r_2 \simeq 0, \; w_{\rm DE} \simeq -1/8$$

The background combined data analysis based onCMB + BAO + SNIa gives

Savvas Nesseris, Antonio De Felice, Shinji Tsujikawa Phys. Rev. D. 82. 124054

$$\alpha = 1.414 \pm 0.056$$

$$\beta = 0.422 \pm 0.022$$

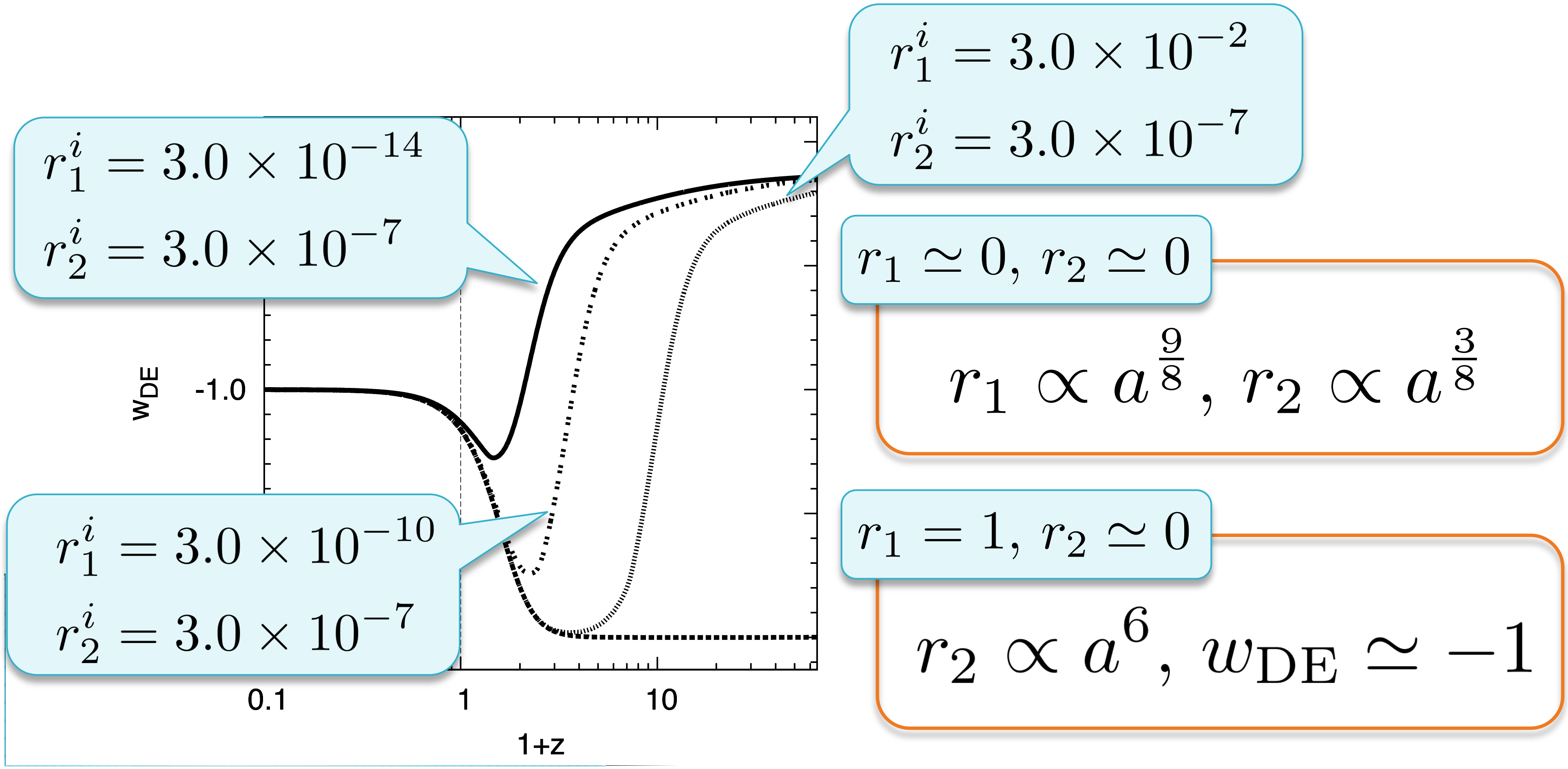
(68% C.L.)

observationally disfavored

tracker solution: $r_1 \simeq 1, \; r_2 \simeq 0, \; w_{\rm DE} \simeq -2$

The late-time tracking solution also looks dangerous, since it's equation of motion become less than −1. However, there is viable model parameter space in which one can avoid the ghosts or the Laplacian instabilities.

