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LOW ENERGY UNIFICATION: PHENOMENOLOGICAL VIABILITY AND ITS IMPLICATIONS

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We investigate the required changes in the MSSM $\beta$-functions in order to achieve intermediate scale unification, working in the traditional logarithmic running of the gauge couplings. A string unified model with the required extra matter is presented. Assuming also a compactification scale (and the appearance of KK states), we check the viability of intermediate scale GUT unification.

1 Introduction

It was recently proposed $^1$, $^2$, $^3$, that in the weakly-coupled Type I string vacua the string scale can naturally lie in some intermediate energy, $10^{10-13}$ GeV, which happens to be the geometrical mean of the Planck scale, $M_P$, and weak scale, $M_W$ (i.e. $M_{\text{string}} \sim \sqrt{M_W M_P} \sim 10^{11}$ GeV). It is a rather interesting fact that the possibility of intermediate scale unification was also shown to appear in the context of Type IIB theories $^4$. This scenario has the advantage that this intermediate scale does not need the power-like running of the gauge couplings in order to achieve unification.

In this talk we would like to investigate the changes that the MSSM $\beta$-functions should suffer in order to achieve gauge coupling unification at $10^{12-13}$ GeV. We further determine the extra matter fields which make gauge couplings merge at an intermediate energy and show that such spectra may appear in the context of specific string unified models which can in principle avoid fast proton decay. Finally, we address the same questions assuming Kaluza-Klein massive states above a compactification scale $M_G$.

2 Renormalization Group Analysis

In this section we will explore the possibility of modifying the MSSM $\beta$-functions in order to implement the intermediate scale unification scenario. We begin by writing down the (one-loop) running of the gauge couplings

$$\frac{1}{\alpha_i(M)} = \frac{1}{\alpha_i(M_U)} + \frac{\beta_i}{2\pi} \log \frac{M_U}{M} + \frac{\beta_i^{\text{SN}}}{2\pi} \log \frac{M_{SB}}{M}$$

where $M_U$ is the unification scale and $M_{SB}$ is the SUSY breaking scale and we have of course $M_U > M_{SB} > M$. In the equation above, we have assumed that i) the three gauge couplings unify at $M_U (\alpha_i(M_U) = \alpha_Y)$, ii) extra matter, possibly remnants of a GUT, appears in the region between $M_U$ and $M_{SB}$: $\beta_i = \beta_i^G + \delta \beta_i$, where $\delta \beta_i = (33/5, 1.1, -3)$ is the MSSM $\beta$ functions, and
iii) in the region between \(M_{SB}\) and \(M_{Z}\) we have the (non-SUSY) SM (although with two higgs instead of one) and the corresponding \(\beta_i^{NS} = (4, 2, -3, -7)\) functions.

By choosing \(M = M_Z\) in (1), we can solve the system of these three equations with respect to \((\alpha_i, M_Z, a_U)\) as functions of the \(\delta \beta_i\)'s, taking the values of \(\alpha_i(M_Z)\) from experiment.

\[
\begin{align*}
\delta \beta_{3U} &= \frac{\delta \beta_{1j}([2\pi\delta(\alpha^{-1} j_1) + \delta \beta_{1S}^N t_Z] - \delta \beta_{1j}(2\pi\delta(\alpha^{-1}) j_k + \delta \beta_{1S}^N t_Z))}{\delta \beta_{1j}([\delta \beta_{1S}^N] - \delta \beta_{1j})} + \delta \beta_{1j}(\delta \beta_{1S}^N - \delta \beta_{1j}) t_{SB} \\
\delta \beta_{3U} &= \frac{2\pi\delta(\alpha^{-1} j_1) + \delta \beta_{1S}^N t_Z - (\delta \beta_{1S}^N - \delta \beta_{1j}) t_{SB}}{\delta \beta_{1j}} \\
\frac{1}{\alpha_i} &= \frac{1}{\alpha_i} - \frac{\beta_i^{NS}}{2\pi} (t_{SB} - t_Z) - \frac{\beta_i}{2\pi} (t_{UV} - t_{SB})
\end{align*}
\]  

(2)

where \(t_{SB, U, Z}\) is the logarithm of the corresponding scales, \(\delta \beta_{1j} = p_j - p_i\) and \(i, j, k\) should be different. It is obvious that \(t_{U}, t_{SB}\) and \(a_U\) depend only on the differences of \(\beta_i\)'s. Therefore, if a certain solution \((t_{U}, t_{SB}, a_U)\) is obtained by using specific values for \((\delta \beta_1, \delta \beta_2, \delta \beta_3)\), the same solution is obtained for \((\delta \beta_1 + c, \delta \beta_2 + c, \delta \beta_3 + c)\) where \(c\) is an arbitrary constant.

