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PREDICTING THE FERMION MASSES IN SUSY GUT's

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ABSTRACT
We present the predictions of a fermion mass ansatz for a wide range of the ratio $tan\beta = \frac{<\bar{h}>}{<h>}$ (i.e. considering cases with large bottom Yukawa coupling). We find two distinct regions for $tan\beta$, predicting fermion masses and mixing angles in agreement with experiment. In the large $tan\beta$ region we find that $m_t \geq 148$GeV while in the small one we get $m_t \geq 125$GeV. We also get an upper bound $m_t \leq 175$GeV from the ratio $\frac{m_u}{m_c}$.

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There is a firm belief among the high energy physicists that in the ultimate theory, all the arbitrary parameters of the standard model will be determined only from a small number of inputs at some unification scale.

Recently, several attempts have been made [1,2,3,4] to determine the possible structures of the fermion mass matrices at the Grand Unification(GUT) scale, which lead to the correct low energy mass spectrum and to the maximal number of predictions. In Ref.[2], a simple ansatz for the fermion mass matrices, at the GUT scale, was proposed. The 13 arbitrary parameters of the low energy were determined by 6 inputs, hence leading to 7 predictions.

All the previous calculations however, have been done for the case of small bottom Yukawa coupling, compared to that of the top quark. This corresponds to a relatively small ratio $\tan\beta$ of the two Higgs vev's $<\bar{h}>$ and $<h>$ which give masses to the up and down quarks respectively. In this particular case one can ignore all but the top Yukawa coupling corrections in the renormalization group equations (RGEs) of the Yukawa couplings. In many unified models however - and in particular in string derived GUTs - it is quite possible for the top and bottom quark Yukawa couplings to be comparable at the GUT scale. In the present work we are exploring this latter case. We start with an overview of the basic features of the proposed framework [2]. It is assumed that there exists some Grand Unified Supersymmetric Model (i.e. SO(10), SU(5), SU(4) etc) with the following form of the mass matrices at the GUT scale

$$M_d = \begin{pmatrix} 0 & fe^{i\phi} & 0 \\ fe^{-i\phi} & d & 2d \\ 0 & 2d & c \end{pmatrix}, \qquad M_e = \begin{pmatrix} 0 & fe^{i\phi} & 0 \\ fe^{-i\phi} & -3d & 2d \\ 0 & 2d & c \end{pmatrix}, \tag{1}$$

$$M_{\nu\nu^c} = \begin{pmatrix} 0 & 0 & b \\ 0 & b & 0 \\ b & 0 & a \end{pmatrix}, \quad M_u = \begin{pmatrix} 0 & 0 & b \\ 0 & b & 0 \\ b & 0 & a \end{pmatrix}, \quad M_{\nu^c\nu^c} = M\, diag(k^{-2}, k^{-1}, 1) \tag{2}$$

The original ansatz for the M_u, M_d and M_e matrices was augmented by a simple assumption for the Dirac, $M_{\nu\nu^c}$, and the heavy Majorana, $M_{\nu^c\nu^c}$, neutrinos mass matrices: $M_{\nu\nu^c}$ is simply taken to be identical to M_u, due to the GUT relations, while $M_{\nu^c\nu^c}$ is taken for simplicity diagonal, whose elements differ by a hierarchy factor $k \approx 10$. The predictions in the neutrino sector have been discussed elsewhere[5], thus we are not going to elaborate them here.

Assuming that the only significant Yukawa terms are the $\{33\}$ entries of the 3×3 Yukawa matrices $\tilde{\lambda}_U$, $\tilde{\lambda}_D$ and $\tilde{\lambda}_E$ (in a suitable basis where $\tilde{\lambda}_U$ is diagonal [2]), we solve the RGE's and obtain the following relations among the masses[6]

$$m_t = \zeta\xi^3 \frac{\eta_u m_c^2}{\eta_c^2 m_u}, \quad m_s \approx \eta_s \frac{\gamma_D}{\gamma_E}\frac{m_\mu}{3}\left(1 - \frac{4}{9}(1 + 3\zeta'^3)\frac{m_\mu}{m_\tau}\right) \tag{3}$$

$$m_b \approx \frac{\gamma_D}{\gamma_E}\frac{\zeta^3}{\zeta'^3}\xi m_\tau \eta_b, \quad m_d \approx \eta_d \frac{\gamma_D}{\gamma_E} 3m_e \left(1 + \frac{4}{9}(1 + 3\zeta'^3)\frac{m_\mu}{m_\tau}\right) \tag{4}$$

where

$$\gamma_\alpha(t) = \exp(-\int_{t_0}^{t} G_\alpha(t)\,dt/(16\pi^2)) \tag{5}$$

$$G_\alpha = \sum_{i=1}^{3} c_\alpha^i g_i^2(t), \qquad g_i^2(t) = \frac{g_i^2(t_0)}{1 - \frac{b_i}{8\pi^2} g_i^2(t_0)(t - t_0)} \tag{6}$$

$$\{c_U^i\}_{i=1,2,3} = \left\{\frac{13}{15}, 3, \frac{16}{3}\right\}, \quad \{c_D^i\}_{i=1,2,3} = \left\{\frac{7}{15}, 3, \frac{16}{3}\right\}, \quad \{c_E^i\}_{i=1,2,3} = \left\{\frac{9}{5}, 3, 0\right\} \tag{7}$$

$$\xi = \exp(\frac{1}{16\pi^2}\int_{t_0}^{t} \tilde{\lambda}_t dt), \quad \zeta = \exp(\frac{1}{16\pi^2}\int_{t_0}^{t} \tilde{\lambda}_b dt), \quad \zeta' = \exp(\frac{1}{16\pi^2}\int_{t_0}^{t} \tilde{\lambda}_\tau dt) \tag{8}$$