Since any solutions which have small initial conditions approach the bottom dotted line at late time, it is called the "tracker" solution and the others called the "late-time tracking" solutions.



3. Cosmological perturbation theories

The modified evolution of matter density perturbations can allow us to distinguish the Galileon model from the LCDM.

$$ds^2 = -(1+2\Psi)dt^2 + a^2(t)(1+2\Phi)\delta_{ij}dx^i dx^j$$

Expansion of the action at second order gives

$\phi(t,x) = \tilde{\phi}(t) + \delta\phi(t,x)$

$$[\sqrt{-g}\mathcal{L}]^{(2)} \equiv L(\Psi, \Phi, \delta\phi, v)$$

velocity potential $v(t,x)$

Variations of the second-order Lagrangian L with respect to above variables gives

$$E_\Psi = 0,\ E_\Phi = 0,\ E_{\delta\phi} = 0,$$

$$\ddot{\delta}_m + 2H\dot{\delta}_m + \frac{k^2}{a^2}\Psi = 3(\ddot{Q} + 2H\dot{Q})$$

where $\delta_m \equiv \delta\rho/\rho + 3Hv,\ Q \equiv Hv - \Phi$

Quasistatic approximation on subhorizon scales

This corresponds to the approximation under which the dominant contributions to the perturbation equations are those including $k^2/a^2,\ \delta_m$.

The full equations for perturbations are very complicated, but they are simplified under this approximation as following

$$\delta_m'' + \left(2 + \frac{H'}{H}\right)\delta_m' - \frac{3}{2}\frac{G_{\rm eff}(t)}{G}\Omega_m\delta_m \simeq 0$$

$$\Phi_{\rm eff} \simeq -\frac{3}{2}\frac{G_{\rm eff}(t)}{G}\frac{1+\eta(t)}{2}\Omega_m\delta_m\left(\frac{aH}{k}\right)^2$$

where $\Phi_{\rm eff} \equiv (\Psi - \Phi)/2,\ \eta(t) \equiv -\Phi/\Psi$.

Unlike LCDM model, the effective gravitational coupling $G_{\rm eff}$ and the anisotropic operator η which describes the difference between the two gravitational potentials depend on time.

- **the approximation in three different regimes**

tracer regime

$$r_1 = 1,\ r_2 \simeq 0$$

$$\frac{G_{\rm eff}}{G} = 1 + \frac{291\alpha^2 + 702\beta^2 - 933\alpha\beta + 20\alpha - 84\beta + 4}{2(10\alpha - 9\beta + 8)} r_2$$

$$\eta = 1 - \frac{3(126\alpha^2 + 306\beta^2 - 405\alpha\beta + 4\alpha - 30\beta)}{2(10\alpha - 9\beta + 8)} r_2$$

late-time tracking regime

$$r_1 \simeq 0,\ r_2 \simeq 0$$

$$\frac{G_{\rm eff}}{G} = 1 + (\frac{255}{8}\beta + \frac{211}{16}\alpha r_1) r_2 \geq 1$$

