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SUPPLEMENTS

Spin-spin correlation functions of spin systems coupled to 2-d quantum gravity for $0 < c < 1$.

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We perform Monte Carlo simulations of 2-d dynamically triangulated surfaces coupled to Ising and three–states Potts model matter. By measuring spin-spin correlation functions as a function of the geodesic distance we provide substantial evidence for a diverging correlation length at β_c. The corresponding scaling exponents are directly related to the KPZ exponents of the matter fields as conjectured in [4].

1. INTRODUCTION

The calculation of the dressed scaling exponents is a milestone in the theory of two-dimensional quantum gravity [1]. Strictly speaking, however, the derivation uses only finite size scaling arguments and involves only matter field correlators integrated over all space–time. The concept of a diverging correlation length at the critical point associated with the correlators is not used in the derivation and was never shown to actually *exist*. It is possible to define a correlation length by defining the two point correlator of a field $\phi(x)$ in terms of the reparametrization invariant geodesic distance R between two marked points:

$$G_\phi(R;\Lambda) = \int \mathcal{D}[g]\mathcal{D}[\phi]\, \mathrm{e}^{-S_G-S_M} \int d^2x d^2y \times\sqrt{g(x)g(y)}\; \phi(x)\phi(y)\delta(D_g(x,y)-R). \quad (1)$$

Even with this definition at hand, it is possible that the correlation length of the system coupled to gravity does not diverge, even though it does in flat space and the transition when coupled to gravity is continuous [2]. The fixed volume correlator $G_\phi(R;V)$ is obtained by Laplace transforming $G_\phi(R;\Lambda)$ and from it one obtains the correlators $S_\phi(R;V) \equiv G_\phi(R;V)/V\,Z(V)$, where $Z(V)$ is the fixed volume partition function. The case $\phi = 1$ corresponds to the volume–volume correlators $G_1(R;V)$, $G_1(R;\Lambda)$ and the average "area" of a spherical shell $S_1(R;V)$ or radius R. In the case of a matter field ϕ with scaling dimension Δ_0 in flat space, $\int_0^\infty dR\; S_\phi(R;V) \sim V^{1-\Delta}$ where Δ is the dressed scaling dimension of the field. From this scaling and the corresponding behaviour of $S_\phi^{(0)}(R;V)$ in flat space

$$S_\phi^{(0)}(R;V) \sim \frac{R}{R^{2\Delta_0}} f(x) = V^{1/2-\Delta_0} F_\phi^{Flat}(x), \quad (2)$$

we may conjecture that [4]

$$S_\phi(R;V) \sim \frac{R^{d_h-1}}{R^{d_h\Delta}} f(x) = V^{1-\Delta-1/d_h} F_\phi(x), \quad (3)$$

where $f(0) > 0$, $F(x) \sim x^{d_h(1-\Delta)-1}$ for small $x = R/V^{1/d_h}$ and d_h is the fractal dimension of space–time as defined for example by the finite size scaling relation $S_1(R;V) \sim V^{1-1/d_h} F_1(x)$. In this article we report on extensive numerical simulations which provide substantial evidence that Eq. (3) holds for matter fields with $0 < c < 1$. This points to the existence of a diverging correlation length of the conformal field theory coupled to gravity if we use the geodesic distance as a measure of length.

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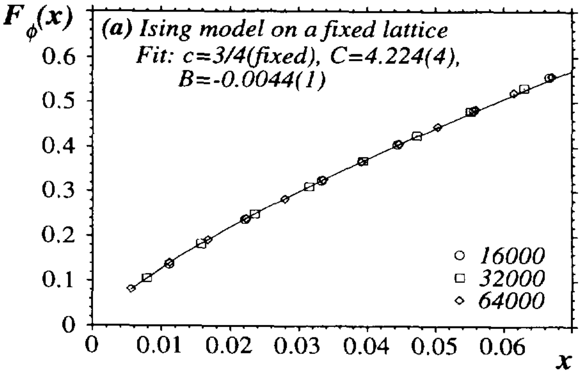

Figure 1. Data for $F_\phi^{Flat}(x)$ as defined in Eq.(2) for small values of $x = r/N^{1/d_h}$, $d_h = 2$. The fit is to Eq.(6a).

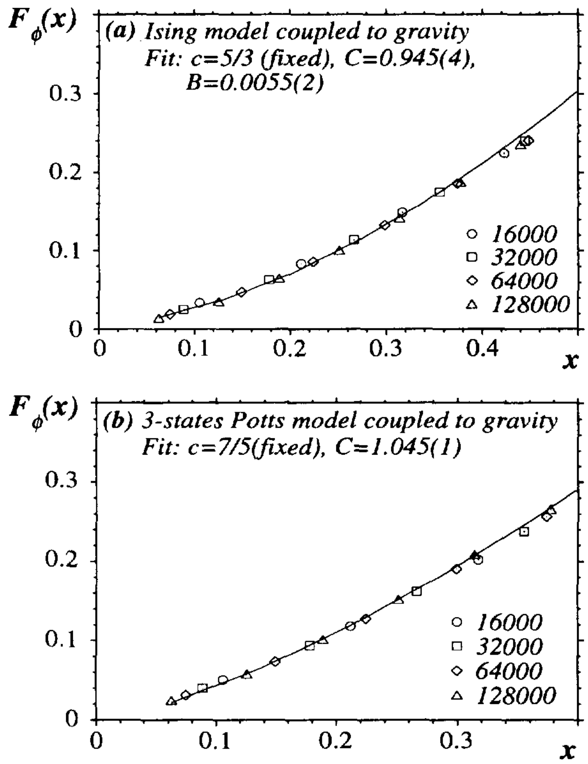

Figure 2. (a) Same as in Fig. 1 for $F_\phi^{Ising}(x)$ for $d_h = 4$. (b) Same as in (a) for $F_\phi^{Potts}(x)$.

