Gauge invariant couplings of fields to torsion: A string inspired model

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Sixth Aegean Summer School
Naxos, Greece

S. Bhattacharjee and A. Chatterjee, Phys. Rev **D 83**, 106007 (2011)

Motivation

- The low energy physics of particle interactions satisfactorily described by the standard model and general relativity.
- At higher energies , it is expected that new degrees of freedom will emerge to play important role $\rightarrow$ available at the early universe or at astrophysical processes.
- New fields might interact with degrees of freedom of the standard model leading to some interesting theoretical predictions and observational signatures.
- String theory is a candidate for a unified description of field interactions even upto the Planck scale.
- We investigate the nature and the specific form of interaction of new fields in string theory with known degrees of freedom.

AIM

- Aim is to construct gauge invariant interactions of gauge fields (electromagnetic and 2 and 3-form gauge fields) to torsion.
- In string theory Kalb-Ramond (KR) field ($B_{\mu\nu}$) acts as a source of torsion.
- KR field is generic to any closed string spectrum but is *not* a degree of freedom of the standard model.
- One has to use standard fields as probes to see any observational effect involving the KR field $\rightarrow$ a window into the otherwise inaccessible world of very high energy physics, predicted by string theories.
- We will look for how EM and gravitational fields couple to KR field and see the effects of those couplings in astrophysical and cosmological processes.

Effective Action

- Effective action incorporating gauge-invariant interactions of KR field to Maxwell and gravitational fields :

$$S[g, A, \Phi_H] = \int_{M_4} \left[\frac{R(g,T)}{16\pi G} - \frac{1}{2} F \wedge {}^*F - \frac{1}{2} d\phi_H \wedge {}^*d\phi_H \right]$$
$$+ \int_{M_4} \frac{\Phi_H}{M_P} \left(F \wedge F + \zeta F \wedge {}^*F - R \wedge R - \zeta R \wedge {}^*R \right)$$

[P. Majumdar and S. SenGupta, Class. Quant. Grav. **16** (1999) L89 [arXiv:gr-qc/9906027]

- Φ_H comes from the local solution of KR Bianchi identity. Since $H = dB$,

$$dH = 0$$

and EOM

$$d^*H = 0 \rightarrow H = -{}^*d\Phi_H$$

- ζ is a parameter which takes values $+1$ or -1.
- The interaction terms are both parity violating and parity conserving depending on the value of ζ.

EM interactions of KR Field

- Parity conserving interaction : $\Phi_H F_{\mu\nu}{}^*F^{\mu\nu}$, Φ_H is parity odd.
- To simplify, assume that the 'axion' field Φ_H is *homogeneous* and provides a background with which the Maxwell field interacts.
- We restrict our attention to lowest order in the inverse Planck mass M_P
- With the ansätz for a plane wave travelling in the z-direction, $\mathbf{B}(\mathbf{x}, t) = \mathbf{B}_0(t)\ \exp ikz$, we obtain, for the left and the right circular polarization states $B_{0\pm} \equiv B_{0x} \pm iB_{0y}$,

$$\frac{d^2 B_{0\pm}}{dt^2} + (k^2 \mp \frac{2f_0 k}{M_P})\, B_{0\pm} = 0 .$$

where $f_0 = \frac{d\Phi_H}{dt}$ is a constant of dimensionality of $(mass)^2$.

- The right and left circular polarization states have different angular frequencies (dispersion)
- Over a time interval Δt, the plane of polarization undergoes a rotation (for large k)
- In FRW spacetime, the value of observed angle of rotation incorporates the scale factor

$$\Delta\Psi_{op} \equiv |\omega_+ - \omega_-| \, \Delta t \;\simeq\; 2\frac{f_0}{a^2(t)M_P}\,\Delta t \equiv \Delta\Psi(z)\ ,$$

- Observationally, for large red shift sources the angle of rotation is less than a degree! this imposes the restriction on the dimensionless quantity $f_0/M_P^2 < 10^{-20}$

- Parity violating interaction : $\Phi_H F_{\mu\nu} F^{\mu\nu}$
- We get the following equation for the left/right circularly polarised light:

$$\frac{d^2 B_{+(-)}}{dt^2} + \frac{\bar{f}_0}{M_P} \frac{dB_{+(-)}}{dt} + k^2 B_{+(-)} = 0,$$

  where, $\bar{f}_0 = \zeta f_0$.
- The effect of parity violation is confined to the second term $\rightarrow$ either an enhancement or an attenuation, of the intensity of the observed electromagnetic wave, depending on the sign of $\bar{f}_0$.