$$\eta = 1 + (\frac{129}{8}\beta + \frac{589}{16}\alpha r_1) r_2 \geq 1$$

de Sitter regime

$$r_1 = 1,\ r_2 = 1$$

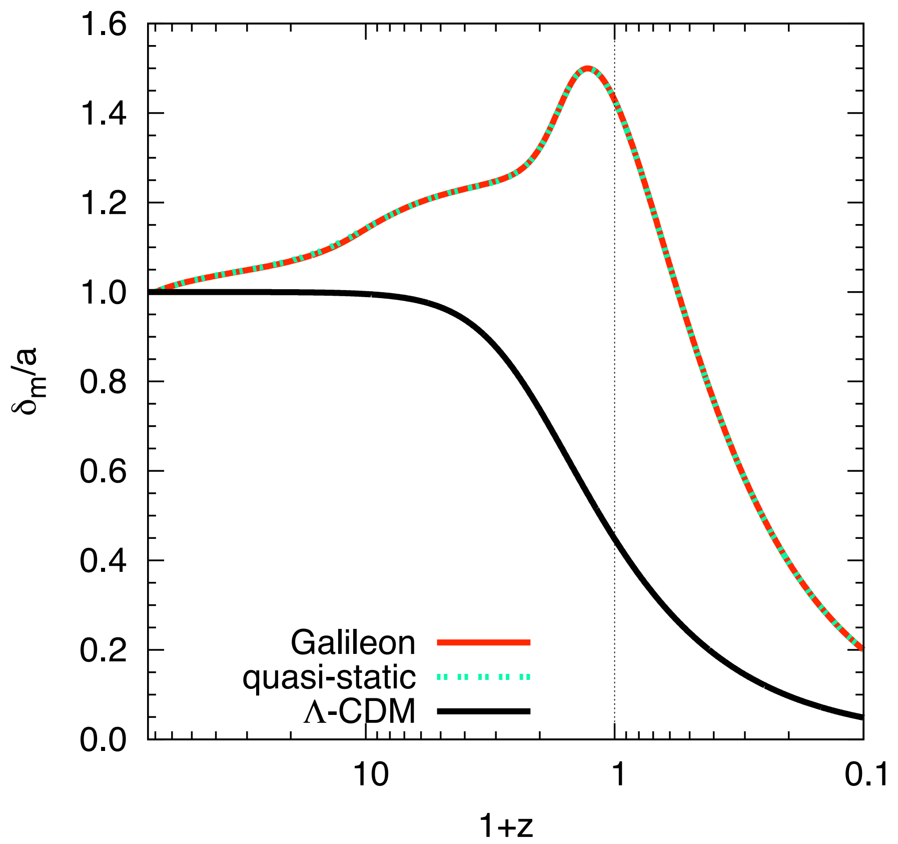
$$\frac{G_{\rm eff}}{G} = \frac{1}{3(\alpha - 2\beta)}$$

$$\eta = 1$$

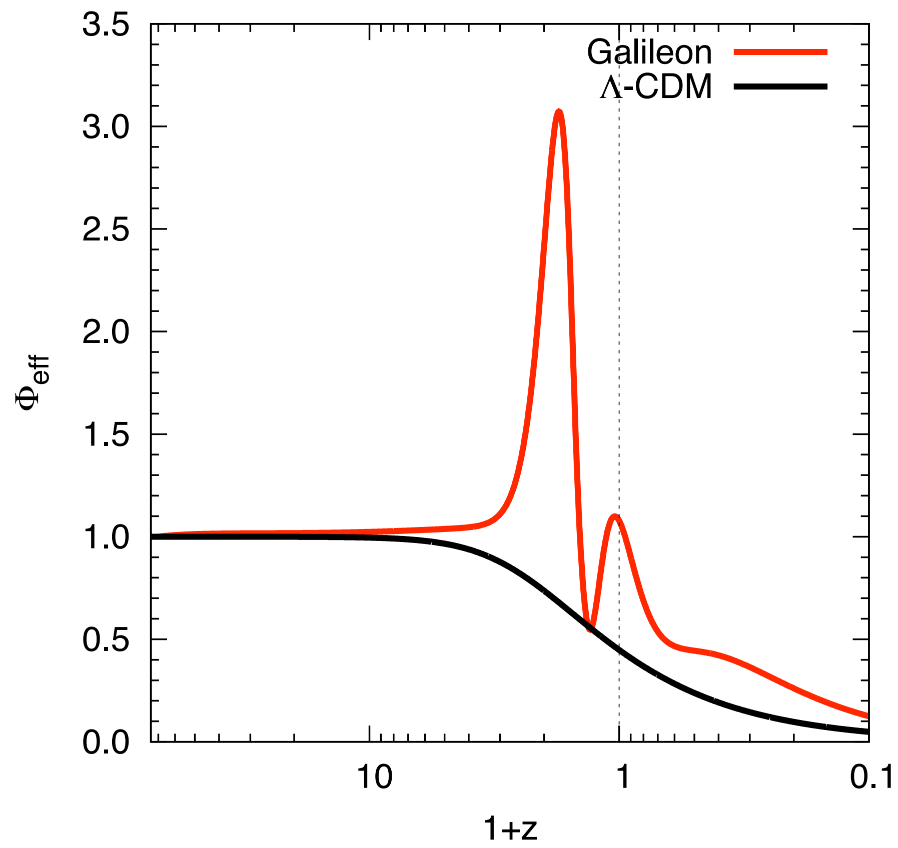
Though in f(R) gravity and in Brans-Dicke theory one has $(G_{\rm eff}/G)(1+\eta)/2 = 1$, in Galileon gravity the unusual behavior of the anisotropic parameter η leads to the nontrivial evolution of perturbations.