2. SIMULATIONS AND RESULTS

We perform numerical simulations on dynamically triangulated surfaces with S^2 topology with tadpoles and self-energy diagrams. We place Ising and three-states Potts model spins on the vertices of the surface which interact with the spins of the neighbouring vertices. The Monte Carlo updating of the triangulations is performed by the so-called flip algorithm and the spins are updated by standard cluster algorithms. The flips are organised in "sweeps" which consist of approximately N_L *accepted* flips where N_L is the number of links of the triangulated surface. After a sweep we update the spin system [5]. The results presented in this paper cover system sizes from 16000 to 128000 triangles and the number of sweeps is 1.7–5.0 × 10^6. The simulations are performed at the analytically known infinite volume β_c [6]. Geodesic distances r on the triangulations are defined as the shortest link distance between two vertices. The discretized volume of the system is the number of triangles N_T and we use the scaled distance $x = R/N^{1/d_h}$, where N is the number of vertices. We measure the discretized distributions corresponding to $S_1(R;V)$ and $S_\phi(R;V)$

$$n_1(r;N) = \langle \sum_j \delta(D_{ij} - r) \rangle , \qquad (4a)$$

$$n_\phi(r;N) = \langle \sum_j \sigma_i \sigma_j \, \delta(D_{ij} - r) \rangle , \qquad (4b)$$

where the indices i and j label vertices: i is a random fixed vertex for each measurement and j runs over all vertices of the given configuration. D_{ij} is the link distance between the vertices labelled by i and j. We determine the fractal dimension d_h by using $n_1(r;N) = N^{1-1/d_h} F(x)$ and $n_\phi(r;N) = N^{1-\Delta-1/d_h} F_\phi(x)$. Our data is in very good agreement with the results obtained in [3,4], namely $d_h = 4$ for both the Ising model and the three-states Potts model coupled to grav-

Table 1
The parameters of the fits to Eq. (6) for the Ising model on a flat T^2 for $N_T = 64000$ ($\Delta_0 = 1/8$), and for the Ising model ($\Delta = 1/3$) and the three-states Potts model ($\Delta = 2/5$) on S^2 coupled to quantum gravity for $N_T = 128000$.

	$\Delta_0 = 1/8$	$\Delta = 1/3$	$\Delta = 2/5$
C_1	4.224(4)	0.945(4)	1.044(4)
B_1	-0.0044(1)	0.0054(2)	0.0003(3)
c	0.765(1)	1.535(6)	1.396(5)
C_2	4.39(2)	0.834(7)	1.040(8)
c	0.753(4)	1.596(6)	1.470(8)
C_3	4.25(4)	0.870(7)	1.10(1)
B_3	-0.0038(7)	0.0036(1)	0.0055(3)

ity.

We test the hypothesis of Eq. (3) by using the values of the dressed scaling dimensions of the Ising and three–states Potts model spin fields $\Delta = 1/3, 2/5$. The prediction for $d_h = 4$ is

$$F_\phi^{Ising}(x) \propto x^{5/3} \quad \text{for} \quad x \ll 1\,, \qquad (5a)$$
$$F_\phi^{Potts}(x) \propto x^{7/5} \quad \text{for} \quad x \ll 1\,. \qquad (5b)$$

In order to calibrate the expected accuracy with which one can determine the exponent $d_h(1-\Delta)-1$ we have performed simulations of an Ising model on a fixed triangular lattice with periodic boundary conditions ($\Delta_0 = 1/8$). We measured $F_\phi^{Flat}(x)$ of Eq. (2) and we expect that $F_\phi^{Flat}(x) \propto x^{3/4}$ for $x \ll 1$.

In Table 2 we show the results of the fits for the largest lattice to the following functional forms:

$$F_\phi(x) = C_1 x^{d_h(1-\Delta)-1} + B_1\,, \qquad (6a)$$
$$= C_2 x^c\,, \qquad (6b)$$
$$= C_3 x^c + B_3\,. \qquad (6c)$$

Our results are consistent with the exponents conjectured in Eqs. (5a) and (5b). We find that simulating the largest lattices, 128000 triangles, is important for obtaining enough data points in the relevant region $x < 0.45$. The fits to the predicted behaviour Eq. (6a) are good and the finite size correction B approaches zero as the volume increases. Even if the exponent c is allowed to vary, as in the fits to Eqs. (6b) and (6c), it approaches its predicted value convincingly as the volume is increased. The small discrepancy is consistent with finite size effects, which clearly are more important than the statistical errors quoted in Table 2. We find that the difference in the values of c obtained from the fits to Eqs. (6b) and (6c) gives a measure of the systematic errors entering from varying the range and type of the fits. For a more complete presentation of the data we refer the reader to [7].

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