$\tilde{\lambda}_t$, $\tilde{\lambda}_b$ and $\tilde{\lambda}_\tau$ stand for $\tilde{\lambda}_{U_{33}}$, $\tilde{\lambda}_{D_{33}}$ and $\tilde{\lambda}_{E_{33}}$ respectively. Note that in the limit where $\tilde{\lambda}_t \gg \tilde{\lambda}_b$, $\tilde{\lambda}_\tau$ we get $\zeta \approx \zeta' \approx 1$ and the above relations reduce to the corresponding ones of Ref[2]. Finally the η's are taking into account the QCD renormalization effects of the corresponding quark mass from m_t down to their masses for b and c quark, and to 1 GeV for u, d and s quark.

In order to compute the various renormalization group parameters which enter the relations given above, we solve numerically the RGEs assuming the initial condition at $M_{GUT} \simeq 10^{16} GeV$, with $\alpha_{GUT} \simeq \frac{1}{25.1}$. We are taking supersymmetric β-function coefficients from M_{GUT} down to m_t, while below m_t we run the system with non-supersymmetric ones. We ensure that the gauge couplings lie in the experimentally accepted region at m_W and we compute the quark masses for a wide range of $tan\beta$, each time using the proper initial values for the couplings $\lambda_{0,t}, \lambda_{0,b}$ (being equal at the GUT svale). Our numerical analysis reproduces the previous results when $tan\beta \leq 5$ and extends the analysis to the case where $tan\beta \gg 1$.

In figure (1) we plot the bottom mass versus the bottom coupling $\lambda_{0,b}$ at the GUT scale, for constant top-mass $m_t = 145 GeV$, for three successive approximations: Contour (I) represents the case where $\lambda_{0,b}, \lambda_{0,\tau}$-corrections are neglected. Contour (II) represents the solution where only $\lambda_{0,\tau}$ correction is neglected, while case (III) is the contour which corresponds to the complete differential system of the RGEs where the corrections from all three couplings are taken into account in the running. The shaded region corresponds to the experimentally accepted value of m_b. In figure(2) we plot contours for $m_t =$ 145GeV and $m_t =$ 170GeV. The shaded area (whose upper bound corresponds to $m_t \simeq (175 \pm 3) GeV$), is prevented due to the bad ratio $\frac{m_u}{m_c}$. Thus, it is remarkable that this ratio which is derived only in terms of the running m_u, m_c masses determined by well known methods[7], can put an upper bound on the top-quark mass. As far as the lower bound on m_t is concerned, in the small $\lambda_{0,b}$ region, and demanding $tan\beta > 1$, we get $m_t \geq$ 125GeV, while in the large $\lambda_{0,b}$ region, the KM matrix entry V_{cb} puts the limit $m_t \geq$ 148GeV (the dashed line cutting the m_t contours corresponds to the maximum allowed $V_{cb} \simeq 5.4 \times 10^{-2}$).

In conclusion, we have reconsidered a proposed [2] *ansatz* for the fermion mass matrices at the GUT scale, and studied in detail the effects of a large bottom Yukawa coupling on the various experimentally measured parameters of the low energy theory. We have shown that the renormalization corrections of the λ_b Yukawa coupling have a significant impact whenever $r = \frac{\lambda_{0,b}}{\lambda_{0,t}} \geq 0.1$, while when $r \ll 1$, they can safely be neglected. Furthermore, for $r \geq 1$, m_b- and V_{cb}- low energy bounds put a lower limit on the top mass $m_t \geq 148GeV$, while in the case of $r \ll 1$, one gets a less restrictive top mass $m_t \geq 125GeV$.

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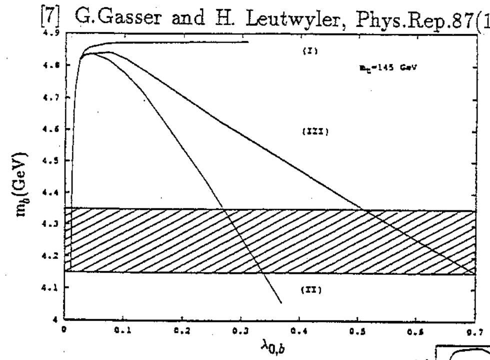

Figure 1. The bottom mass m_t as a function of $\lambda_b(M_{GUT})$, for m_t = 145GeV. The curves I, II and III correspond to the cases where i) only λ_t, ii) λ_t and λ_b and iii) all three λ_t, λ_b and λ_τ are taken into account. The shaded region shows the experimentally accepted region of m_b.

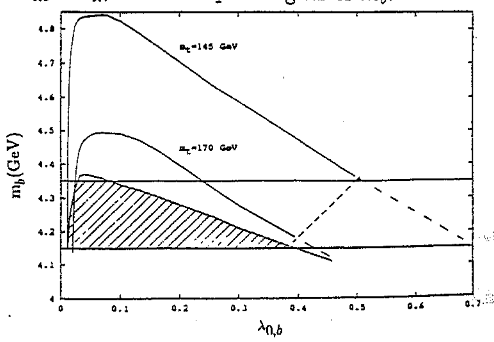

Figure 2. Same as the curve II in Fig.1, for m_t = 145 and 170GeV. The shaded area corresponds to unacceptable $\frac{m_u}{m_c}$ ratio, while the lower right region (dashed curves) are prevented from V_{cb} bounds.