- Parity conserving term : $\Phi_H \mathrm{tr}\,(R \wedge R)$
- EOM

$$\mathcal{G}_{\mu\nu} = \frac{8\pi}{M_P^2} T_{\mu\nu} + \frac{16\pi}{M_P^3} \frac{1}{\sqrt{-g}} \frac{\delta}{\delta g^{\mu\nu}} \int d^4x' \, \sqrt{-g}(x') \, \Phi_H(x') \, R_{\rho\lambda\sigma\eta}(x') \, {}^*R^{\rho\lambda\sigma\eta}(x')$$

- E.M. tensor

$$T_{\mu\nu} = H_{(\mu|\tau\rho} \, H_{\nu)}{}^{\tau\rho} - \frac{1}{6} g_{\mu\nu} H^2$$

- Decompose the metric $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ where $h_{\mu\nu}$ is small

- We consider large k but in Planckian regime $k < M_P$:
- EOM :

$$\left[\frac{d^2}{dt^2} + k^2 + 8\pi f_0^2/M_P^2 \mp 1024\pi^2 k f_0^3/M_P^5\right]\epsilon_\pm$$
$$\simeq \quad - 8\pi f_0^2 \left(1 \mp 16\pi k f_0/M_P^3\right)/M_P^2 \ .$$

- Rotation of the polarization plane for gravitational waves :

$$\Delta\Psi_{grav} \simeq 1024\pi^2 \frac{f_0^3}{M_P^5} \Delta t \ .$$

- The effect is tiny; with the limits on f_0 given by $O(10^{-30})$.
- Since the tensor perturbations characterizing the gravitational wave do not get randomized, effect is in principle observable.

- GW for parity violating interaction : $\Phi_H \, \mathrm{tr}(R \wedge {}^*R)$
- Additional term in the wave equation :

$$\Phi_H(x') \, R_{\rho\lambda\sigma\eta}(x') \, R^{\rho\lambda\sigma\eta}(x')$$

- EOM :

$$\frac{d^2\epsilon_{ij}}{dt^2} + 16\pi \, \alpha\zeta \, \beta \, k \frac{d \, \epsilon_{ij}}{dt} + k^2 (1 - \frac{16\pi\alpha^2}{\beta}) \, \epsilon_{ij} = \frac{16\pi \, f_0^2}{M_P^2} \, \eta_{ij}$$

- $\alpha := (f_0/M_P^2) << 1$ and $\beta := k/M_P$ two dimensionless quantities.
- This is an equation for a damped oscillator with a forcing term.

- $\alpha^2/\beta \geq 1$, *i.e.*, small values of k we get the scenario where the gravity waves dampen and is not observed:

$$h_{ij}(t,z) = \exp(-\frac{16\pi\alpha\,\zeta k}{M_P})\,[A_{ij}\,e^{\bar{k}t-ikz} + B_{ij}\,e^{-\bar{k}t-ikz}]$$

- $\alpha^2/\beta < 1$ (*i.e.* large values of k).

$$h_{ij}(t,z) = \exp(-\frac{16\pi\,\zeta\,\alpha\,k}{M_P})\,[\,(A_{ij}\,e^{ikt-ikz} + B_{ij}\,e^{-ikt-ikz}]$$

- This equation can give attenuation/amplification of amplitude of gravity waves.
- $\zeta = +1 \rightarrow$ attenuation or $\zeta = -1 \rightarrow$ amplification.
- We may get bounds on the strength of such parity violating terms by calculating cross correlations in multipole moments C_l^{EB} and C_l^{TB}.

Effective Potential

- Effective-potential serves as a useful tool to investigate the vacuum instability of a theory.
- Aim is to study whether quantum fluctuations of the metric generates any potential for such scalar(pseudoscalar) fields.
- Important in inflationry cosmological scenario also. Quantum effects also affect the CMB spectrum.
- We consider a generic action to study the quantum effective potential.

$$\begin{aligned}\mathcal{L} &= \mathcal{L}_{g1} + \mathcal{L}_{g2} + \mathcal{L}_{g3} + \mathcal{L}_m \\ &= -\frac{1}{\kappa^2} R + a\,\phi\,R_{\mu\nu\alpha\beta}\,R^{\mu\nu\alpha\beta} + b\,\phi_A\,R_{\mu\nu\alpha\beta}\,{}^*R^{\mu\nu\alpha\beta} \\ &+ \frac{1}{2} g^{\mu\nu}\,\partial_\mu\phi\,\partial_\nu\phi + \frac{1}{2} g^{\mu\nu}\,\partial_\mu\phi_A\,\partial_\nu\phi_A + \frac{1}{2} g^{\mu\nu}\,\partial_\mu\phi_S\,\partial_\nu\phi_S + V(\phi_S)\end{aligned}$$

- $\kappa^2 = 16\pi G$ and a, b are coupling constants which can be specified later.

EP contd.