In Fig.1 we plot the acceptable values of \((\delta \beta_1, \delta \beta_2)\) for \(\delta \beta_3 = 0\), demanding the unification scale to be in the region \((10^{10} - 10^{12})\text{GeV}\) while the SUSY breaking scale is in the region \((1 - 3)\times 10^5\text{GeV}\). The four "lines" correspond to the four indicated combinations of \((a_U(M_Z), \delta \beta_1, \delta \beta_3)\).

Translating the "lines" by an amount \(c\) in both directions, the corresponding figure for \(\delta \beta_3 = c\) appears.

Let us try to find the acceptable values for a specific GUT model, namely the \(SU(4) \times SU(2)_L \times SU(2)_R\). In this case, we assume that the breaking to the standard model occurs directly at the string scale, so that the gauge couplings \(g_L, g_R, g_4\) attain a common value \(g_L\). The massless spectrum of the string model – in addition to the three families and the standard higgs fields – leaves some extra matter fields which change the MSSM \(\beta\)-functions. Taking into account these remnants and the extra constraints of the specific GUT model (requiring even number of these extra states), only the following 3 points are acceptable in all the region allowed by the constraints on \(\sin^2\theta_W(M_Z)\) and \(a_3(M_Z)\) having put earlier (keeping \(\delta \beta_3 = 0\))

\((\delta \beta_1, \delta \beta_2, \delta \beta_3) = (4, 2, 0), \ (6.5, 3, 0), \ (8.5, 4, 0)\)

Of course, several possible sets of the extra states can generate the above changes in the \(\beta\)-functions.
3 Inclusion of KK-states

Here we assume the existence of a compactification scale \( M_C \) (smaller than the would be unification scale if \( M_C \) had not existed) above which KK-excitations are considered. Although the tower of the massive KK-states dictates a power law running of the gauge couplings, we use the successful approximation of incorporating the massive KK-states with masses less than the running scale, working therefore with the conventional logarithmic running.

In ref (6) it was stated that in the MSSM, whatever the chosen \( M_C \) is, unification of the gauge couplings always occurs. This statement can be generalized in the form that: if \( i \) There exists an energy scale \( M_H \) where the coupling constants \( q_i \)'s unify, assuming conventional logarithmic running (no-compactification scenario) and \( ii \) The ratio of the differences of the \( \beta \)-functions \( \beta_{KK} \) (above the compactification scale \( M_C \)) to the corresponding difference \( \beta_U \) (below the compactification scale \( M_C \)) is the same for all \( (ij) \) combinations, then it can be shown that the gauge couplings do unify, whatever energy scale we choose as a compactification scale \( M_C \), above which the massive KK-states contribute to the running. MSSM with the standard matter content, while only gauge bosons and higgs acquire KK-states, satisfies with a good approximation these two requirements. Therefore, with any chosen \( M_C \leq 10^{14} \text{GeV} \), gauge coupling unification is ensured.

Working in the \( SU(4) \times SU(2)_L \times SU(2)_R \) GUT model, we find numerous matter content above the GUT scale \( M_G \) (which is taken to be in the intermediate region \( \sim 10^{13} \text{GeV} \)) and several possibilities for the states acquiring (or not) KK-states (above \( M_C \)), where the \( \beta \)-functions obey the condition in order to achieve gauge coupling unification whatever the compactification scale is. In Fig 2, we show the running of the gauge couplings for three different compactification scales and for a specific matter content, while we have chosen the GUT scale to be \( 10^{13} \text{GeV} \).

Concluding, we have checked the possibility of intermediate scale (\( 10^{10} - 13 \text{ GeV} \)) gauge coupling unification, using the traditional logarithmic running, i.e. without incorporating the power-law dependence on the scale coming from the Kaluza-Klein tower of states. We have showed that this kind of unification can be achieved with small changes of the \( \beta \)-functions of the MSSM gauge couplings, which can be attributed to matter remnants of superstring models. We have applied the above to the successful \( SU(4) \times SU(2)_L \times SU(2)_R \) model (which is safe against proton decay even in this intermediate scale), and found the necessary extra massless matter and higgs fields needed. Finally, incorporating KK-states, we have shown that there exist numerous cases of massless spectra (which can be derived from the superstring), implying naturally intermediate scale unification.

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References