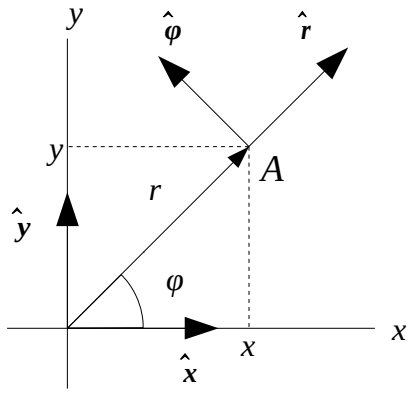
# ΠΟΛΙΚΕΣ ΣΥΝΤΕΤΑΓΜΕΝΕΣ




$A(x,y)$ $A(r,\varphi)$

$x=r\cos(\varphi), y=r\sin(\varphi)$

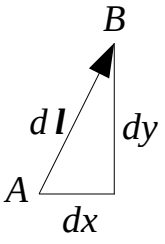
$\varphi: 0 \rightarrow 2\pi$

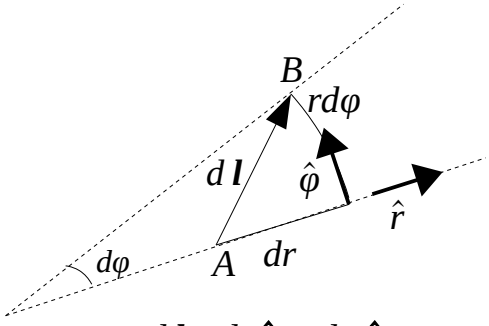
## ΜΟΝΑΔΙΑΙΑ ΔΙΑΝΥΣΜΑΤΑ

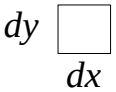
$(\hat{\boldsymbol{x}},\hat{\boldsymbol{y}})$ $(\hat{\boldsymbol{r}},\hat{\boldsymbol{\varphi}})$

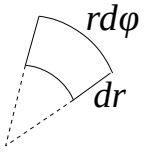
$\hat{\boldsymbol{r}}=\cos(\varphi)\hat{\boldsymbol{x}}+\sin(\varphi)\hat{\boldsymbol{y}}$

$\hat{\boldsymbol{\varphi}}=-\sin(\varphi)\hat{\boldsymbol{x}}+\cos(\varphi)\hat{\boldsymbol{y}}$



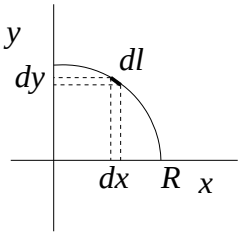




$d\boldsymbol{l}=dx\,\hat{\boldsymbol{x}}+dy\,\hat{\boldsymbol{y}}$ $d\boldsymbol{l}=dr\,\hat{\boldsymbol{r}}+rd\varphi\,\hat{\boldsymbol{\varphi}}$ $da=(dx)(dy)$ $da=(dr)(rd\varphi)$

## ΠΕΡΙΦΕΡΕΙΑ ΚΥΚΛΟΥ

### Καρτεσιανές σ.

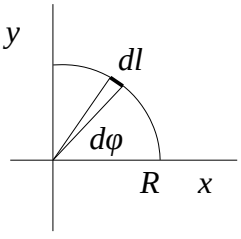



$y=\sqrt{R^2-x^2} \rightarrow dy=\frac{xdx}{\sqrt{R^2-x^2}}$ $(dl)^2=(dx)^2+(dy)^2 \rightarrow dl=dx\sqrt{1+\frac{x^2}{R^2-x^2}}=dx\frac{R}{\sqrt{R^2-x^2}}$

$1/4\,\pi\varepsilon\rho\iota\varphi=\int dl=\int_0^R dx\frac{R}{\sqrt{R^2-x^2}}=\int_0^R \frac{dx}{\sqrt{1-x^2/R^2}}$ $\frac{x}{r}=\sin\theta \rightarrow dx=R\cos\theta\,d\theta$

$\int_0^{\pi/2} \frac{R\cos\theta\,d\theta}{\cos\theta}=\frac{\pi}{2}R$

### Πολικές σ.

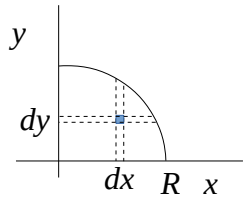



$dl=R\,d\varphi$

$1/4\,\pi\varepsilon\rho\iota\varphi=\int dl=\int_0^{\pi/2} R\,d\varphi=\frac{\pi}{2}R$

## ΕΜΒΑΔΟΝ ΚΥΚΛΟΥ

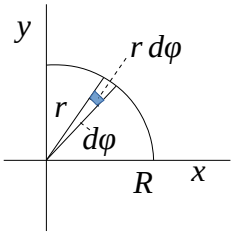
### Καρτεσιανές σ.




$$1/4\,\varepsilon\mu\beta=\int_0^R dx\int_0^{\sqrt{R^2-x^2}} dy=\int_0^R dx\sqrt{R^2-x^2}=R\int_0^R dx\sqrt{1-\frac{x^2}{R^2}}$$