- Expanding the Lagrangians only upto quadratic order in the $h_{\mu\nu}$ and taking space time independent saddle points for the scalar (and pseudo-scalar) fields;

$$\phi(x) = \phi_0 + \Phi(x);\ \ \phi_A(x) = \phi_{A0} + \Phi_A(x);\ \ \phi_S(x) = \phi_{S0} + \Phi_S(x)$$

we get the following Lagrangian with the choice of gauge $\partial_\mu h^{\mu\nu} = 0$ and $h = 0$:

$$\begin{aligned}
\mathcal{L}_{rel} &= -V(\phi_{S0}) - \frac{1}{2}\Phi_S(-\Box_E + V''(\phi_{S0}))\Phi_S \\
&+ \frac{1}{4}h_{\mu\nu}(-\Box_E)\,h^{\mu\nu} + a\,\kappa^2\,\phi_0\,h_{\mu\nu}\,\Box_E\,\Box_E\,h^{\mu\nu} \\
&- \frac{1}{4}\,\kappa^2\,h_{\mu\nu}Vh^{\mu\nu} + \frac{1}{2}\Phi\,(-\Box_E)\Phi + \frac{1}{2}\Phi_A\,(-\Box_E)\Phi_A
\end{aligned}$$

EP contd.

- The one-loop effective potential in momentum space :

$$V_{eff}^{(1)} = V(\phi_{S0}) + \frac{1}{2} Tr \ln(k^2 + V^{''}) + \sum_{i=1}^{10} \frac{1}{2} Tr \ln \lambda_i(k^2)$$

EP contd.

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$$V_{\rm eff}(\phi_{S0}, \phi_0) = \frac{5}{16\pi^2}\left[\left(\frac{1+8\kappa^4\phi_0 aV}{64\kappa^4\phi_0^2 a^2} - \frac{\Lambda^4}{2}\right)\ln\frac{V}{\Lambda^4} + \frac{\Lambda^2}{8\kappa^4\phi_0 a^2} - \frac{V}{2} - \frac{1}{64\kappa^4\phi_0^2 a^2}\right.$$
$$\left. + \frac{\sqrt{1+8\kappa^4\phi_0 aV}}{64\kappa^4\phi_0^2 a^2}\ln\left(\frac{1+\sqrt{1+8\kappa^4\phi_0 aV}}{1-\sqrt{1+8\kappa^4\phi_0 aV}}\right)\right] + \frac{5i}{16\pi}\left(\frac{1+8\kappa^4\phi_0 aV}{64\kappa^4\phi_0^2 a^2} - \frac{\Lambda^4}{2}\right)$$
$$+ \frac{\Lambda^2 V''}{32\pi^2} + \frac{V''^2}{64\pi^2}\left(\ln\frac{V''}{\Lambda^2} - \frac{1}{2}\right) + V(\phi_{S0})$$

- No contribution from axion field!

Interpretation of the Imaginary part

- The imagianry part signifies that flat space is not a stable vacuum of the theory.
- The value of V_{eff} at the asymmetric minimum serves as a cosmological constant at the tree level.
- Let V_{eff} develops a symmetry breaking minima at the value of $\phi_{S0} = \phi_{S_{min}}$ and $V_{eff}(\phi_{S_{min}}) \neq 0$ then $V_{eff}(\phi_{S_{min}})$ will act as a cosmological constant at the tree level.
- If we include a cosmological constant to this theory, we have a different vacuum state not a flat space but a de Sitter space.
- Now fine tune the cosmological constant so that the imaginary part vanishes with the cosmolgical constant at the chosen vacuum.

Summary and Scope

- In string theory, the Kalb-Ramond field acts as a source term for torsion which has various interactions with gauge fields.
- In order that the interactions are gauge invariant, the Kalb-Ramond field $B_{\mu\nu}$ must be endowed with non-trivial transformations under gauge fields.
- This leads to some interesting interactions with observable consequences. One of them is the rotation of plane of polarisation for electromagnetic and gravity waves.
- However, these interactions are not the only possible ones. One can have additional ones which arise from the gauge invariant coupling of higher form fields to torsion.

- Observational consequences of such interactions are alltogether different. They lead to amplification/attenuation of electromagnetic or gravity waves and have important implications for anisotropy of the Cosmic Microwave Background (CMB) by spatial parity violation.
- Constraint on the coefficient ζ of such parity violating interactions can be given comparing with the WMAP or PLANCK data.
- We also study the Coleman-Weinberg potential for such extended theory. The axion field doesn't have any contribution to the potential. There is an imaginary part in the EP which signifies that flat space is not a stable vacuum of the theory and the cosmological constant becomes dynamical.
- Calculating EP in Vilkovisky-DeWitt approach to get a unique gauge and reparametrization invariant result is to be taken as a future project.

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Thank You