4. Full Numerical result

- Considering the case in which the solutions reach the tracker at late times.



The wave numbers relevant to the linear regime of the galaxy power spectrum:
$$k = 300 a_0 H_0$$

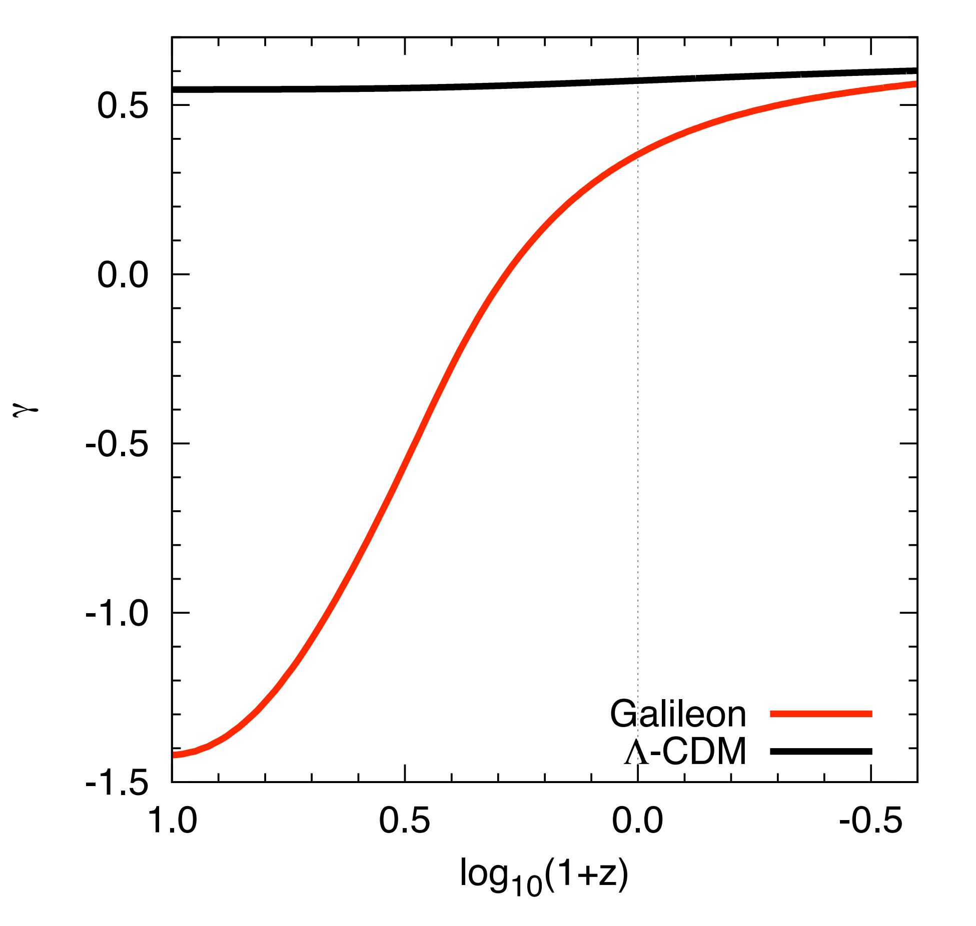


The wave numbers relevant to the ISW effect in CMB anisotropies:
$$k = 5 a_0 H_0$$

The quasi-static approximation (the dotted green line) agrees with full numerical results for the modes deep inside the Hubble radius.

- The evolution of growth rate γ defined as $\delta'_m/\delta_m = (\Omega_m)^\gamma$

Limin Wang, Paul J. Steinhardt
Astrophys.J.508:483-490,1998



LCDM model

nearly constant: $\gamma \simeq 0.55$

Galileon model

For the modes relevant to large-scale structure, we find that γ varies in time with the present value:

$$\gamma_0 \simeq 0.35$$

This property can be distinguished from the LCDM.

5. Conclusion

- **We have studied the dynamics of cosmological perturbations in the Galileon model, and it can allow us to distinguish the Galileon model from the LCDM model further.**

- **Unusual behavior of the two important quantities, effective gravitational coupling $G_{\rm eff}$ and the anisotropic parameter η , give rise to the nontrivial evolution of perturbations relative to those in LCDM model.**

- **The modified growth of perturbations affects the large-scale structure, the ISW effect in CMB, and weak lensing.**

It will be of interest to find some signatures of the Galileon model and its generalizations in future high-precision observations.