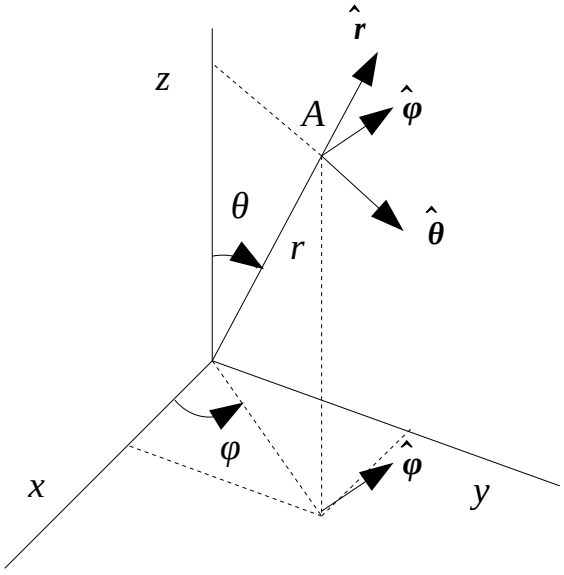
$$R^2\int_0^{\pi/2}\cos\varphi\cos\varphi\,d\varphi=R^2\frac{1}{2}\frac{\pi}{2}=\frac{\pi}{4}R^2$$

### Πολικές σ.




$$1/4\,\varepsilon\mu\beta=\int_0^R dr\int_0^{\pi/2} rd\varphi=\int_0^R dr\,r\int_0^{\pi/2} d\varphi=\frac{R^2}{2}\frac{\pi}{2}=\frac{\pi}{4}R^2$$

# ΣΦΑΙΡΙΚΕΣ ΣΥΝΤΕΤΑΓΜΕΝΕΣ




$A(x,y,z)$ $\qquad$ $A(r,\theta,\varphi)$

$$z=r\cos(\theta),\ x=r\sin(\theta)\cos(\varphi),\ y=r\sin(\theta)\sin(\varphi)$$

$$\theta:0\to\pi,\ \varphi:0\to 2\pi$$

### ΜΟΝΑΔΙΑΙΑ ΔΙΑΝΥΣΜΑΤΑ

$(\hat{\boldsymbol{x}},\hat{\boldsymbol{y}},\hat{\boldsymbol{z}})$ $\qquad$ $(\hat{\boldsymbol{r}},\hat{\boldsymbol{\theta}},\hat{\boldsymbol{\varphi}})$

$$\hat{\boldsymbol{r}}=\sin(\theta)\cos(\varphi)\hat{\boldsymbol{x}}+\sin(\theta)\sin(\varphi)\hat{\boldsymbol{y}}+\cos(\theta)\hat{\boldsymbol{z}}$$

$$\hat{\boldsymbol{\theta}}=\cos(\theta)\cos(\varphi)\hat{\boldsymbol{x}}+\cos(\theta)\sin(\varphi)\hat{\boldsymbol{y}}-\sin(\theta)\hat{\boldsymbol{z}}$$

$$\hat{\boldsymbol{\varphi}}=-\sin(\varphi)\hat{\boldsymbol{x}}+\cos(\varphi)\hat{\boldsymbol{y}}$$

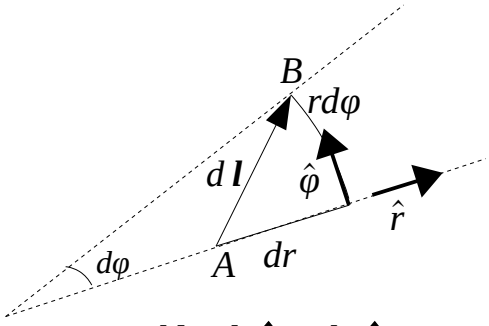
$$d\boldsymbol{l}=dx\,\hat{\boldsymbol{x}}+dy\,\hat{\boldsymbol{y}}+dz\,\hat{\boldsymbol{z}} \qquad d\boldsymbol{l}=dr\,\hat{\boldsymbol{r}}+rd\theta\,\hat{\boldsymbol{\theta}}+r\sin(\theta)\,d\varphi\,\hat{\boldsymbol{\varphi}}$$

### ΕΠΙΦΑΝΕΙΑ ΣΦΑΙΡΑΣ

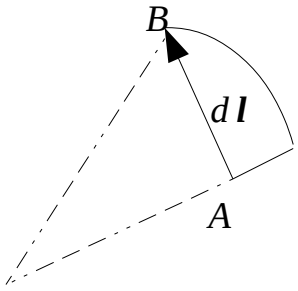
$$\int_{\theta=0}^{\pi} R\,d\theta\int_{\varphi=0}^{2\pi} R\sin(\theta)\,d\varphi=R^2\int_{\theta=0}^{\pi}\sin(\theta)d\theta\int_{\varphi=0}^{2\pi}d\varphi=R^2(2)(2\pi)=4\pi R^2$$

### ΟΓΚΟΣ ΣΦΑΙΡΑΣ

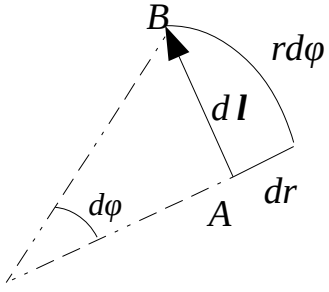
$$\int_0^R dr\int_{\theta=0}^{\pi} r\,d\theta\int_{\varphi=0}^{2\pi} r\sin(\theta)\,d\varphi=\int_0^R dr\,4\pi r^2=\frac{4}{3}\pi R^3$$




$$d\boldsymbol{l} = dr\,\hat{\boldsymbol{r}} + r d\varphi\,\hat{\boldsymbol{\varphi}}$$




$$d\boldsymbol{l} = dx\,\hat{\boldsymbol{x}} + dy\,\hat{\boldsymbol{y}}$$




$$d\boldsymbol{l} = dr\,\hat{\boldsymbol{r}} + r d\varphi\,\hat{\boldsymbol{\varphi}}$$